Finding the Cold and Lonely Planets with Gravitational Microlensing



Lecture 1 Outline

- Planet Discovery History and Planet Formation Theories
 - Gravitational Instatibility
 - Core Accretion Model
 - Scattering and Migration
 - Theory follows observations
- Exoplanet Detection Methods
 - Doppler Radial Velocity and Transit methods favor short period planets
 - Direct detection and Microlensing Favor longer period planets
- Gravitational Microlensing
 - Single lens events
 - MOA Search for Isolated Planets

History of Observed Planetary Systems



1543: Copernicus: *Revolutionibus*1600: Bruno burned
1604: Kepler's Supernova
1609: Galileo's telescope
1618: Kepler's 3rd law

1687: Newton: *Principia*1755: Kant on planet formation1781: Herschel: Uranus1796: Laplace on planet formation

1838: Parallax measured1846: Adams & Le Verrier: Neptune1855: 70 Ophiuchi b (false detection)

1925: Hubble: Cepheids in "nebulae"
1930: Tombaugh: Pluto 1963: Barnard's Star b (false detection)



Planet Formation "Theories"

- Closely tied to observations
 - Calculations from first principles do not predict that planets will form!
 - The physics of planet formation is very complicated
- Until 1995, the theories were only compared to the properties of our Solar System
- Since 1995, observations regularly reveal flaws in theory
- Two Leading Theories
 - Core Accretion: the leading theory
 - Gravitational Instability: main challenger probably works in some cases

Gravitational Instability Pros:

Mayer et al. 2002



TreeSPH, isothermal EOS,

- allows planets to form quickly (<10³ yr)
- explains distant planetary companions

Cons:

•does not naturally explain cores (and high-Z element enhancements) of Jupiter and Saturn*

•does extremely poor job accounting for the cores of Neptune and Uranus

•doesn't explain terrestrial planets

- requires extremely massive protoplanetary disks between 4 and 20 AU (typical observed disk masses are within 100 AU)
- has been demonstrated to robustly operate only in simulations using isothermal equation of state
 - otherwise, protoplanets don't collapse due to thermal pressure

* Alan Boss begs to differ!

Adapted from R. Rafikov

The core accretion hypothesis

- forming Sun is surrounded by a gas disk (like nebular hypothesis)
- planets form by multi-stage process:
 - 1. as the disk cools, rock and ice grains condense out and settle to the midplane of the disk chemistry and gas drag are dominant processes
 - small solid bodies grow from the thin dust layer to form km-sized bodies ("planetesimals") - gas drag, gravity and chemical bonding are dominant processes
 - 3. planetesimals collide and grow gravitational scattering and solar gravity are dominant processes. "Molecular chaos" applies and evolution is described by statistical mechanics

requires growth by ~45 orders of magnitude in mass through ~6 different physical processes!

Stages of Planet Formation by Core Accretion

- From dust (~µm-cm) to pebbles (~cm)
 - Myriads of microscopic dust & ice particles merging together Motion of solid objects is strongly coupled to gas

• From pebbles to boulders (~10m)

Many bodies, must have rapid growth (<100yr), but how? Motion of solid objects is weakly coupled to gas

• From boulders to planetesimals (>10km)

Orderly growth through collisions, mergers, & fragmentation

• From planetesimals to embryos (~1000km, Moon-sized)

Runaway growth of a small number of separated embryos

• From embryos to planet cores

Gravitational interactions stir and reduce gravity focusing Oligarchic growth up to isolation mass (0.1-10M_{Earth}) Gravitational perturbations cause orbits to cross

 \rightarrow chaotic growth via giant impacts or ejections.

- Dominant Planets form beyond "snow-line": $\rho \rightarrow 5 \times \rho$ dust + ice!
- Possible accretion of gas and transition to gas giants

Adapted from R. Rafikov



Core Accretion predicts failed Jupiters, especially around low-mass stars

Don't Stop Here!

- Pre-1995 this was the end of planet formation
- But many exoplanet systems have hot Jupiters
 Should form outside the "snow-line" not at a < 0.05 AU !
- Many exoplanet systems have massive planets on eccentric orbits
- Planet-planet scattering and migration determine the final planetary system configuration

Early Planet Formation



Illustration by E. Chiang

Planet Scattering



Predicts many free-floating planets

Illustration by E. Chiang

Orbital Migration



Illustration by E. Chiang

Planet-disk interaction



Lubow et al. (2002b)

- Tidal interaction of planet with the disk leads to the formation of spiral density perturbation which, because of the differential rotation, leads (trails) planet in the inner (outer) disk.
- As a result, planet is pulled forward (backward) and its angular momentum increases (decreases).
- Inner (outer) disk loses (gains) angular momentum.

Thus, planet repels disk. This might lead to a gap formation.

Slight imbalance between the torques exerted on the inner/outer disk leads to **planet migration**. Its direction is usually **inward**.

Planet Formation Theory

- A combination of complicated physical processes
 - many of these cannot be reliably calculated
 - Some parts of the process can be calculated
 - theory is just too hard for the theorists!
- Observations of extrasolar planetary systems are the key to progress!

Planet Discoveries by Method

masses

Mass

- >400 Doppler discoveries in black
- Transit discoveries are blue squares
- Gravitational microlensing discoveries in red
 - discoveries in red
- Direct detection, and timing are magenta and green triangles
- Kepler candidates are cyan spots



Exoplanet Search Techniques

Reflex Motion - Orbit of Star due to planet

- sensitive to planetary mass
- must observe for ~1 full period
- Precision Radial Velocities
 - ~300 planets Discovered
 - M_{planet} > 0.3 M_{Jup} or a < 1AU
 - mass ambiguity due to inclination
 - v_r = 13 km/sec for Jupiter,
 13 m/sec for Sun
- Astrometry transverse motion
 - ground based: Keck, VLTI
 - space based missions: SIMC
 GAIA



Astrometric motion of the Sun over 30 years as seen from North ecliptic pole



http://certificate.ulo.ucl.ac.uk/modules/year_one/NASA_SIM/ finding_planets.html

Indirect Detection 1: Reflex Motion

- Planets don't really orbit their host stars
- Instead both the star and planet orbit their center of mass
- Jupiter orbits the sun at ~13 km/sec
- Sun is 1000× Jupiter's mass
 - orbits at ~13 m/sec
 = 29 mph (you can drive faster than this!)





Astrometric Wobble

- Star wobbles back & forth on the sky relative to more distant background stars.
- Problem:
 - The wobble is very small
 - Best seen looking down on the orbital plane.
- From 5 parsecs away, the
 Sun's astrometric wobble is ≈
 0.001 arcseconds
- Longer period => larger signal
 - ~20 year data sets



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Astrometry's Checkered History

- "planets" orbiting 70 Ophiuchi
 - 1855 W.S. Jacob of Madras Obs. planet "highly probable"
 - MNRAS, 15, 228
 - 1899 T.S.S. See
 - 1943 Reuyl & Holberg
- Barnard's Star
 - van de Kamp 1963 1 planet, 1969 2-planet system
 - refuted in 1973 by Hershey and by Gatewood & Eichhorn
- Lalande 21185
 - 1951 van de Kamp & Lippencott claim a planet
 - 1960 Lippencott revises parameters
 - 1974 Gatewood refutes these claims
 - 1996 Gatewood claims a planet
 - no evidence in radial velocities not believed
- VB 10
 - 2009 Pravdo & Shaklan claim a planet
 - 2010 Bean et al. refute the claim

Solar System with the RV Doppler Technique



planet must be observed for 1 orbital period for detection

Doppler Technique: background

Echelle spectrum

The spectrum is extracted from alund make a low bar bar bar the 2-D echelle image, to give an Hα array of intensity vs wavelength 100 NaD for each spectral order. 200 1.5×10⁴ 1.0×10⁴ Intensity 5.0×10³ O 5885 5890 5895 5900 Wavelength

Doppler RV material from Debra Fischer & Michel Mayor

Doppler Technique: background

Atomic and molecular lines in the atmosphere of the star absorb light at particular wavelengths.



- 1) the intensity of spectral lines is determined by the optical depth of the absorbing species, regulated by temperature and element abundance.
- Exploiting the Doppler effect, the relative velocity between the telescope and the star can be measured by measuring shifts in spectral lines:

 $\Delta\lambda/\lambda=v/c$



Physical dimension:	15 µ
Dispersion λ :	0.05 A
Velocity:	3000 m/s

For typical high resolution spectrographs with 15 μ -pixel CCD detectors (HDS on Subaru or HIRES on Keck), $\delta\lambda = 0.0002$ A corresponds to 0.004 pixel shift. To measure this signal, you need a precision that is many times better.

Note: typical spectral lines are 0.1 - 0.2 Angstroms (or a few CCD pixels) in width, so we need to detect shifts approaching 1/1000 the width of the lines we model.

Astrophysical False Positives: Starspots



These astrophysical challenges to Doppler precision are likely to be problems for other detection techniques (astrometry, transits), too.

Exoplanet Search Methods: Transits



• Transits

- -Detect size, not mass
- ~170 discoveries to date
- -Short period orbits strongly favored
- -Several Jupiter mass planets discovered
 - With radial velocity: planetary mass & radius
- –Jupiter size planets: ~1% signal
- -Earth size planets: ~0.01% signal
 - only from space: Kepler
 - many orbits needed
 - image blending w/ faint eclipsing binaries = background



http://certificate.ulo.ucl.ac.uk/ modules/year_one/NASA_SIM/ finding_planets.html

Exoplanetary Transits

- Except for one "crackpot" advocating detections by transits in the 1980's, this was not considered a serious detection method
- Discovery of hot-Jupiters in 1995
 - geometrical transit probability ~ 10%, instead of 0.5% for Earth
- first discoveries in 2003 from OGLE candidate list using methods developed for microlensing
- modern surveys use dedicated small telescopes to cover wide FOV for brighter stars
- false alarm probability ~90% from ground
 - $R_{Jup} \approx R_{BD} \approx R_{M-dwarf}$; faint eclipsing binary blends, grazing eclipsing binaries
 - RV follow-up for confirmation Exoplanet Encyclopedia says RV discoveries
- transits + RV provide exoplanet masses and radii
- Space-based transits (COROT & Kepler) should discover Earths

Measuring Exoplanet Inclinations

- Rossiter-McLaughlin effect: radial velocities during transit
- Strong scattering predicts some planets with large inclinations (Chatterjee et al. 2008; Fabrycky & Tremaine 2008; Nagasawa et al. 2008)
- Scattering + tides may produce inclined hot-Jupiters



Retrograde Hot-Jupiters



Inclination vs. Discovery Date & T_{eff}



Winn et al. (2010): inclination (or obliquity) depends on stellar temperature

•Connection to depth of stellar convection zone

•Initial obliquity random, but aligned through tidal effects?



BRIGHTNESS

Transits from Space



http://kepler.nasa.gov/



TIME IN HOURS



Kepler Mission PI: W.J. Borucki

Kepler Mission Concept

- *Kepler* is optimized to find transiting Earth-like planets
 - Radius down to 1 R_{\oplus}
 - Sun-like host star
 - Orbit out to 1 AU = 1 year
- Mission characteristics
 - 150,000 selected targets
 - Earth-trailing orbit for stability
 - Stare at one FOV for 3.5 years
 - or 7 years with mission extension



32

http://kepler.nasa.gov/

Kepler "Failure" and Unpredicted Success

- The Sun has less photometric noise than the typical Kepler star
 - Kepler stars have 50% more photometric noise than expected
 - A 7-year extended mission is needed to achieve required sensitivity to Earths at 1 AU
- The number of systems with multiple transiting planets is much higher than expected
 - Transit timing variations can be observed
 - 3rd body implies that the center of mass of bodies 1+2 changes with time
 - Analysis of transit timing variations yields planet masses
 - This is good because radial velocities cannot get the masses of most of the Kepler discoveries



Transiting Circumbinary Planet (Kepler-16)!



Validation of Discoveries

- SNR > 7 to rule out statistical fluctuations
- Three or more transits to confirm orbital periodicity
- Light curve depth, shape, and duration
- Image subtraction to identify signals from background stars
- Radial velocity
 - Medium precision to rule out stellar companions
 - High precision to measure mass of super-Earths and giant planets
 - R-M effect to confirm orbiting planet
- High spatial resolution to identify extremely close background stars. Then observe transits of background stars.
- Check for color change during transit
- Measure number of background binaries & compute reliability
- Detect transit timing variations in systems with >2 bodies!
Hot Planets: Radial Velocities & Transits



The Doppler radial velocity and transit methods have discovered almost all the known exoplanets, and this will only increase with the coming flood of Kepler discoveries. But these are mostly hot, inner planets. Solar Array Star Trackers (2) Thruster Modules (4) High Gain Antenna

http://www.nasa.gov/mission_pages/kepler/overview/index.html

Microlensing & Direct Detection Find (Marois, et al. 2010) the Cool Planets



- Thus far, only massive, young, self-luminous giant planets have been directly detected at very large orbital separations
- But microlensing has found that cold Neptunes and Saturns appear to be quite common beyond the snow line

http://wfirst.gsfc.nasa.gov/about/microlensing.jpg

The Physics of Microlensing

- Foreground "lens" star + planet bend light of "source" star
- Multiple distorted images
 - Only total brightness change is observable
- Sensitive to planetary mass
- Low mass planet signals are rare – not weak
- Stellar lensing probability ~a few ×10⁻⁶
 - Planetary lensing probability ~0.001-1 depending on event details
- Peak sensitivity is at 2-3 AU: the Einstein ring radius, R_E



Microlensing Target Fields are in the Galactic Bulge



10s of millions of stars in the Galactic bulge in order to detect planetary companions to stars in the Galactic disk and bulge.

Planet Discoveries by Method

- ~400 Doppler discoveries in black
- Transit discoveries are blue squares
- Gravitational microlensing discoveries in red
 - cool, low-mass planets
- Direct detection, and timing are magenta and green triangles
- Kepler candidates are cyan spots



Planet mass vs. semi-major axis/snow-line

- "snow-line" defined to be 2.7 AU (*M*/*M*_☉)
 - since $L \propto M^2$ during planet formation
- Microlensing discoveries in red.
- Doppler discoveries in black
- Transit discoveries shown as blue circles
- Kepler candidates are cyan spots
- Super-Earth planets beyond the snow-line appear to be the most common type yet discovered

Exoplanet Discoveries vs. Snow Line 10^{3} masses 10² Most planets N (Earth 10 were here! Mass VΕ 1 Μ 0.1 Μ 1 1 1 1 1 1 1 1 1 1 1 0.01 0.1 10(semi-major axis)/(snow-line) Muraki et al., 2011

Bohdan Paczyński 1986

THE ASTROPHYSICAL JOURNAL, 304: 1-5, 1986 May 1 © 1986. The American Astronomical Society. All rights reserved. Printed in U.S.A.



GRAVITATIONAL MICROLENSING BY THE GALACTIC HALO

BOHDAN PACZYŃSKI¹ Princeton University Observatory Received 1985 August 1; accepted 1985 October 23

ABSTRACT

The massive halo of our Galaxy has an optical depth to gravitational microlensing $\tau \approx 10^{-6}$. If the halo is made of objects more massive than $\sim 10^{-8} M_{\odot}$, then any star in a nearby galaxy has a probability of 10⁻⁶ to be strongly microlensed at any time. The lensing events last ~ 2 hr if a typical "dark halo" object has a mass of $10^{-6} M_{\odot}$, and they last ~ 2 yr for objects of 100 M_{\odot} . Monitoring the brightness of a few million stars in the Magellanic Clouds over a time scale between 2 hr and 2 yr may lead to a discovery of "dark halo" objects in the mass range $10^{-6}-10^2 M_{\odot}$ or it may put strong upper limits on the number of such objects. Subject headings: galaxies: Magellanic Clouds gravitation — stars: variables

Proposed Gravitational Microlensing Dark Matter Search

Gravitational Lensing



be resolved (even with HST). This is referred to as microlensing.

A =
$$\frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$
, then
A = $\frac{u^2 + 2}{u\sqrt{u^2 + 4}}$

Lensed Images (Einstein 1936)



When source is distant, we see distorted, magnified images. If the alignment is perfect, we see an "Einstein Ring". Einstein said, "there is no great chance of observing this effect". The probability at any one time is \sim 1 in a million, but we see \sim 800 per year.

How Likely is This?



Gravitational Microlensing Images

- top view is image plane
- circle = Einstein Ring, typically ≤ 1 mas
- red dot = lens
- green = unlensed image
- blue = lensed images
- bottom panel shows light curve for both images = blue/green





Private communication, Han

"Normal Microlensing Light Curve"



- Assume point source & lens, plus constant velocities
- 3 measurable parameters: $t_{\rm E}$, t_0 , and u_0 (or $A_{\rm max}$)
- only Einstein diameter crossing time, t_E, yields information on M_{lens}, v_⊥, and distance

A Convenient Collection of Source Stars Down South

Microlensing optical depth $\tau = 5 \times 10^{-7}$

The Large Magellanic Cloud (photo by David Malin, AAO)

The MACHO Project (1990-2000)



A Project Search for Old, Isolated planets

- High-cadence survey allows the detection of very short events due to Isolated planets
- Einstein radius crossing time, $t_E \sim \sqrt{M/M_{Jup}}$ days
- MOA-II : 1.8m telescope, 2.2 sq. deg. FOV
 - Allows high cadence surveys, with sampling every 10-60 minutes
 - Analysis of 2006-2007 MOA-II Galactic bulge survey data
 - Searched for single lens events
 - 474 events with well defined event parameters
 - 10 events with $t_{\rm E}$ < 2 days



http://www.phys.canterbury.ac.nz/moa/

MOA-II 2006-2007 Observing Strategy





- 50 deg.²(20Mstars)
 - monitor all events for planets
- •1obs./hr (M_{Jup})
- \bigcirc 10bs./10min. (M_{\oplus})



http://www.massey.ac.nz/~iabond/alert/alert.html

10 events with t_E < 2 days from 2006-2007 (events 1, 2)



MOA data in black, confirmed by OGLE data in red

10 events with t_E < 2 days from 2006-2007 (events 3, 4)



MOA data in black, confirmed by OGLE data in red

10 events with t_E < 2 days from 2006-2007 (events 5, 6)



MOA data in black, confirmed by OGLE data in red

10 events with t_E < 2 days from 2006-2007 (events 7, 8)



MOA data in black, confirmed by OGLE data in red

10 events with t_{E} < 2 days from 2006-2007 (events 9,10)



Binary Lens Background Rejection

- Both close ($d < R_E$) and wide ($d > R_E$) binary lens events can give rise to brief microlensing magnifications
- All short events can be fit by a wide binary model, because a wide binary approaches a single lens as d -> ∞
 - host stars must be at a distance > 3-15 R_E , depending on the event
 - high magnification events have the tightest limits
 - 2 wide binaries fail light curve shape cuts
- Close binaries have small external caustics that can also give short events
 - 1 such event passed all cuts but the light curve fit.
 - Close binary models have different, usually asymmetric, light curves
 - Close binary models can be rejected for all t_E < 2 day events, except for event 5
 - Since only 1 of 13 short events is a close binary, event 5 is probably a single lens event

Background: Short Binary Events



Background: Short Binary & CV



CV Background Rejection

- Poor fit to microlensing event or unphysical source brightness
- Repeating
- 208 of 418 CV light curves in 2006-2007 data have a 2nd outburst in 2006-2010
 - Classified by eye from rejected events
 - 421 multiple outbursts fit to microlensing from multiple outburst events
 - All 421 failed to pass the cuts
- after analysis was complete, OGLE-III, II, I, and MACHO databases were checked
 - OGLE-III data confirms lens models for events 2, 3, 4, 6, 7, 8 and 9
 - OGLE-III 2002-2008 data shows no additional outburst back to 2002 for events 2, 3, 4, 5, 6, 7, 8, and 9
 - Events 3, 5, 6, and 8 show no outburst in 1990s MACHO

Detection Efficiency



Private communication, Sumi

Fit to efficiency corrected t_E distribution





Mass Function Models

- Stars >1 M_{\odot} have become stellar remnants
- Assume Salpeter-like slope (α = -2) for initial >1 M_{\odot} stars
- Two choices at < 1 M_{\odot}
 - Broken power law
 - α = -2 for M > 0.7 M_{\odot}
 - $\alpha = -1.3$ for 0.7 $M_{\odot} > M > 0.08 M_{\odot}$
 - $\alpha = -0.52$ for 0.08 $M_{\odot} > M > 0.01 M_{\odot}$
 - Chabrier log-normal
 - $M_{\rm c}$ = 0.12 M_{\odot} , $\sigma_{\rm c}$ = 0.76 $dN/d\log M = \exp[(\log M \log M_c)^2/(2\sigma_c^2)]$
 - Planetary δ -function in mass
 - mass resolution limited by factor of 2-3 precision in t_E mass relation

Planetary Mass Function Parameters



Final Mass Function Models

#	$egin{array}{c} { m Mass} \ (M_{\odot}) \end{array}$	Function	parameter $(M \text{ and } \sigma \text{ are in } M_{\odot})$	Fraction (N_*)
1	$40.0 \le M$	Gaussian	Black hole $(M_r = 5, \sigma_r = 1)$	0.0031
	$8.00 \le M \le 40.0$	Gaussian	Neutron star $(M_r = 1.35, \sigma_r = 0.04)$	0.021
	$1.00 \le M \le 8.00$	Gaussian	White dwarf $(M_r = 0.6, \sigma_r = 0.16)$	0.18
	$0.70 \le M \le 1.00$	Power-law	$\alpha_1 = 2.0$	1.0
	$0.08 \le M \le 0.70$	Power-law	$\alpha_2 = 1.3$	
	$0.01 \le M \le 0.08$	Power-law*	$\alpha_3 = 0.48^{+0.29}_{-0.37} \text{ w/o PL}$	$0.73_{-0.19}^{+0.22}$
	$0.01 \le M \le 0.08$	Power-law ^{**}	$\alpha_3 = 0.50^{+0.36}_{-0.60} \text{ w/ PL}$	$0.74_{-0.27}^{+0.30}$
	$M = M_{\rm PL}$	δ -function**	$M_{\rm PL} = 1.1^{+1.2}_{-0.6} \times 10^{-3}, \mathbf{\Phi}_{\rm PL} = 0.49^{+0.13}_{-0.13}$	$1.9^{+1.3}_{-0.8}$
2	$40.0 \le M$	Gaussian	Black hole $(M_r = 5, \sigma_r = 1)$	0.0031
	$8.00 \le M \le 40.0$	Gaussian	Neutron star $(M_r = 1.35, \sigma_r = 0.04)$	0.021
	$1.00 \le M \le 8.00$	Gaussian	White dwarf $(M_r = 0.6, \sigma_r = 0.16)$	0.18
	$0.08 \le M \le 1.00$	$Log-normal^*$	$M_c = 0.12^{+0.03}_{-0.03}, \sigma_c = 0.76^{+0.27}_{-0.16}$	1.0
	$0.01 \le M \le 0.08$	$Log-normal^*$	$M_c = 0.12^{+0.03}_{-0.03}, \sigma_c = 0.76^{+0.27}_{-0.16}$	$0.70\substack{+0.19\\-0.30}$
	$0.00 \le M \le 0.01$	$Log-normal^*$	$M_c = 0.12_{-0.03}^{+0.03}, \sigma_c = 0.76_{-0.16}^{+0.27}$	$0.17\substack{+0.24\\-0.15}$
	$M = M_{\rm PL}$	δ -function***	$M_{\rm PL} = 0.83^{+0.96}_{-0.51} \times 10^{-3}, \mathbf{\Phi}_{\rm PL} = 0.46^{+0.17}_{-0.15}$	$1.8^{+1.7}_{-0.8}$
4	$0.08 \le M$		same as model (1)	
	$0.01 \le M \le 0.08$	Power-law ^{**}	$\alpha_3 = 0.49^{+0.24}_{-0.27} \text{ w/ PL}$	$0.73_{-0.15}^{+0.17}$
	$10^{-5} \le M \le 0.01$	Power-law ^{**}	$\alpha_{\rm PL} = 1.3^{+0.3}_{-0.4} \text{ w/ PL}$	$5.5^{+18.1}_{-4.3}$

Are Isolated Planets Unbound?



- Microlensing data only sets a lower limit on the separation
- HST follow-up can set tighter limits or detect host
- GDPS direct detection limits from Lafreniere et al. (2007)
 - < 40% of stars have 1 Jupiter-mass planet at 12 AU < a < 500 AU</p>
- We find 1.8 planets per star, so at least 75% of these should be free
- If the "isolated" population consists of mostly Jupiter-mass planets, then most are free-floating
 - But if they are Saturns, then we have no constraint

~1.8× as many Isolated Planets as Stars!

- Isolated means no detectable host either free-floating or in a distant orbit > 7-45 AU depending on the event
- Log-normal mass function implies 8 planets (plus 3 planetary mass brown dwarfs)
- Also, 5 planet+star events in the sample
 - Efficiency is higher for $d \sim R_E$, but -> 0 for $d << R_E$
 - So, a isolated:bound ratio of 8/5 = 1.6 might be about right
- We can also compare to measurements of Cumming et al. (2008) and Gould et al. (2010) inside and outside the snow-line
 - Implies 1.2 Saturn-Jupiter mass planets per star at 0.03-10 AU
 - So, isolated:bound ratio ~ 1.8/1.2 = 1.5
 - Since some may be bound, bound:unbound ratio ~1

Formation Scenarios

- 1. Formed like stars through gas cloud collapse (sub-brown dwarfs)
 - Hard to form Jupiter-mass objects
 - Planetary-mass sub brown dwarf can explain only 1 or 2 short events.
 - Abrupt change in mass function at Jupiter
 - Unlikely
- 2. Formed around a host star, and then removed from orbit
 - Stellar death mass loss





- By a star binary system or dense cluster
- by a planet
- Evidence:
 - Hot Jupiters orbiting hot stars have high obliquities (Winn et al. 2010, Triaud et al. 2010)
 - Hot Jupiters are alone (Latham et al. 2011)
 - No desert for short-period super-earths (Howard et al. 2010)
 - scattering more important than planet-disk interactions

HR 8799 Planetary System Doomed



- Young (10⁸ yr) planetary system found by direct detection (Marois et al. 2008)
- Planets of 10, 10, 7 *M*_{Jup} at *a* = 24, 37, 67 AU
- Simulations indicate that half of all such planets will be ejected within 10⁸ years
More Events in 2008-2010 data future analysis will focus on mass function



The alert system has revealed several events with $t_{E} \sim 0.4$ days, likely to be Neptune-mass planets. The future analysis will focus measuring the slope of the isolated planet mass function.

Theorists Speculate that Free-Floating Earths Could Be Habitable

- Stevenson (1999) a free floating Earth that is ejected early could have high density H₂ in its upper atmosphere, and this could allow T = 300K at the surface, due to radioactive geothermal heat.
- Abbot & Switzer (2011) Steppenwolf planets an deep ice covered ocean, like Europa.
- But, we will need WFIRST to determine the frequency of free floating earths.





Lecture 2 Outline

- Gravitational Microlensing by Planets with Host Stars
 - Basic multiple lens physics
 - planetary signals at high and low magnification
- Statistical results from exoplanetary microlensing
 - Cold Saturns are common
- Lens System Properties
 - Mass measurements from microlensing parallax (orbital motion of the Earth)
 - Host (lens) star detection => masses of planet and host star
- Space-based microlensing survey
 - Finds sub-Earth mass planets at all separations > 0.5 AU.
 - NASA's WFIRST (Wide Field Infrared Survey Telescope)
 - ESA's Euclid

Bound Exoplanets via Gravitational Microlensing

- 13 published discoveries and a similar number in preparation
- Sensitive to low-mass planets at a few AU
- Sensitive to planetary mass
- Planetary signal strength independent of mass
 - if $M_{planet} > 0.1 M_{\oplus}$ for main sequence source stars
 - low-mass planet signals are brief and rare
- ~10% photometric variations
 - required photometric accuracy demonstrated
- Prime sensitivity near Einstein radius at ~2-3 AU
 - High sensitivity near "snow line" important for testing planet formation theories
- M_{planet}/M_{*}, separation/(Einstein radius) from light curve
- follow-up observations measure M_{planet} , M_{\star}
- Potentially finds free-floating planets, too

Lensed Images (Einstein 1936)



When source is distant, we see distorted, magnified images. If the alignment is perfect, we see an "Einstein Ring". Einstein said, "there is no great chance of observing this effect". The probability at any one time is \sim 1 in a million, but we see \sim 800 per year.

Lensed images at µarcsec resolution

View from telescope

A planet can be discovered when one of the lensed images approaches its projected position.



Private Communication, Gaudi

Simulated Lightcurve of 1st Planetary Event

OGLE 2003-BLG-235/ 12 MOA 2008-BLG-58 Brightness 2840ann 2920 Time [days]

Simulated version of actual data



Best fit light curve simulated on an OGLE image

OGLE-2005-BLG-390Lb - "lowest" mass exoplanet



OGLE-2005-BLG-390Lb at high resolution



- Simulated view from 10,000 km aperture space telescope
- H- α filter Solar images generate cool videos!

http://planet.iap.fr/OB05390.html

OGLE-2005-BLG-390Lb at high resolution



5.5 Earth-mass planet vs. 16.5 Earth-mass planet. http://planet.iap.fr/ Only the total image area is observable. 5.5 Earth-mass is near limit for giant source. OB05390.html



low-mass planet signals are rare and brief, but not weak Mars-mass planets detectable if solar-type sources can be monitored!

Ground-based Microlensing Exoplanet Searches

- At any given time in the Galactic bulge, ~2 stars in a million are being microlensed
 - So, we'd like to monitor ~100 million star to look for microlensing events
 - The OGLE and MOA projects survey many 10s of millions of Galactic bulge stars and announce events in progress on the web.
- Stellar microlensing events typically last 1-2 months
- Planetary microlensing events have durations from several hours to several days (duration ~ \sqrt{mass})
- ~24 hour light curve coverage is needed
 - Global telescope networks
 - PLANET (Probing Lensing Anomalies NETwork)
 - MicroFUN (Microlensing Follow-Up Network)
 - Includes amateurs
 - RoboNet, MINDSTEP

Microlensing Observation Network



Anyone who wants alert is welcome to sign up on the websites.

Statistical Results from Microlensing

- Microlensing surveys all "stars" including brown dwarfs and stellar remnants
- Microlensing probability scales as $M^{1/2}$
- Longer events due to larger mass or location in disk instead of bulge have a higher detection efficiency
- Gould et al. (2010) 13 high-mag events w/ 6 planets
 - excellent light curve coverage due to high sensitivity not planetary signals: d²N/d(log q) d(log a) = 0.36 ± 0.15
 - -q = mass ratio
- Sumi et al. (2010) 10 planets with relative efficiencies – dN/d(log q) ~ $q^{-0.7\pm0.2}$ – Joint analysis (11 events) $\frac{d^2 N_{\rm pl}}{d\log(s) d\log(q)} = (0.40 \pm 0.16) \left(\frac{q}{5 \times 10^{-4}}\right)^{-0.68\pm0.20}$
- Sumi et al. (2011) excess of events with $t_{\rm E}$ < 2 days

~1.8 isolated jupiter mass lenses per main sequence star

Magnification as a Function of Source Position



Deviation from single-lens is largely determined by "caustics". Multiple planet sensitivity in high magnification events.

Private Communication, Kubas

OGLE-05-390 – Planetary Caustic



High-magnification: Low-mass planets

OGLE-2005-BLG-169Lb

- Detection of a ~13 M_⊕ planet in a A_{max}= 800 event
- Caustic crossing signal is obvious when light curve is divided by a single lens curve.
- Detection efficiency for ~10
 M_⊕ planets is << than for Jupiter-mass planets
- Competing models with an Earth-mass planet had a signal of similar amplitude
- So, an Earth-mass planet could have been detected in this event!





Light curve and exclusion diagram for OGLE-2007-BLG-050: intensive observations of high magnification events leads to high sensitivity

Batista et al., 2009

Statistical Results from Microlensing

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- Sumi et al. (2011) excess of events with $t_{\rm E}$ < 2 days

~1.8 isolated jupiter mass lenses per main sequence star

Gould et al. (2010)

- 13 high-mag events with A_{max} > 200
- 5/13 events have planets
- 6 planets in sample
- Median mass ratio $q = 5 \times 10^{-4}$
- Typical host star mass $M \sim 0.5 M_{\odot}$
- Low-mass gas giants are common around early M-dwarfs



Sumi et al. (2010)

- Detection efficiencies for 10 planetary microlensing events
- Efficiencies not yet calculated for 100's of events without planetary signals
 - at log magnification, detection probability per event can be low ~ 1-10%
- Null detections needed for full abundance
- But sample of events with planets is a fair sample for determining planet frequency as a function of mass ratio, *q*

$$\frac{dN_{pl}}{d\log q} \propto q^{-0.68\pm0.20}$$

Comparison of Statistical Results (for planetary masses)



Detection Efficiencies for Events w/ Planets



Detection efficiencies assuming 1 planet with separation $0.1 < d/R_{\rm E} < 10$

Distribution seems nearly uniform down to $q \sim 5 \times 10^{-5}$

Full efficiency analysis for low-mag & survey sample requires calculation for a large number of low-efficiency events.

Combined Analysis

Assume a power-law mass function for $q_{min} < q < q_{max}$ $q_{max} = 0.01$

Multiply Gould et al (2010) probability function by a probability function similar to that of Sumi et al (2010), ⊲ but excluding common events and using full efficiencies instead of power-law approximation.

Result is very similar to the separate Gould et al (2010) and Sumi et al (2010) results.



Dependence on q_{\min}

$oldsymbol{q}_{min}$	N events	Α	n
6×10⁻⁵	11	$0.42^{\mathrm{+0.19}}_{\mathrm{-0.14}}$	-0.75 ± 0.22
4×10 ⁻⁵	11	$0.40^{+0.18}_{-0.14}$	-0.68 ± 0.20
2×10 ⁻⁵	11	$0.36^{+0.17}_{-0.12}$	-0.56 ± 0.18
1×10 ⁻⁵	11	$0.34^{\tiny +0.15}_{\tiny -0.12}$	-0.47 ± 0.17
5×10 ⁻⁶	11	$0.33_{-0.12}^{+0.15}$	-0.44 ± 0.16
5×10 ⁻⁶	12*	$0.36^{+0.16}_{-0.12}$	-0.52 ± 0.15

* includes $q = 9 \times 10^{-6} 2^{nd}$ planet for OGLE-2007-BLG-349 (may not be real)

dependence on q_{min} may be due shallowing of slope below 10 Earth-masses - fewer cold Earths?

Characterization of Microlensing Planets and Their Host Stars

"I don't understand. You are looking for planets you can't see around stars you can't see."



Debra Fischer
 RV planet hunter
 2000 Microlensing Workshop

Microlensing events might only give mass ratio, q, and separation, d/R_E , in Einstein radius units. We want more info on the planetary events than this!

Lens System Properties

- For a single lens event, 3 parameters (lens mass, distance, and velocity) are constrained by the Einstein radius crossing time, $t_{\rm E}$
- There are two ways to improve upon this with light curve data:
 - Determine the angular Einstein radius : $\theta_E = \theta_* t_E / t_* = t_E \mu_{rel}$ where θ_* is the angular radius of the star and μ_{rel} is the relative lens-source proper motion
 - Measure the projected Einstein radius, $\tilde{r}_{\rm E}$, with the microlensing parallax effect (due to Earth's orbital motion).



- Einstein radius : $\theta_{\rm E} = \theta_* t_{\rm E} / t_*$ and projected Einstein radius, $\tilde{r}_{\rm E}$
 - θ_* = the angular radius of the star
 - $-\tilde{r}_{\rm E}$ from the microlensing parallax effect (due to Earth's orbital motion).

$$R_E = \theta_E D_L$$
, so $\alpha = \frac{\tilde{r}_E}{D_L} = \frac{4GM}{c^2 \theta_E D_L}$. Hence $M = \frac{c^2}{4G} \theta_E \tilde{r}_E$

Finite Source Effects & Microlensing Parallax Yield Lens System Mass

- If only θ_E or \tilde{r}_E is measured, then we have a mass-distance relation.
- Such a relation can be solved if we detect the lens star and use a mass-luminosity relation
 - This requires ground-based adaptive optics or space-based observations
- With θ_E , \tilde{r}_E , and lens star brightness, we have more constraints than parameters

mass-distance relations:

$$M_{L} = \frac{c^{2}}{4G}\theta_{E}^{2}\frac{D_{S}D_{L}}{D_{S} - D_{L}}$$
$$M_{L} = \frac{c^{2}}{4G}\tilde{r}_{E}^{2}\frac{D_{S} - D_{L}}{D_{S}D_{L}}$$
$$M_{L} = \frac{c^{2}}{4G}\tilde{r}_{E}\theta_{E}$$

Double-Planet Event: OGLE-2006-BLG-109

- •5 distinct planetary light curve features
- •OGLE alerted 1st feature as potential planetary signal
- High magnification
- Feature #4 requires an additional planet
- Planetary signals visible for 11 days
- Features #1 & #5 require the orbital motion of the Saturnmass planet



OGLE-2006-BLG-109 Light Curve Features

- The basic 2-planet nature of the event was identified during the event,
- But the final model required inclusion of orbital motion, microlensing parallax and computational improvements (by Bennett).





Gaudi et al., 2008

OGLE-2006-BLG-109 Source Star



The model indicates that the source is much fainter than the apparent star at the position of the source. Could the brighter star be the lens star?



Bennett et al., 2010

OGLE-2006-BLG-109Lb,c Host Star



- OGLE images show that the source is offset from the bright star by 350 mas
- B. Macintosh: Keck AO images resolve lens+source stars from the brighter star.
- But, source+lens blend is 6× brighter than the source (from CTIO H-band light curve), so the lens star is 5× brighter than source.
 - H-band observations of the light curve are critical because the lens and source and not resolved
- Planet host (lens) star magnitude H ≈ 17.17
 - JHK observations will help to constrain the extinction toward the lens star

Bennett et al., 2010

Only Multiplanet System with Measured Masses

Host star mass: $M_L = 0.52^{+0.18}_{-0.07} M_{\odot}$ from light curve model.

- Apply lens brightness constraint: $H_L \approx 17.17$.
- Correcting for extinction: H_{L0} = 16.93 ± 0.25
 - Extinction correction is based on H_L - K_L color
 - Error bar includes both extinction and photometric uncertainties
- Lens system distance: D_L = 1.54 ± 0.13 kpc

Host star mass: $M_L = 0.51 \pm 0.05 M_{\odot}$ from light curve and

lens H-magnitude. Other parameter values:

- "Jupiter" mass: semi-major axis:
- "Saturn" mass: semi-major axis:
- "Saturn" orbital velocity eccentricity inclination

S: $m_b = 0.73 \pm 0.06 M_{Jup}$ $a_b = 2.3 \pm 0.5 AU$ $m_c = 0.27 \pm 0.03 M_{Jup} = 0.90 M_{Sat}$ S: $a_c = 4.5^{+2.2}_{-1.0} AU$ y $v_t = 9.5 \pm 0.5 \text{ km/sec}$ $\epsilon = 0.15^{+0.17}_{-0.10}$ $i = 63 \pm 6^{\circ}$
Orbital Motion Modeling



- 4 orbital parameters are well determined from the light curve
 - 2-d positions and velocities
 - Slight dependence on distance to the source star when converting to physical from Einstein Radii units
- Masses of the host star and planets are determined directly from the light curve
 - So a full orbit is described by 6 parameters (3 relative positions & 3 relative velocities)
 - A circular orbit is described by 5 parameters
- Models assume planetary circular motion
 - 2-d positions and velocities are well determined
 - Orbital period is constrained, but not fixed by the light curve
 - The orbital period parameter can be interpreted as acceleration or 3-d Star-Saturn distance (via $a = GM/r^2$)
- Details in Bennett et al (2010)

Full Orbit Determination for OGLE-2006-BLG-109Lc



Future Doppler Radial Velocity Confirmation



A high throughput, high resolution spectrograph on a 22-40m aperture telescope can measure the 19 m/s RV signal

OGLE-2006-BLG-Lb,c Discovery Implications

- OGLE-2006-BLG-109L is the first lens system with a Jovian Planet which has very high sensitivity to additional Saturn-mass planets
 - OGLE-2003-BLG-235 and OGLE-2005-BLG-71 had much lower magnification
 - OGLE-2005-BLG-169 had only a Neptune (or Super-earth)
- Jupiter + Saturn systems may be common among systems with gas-giant planets
 - Radial velocity planets 47 UMa & 14 Her are similar systems with more massive planets.

Survey Discovery: MOA-2009-BLG-266

- Planet discovered by MOA on Sept. 11, 2009
- Only cold super-Earth with a mass measurement

$$m_p = 10.4 \pm 1.7 M_{\oplus}$$

 $M_* = 0.56 \pm 0.09 M_{\odot}$
 $a = 3.2^{+1.9}_{-1.5} \text{ AU}$
 $D_L = 3.0 \pm 0.3 \text{ kpc}$



Space-Based Microlensing Parallax

2004: study LMC microlensing w/ DI imaging (proposed)

2009: Geometric exoplanet and host star mass measurements with DI





EPOXI PSF!



Satellite Observations of Exoplanet Microlensing events



Galactic disk lens system

Galactic bulge lens system

Why Space-based Microlensing?

- Microlensing requires extremely crowded fields
- Source stars only resolvable from space
- Ground-based surveys need high lensing magnification to resolve most source stars
 - Limits sensitivity to near the Einstein ring
 - Space-based microlensing sensitive from 0.5 AU ∞
- Space-based microlensing allows detection of most lens stars
 - Allows direct determination of star and planet masses
- Simulations from Bennett & Rhie (2002)
- Basic results confirmed by independent simulations (Gaudi)
- Microlensing Planet Finder (MPF) -> WFIRST

Ground-based confusion, space-based resolution



- Space-based imaging needed for high precision photometry of main sequence source stars (at low magnification) and lens star detection
- High Resolution + large field + 24hr duty cycle => WFIRST Microlensing program
- Space observations needed for sensitivity at a range of separations and mass determinations



Infrared Observations Are Best

The central Milky Way:

near infrared



optical

Dust obscures the best microlensing fields toward the center of the Galaxy



The spectrum of a typical reddened source star is compared to the QE curves of CCDs and Si-PIN detector arrays. The HgCdTe detectors developed for HST's WFC3 instrument can detect twice as many photons as the most IR sensitive Si detectors (CCDs or CMOS). MPF will employ 35 HgCdTe detectors. 3 filters: "clear" 600-1700nm, "visible" 600-900nm, and "IR" 1300-1700nm.

Astro-2010 Decadal Survey

"WFIRST designed to settle important questions in both exoplanet and dark energy research"

"the Kepler satellite ... should be capable of detecting Earth-size planets out to almost Earth-like orbits."

"As microlensing is sensitive to planets of all masses having orbits larger than about half of Earth's, WFIRST would be able to complement and complete the statistical task underway with Kepler, resulting in an unbiased survey of the properties of distant planetary systems.

WFIRST does a microlensing planet search, multiple dark energy studies plus IR surveys and GO observations



WFIRST vs. Kepler



Figures from B. MacIntosh of the ExoPlanet Task Force

WFIRST's Predicted Discoveries



The number of expected WFIRST planet discoveries per 9-month observing season as a function of planet mass.

Lens Star Detection in WFIRST Images

- The typical lens-source relative proper motion is μ_{rel}~ 5 mas/yr
- This gives a total motion of >0.05 pixels over 3 years
- This is directly detectable in co-added WFIRST images due to WFIRST's stable PSF and large number of images of each of the target fields.
- μ_{rel} is also determined from the light curve fit.
- A color difference between the source and lens stars provides a signal of μ_{rel} in the color dependence of the source+lens centroid position



A 3× super-sampled, drizzled 4-month WFIRST image stack showing a lenssource blend with a separation of 0.07 pixel, is very similar to a point source (left). But with PSF subtraction, the image elongation becomes clear, indicating measurable relative proper motion.

Lens Star Identification from Space

- Lens-source proper motion gives $\theta_E = \mu_{rel} t_E$
- μ_{rel}= 8.4±0.6 mas/yr for OGLE-2005-BLG-169
- Simulated HST ACS/HRC F814W (*I*-band) single orbit image "stacks" taken 2.4 years after peak magnification
 - 2× native resolution
 - also detectable with HST WFPC2/PC & NICMOS/NIC1
- Stable HST PSF allows clear detection of PSF elongation signal
- A main sequence lens of any mass is easily detected (for this event)

Simulated HST images: $M_1 = 0.35 M_2$ = 0.63 *M*/ binned **PSF** subtracted raw image

Bennett, 2009

Color Dependent Image Center Shift



Source & Planetary Host stars usually have different colors, so lenssource separation is revealed by different centroids in different passbands

HST Observation Predictions for OGLE-2003-BLG-235L/MOA-2003-BLG-53L



Bennett, 2009

Lens Detection Provides Complete Lens Solution



- The observed brightness of the lens can be combined with a mass-luminosity relation, plus the mass-distance relation that comes from the μ_{rel} measurement, to yield a complete lens solution.
- The resulting uncertainties in the absolute planet and star masses and projected separation are shown above.
- Multiple methods to determine μ_{rel} and masses (such as lens star color and microlensing parallax) imply that complications like source star binarity are not a problem.



- Microlensing Planet Finder combined with JDEM-Omega and NIRSS by decadal survey to make WFIRST
- WFIRST Science Definition Team formed
- Charge to SDT
 - Design WFIRST
 - Look at low-cost options
 - Advice to NASA for possible merger with Euclid



Wide-Field InfraRed Survey Telescope WFIRST Interim Report

Science Definition Team

nergy - Exoplanets

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Digitally signed by Paul Schechter DN: cn-Paul Schechter, c=US, o=Massachusetts Institute of Paul Schechter schnology, ou=Kanti Institute for Astrophysics and Space earch, email=schech@mit.edu 7-11-11 ocation: MIT Cambridge MA 02139 Date: 2011.07.11 12:48:52 -04'00' James Green, SDT-Co-Chair Date Paul Schechter, SDT Co-Chair Date

Green et al., 2011

International Situation

- ESA's Euclid Mission
 - Focuses on Dark Energy
 - Selection announced next month
 - In competition with Solar Orbiter and Plato for 2 slots
 - A small microlensing exoplanet program
 - could be expanded
- NASA is interested in international partners for WFIRST
 - possible joint mission or joint program with Euclid
 - JAXA participation?

Videos by D. Bennett & A. Williams

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