Detailed Radiative Transfer Schemes
in the 3-D Hydrodynamical Solar Surface

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ABSTRACT. We have investigated the detailed non-grey radiative transfer scheme in the three dimensional hydrodynamical solar surface. Outer convection zone is extremely turbulent region composed of partly ionized compressible gases in high temperature. Especially, super-adiabatic layer (SAL) is the transition region where the transport of energy changes drastically from convection to radiation. In order to describe physical processes in SAL accurately, a realistic treatment of radiation should be considered as well as convection. For a detailed computation of radiative transfer, the Accelerated Lambda Iteration (ALI) methods have been applied to Large-Eddy Simulation (LES) with non-grey opacity schemes using the Opacity Distribution Function (ODF). Our computational domain is the rectangular box of dimensions $4^2 \times 3$ Mm with the resolution of $117^2 \times 190$ meshed grids, which covers several granules horizontally and 8~9 pressure scale heights vertically. As the result of numerical simulation, we present the time-dependent variation of radiation fields and thermodynamic structures in the solar outer convection zone. In addition, our radiation-hydrodynamical computation has been compared with the classical approximations such as grey atmosphere and Eddington approximation.

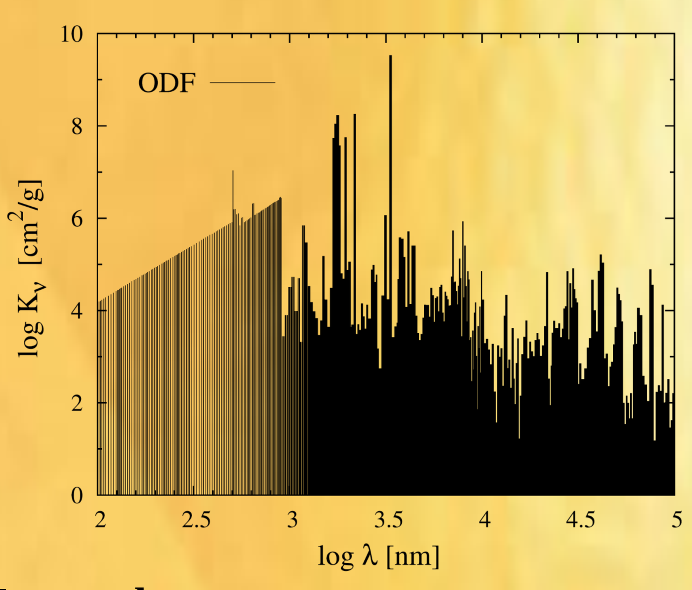
Solar Calibration

Initial Configuration The starting model for the 3-D simulation has been obtained using the 1-D stellar structure & evolution code, **YREC**. With well-defined observables such as solar effective temperature and luminosity, solar calibration is to find modeling parameter set. In this study, the standard solar model has been constructed based on the GS98 solar abundance.

Parameters	Values
(X_0, Y_0, Z_0)	(0.7085, 0.2726, 0.0188)
(X_s, Y_s, Z_s)	(0.7399, 0.2432, 0.0169)
$(Z/X)_\odot$	0.02292 (GS98)
α_{MLT}	1.87
Age	45.5 Gyr
Input microphysics	
Microscopic Diffusion	*Y, Z*–diffusion
Atmospheric approximation	Eddington T – τ
Solar mixture	Grevesse & Sauval (1998)
Opacities	OPAL Opacity (Iglesias & Rogers, 1996, updated 2001)
Low temperature opacities	(Alexander & Ferguson, 1994, updated 2005)
Equation of states	OPAL EOS (Rogers, Swenson, & Iglesias, 1996, updated 2006)
Core overshooting	Woo & Demarque (2001)

Opacities

ODF The opacity distribution function (ODF) as a non-grey treatment has been employed to our transfer problem (Kurucz, 1993). The key idea is that the transport of radiant energy can be calculated from the probability distribution of opacities composed of a series of rectangles.



Grey Atmosphere The Rosseland mean opacity has been considered reasonable representation for grey opacity in deep layers. In this study, classical approximation such as grey atmosphere and Eddington approximation have been compared in 3-D HD computation.

$$\frac{1}{\kappa_{ross}} \equiv \frac{\int_0^\infty \kappa_\nu{}^{-1}(\partial B_\nu/\partial T)d\nu}{\int_0^\infty (\partial B_\nu/\partial T)d\nu}$$

Model : (i) HD + ALI + ODF (ii) HD + EDD + ROSS

Hydrodynamics

Large-Eddy Simulation as a numerical tool for turbulent flows of stellar convection has been applied to a fully compressible Newtonian fluid. In order to describe stellar turbulent convection, the full set of Navier-Stokes equations should be solved.

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot\rho\mathbf{v}$$

$$\frac{\partial \rho\mathbf{v}}{\partial t} = -\nabla\cdot\rho\mathbf{v}\mathbf{v} - \nabla P + \nabla\cdot\mathbf{\Sigma} + \rho\mathbf{g}$$

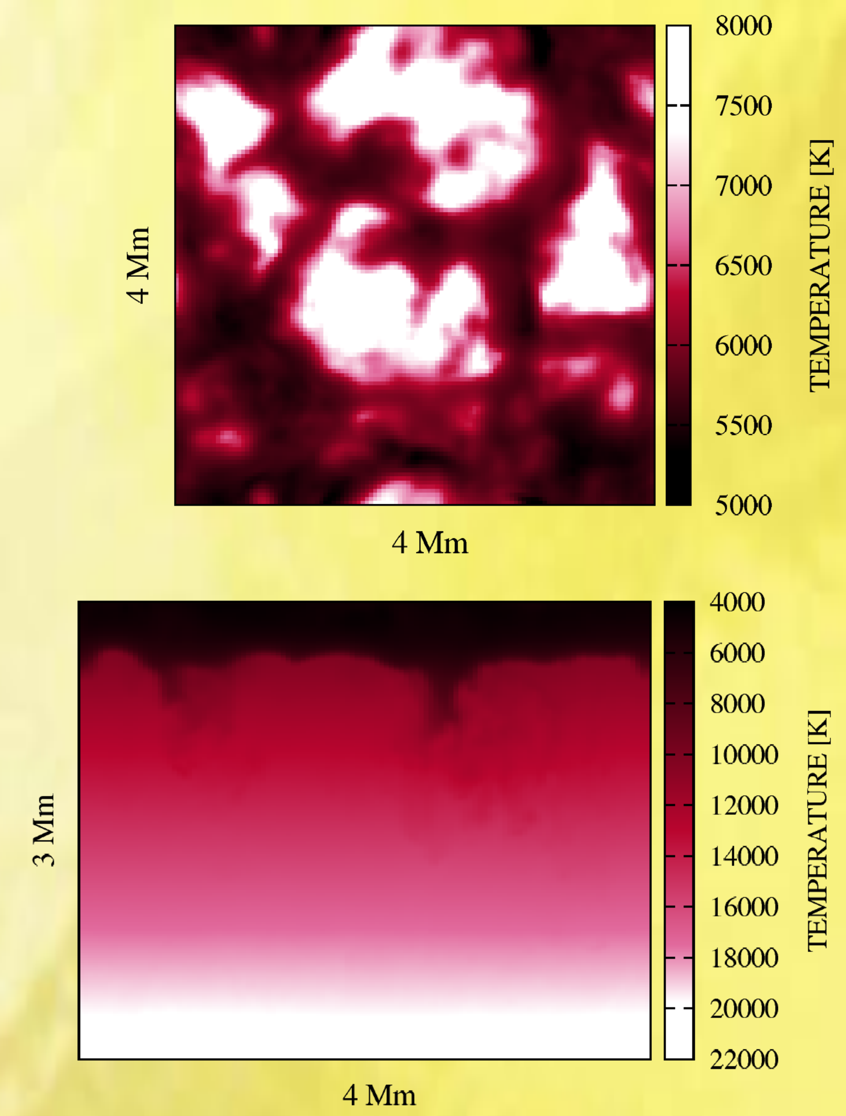
$$\frac{\partial E}{\partial t} = -\nabla\cdot[(E+P)\mathbf{v} - \mathbf{v}\cdot\mathbf{\Sigma} + f] + \rho\mathbf{v}\cdot\mathbf{g} + Q_{rad}$$

Domain is set to be a plain-parallel, closed box with stress-free top & bottom and periodic sides. Computational domain extends $4^2 \times 3$ Mm covering several granules and 8~9 pressure scale heights with the resolution of $117^2 \times 190$ staggered mesh grids (Chan & Wolff 1982). 3-D Snapshots has been accumulated during 800min in real time scale, which covers sufficiently the typical convective turn-over time.

Numerical Scheme consists of two steps : (i) An alternating direction implicit (ADI) with large time steps & first order accuracy and (ii) an explicit method (ADE) with second order accuracy. When the flow reaches statistical relaxation, simulation is switched to the explicit schemes incorporating the second order predictor-corrector time integration.

Simulations

Fig 1. Snapshots A horizontal slice **(top)** and a vertical slice **(bottom)** of the 3D thermodynamic structure are presented.



Radiative Transfer

ALI Accelerated Lambda Iteration method has been applied to 3-D HD medium as an optically thin regime ($\tau \leq 10^4$).

$$J_\nu \equiv \Lambda_\nu S_\nu$$

$$\Lambda = \left(\Lambda - \Lambda^*\right) + \Lambda^*$$

$$J^{n+1} = \Lambda^*[S^{n+1}] + \left(\Lambda - \Lambda^*\right)[S^n]$$

$$S^{n+1} - S^n = [1-(1-\varepsilon)\Lambda^*]^{-1}[S^{FS} - S^n]$$

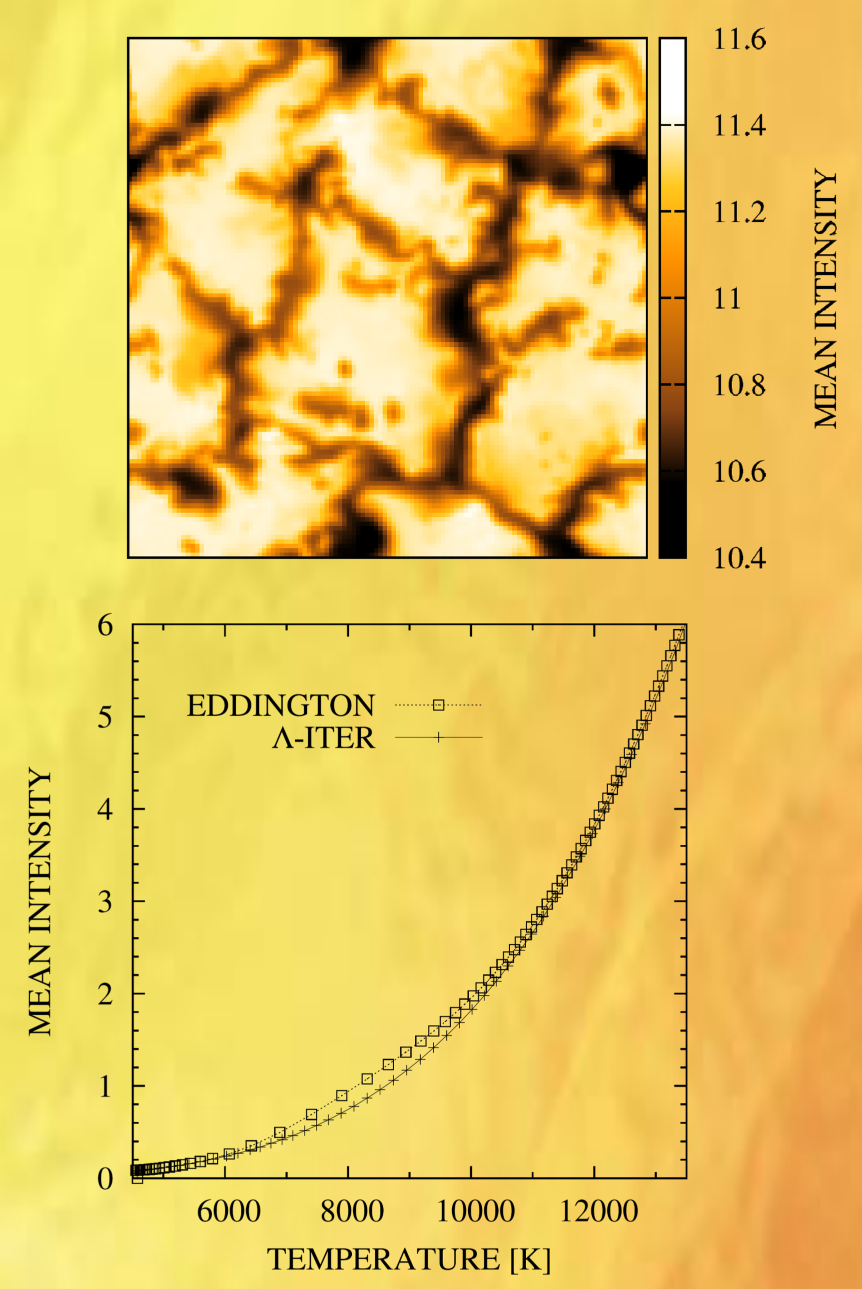
Eddington Approximation In previous version of RHD code, radiation part has been constructed using the generalized 3-D Eddington approximation as anisotropic diffusion in the upper region (Unno & Spiegel, 1966).

$$\nabla \circ \left(\frac{1}{3\kappa\rho}\nabla J\right) - \kappa\rho J + \kappa\rho B = 0$$

Diffusion In deep layers, the diffusion approximation has been considered as an optically thick regime ($\tau \geq 10^4$).

$$Q_{rad} = \nabla\cdot\left[\frac{4acT^3}{3\kappa\rho}\nabla T\right]$$

Fig 2. Radiation Fields (top) The frequency averaged intensities near the solar surface. The mean intensities are scaled by 1×10^{10} in cgs unit. **(bottom) Vertical distribution of mean intensity.** The frequency integrated mean intensities have been averaged temporally and spatially in computational domain.



Discussion

The detailed radiative transfer schemes in the three dimensional hydrodynamical stellar surfaces are investigated. In order to describe radiation fields accurately, direct computation using the ALI method has been applied to the 3-D HD solar surface with a non-grey treatment of opacities. From our RHD simulation for the solar surface convection, thermodynamic structures including the topology and life time of the solar granules have been reconstructed. In surfaces and deep layers, the classical approximations are in a good agreement with the non-grey transfer computation. It implies that the Eddington approximation is a reasonable prescription approaching two limits : the streaming limit ($\tau \sim 0$) and the diffusion limit ($\tau \gg 1$). However, there is a discrepancy of about 5% of radiant energy in the intermediate region of the super-adiabatic layers. The Rosseland mean underestimates the strength of absorbers in transition region. Now we are computing the other solar simulation incorporating the recent solar mixture (Asplund et al. 2009). We believe that a qualitative analysis of two simulations will provide better discrimination in the recent solar abundance problem. Convection and radiation are fundamental processes in the stellar astrophysics. Detailed information of radiation fields and thermodynamic properties from the direct numerical computation will provide deeper insight of physical processes in the Sun and stars.

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