EVOLUTIONS OF SMALL BODIES IN OUR SOLAR SYSTEM

Dynamics and collisional processes

Dr. Patrick MICHEL
Côte d'Azur Observatory
Cassiopeé Laboratory, CNRS
Nice, France
Plan

- **Chapter I:**
  A few concepts on dynamics and transport mechanisms in the Solar System; application to the origin of Near-Earth Objects (NEOs)

- **Chapter II:**
  On the strength of rocks and implication on the tidal and collisional disruption of small bodies
Preliminaries: orbital elements

- $a =$ semi major axis
- $e =$ eccentricity
- $f =$ true anomaly
- $E =$ eccentric anomaly

Mean anomaly:

$$M = E - e \sin E = n \, t$$

with

$$n = \left(\frac{GM_*}{a^3}\right)^{1/2}$$

(orbital frequency)
Preliminaries: orbital elements

\[ i = \text{inclination} \]
\[ \Omega = \text{longitude of node} \]
\[ \omega = \text{argument of pericenter} \]

Longitude of pericenter:
\[ \varpi = \omega + \Omega \]

Mean longitude:
\[ \lambda = M + \varpi \]
The small body populations in the Inner Solar System

Green: Asteroid Main Belt
Blue squares: Comets
Red: objects with perihelion distance $q<1.3$ AU
The NEO population

- **Amors**: $a > 1$ AU
  $1.017 < q < 1.3$ AU

- **Apollos**: $a > 1$ AU
  $q < 1.017$ AU

- **Atens**: $a < 1$ AU
  $Q > 0.987$ AU

- **IEOs**: $a < 1$ AU
  $Q < 0.987$

1000 Objects with $D > 1$ km,
$\approx 500$ discovered
The NEO threat!

Impacts are real facts!

Moon

Venus

Earth: Tunguska, 1908

The least likely natural disaster BUT the only that may be predicted and avoided!
Main transport mechanisms in the Solar System

- **Fast mechanisms:**
  - Mean motion resonances with planets
  - First-order secular resonances with planets
- **Slow diffusions (not described in this lecture):**
  - Non-gravitational effects (Yarkovsky)
  - High-order and three-body resonances
The Kirkwood gaps in the asteroid Main Belt

![Diagram showing the Kirkwood gaps in the asteroid Main Belt with plots of eccentricity and inclination against semimajor axis.](image-url)
Collisions produce asteroid families!
This will be addressed in Chapter II
MM Resonance

\( (e, \sigma) \) surface plot

\[ \sigma_{ij} = i\lambda_p - j\lambda - (i-j)\omega \]

Planet collision line

Resonance width
Trace of $e, \sigma$ surface on an $(a,e)$ plot
SECULAR RESONANCES

Resonance:

\[ g = g_n \] (perihelion: affects \( e \))

\[ s = s_n \] (nodal: affects \( i \))

\( g \): frequency longitude of perihelion

\( s \): frequency longitude of node
Main principle

At first order in planetary mass ($j =$ planet index), the hamiltonian of a massless body expresses as:

$$H(\mathbf{\dot{r}}, \mathbf{\dot{r}}_j; \mathbf{r}, \mathbf{r}_j) = \frac{1}{2} \|\mathbf{\dot{r}}\|^2 - \frac{1}{\|\mathbf{r}\|} - \sum_{j=1}^{N_p} m_j \left[ \frac{1}{\|\Delta_j\|} - \frac{\mathbf{r}_j \cdot \mathbf{r}}{\|\mathbf{r}_j\|^3} \right]$$

**Keplerian part**

- \( L = \sqrt{a} \)
- \( l = M \)

**Planetary perturbations**

- \( G = \sqrt{a(1 - e^2)} \)
- \( g = \omega \)

**Delaunay variables**

- \( H = \sqrt{a(1 - e^2)} \cos i \)
- \( h = \Omega \)
Assumption: the small body is not in a mean motion resonance

The Hamiltonian (at 1st order in planet masses) can be averaged over all mean anomalies $l$ and $l_j$ (fast angles)

$$
\bar{H} = - \frac{1}{2L^2} - \sum_{j=2}^{N_p} m_j \bar{P}_j (-, G, H, -, G_j, H_j; -, g, h, -, g_j, h_j)
$$

$L = \text{cste}$, so we omit the first term and expand the perturbation w.r.t. planetary eccentricities and inclinations:

$$
\bar{H} = - \sum_{j=2}^{N_p} m_j \left[ K_0^j + (e_j, i_j) K_1^j + (e_j, i_j)^2 K_2^j + \ldots \right]
$$

$(e_j, i_j)^r$ are terms prop. to $e_j^a i_j^b$, with $a+b = r$ and $a, b \geq 0$
Isolate the first term \( K_0 \)

It can be shown that:

\[
K_0 = \sum_{p,q \in \mathbb{N}} c_{0,-v,v,0,p,q,0,0} e^{2\nu + 2p} i^{2\nu + 2q} \cos(2\nu(\nu - \Omega)) \frac{1}{\nu}
\]

Thus, \( K_0 = f(\nu - \Omega) = f(\nu) \)

\( K_0 = 1 \) degree of freedom integrable hamiltonian in the variables \( G, g \) as it depends only on the angle \( g (= \omega) \).

It is parametrized by the constants actions \( L \) and \( H \).

Its highly non-linear dynamics can be studied in details (Kozai 1962) by drawing level curves in the \((e, \omega)\) plane on a surface \( H = \text{constant} \).
Dynamics of $K_0$ at $a=0.98$ AU on 3 different surfaces of $H=cst$, each characterized by a value of $i_{max}$ ($1^\circ$, $20^\circ$, $60^\circ$)

- Polar diagram $(e, \omega)$

From Michel & Thomas (1996, AA 307)
Location of secular resonances

The free frequencies of $\omega$ and $\Omega$ of the asteroid’s orbit in the (a,e,i) space are obtained by integrating wrt time:

\[
\begin{align*}
\dot{G} &= -\left( \frac{\partial K_0}{\partial g} \right), \\
\dot{g} &= \left( \frac{\partial K_0}{\partial G} \right), \\
\dot{h} &= \left( \frac{\partial K_0}{\partial H} \right),
\end{align*}
\]

Proper frequencies = average values over a complete cycle of the free oscillations with period $T$ (from $g=0$ to $g=g(T)=2\pi$)

Secular resonance: (a,e,i) for which:

\[< \omega > = g_{\text{planet}} \quad \text{or} \quad < \dot{\Omega} > = s_{\text{planet}}\]

Ex: $\nu_6 \rightarrow g_6 \approx 28.22 \degree/\text{yr}$
Some secular resonance locations
(left: main belt, right: NEO region)

From Michel & Froeschlé (1997, Icarus 128)
Effect of secular resonances

Needs to consider the first-order term in $e_j$ and $i_j$ of the Hamiltonian ($K_j$)

Ex: effect on eccentricity of $\nu_4$ at $a=1.2$ AU
Effect of secular resonances (II)

Semi-analytical theory

a = 1.2 AU

Numerical integration

(i, σ_{14})
Effect of resonance overlapping

Overlapping of $\nu_{13}$ and $\nu_{14}$

Surface of section at $\sigma_{14} = \pi$

$a = 1.2$ AU

$\sigma_{13}$

$l$

$\pm 30$
Example: Dynamics of the $g=g_8$ resonance at 41 AU

\[ \sigma_8 = \omega - \omega_N \]
Simulation of the evolution of a body in the $g=g_8$ resonance by Holman and Wisdom, 1993

Secular resonance driven slow oscillations
Origin of NEOs

Asteroids from different regions of the Main Belt (MB) are injected into resonances which transport them on Earth-crossing orbits.

Hamiltonian describing the evolution of a massless body:

\[ H(\dot{\mathbf{r}}, \dot{\mathbf{r}}_j; \mathbf{r}, \mathbf{r}_j) = \frac{1}{2} \left\| \dot{\mathbf{r}} \right\|^2 - \frac{1}{\left\| \mathbf{r} \right\|} - \sum_{j=1}^{N_p} m_j \left[ \frac{1}{\left\| \Delta_j \right\|} - \frac{\mathbf{r}_j \cdot \mathbf{r}}{\left\| \mathbf{r}_j \right\|^3} \right] \]

(heliocentric frame)

\[ \Delta_j = \mathbf{r}_j - \mathbf{r} \quad \quad G = M_{\text{sol}} = 1 \]
SPECIFIC SOURCES OF NEOs:

- Nu6
- MC
- JFC
- OB
- MC
- 3:1
Fast resonances: Main Belt Asteroids become rapidly NEOs by dynamical transport from a source region (in a few million years)

Numerical simulations:
Several 1000 particles
Combine the sources of NEOs so that applying observational biases on the total distribution reproduces the observed distribution.

Mars Crossers

3:1

External Belt

Combine NEO Sources

R (a,e,i)

nu6

IMC

3:1

Outer MB

JFCs
Comparison between the biased model of NEOs and real data

1. Combine NEO Sources
   - nu6
   - IMC
   - 3:1
   - Outer MB
   - JFCs

2. Abs. Mag. Distribution
   - N (H)

3. Debiased NEO Orbits
   - Model (a,e,i,H)

4. "Observed" NEO Distribution
   - n (a,e,i,H)

5. Compare with Spacewatch NEO Data
   - n (a,e,i,H) = "Known NEOs"?

Continue Until "Best-Fit" Found
Comparison Between Discovered NEOs and Best-Fit Model

Weighting factors

<table>
<thead>
<tr>
<th>Model</th>
<th>Weighting Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν₆</td>
<td>0.36 ± 0.09</td>
</tr>
<tr>
<td>IMC</td>
<td>0.29 ± 0.03</td>
</tr>
<tr>
<td>3:1</td>
<td>0.22 ± 0.09</td>
</tr>
<tr>
<td>Outer MB</td>
<td>0.06 ± 0.01</td>
</tr>
<tr>
<td>JFC</td>
<td>0.07 ± 0.05</td>
</tr>
</tbody>
</table>

Model fit to 138 Spacewatch NEOs with $H < 22$
Our model of real orbital and absolute magnitude distributions of Near Earth Objects

~1000 NEOs with H<18 and a<7.4 AU
32% Amors
61% Apollos
6% Atens
94% of asteroidal origin
6% dormant comets (Jupiter family)

(Bóttek et al., 2000, 2002)

White = model; Gray = observations
Debiased NEO Orbital Distribution

• The NEO population having $H < 22$ and $a < 7.4$ AU consists of:
  → 32% Amors.
  → 61% Apollos.
  → 6% Atens.

• 2% are IEOs (Inside Earth’s Orbit).

Bottke et al. 2002
Estimate of 1 impact with energy > 1,000MT per 64,000 years

Known NEOs carry only 18% of this total collision probability \((H<20.5)\)

<table>
<thead>
<tr>
<th>Impact Energy</th>
<th>Mean Frequency (years)</th>
<th>Mean projectile’s size</th>
<th>Completeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000 MT</td>
<td>63,000</td>
<td>277 m ((H=20.5))</td>
<td>16%</td>
</tr>
<tr>
<td>10,000 MT</td>
<td>241,000</td>
<td>597 m ((H=18.9))</td>
<td>35%</td>
</tr>
<tr>
<td>100,000 MT</td>
<td>935,000</td>
<td>1,287 m ((H=17.5))</td>
<td>50%</td>
</tr>
<tr>
<td>1,000,000MT</td>
<td>3,850,000</td>
<td>2,774 m ((H=15.6))</td>
<td>70%</td>
</tr>
</tbody>
</table>

Morbidelli et al. 2002
The Lyapunov exponent: a tool to characterize the chaotic nature of an evolution

\[ L = \lim_{t \to \infty} \frac{\log(d_t)}{t} \]

NEOs have positive Lyapunov exponent indicating chaotic evolutions

⇒ impossible to make long term predictions of individual trajectories
NEOs have chaotic evolutions

Example of Itokawa

Computation of the evolutions of 100 initially very close orbits

Expected timescale for a Collision of Itokawa with the Earth: 1 Myr

On a shorter term: the threatening object Apophis (size: 300 m)

Trajectory uncertainty: 600 m within which a solution leads to a collision in 2036

In 2029: approach within 32,000 km!!
Origin of the Late Heavy Bombardment (3.9 Byr ago)

External Solar System (in red: Jupiter)
In green: disk of planetesimals
1st scenario which simultaneously explains: giant planet excentricities, origin of Trojans, LHB, and structure of the Kuiper Belt!
Conclusion I

Mean motion and secular resonances = efficient transport mechanisms by increasing eccentricities or inclinations

Most NEOs come from the main belt through resonance channels

LHB can be explained by passage of Jupiter and Saturn in the \( \frac{1}{2} \) MM resonance
sand and water: 2 cohesionless materials!

The Alpes in the snow (another material)!
Both *dynamical AND physical properties must be characterized*

- To determine the global (collisional and dynamical) evolution of small body populations
- To determine the origin of observed properties (e.g., existence of binaries)
- To define efficient mitigation strategies
Rocks:

A Modeling Challenge
The modeling of material behavior is the biggest shortcoming in code calculations, and the primary reason for bad results.
What I won’t talk about, but are important:

Eulerian v. Lagrangian codes
Handling Mixtures in Eulerian codes
Boundaries in Eulerian codes
Grid distortion in Lagrangian
Equations of States of rock materials
Understanding the process: Regions of Impact Process

1. \( r \sim 0 \to a \): Coupling of the energy and momentum of the impactor into the asteroid
2. \( r \sim a \to 2a \): Transition into point source solution, shock breakaway.
3. \( r \sim 2a \to +\infty \): Shock decays with distance, strength (& gravity) become important
Strength v. Strength v. Strength

Rock Strength

Sand Strength

Water Strength
Strength:

The Mohr-Coulomb (or Drucker-Prager) model:

- Shear strength
- Cohesion
- Rocks
- Sand
- Water
- Pressure

Tensile region
Compressive region

“Angle of Friction”
Yield depends on pressure

Some real data

Sandstone data (after Hoek & Brown, 1980)

Cohesion
Tensile strength

ANGLE OF FRICTION

Yield

Pressure

Tensile strength

Cohesion

Sandstone data (after Hoek & Brown, 1980)
**Strength**

- A rock has each of:
  - Tensile strength
  - Shear strength (cohesion) ~same as tensile
  - Compressive strength ~5-7* tensile

- But at large pressure, the cohesion can be ignored...
And we have the model for large cohesion-less bodies or rubble piles:

- Shear strength
- Cohesion
- Pressure
- Rocks
- Sand

The ‘strength’ is due to the pressure, which is a result of self gravity holding the body together (*but it has no tensile strength*)
First application:
Roche limit of cohesionless bodies

- The Roche limit is a well known feature for small orbiting (or passing) bodies.

- But:
  - It assumes a fluid body
  - It requires an almost prolate shape with aspect ratios ~2.1:1
So here is the **fluid tidal disruption problem**

Semi-major Axes: $a$, $b$, $c$

Aspect ratios:

\[ \alpha = \frac{c}{a} \]

\[ \beta = \frac{b}{a} \]

Aspect ratio $\alpha$
An Example (Phobos):

Does this look fluid to You??

Is it anywhere near the required shape for a fluid body?? (No: $a=0.7$, not 0.49)

A fluid model is not mandatory for any solid body, even when dominated by gravity (see further)
So:

“Satellites can orbit within their Roche limit because they have non-zero strength”

But, what is “strength”?
Here, we do not mean cohesion
BUT shear strength under pressure
The Problem Solved:

- Determine the tidal disruption limits for a geological material such as rock or sand.
  
  - Rubble Piles (Ignore cohesion)
  
  - Then what are the limit tidal disruption distances?

Step 1: Determine the stress state

- Include spin, gravity, and tidal forces

But there are different ways to do this:

- Elastic Theory: Can determine state for “first yield”
  (but that depends on residual stresses, which cannot be known)

- Plastic Limit Theory: Can determine states for “final failure” irrespective of past history.
The stresses at ‘final failure’ in an ellipsoidal body

\[
\begin{align*}
\sigma_x &= -\rho k_x a^2 \left[ 1 - \left( \frac{x}{a} \right)^2 - \left( \frac{y}{b} \right)^2 - \left( \frac{z}{c} \right)^2 \right] \\
\sigma_y &= -\rho k_y b^2 \left[ 1 - \left( \frac{x}{a} \right)^2 - \left( \frac{y}{b} \right)^2 - \left( \frac{z}{c} \right)^2 \right] \\
\sigma_z &= -\rho k_z c^2 \left[ 1 - \left( \frac{x}{a} \right)^2 - \left( \frac{y}{b} \right)^2 - \left( \frac{z}{c} \right)^2 \right]
\end{align*}
\]

The magnitudes are determined by \(k_x, k_y\) and \(k_z\) which have specific components from each of gravity, spin and tidal forces. For the long x-axis is pointed toward the primary center:

\[
\begin{align*}
k_x &= \left( -2\pi \rho GA_x + \omega^2 + 2 \frac{GM}{d^3} \right) x, \\
k_y &= \left( -2\pi \rho GA_y + \omega^2 - \frac{GM}{d^3} \right) y, \\
k_z &= \left( -2\pi \rho GA_z - \frac{GM}{d^3} \right) z
\end{align*}
\]

Ax=Ay=Az=2/3 for a sphere and are expressed in terms of elliptic integrals for an ellipsoid

\(\omega\) = spin magnitude (about z)

M = primary’s mass, \(\rho\) = body’s density

d = distance (primary’s and body’s center)
Step 2: Solve the failure criterion for the tidal disruption limit distance

The failure criterion is the Druker-Prager one (zero-cohesion):

\[
\frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_2)^2 \right] = s^2 \left[ \sigma_1 + \sigma_2 + \sigma_3 \right]^2
\]

Define: \( \Omega = \frac{\omega}{\sqrt{\pi \rho G}} \)

and solve for the dimensionless distance:

\[
\delta = \left( \frac{\rho}{\rho_p} \right)^{1/3} \frac{d}{R} = F[\alpha, \beta, \rho, \phi, \Omega]
\]

\( s = \frac{2 \sin \phi}{\sqrt{3(3 - \sin \phi)}} \)

\( \rho = \frac{m}{M} \)

\( \phi = \text{angle of friction} \)

\( \rho_p, R: \text{primary density and radius} \)

\( \alpha, \beta: \text{aspect ratios} \)
This corresponds to solve for “LIMIT” State where no further plastic re-adjustments are possible.

We have done that for many combinations of distance, spin, shape, and secondary size.

as a function of the angle of friction…
(so the fluid case with zero angle of friction is a special case)
**Example: prolate bodies, spin-locked**

**Aspects**

- Semi-major Axes: $a, b, c$
- Aspect ratios:
  - $\alpha = \frac{c}{a}$, (equal 0.6 here)
  - $\beta = \frac{b}{a}$

**Equation**

\[ \delta = \frac{d}{R} \quad \text{(for } \rho = \rho_p) \]

**Diagram**

- **Roche Fluid**
- **Rubble-Pile**

**Aspect Ratio $\alpha = 0.6, \rho = 0$**

**Aspect Ratio $\beta$**

**Limits on Distance, $d/R$**

**COE Planetary School 12/4/2006**
And finally, what if it does have ‘final failure’?

Prolate passing body, Long axis ‘down’, No spin, 30° friction angle
Application: 99942 Apophis (2004 MN4)

- In 2029: approach within 5.6 +/- 1.4 Earth’s radii from Earth’s center.

- Ellipsoid with aspect ratios $a=0.57$, $b=0.71$ (Scheeres et al. 2005), rotation period=30 h.

- We can determine the bulk density for tidal disruption or reshaping vs. the angle of friction $f$
Minimum bulk density of Apophis for survival without tidal breakup during the passage by the Earth at $d/R=5.6$, 4.2, and 7.0, for various angles of friction (assumes the worst-case orientation of the longest axis pointed down).
Application: (25143) Itokawa

Minimal distance to Earth for tidal effects

Scaled Distance, d/R vs. Angle of Friction, degrees

20 40 60 80

2 3 4 5 6 7 8
FUTURE STEP: ADD COHESION

« Ostriches trying to stick their heads in the sand »
Simulating the collisional disruption of a small body: what do we need to know?

- Balance Laws (easy: continuum mechanics: balance of mass, momentum, energy)
- Material behavior (very hard: 100 Mbar down to partial bars!)
- Robust computer codes
Comparison of scaling models
Material Behavior: Three regimes

**EOS**
- Single species EOS: $e(P, \rho)$
- Single species EOS: $e(P, \rho)$

**Solids**
- Mixture Theory (including porosity)
  - Stress-Strain Equations

**Flow, fracture, failure**
- Yield
  - Yield surface, Flow rule
- Flow & Failure
  - Fracture
  - Fracture Criteria

$P >> \rho c^2$
$P \sim \rho c^2$
$P << \rho c^2$
Stress-Strain behavior

• When $P \approx \rho c^2$ the material no longer behaves as a fluid.

• Then we need a constitutive equation for the stress-strain behavior

• Almost always, in wave codes that is simply an isotropic linear elastic relation (which is undoubtedly extremely crude).
Which brings us to the strength parts..

**EOS**
- Single species EOS $e(P, \rho)$
- Single species EOS $e(P, \rho)$
- Mixture Theory (including porosity)

**Solids**
- Stress-Strain Equations

**Flow, fracture, failure**
- Yield
- Flow & Failure
- Fracture
  - Yield surface, Flow rule
  - Fracture Criteria

**Equations**
- $P >> \rho c^2$
- $P \sim \rho c^2$
- $P << \rho c^2$
The “F” words: Flow, Fracture and Failure

Models for these fall into three groups:

• “Degraded Stiffness”, no explicit flow or fracture.

• “Flow” including plasticity and damage, used to model microscopic voids and cracks leading to an inability to resist stress.

• “Fracture”, involving actual macroscopic cracks and voids which are tracked, leading to an inability to resist stress.
In a continuum theory, the first two can be included directly, the latter is difficult, unless some statistical approach is used to smear them out.
Damage and degradation leading to ultimate failure occur at some limiting strain.
Flow and Fracture: Yielding and Cracking

Initial Yield = F(stresses) or G(strains)

- Isotropic => $\sigma_1$, $\sigma_2$, $\sigma_3$
  (Or three stress invariants)
- Commonly only 2, e.g.
  $J_2 = F(P)$
  Or max shear = f(pressure)
The Grady-Kipp Model

- It is a Tensile Brittle Fracture Mechanism
- For fragmentation in mining
- One-Dimensional Model
- Synthesized for constant strain rate histories only
- Governed by Crack Distributions (Weibull) and growth
- Implies rate and size-dependent strength

But Attractive Physics
There exists an initial distribution of incipient flaws in the target

\[ N(\varepsilon) = k \varepsilon^m \]

where:
- \( N \) = density number of flaws activating at or below the strain \( \varepsilon \)
- \( k, m \): Weibull parameters (large \( m \) = more homogeneous material)

\[ \varepsilon_{\text{min}} = \left(\frac{1}{kV}\right)^{-m} \]

Larger targets (volume \( V \)) activate largest crack at lower strain

\[ \Rightarrow \text{Larger targets are weaker} \]
**Tensile fracture depends strongly on strain rate**

Strength v. Strain Rate from Various Studies

- Concrete (plain)
- Concrete (polyester)
- Limestone (Oakhall)
- Oil Shale (80ml/kg)
- Arkansas Novaculite
- Westerly Granite (Lipkin)
- H&H Granite (Crack Distribution)
- Fully Cracked, Large (Various Materials)
- Melosh et al. (Basalt)
- Dresser Basalt
- Benz and Asphaug, 1994

**Low strain rate**

**High strain rate**

(From Asphaug)
• Damage is isotropic, so that when a crack is formed in one direction, all directions lose stiffness.

• As damage accumulates, the stiffness in both tension and in shear decrease, eventually to zero.

• Therefore, material failed by the outgoing shock behaves as water.

• *Calibrated to disruption test, by adjusting the strength (Weibull) parameters*
The Grady-Kipp Approach

Shear stress at a given shear strain

Tensile stress at a given tensile strain

Fully damaged material

Pressure

Damage Affect

Shear
**Equation of state**

\[ P = f(E, \rho) \]

**Model of brittle Failure**

**Stress tensor**

\[ \sigma_{\alpha\beta} = -P \delta_{\alpha\beta} + S_{\alpha\beta} \]

\[ S_{\alpha\beta} = \mu(\epsilon_{\alpha\beta} - 1/3 \epsilon_{YY} \delta_{\alpha\beta}) \]

**Yielding criterion:**

\[ S_{\alpha\beta} \rightarrow f S_{\alpha\beta} \]

**Conservation equations**

**SPH techniques**
Fragmentation Phase

Shock wave Propagation

Impact velocity: 5 km/s

Impact angle: 45°

P. Michel & W. Benz
From Asphaug et al. 1998, Nature **393**.
Impact angle: $66^\circ$, $V = 5$ km/s  

D=164 km

Velocity distribution
At the end of the fragmentation phase

Colors from **Yellow** to **Blue** indicate velocities from **large** to **small**

**Intermediate impact regime**
Gravitational Phase: parallel N-Body simulations

Several hundreds of thousands km-size fragments can be generated by the fragmentation phase.

Impossible to compute their gravitational interaction by classical methods:

The CPU time required to compute N interactions between N particles is of $O(N \times N)$!!

Using the so-called hierarchical tree method (tree code):

CPU Time = $O(N \log N)$
Gravitational Phase: parallel N-Body simulations

- Parallel N-Body code: *pkdgrav* (Parallel K-D tree GRAVity code); developed at UW by T. Quinn, J. Stadel, D.C. Richardson

  - Detects and handles collisions between massive particles. Several options:

    1. Systematic particle merging
    2. Merging/Bouncing of particles depending on impact speed and spins.

Particle shape: spherical
Simulations of Collisions in the Gravity Regime

- SPH hydrocode → crack propagation through the target
- Nbody code → gravitational interaction between intact fragments

Simulation of target shattering + fragment dispersion and/or reaccumulation

Michel et al. (2001), Science Vol. 294, pp 1696-1700.
Simulations of asteroid disruptions have
1. successfully reproduced asteroid families
2. suggest that most kilometer-sized objects
   are gravitational aggregates

Impact energies and collisional outcomes depend highly on the internal structure of the parent body

Michel et al., Science 294 (2001)

Different phases of the reaccumulation process


T=84 minutes

T=2 minutes

T=2 seconds
Implication: most asteroids originating from the disruption of a larger one - such as most NEOs - should be rubble piles.

The Japanese mission Hayabusa brought us some evidence in this direction: where are the craters?? why so many debris ?? What about the small bulk density (< 2 g/cm3)
From Velocities to Orbital Elements

Gauss Formulae: transformation velocities to orbital elements

\[
\frac{\partial a}{a} = \frac{2}{n a \sqrt{1 - e^2}} \left[ (1 + e \cos f) V_t + e \sin f V_r \right]
\]

\[
\frac{\partial e}{n a} = \frac{\sqrt{1 - e^2}}{n a} \left[ \frac{e + 2 \cos f + e \cos^2 f}{1 + e \cos f} V_t + \sin f V_r \right]
\]

\[
\frac{\partial i}{n a} = \frac{\sqrt{1 - e^2}}{n a} \left[ \frac{\cos(\omega + f)}{1 + e \cos f} V_w \right]
\]

(a, e, i, w, f, n) = orbital elements of Parent body (family barycenter)

Requires to assume *a priori* ω and f of the parent body at the impact instant
Effect of the Parent Body’s Internal Structure

- Previous simulations assumed monolithic parent bodies
- Large asteroids are likely to undergo shattering events before disruptive ones
- What is the outcome of the disruption of a pre-shattered parent body?
Yellow zones = fragments

Red zones = damage (separation between fragments)

Black points = void

W. Benz & P. Michel
Two types of pre-shattered internal structures

Presence of damage zones

Presence of damage zones + voids
Monolithic/Pre-shattered Parent Body

Monolithic Parent Body

N>D vs D (km)

Pre-shattered Parent Body
**Monolithic/Pre-shattered Parent Body**

Ellipses = spreading of the real family

Crosses = simulation

I (rad) vs a (UA)
So how can we improve the models?

Compare, Compare, Compare

- to real experiments
  - Large explosive field tests
  - Carefully controlled lab tests

- to impact craters
  - (but what was the impactor?)

Test, Test, Test

- real materials
  - Crushability
  - Strength in different states
Experiments = first and crucial step for code validation

Example:

Simulation by an Hydrocode of the Impact experiment on basalt of Nakamura & Fujiwara in 1992

The core fragment is successfully reproduced
“Some” Current Shortcomings:

• Most strength models do not address all types of “strength”

• Codes often have “hidden features”

• Equations of state of some materials are still uncertain

• We do not often enough make comparisons to any experiments
Some more specific shortcomings

• We cannot model well enough to distinguish details for a particular crater

• We cannot handle mixtures well

• Mixing rocks and atmospheres, and porosity makes for very difficult code calculations

• We don’t do chemistry
However, on the positive side

- **In the gravity regime:** we were able to reproduce qualitatively the main properties of asteroid families → reaccumulation processes may dominate and « accurate » modeling of fragmentation may not be so crucial (needs to be checked) for qualitative studies.

- **In the strength regime:** the SPH hydrocode including a model of brittle failure has at least reproduced successfully some experiments on basalt targets.

- **Future challenge:** characterizing the behavior of porous materials and differentiated objects, first in the strength regime (with confrontation to experiments) and then in the gravity regime (formation of C-type asteroid families, impact response of comets, KBOs ...).
Arigato Gozai-Masu
Thank you for your attention
Merci beaucoup ...