Boussinesq thermal convection in a rotating spherical shell with an outer stably stratified layer

Shin-ichi Takehiro[1*], Michio Yamada[1] and Yoshi-Yuki Hayashi[2]

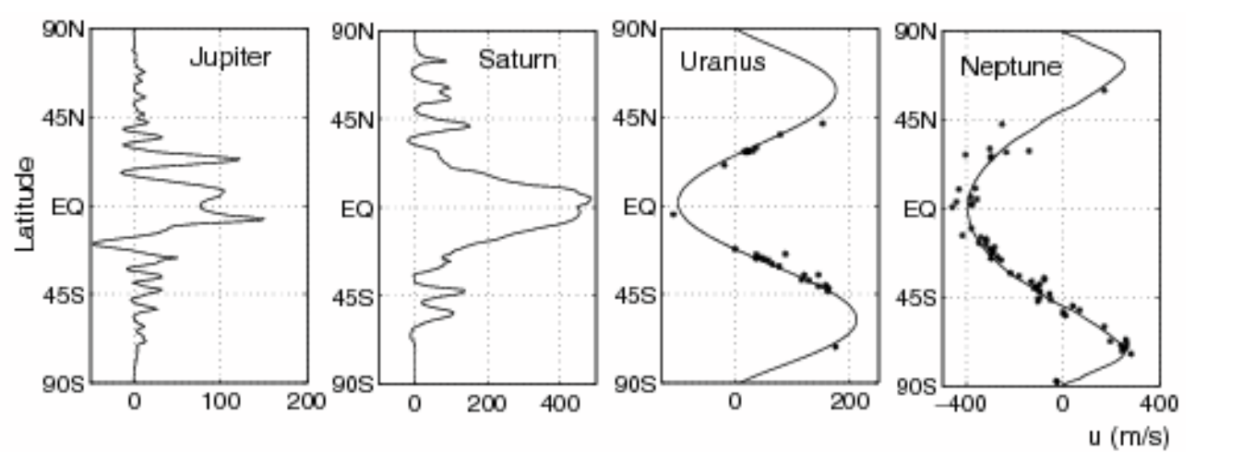
[1]Research Institute for Mathematical Sciences, Kyoto University, Japan
[2]Department of Earth and Planetary Sciences, Kobe University, Japan
*contact : takepiro@gfd-dennou.org

Abstract

Finite amplitude thermal convection in a rapidly rotating spherical shell associated with a stably stratified layer placed near the outer surface is investigated. Systematic numerical experiments are performed with an Ekman number of $E=10^{-3}$, a Prandtl number of $P=1$ and an inner/outer radius ratio of $\eta=0.4$, and the existence of a strongly stratified upper layer is shown to enhance the generation of equatorial surface retrograde flows when the Rayleigh number is approximately ten times larger than the critical value. The existence of the stable layer causes the bottom of the stable layer to behave as a virtual boundary for the convective motion underneath. Its effective dynamic condition varies from the free-slip condition to the no-slip condition as the Rayleigh number increases. Budget analysis of angular momentum shows that the Reynolds stress of the convective vortices beneath the stable layer is weakened and is dominated by the transport of the planetary angular momentum when the Rayleigh number is increased. As a result, the latitudinal temperature gradient produced at the bottom of the stable layer induces the equatorial retrograde flow through the thermal wind balance. This diffuses through the stable layer by viscosity and produces the equatorial surface retrograde flow.

1. Introduction

○Gas giant planets : equatorial superrotation

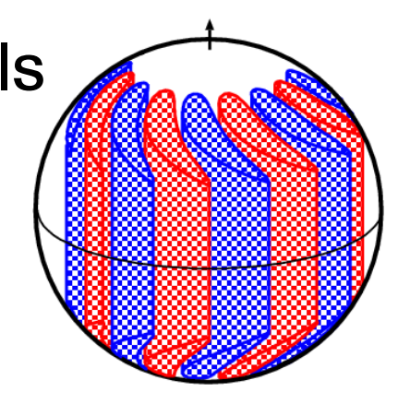


Surface zonal flows of the gas and icy planets (Sukoriansky, et al. 2002)

- Jupter, Saturn : equatorial superrotation
- Equatorial superrotation : dynamically interesting
  - Simple angular mom.transport → equatorial surface retrograde flow
  - Some acceleration mechanisms needed

○Thermal convection in rotating spherical shells

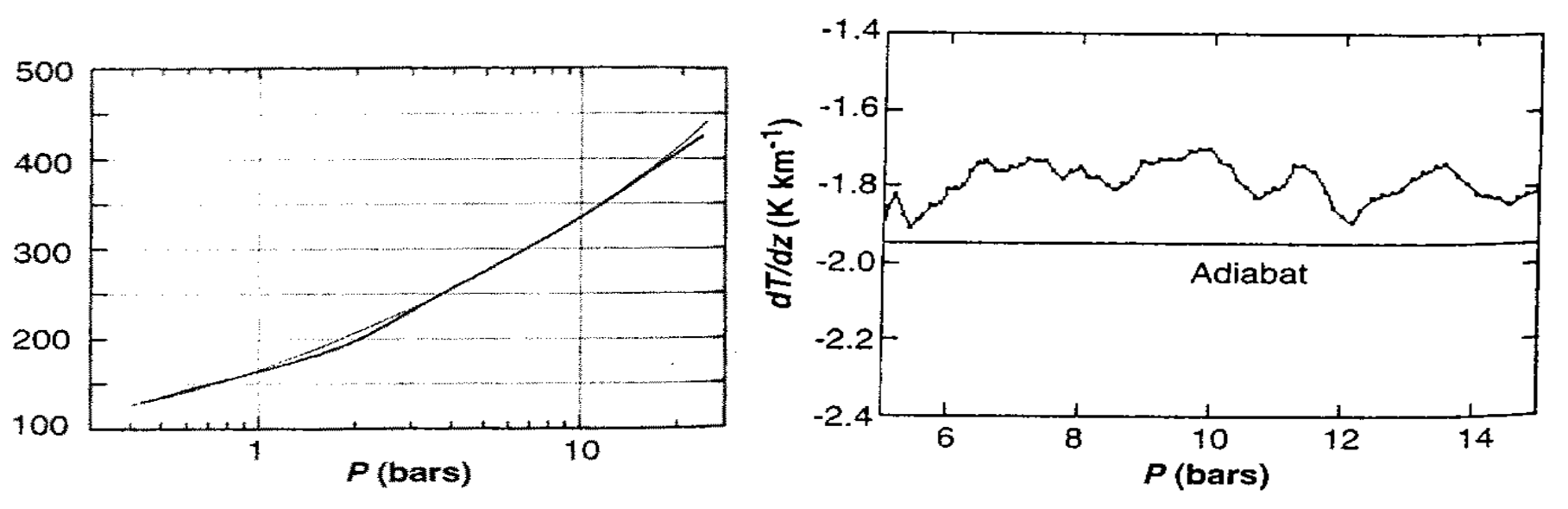
- A model for explaining equatorial superrotaion
- In rapidly rotating spherical shells → Taylor-column type convection uniform along the rotating axis
- prograde- outward tilting of the convection cells → momentum transport by Reynolds stress → Equatorial superrotation
- When the Rayleigh number is increased → mixing of angular mom. → retrograde equatorial flows



A schematic picture of thermal convection cells in a rapidly rotating spherical shells

○A stably stratified layer near the top boundary?

- Most of studies : the whole layer is unstable
- Stably stratified layers in the upper part of the actual planetary atmospheres (stratospheres, tropospheres)
- The existence of stable layers inhibits columnar convection → angular mom.transport possible?



Vertical temperature profile (left) and temperature gradient (right). Note that stable stratification. Seiff et al. (1996)

○Previous studies

- Zhang and Schubert (1996,1997)
  Critical convection. Columnar convection completely penetrating a stable layer
- Takehiro and Lister (2001)
  The penetration thickness is in proportion to the ratio between the angular velocity and Brunt-Vaisala frequency Ω/N
- Takehiro and Lister (2002)
  Finite amplitude convection with the Rayleigh numbers several times of the critical. Mean zonal flows penetrates by viscosity, resluting equatorial superrotation.

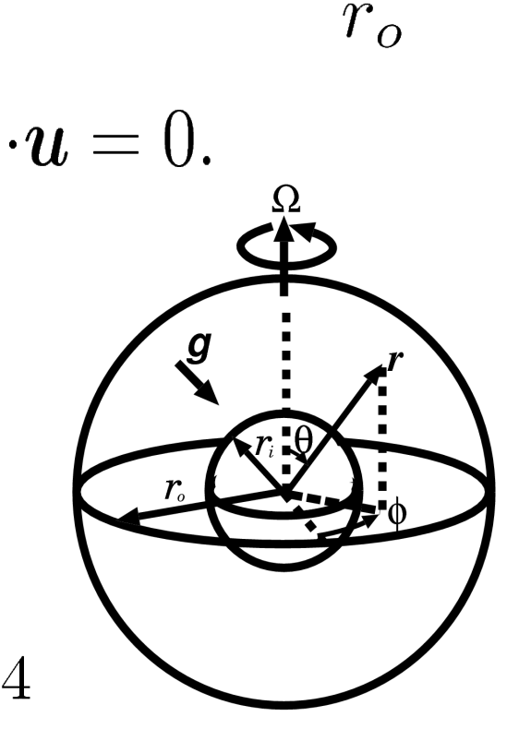
○This study

- The dependency of equatorial surface mean zonal flows on the Rayleigh number and strength of stably stratified layers?
- The Rayleigh number is further increased.
- The stability of stable layer is varied more widely.

2. Model

- System : Boussinesq fluid in a rotating spherical shell

$$E\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} - \nabla^2\boldsymbol{u}\right) + 2\boldsymbol{k}\times\boldsymbol{u} + \nabla p = R\frac{\boldsymbol{r}}{r_o}T,$$
$$\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T + u_r\cdot\frac{dT_B}{dr} = \frac{1}{P}\nabla^2 T, \quad \nabla\cdot\boldsymbol{u} = 0.$$



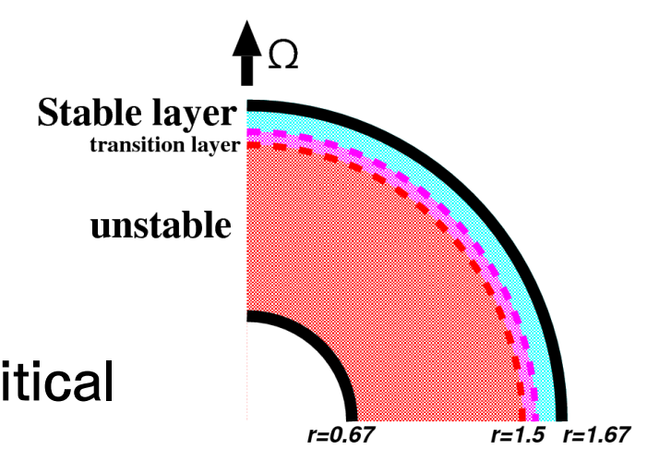
- Parameters
  - The Rayleigh number: R
  - The Ekman number: $E = 10^{-3}$
  - The Prandtl number: $P = 1$
  - The ratio of the inner and outer radii: $\eta = 0.4$
  - The basic temperature (Stability of the stable layer): $T_B(r)$

○Experimental setup

- The basic temperature profile

$$\frac{dT_B}{dr} = -\frac{1}{2}(r+\Gamma)\left[1-\tanh\frac{r-r_b}{a}\right] + \Gamma$$

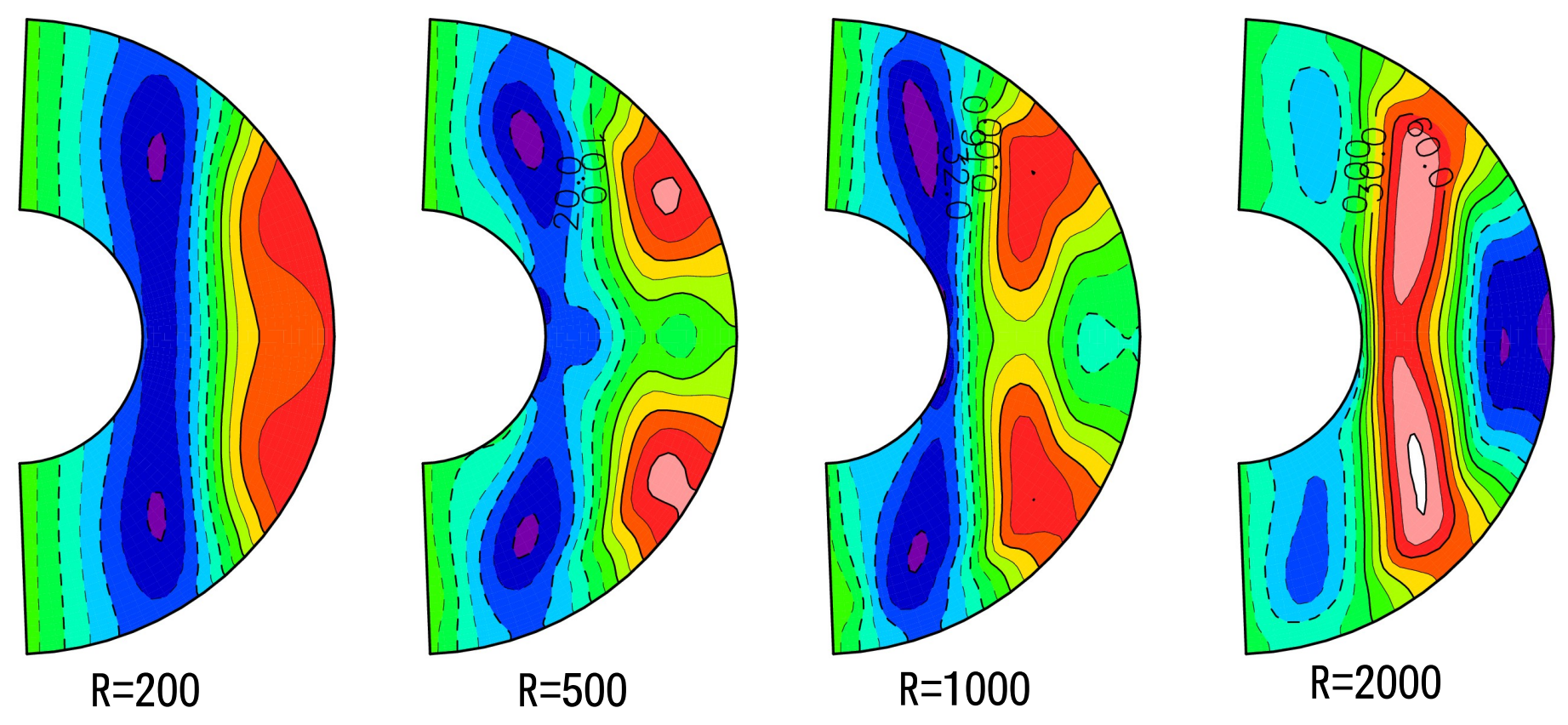
  - The outer part : stable, constant temp.grad.: Γ
  - The inner part : unstable, homogeneous heat source
- The boundary conditions :
  - fixed temperature, stress free
- The parameters for experimens
  - The Rayleigh number : several to 10 times critical
  - Stable stratification : $\Gamma = 1 \sim 10^4$



3. Results

○Mean zonal flows : Rayleigh number dependency

Red : prograde. Blue : retrograde. The cases with Γ=100.

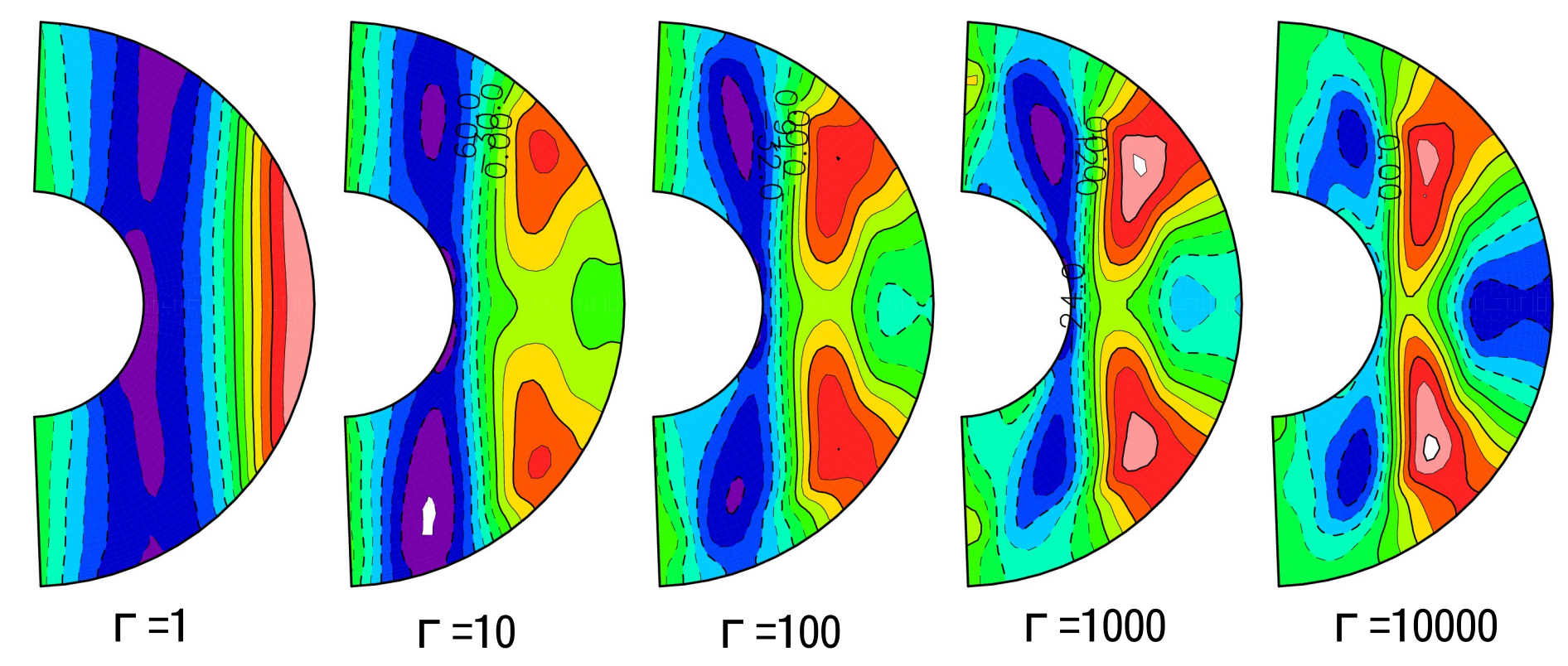


R=200 R=500 R=1000 R=2000

- Small Rayleigh numbers : equatorial prograde at the top
- Large Rayleigh numbers : equatorial retrograde at the top

○Mean zonal flows : dependence of the stable stratification

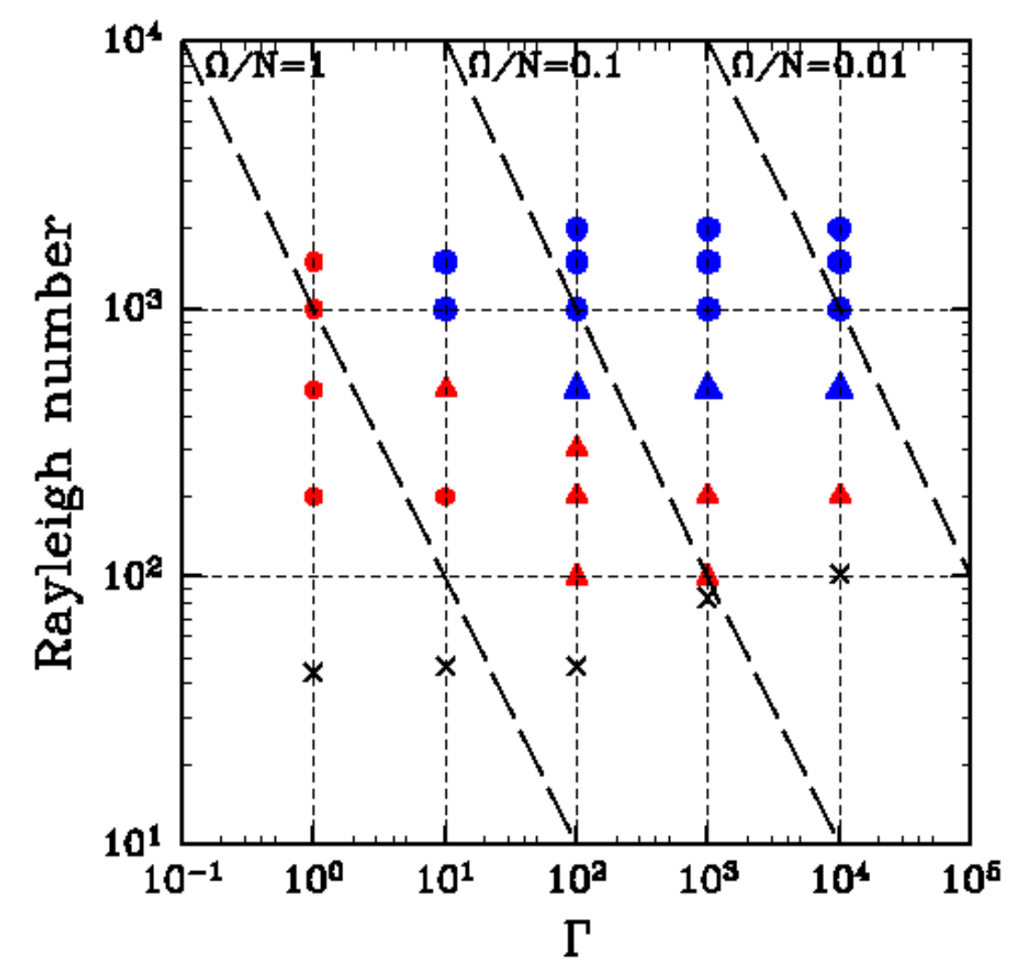
Red : prograde. Blue : retrograde. The cases with R=1000.



Γ=1 Γ=10 Γ=100 Γ=1000 Γ=10000

- Weak stratification : equatorial prograde at the top
- Strong stratification : equatorial retrograde at the top

○Direction of equatirial surface zonal flows

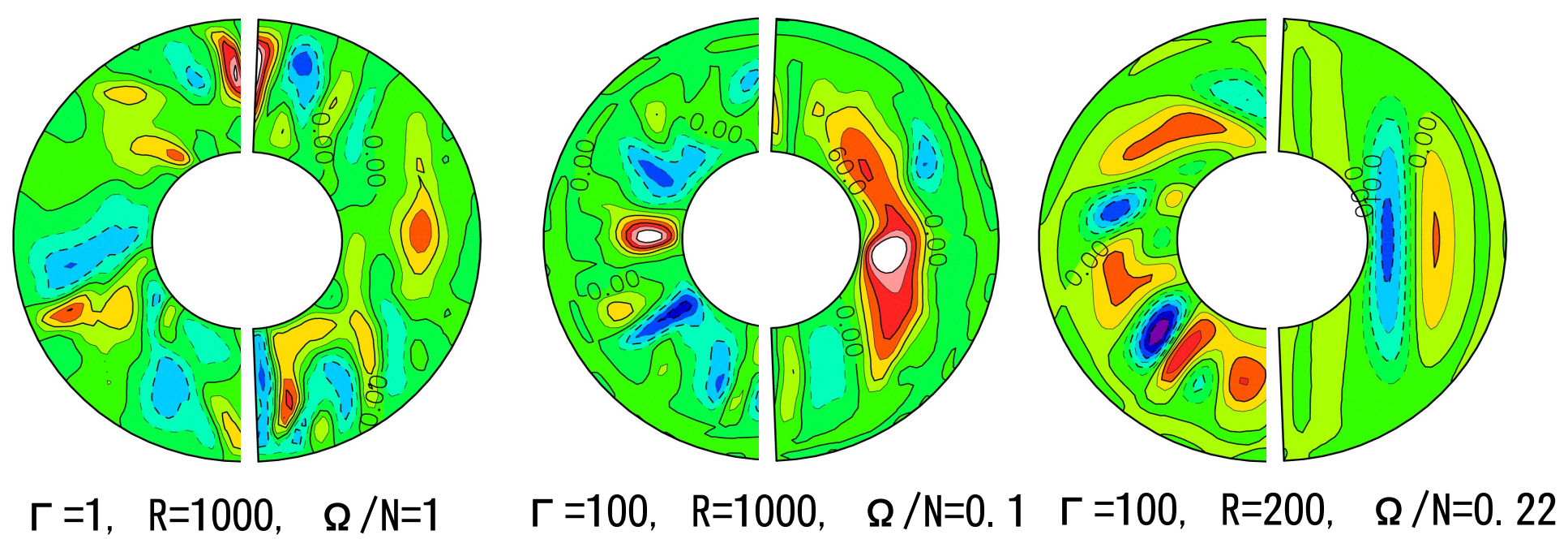


Red : equatorial prograde at the top.
Blue : equatorial retrograde at the top.

- Rayleigh number is several times critical → equatorial prograde at the top
- Large Rayleigh number, strong stratification → equatorial retrograde at the top

○Convective flows

Left : equatorial cross section. Right : meridonal cross section
Red : upward flows. Blue : downward flows

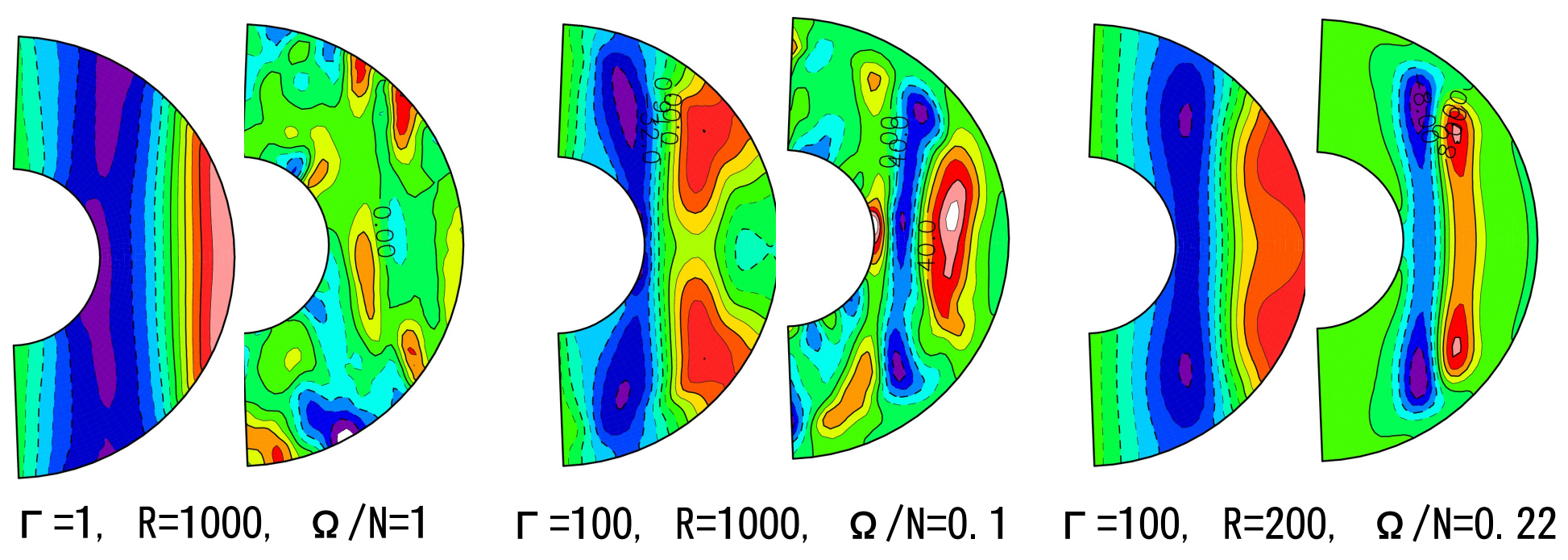


Γ=1, R=1000, Ω/N=1 Γ=100, R=1000, Ω/N=0.1 Γ=100, R=200, Ω/N=0.22

- Weak stratification : convective motions penetrate the stable layer
- Strong stratification, large Rayleigh numbers : trapped below the stable layer
- Strong stratification, small Rayleigh numbers : columnar convection below the stable layer

○Mean zonal flows and disturbances

Left: mean zonal flows. Right : disturbances. Red : prograde. Blue : retrograde.



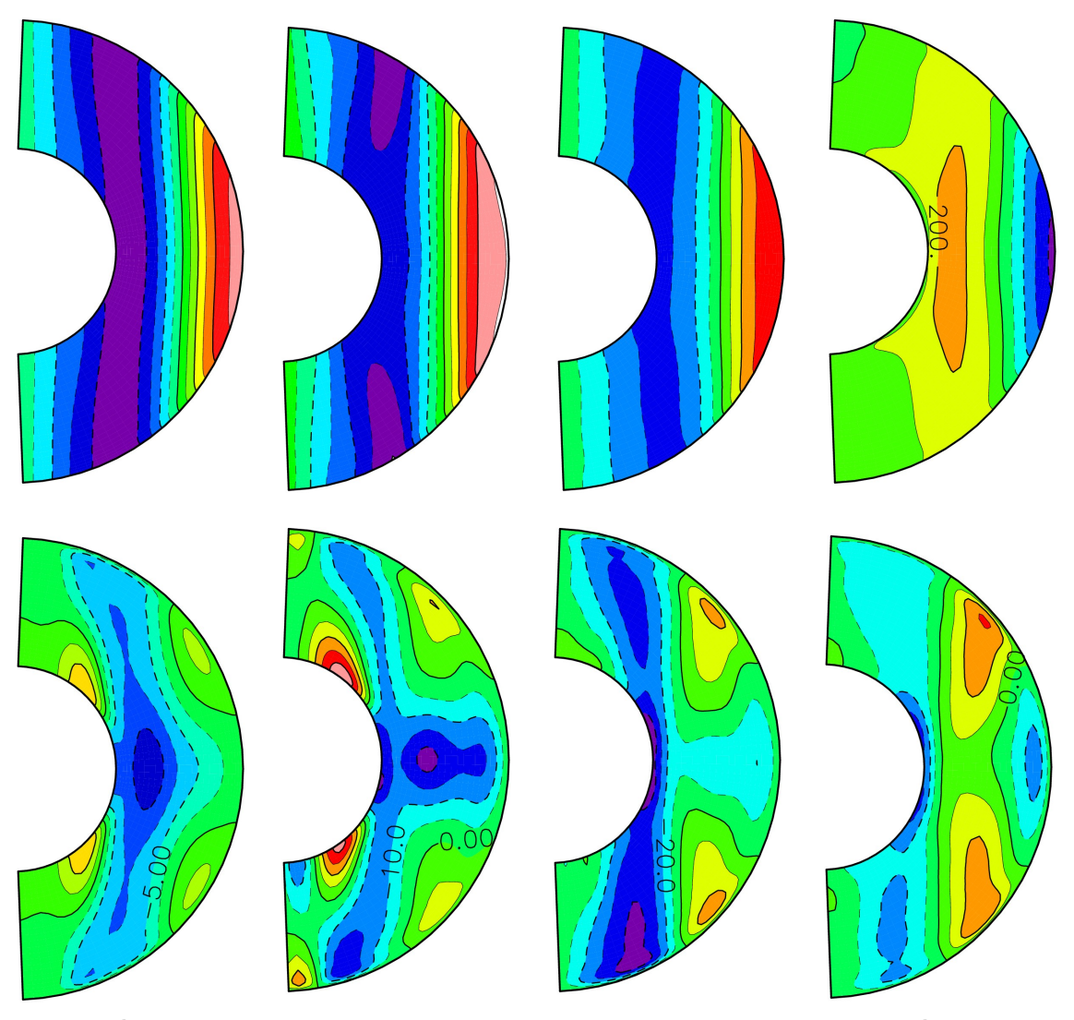
Γ=1, R=1000, Ω/N=1 Γ=100, R=1000, Ω/N=0.1 Γ=100, R=200, Ω/N=0.22

- Weak stratification : disturbances erode the stable layer
- Strong stratification : disturbances are trapped below the stable layer

4. Discussions

○Experiments with a boundary just beneath the stable layer

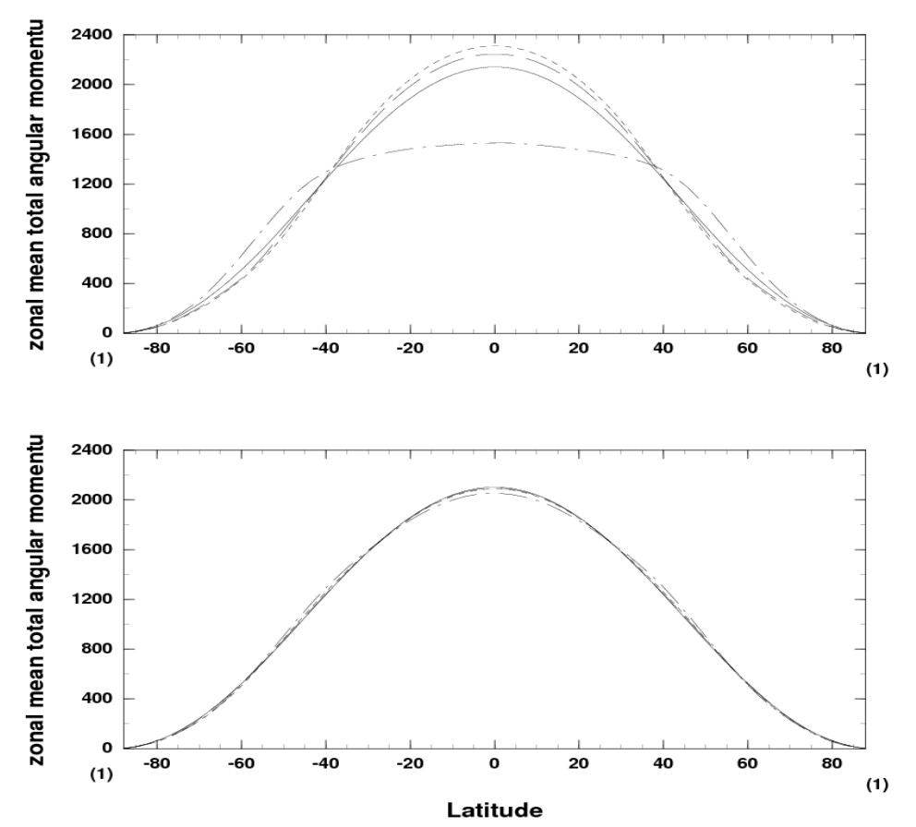
Red : prograde. Blue : retrograde.
Upper : free-slip at the top. Lower : no-slip at the top.



R=200 R=500 R=1000 R=2000

- Free-slip at the top : equatorial retrograde at large Rayleigh numbers
- No-slip at the top : equatorial retrograde at small Rayleigh numbers
- R≧500 : the distributions are similar to the no-slip boundary cases
- R=200 : similar to the free-slip boundary cases → the effect of the stable layer is a medium condition between no-slip and free-slip

○Distribution of angular momentum



Distributions of angular momentum near the top boundary.
Stress-free cases (upper) and no-slip cases (lower).
Solid, dashed, dotted, and dash-dotted lines are the cases with $R = 2\times10^2, 5\times10^2, 1\times10^3, 2\times10^3$, respectively.

- Retrograde with free-slip boundary ← angular mom.homogenization
- Retrograde with no-slip boundary : inhomogeneous angular mom. ← weakening of Reynolds stresses by viscosity?

○Angular momentum budget analysis

- Conservation of angular momentum

$$\frac{\partial}{\partial t}(\overline{u}_\phi r\sin\theta) + \nabla\cdot\mathbf{F}_p + \nabla\cdot\mathbf{F}_r + \nabla\cdot\mathbf{F}_d + \nabla\cdot\mathbf{F}_v = 0.$$

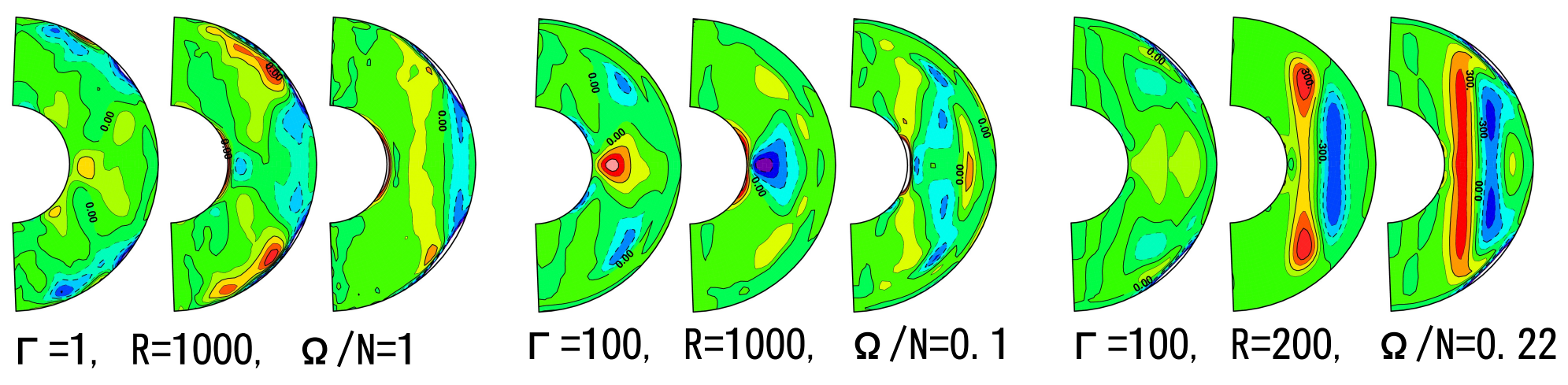
$$\mathbf{F}_p = \frac{1}{E}r^2\sin^2\theta(\overline{u}_\theta\mathbf{e}_\theta + \overline{u}_r\mathbf{e}_r)$$ Planetary angular mom. flux

$$\mathbf{F}_r = \overline{u}_\phi\overline{u}_\theta r\sin\theta\mathbf{e}_\theta + \overline{u}_\phi\overline{u}_r r\sin\theta\mathbf{e}_r$$ Relative angular mom. flux

$$\mathbf{F}_d = \overline{u'_\phi u'_\theta}r\sin\theta\mathbf{e}_\theta + \overline{u'_\phi u'_r}r\sin\theta\mathbf{e}_r$$ Reynolds stresses

$$\mathbf{F}_v = -r^2\sin^2\theta\nabla\left(\frac{\overline{u}_\phi\sin\theta}{r^2\sin^2\theta}\right)$$ Viscous angular mom. flux

Distributions of zonally and temporally averaged angular momentum flux divergences.
Left, center, and right panels are $\nabla\cdot\mathbf{F}_p, \nabla\cdot\mathbf{F}_d, \nabla\cdot\mathbf{F}_p+\nabla\cdot\mathbf{F}_r+\nabla\cdot\mathbf{F}_d$, respectively.
Red : divergence. Blue : convergence.



Γ=1, R=1000, Ω/N=1 Γ=100, R=1000, Ω/N=0.1 Γ=100, R=200, Ω/N=0.22

- Weak stratification, large Rayleigh numbers: Reynolds stress dominates → equatorial prograde
- Strong stratification, large Rayleigh numbers : Planetary angular momentum flux dominates → equatorial retrograde
- Strong stratification, small Rayleigh numbers : Reynolds stress dominates → equatorial prograde

5. Summary

- Large Rayleigh numbers, strong stratification → equatorial retrograde flows
- Deep convection is turbulent rather than regular and columnar, but does not penetrate the stable layer
- The existence of the stable layer
  - Changes the radius ratio of the convective region
  - Boundary condition at the top of the convective region → free-slip to no-slip as the Rayleigh number is increased
  - Reynolds stress is weakened and transport by mean meridional circulation dominates

References

Seiff, A., Kirk, D.B., Knight, T.C.D., Mihalov, J D.,Blanchard, R.C., Young, R.E., Schubert, G., Zhang, U.,Lehmacher, G., Milos, F.S. and Wang, J., 1996 : Structure of the atmospheric of Jupiter: Galileo probe measurements,Science, 272, 844--845.

Sukoriansky, S., Galperin, B., Dikovskaya, N., 2002 : Universal spectrum of two-dimensional turbulence on a rotating sphere and some basic features of atmospheric circulation on giant planets.Phys. Rev. Lett., 89, 124501-1--4.

Takehiro S. and Lister, J. R., 2001 : Penetration of columnar convection in an outer stably stratified layer in rapidly rotating spherical fluid shells, Earth Planet. Sci. Lett., 187, 357--366.

Takehiro S. and Lister, J. R., 2002 : Surface zonal flows induced by thermal convection trapped below a stably stratified layer in a rapidly rotating spherical shell, Geophy. Res. Lett., 29, 10.1029/2002GL015450

Takehiro, S., Yamada, M., and Hayashi, Y.-Y., 2011 : Retrograde equatorial surface flows generated by thermal convection confined under a stably stratified layer in a rapidly rotating spherical shell,Geophys. Astrophys. Fluid Dyn., 105, 61--81.

Zhang, K. and Schubert, G., 1996 : Penetrative convection and zonal flow on Jupiter, Science, 273, 941--943.

Zhang, K. and Schubert, G., 1997 : Linear penetrative spherical rotating convection, J. Atmos. Sci., 54, 2509--2518.

CPS 7th International School of Planetary Sciences, ``Theory of Stellar Evolution and Its Applications -- From the First Stars to Planet-Hosting Stars and Gas Giant Planets'', 10 -- 15 January 2011, Seapal Suma, Kobe, Japan.