



# Entropy Changes in the Clustering of Galaxies in an Expanding Universe

<sup>1</sup>Tabasum Masood, <sup>1,2</sup> Naseer Iqbal, <sup>1</sup> Mohammad Shafi Khan

1 Department of Physics, University Of Kashmir, Srinagar, India.  
2 Interuniversity Centre for Astronomy and Astrophysics, Pune, India.

## Abstract:

The approach thermodynamics and statistical mechanics of gravitating systems is applied to study the entropy change in gravitational clustering of galaxies in an expanding universe. We derive analytically the expressions for gravitational entropy in terms of temperature T and average density n of the particles (galaxies) in the given phase space cell. It is found that during initial stage of clustering of galaxies, the entropy decreases and finally seems to be increasing when the system attains virial equilibrium. The entropy changes are studied for different range of measuring correlation parameter b. The entropy results for a system consisting of extended mass (non-point mass) particles show a similar behavior with that of point mass particles clustering gravitationally in an expanding universe.

## 1. Introduction:

Galaxy groups and clusters are the largest known gravitationally bound objects to have arisen so far in the process of cosmic structure formation (Voit 2005). They form the densest part of the large scale structure of the universe. Cluster of galaxies are the most massive object to have arisen in the hierarchical structure formation of the universe and the study of clusters tells one about the way galaxies form and evolve. The average density n and the temperature T of a gravitating system discuss some thermal history of cluster formation. For a better large understanding of this thermal history it is important to study the entropy change resulting during the clustering phenomena because the entropy is the quantity most directly changed by decreasing or increasing thermal energy of intracluster gas. Entropy is a measure of how disorganized a system is. The concept of entropy is generally not well understood. For erupting stars, colliding galaxies, collapsing black holes –the cosmos is a surprisingly orderly place. Super massive black holes, dark matter and stars are some of the contributors to the overall entropy of the universe. The microscopic explanation of entropy has been challenged both from the experimental and theoretical point of view (Freud 1970 and Khinchin 1949). Standard calculations have shown that the entropy of our universe is dominated by black holes, whose entropy is of the order of their area in Planck units (Frampton 2009). Statistical mechanics explains entropy as the amount of uncertainty which remains about a system after its observable macroscopic properties have been taken into account (Rief 1965). In real experiments, it is quite difficult to measure the entropy of a system. The technique for doing so is based on the thermodynamic definition of entropy. Here we explain in what sense the entropy change S-S<sub>0</sub> shows changing behavior with respect to a measuring correlation parameter b= 0-1.

## 2. Thermodynamic Description of Galaxy Clusters:

The universe is considered to be an infinite gas in which each gas molecule is treated to be a galaxy. The gravitational force is a binary interaction and as a result a number of particles cluster together. We use the same approximation of binary interaction for our universe (system) consisting of large number of galaxies clustering together under the influence of gravitational force.. We know equations of state for internal energy U and pressure P are of the form (Hill 1956).

$$U = \frac{3NT}{2} (1 - 2b) \quad (1)$$

$$P = \frac{NT}{V} (1 - b) \quad (2)$$

Where b defines the measuring correlation parameter given by (Saslaw 1984)

$$b = -\frac{W}{2K} = 2\tau Gm^2 \frac{n}{3T} \int_0^\infty \xi(\bar{n}, T, r) r dr \quad (3)$$

For an ideal gas behavior b=0 and for non ideal gas system b varies from 0 and 1. b is also defined as (Saslaw 1984 and Iqbal 2006)

$$b = \frac{\beta \bar{n} T^{-3}}{1 + \beta \bar{n} T^{-3}} \quad (4)$$

This indicates b has a specific dependence on the combination  $\bar{n} T^{-3}$

## 3. Entropy Calculations:

The concept of entropy evolved in order to explain why some processes are spontaneous and others are not; systems tend to progress in the direction of increasing entropy. Following statistical mechanics (Ahmad 2002), the grand canonical partition function is given by

$$Z_N(T, V) = \frac{1}{N!} \left( \frac{2\pi m k T}{A^2} \right)^{\frac{3N}{2}} V^N [1 + \beta \bar{n} T^{-3}] \quad (5)$$

The Helmholtz free energy is given by

$$A = -T \ln Z_N \quad (6)$$

Thermodynamic description of entropy can be calculated

$$S = - \left( \frac{\partial A}{\partial T} \right)_{N, V} \quad (7)$$

Using equations (5), (6) and (7), we have

$$S - S_0 = \ln(\bar{n}^{-1} T^2) - \ln(1 - b) - 3b \quad (8)$$

Where S<sub>0</sub> is an arbitrary constant. From equation (4), we write

$$\bar{n} = \frac{b}{(1 - b)\beta T^{-3}} \quad (9)$$

Using equation (9), we can write equation (8) as

$$S - S_0 = - \left[ 3b + \ln(b T^2) \right] \quad (10)$$

Again from equation (4),

$$T^{3/2} = \left[ \frac{\beta n (1 - b)}{b} \right]^{1/2} \quad (11)$$

Using eq. (11), equation (10) modifies to

$$S - S_0 = - \left[ \frac{1}{2} \ln \bar{n} + \frac{1}{2} \ln [b(1 - b)] + 3b \right] \quad (12)$$

This is expression for entropy of a system consisting of point mass particles, but actually galaxies have extended structures. So for extended structures, we make use of softening parameter  $\epsilon$  whose value is taken between 0.01 and 0.05 (in the units of total radius). Following the same procedure as above, equations for extended masses gets modified to

$$S - S_0 = N \ln \left[ \frac{N}{V} T^2 \right] - N \ln(1 - b_\epsilon) - 3N b_\epsilon \quad (13)$$

Using relation between b and b $\epsilon$ , equations take the form as

$$b_\epsilon = \left[ \frac{\beta n T^{-3} \alpha(\epsilon/R)}{1 + \beta n T^{-3} \alpha(\epsilon/R)} \right] \quad (14)$$

$$b_\epsilon = \frac{b\alpha}{1 + b(\alpha - 1)} \quad (15)$$

Equations 10 and 12 takes the form respectively as

$$S - S_0 = - \left[ \ln \frac{b T^2}{1 + b(\alpha - 1)} + \frac{3b\alpha}{1 + b(\alpha - 1)} \right] \quad (16)$$

$$S - S_0 = - \left[ \frac{1}{2} \ln \bar{n} + \ln \frac{[b(1 - b)]^{1/2}}{1 + b(\alpha - 1)} + \frac{3b\alpha}{1 + b(\alpha - 1)} \right] \quad (17)$$

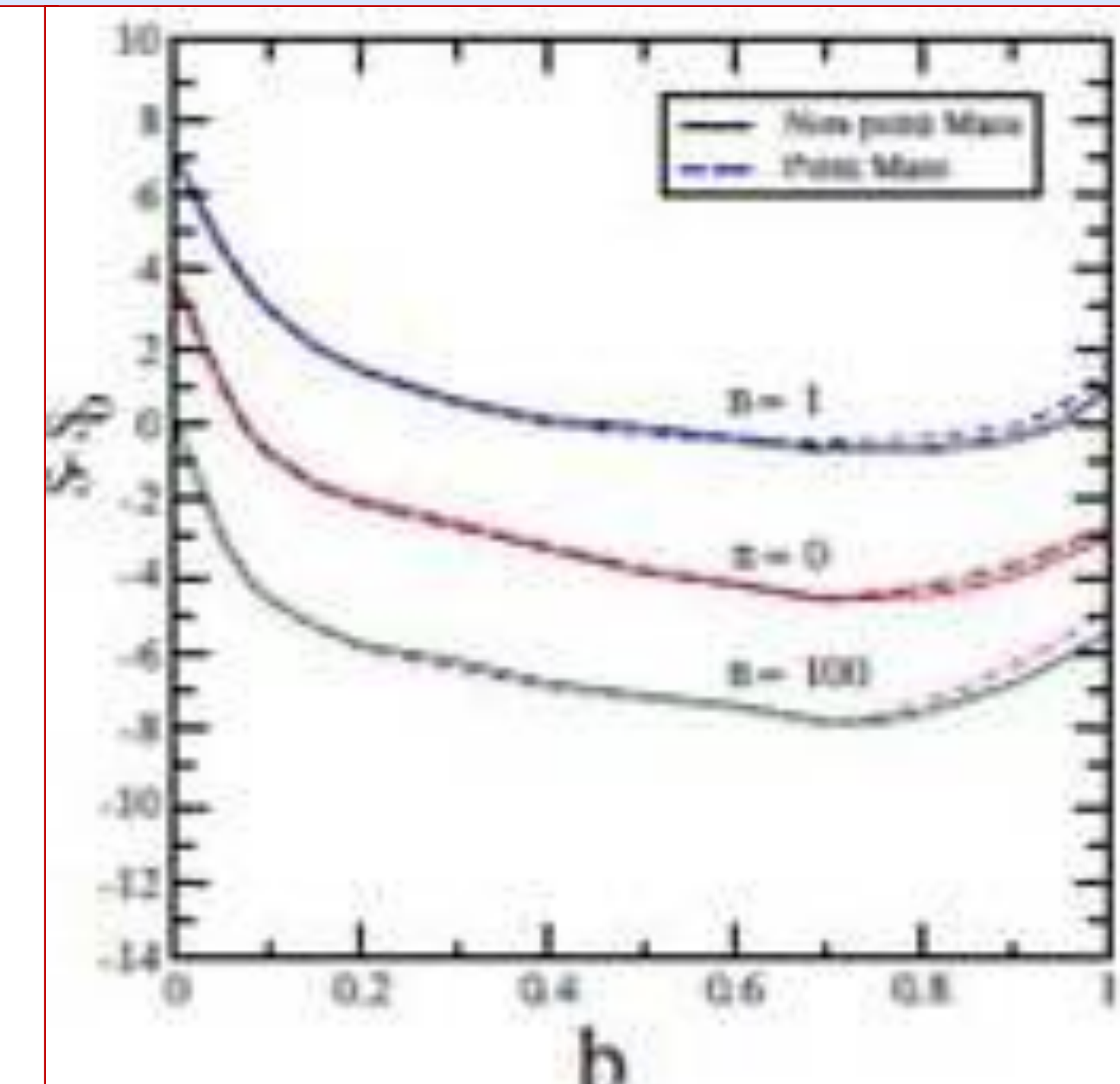
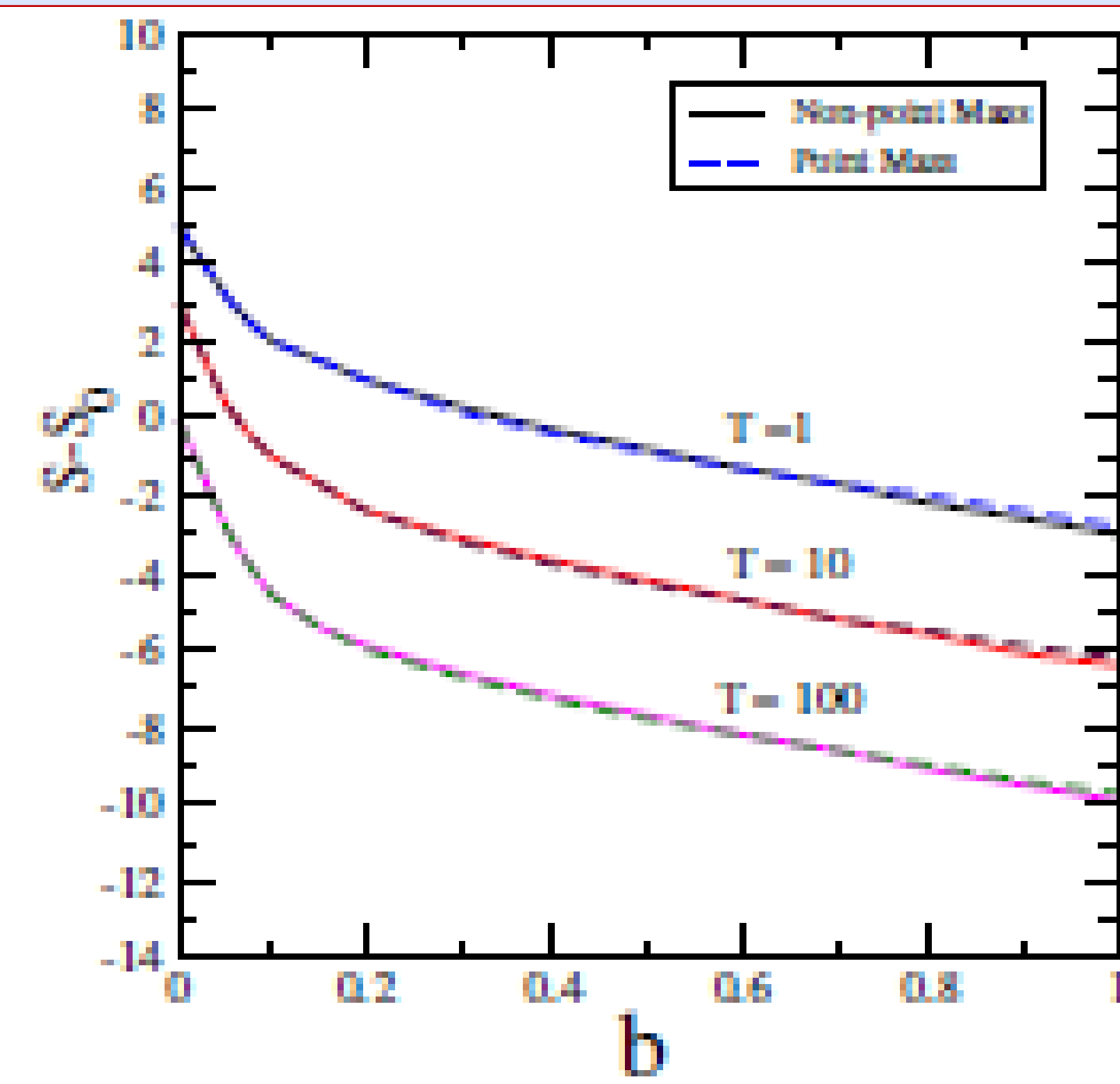
where

$$\alpha\left(\frac{\epsilon}{R}\right) = \sqrt{1 + \left(\frac{\epsilon^2}{R}\right) + \left(\frac{\epsilon^2}{R^2}\right) \ln \frac{\frac{\epsilon}{R}}{1 + \sqrt{1 + \left(\frac{\epsilon^2}{R^2}\right)}}} \quad (18)$$

If  $\epsilon = 0$ ,  $\alpha = 1$ , the entropy equations for extended mass galaxies are exactly same with that of a system of point mass galaxies approximation. Equation 10, 12, 16 and 17 are used here to study the entropy changes in the cosmological many body problem.

## 4. Figures:

We study the variations of entropy changes S-S<sub>0</sub> with the changing parameter b for different values of n and T. Some graphical variations for S-S<sub>0</sub> with b for different values of n=0,1,100 and average temperature T= 1,10,100 and by fixing value of cell size R = 0.04 and 0.06.



## 5. Results

The formula for entropy calculated here has provided a convenient way to study the entropy changes in gravitational galaxy clusters in an expanding universe. Gravity changes things that we have witnessed in this research. Clustering of galaxies in an expanding universe, which is like that of a self gravitating gas increases the gases volume which increases the entropy, but it also increases the potential energy and thus decreases the kinetic energy as particles must work against the attractive gravitational field. So we expect expanding gases to cool down, and therefore there is a probability that the entropy has to decrease which gets confirmed from our theoretical calculations as shown in figures.

Every thing from gravitational clustering to supernova are contributors to entropy budget of the universe. A new calculation and study of entropy results given by equations 13 shows that the entropy of universe decreases first with the clustering rate of the particles and then gradually increases as the system attains virial equilibrium.

## References:

- [1.] Voit, G.M. (2005) *Tracing cosmic evolution with clusters of galaxies. Review of modern physics*, 77, 207-248
- [2.] Hill, T.L. (1956) *Statistical mechanics: Principles and statistical applications. McGraw-Hill, New York.*
- [3.] Frampton, P., Stephen, D.H., Kephart, T.W. and Reeb, D. (2009) *Classical Quantum Gravity*, 26, 145005
- [4.] Ahmad, F., Saslaw, W.C., and Bhat, N.I., (2002) *Statistical mechanics of cosmological many body problem. Astrophysical journal*, 571, 576-584
- [5.] Rief, F., (1965) *Fundamentals of statistical and Thermal physics. McGraw-Hill, Tokyo*
- [6.] Freud, P.G. (1970) *Physics: A contemporary Perspective. Taylor and Francis Group*
- [7.] Khinchin, A.I. (1949) *Mathematical foundation of statistical mechanics. Dover Publications, New York*
- [8.] Saslaw, W.C. and Hamilton, A.J.S (1984) *Thermodynamics and galaxy clustering. Astrophysical journal*, 276, 13-25.
- [9.] Iqbal, N., Ahmad, F. and Khan, M.S. (2006) *Gravitational Clustering of galaxies in an expanding universe. Journal of Astronomy and Astrophysics*, 27, 373-379.