Memorial lecture for Prof. Hayashi
Discovery of Hayashi Phase
and
his way of thinking

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Daiichiro Sugimoto
former affiliations: Univ. Tokyo
& Open University of Japan
Jan 15, 2011
Obituaries

Chushiro Hayashi
1920–2010

Honorary Fellow of the RAS, renowned for his stellar and solar system modeling, powerful advocate of the use of computers in astrophysics and mentor to many Japanese astrophysicists.

Chushiro Hayashi was born on 25 July 1920 in Kyoto, and passed away on 28 February 2010. As his name implies (chu, loyal; shiro, the fourth son), he grew up known as HHS, became one of the most frequently cited papers in the community. In 1959 Chushiro was appointed as the first NAS/NASA foreign research associate to reside track. It

Astronomy & Geophysics, 51: 3.36.
24 MAY 2010, by D. Sugimoto
Fundamental Textbooks

A.S. Eddington 1926

S. Chandrasekhar 1939
### Numerical Integrations for the Stellar Interior


#### Equations & Parameters

Integration of the differential equations for the stellar interior:

**Parameter ψ listed in first line of tabulation of each integration.**

#### Table

<table>
<thead>
<tr>
<th>Int.</th>
<th>( r )</th>
<th>Int.</th>
<th>( r )</th>
<th>Int.</th>
<th>( r )</th>
<th>Int.</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.1</td>
<td>-0.8</td>
<td>13.1</td>
<td>-0.8</td>
<td>13.1</td>
<td>-0.8</td>
<td>13.1</td>
<td>-0.8</td>
</tr>
<tr>
<td>13.2</td>
<td>-0.6</td>
<td>13.3</td>
<td>-0.7</td>
<td>13.4</td>
<td>-0.9</td>
<td>13.5</td>
<td>-1.2</td>
</tr>
<tr>
<td>13.3</td>
<td>-0.7</td>
<td>13.6</td>
<td>-1.0</td>
<td>13.9</td>
<td>-1.2</td>
<td>13.8</td>
<td>-1.2</td>
</tr>
</tbody>
</table>


**Differential equations, etc.:**

See Wares (1944), particularly his equations (4) and (39).

**Homology invariants:**

\[
U = F_{1/2} (\psi) \left( -\frac{d\psi}{d\xi} \right)^{-1}, \quad V = \frac{d}{d\xi} \left[ \frac{1}{2} F_{1/2} (\psi) \frac{d\psi}{d\xi} \right] \left( -\frac{d\psi}{d\xi} \right).
\]

**Initial conditions:**

\[
\psi = \psi_0, \quad \frac{d\psi}{d\xi} = 0 \quad \text{at} \quad \xi = 0.
\]

**Starting values near center:**

\[
\psi = \psi_0 - \frac{1}{2} F_{1/2} (\psi_0) \xi^2 + \ldots
\]
Synthesis of the Elements in Stars

E. Margaret Burbidge, G. R. Burbidge, William A. Fowler, and F. Hoyle

Kellogg Radiation Laboratory, California Institute of Technology, and
Mount Wilson and Palomar Observatories, Carnegie Institution of Washington,
California Institute of Technology, Pasadena, California

"It is the stars, The stars above us, govern our conditions";
(King Lear, Act IV, Scene 3)

but perhaps

"The fault, dear Brutus, is not in our stars, But in ourselves;"
(Julius Caesar, Act I, Scene 2)

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A. Modes of Element Synthesis ........................................ 551
Classification of processes for nucleosynthesis, Rev. Mod. Phys. 1957

H-burning

Hydrogen burning is responsible for the majority of the energy production in the stars. By hydrogen burning in element synthesis we shall mean the cycles which synthesize helium from hydrogen and which synthesize the isotopes of carbon, nitrogen, oxygen, fluorine, neon, and sodium which are not produced by processes (ii) and (iii). A detailed discussion of hydrogen burning is given in Sec. III.

He-burning

These processes are responsible for the synthesis of carbon from helium, and by further $\alpha$-particle addition for the production of $^{12}$C, $^{13}$C, and perhaps $^{14}$N. They are described in detail in Sec. V.

$\alpha$-process

The source of the $\alpha$ particles is different in the $\alpha$ process than in helium burning.

$\rho$ Process

This is the process of proton capture with the emission of gamma radiation ($\gamma$, $\rho$), or the emission of a neutron following gamma-ray absorption ($\gamma$, $n$), which is responsible for the synthesis of a number of proton-rich isotopes having low abundances as compared with the nearby normal and neutron-rich isotopes. It is discussed in Sec. IX.

$x$-Process

This process is responsible for the synthesis of deuterium, lithium, beryllium, and boron. More than one type of process may be demanded here (described collectively as the $x$ process), but the characteristic of all of these elements is that they are very unstable at the temperatures of stellar interiors, so that it appears probable that they have been produced in regions of low density and temperature. There is, however, some observational evidence against this which is discussed in Sec. X together with the details of the possible synthesizing processes.
M. Schwarzschild, 1958
HHS for Evolution of the Stars
C. Hayashi, R. Hoshi & Sugimoto 1962

SUPPLEMENT
OF THE
PROGRESS OF THEORETICAL PHYSICS

NUMBER 32 1962

Evolution of the Stars

Published by the
Research Institute for Fundamental Physics

Evolution of the Stars

Chushin HAYASHI, Reum HOSHI and Daichiro SUGIMOTO
Department of Nuclear Science, Kyoto University, Kyoto

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4) Temperature gradient
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2) Solutions of centrally condensed type

§4. Solutions in the Envelope, in the Core and near the Surface
4A. General Outline
TOOLS

slide rule

2 x 3 = 6

abacus
TOOLS-2
Machine (gear) calculators

TIGER
by hand

MONROE
electrically driven
Hayashi’s sayings (aphorisms)

林 語録

- Extend the problem as wide as possible
- Then, concentrate to the central problem
- Avoid unclear assumptions
- Return to physics (esp. to elementary processes)
- Construct a system from elementary processes
  (René Descartes: Discours de la méthode pour bien conduire sa raison, et chercher la vérité dans les sciences 方法序説)
- Construct a whole story (of evolution)
According to the shell source model of Gamow and Keller,\(^{(1)}\) red giant stars are considered as being at the evolitional stages of the main sequence stars and generating energy by \(C-N\) reactions only in a shell inside which all the hydrogen contents have already been consumed. However, their results about the radii and luminosities of stars with large mass are not definite, because the consistency between the luminosity and energy liberation is not taken into account.
1945: fixed $T^* = 2 \times 10^7$K

CONCLUSIONS

The results obtained in the previous section indicate that the growth of the energy producing shell within a sufficiently massive star may lead to a very large increase of stellar radius, thus bringing the star into the region of the Hertzsprung-Russell diagram occupied by the red giant and supergiant stars. It is tempting, therefore, to consider the stars of these groups as representing various stages of hydrogen shell source evolution, particularly in view of the fact that there is, as it seems, no other adequate explanation of their existence. In fact, it is not possible to consider stars of the red giant branch as still being in the stage of gravitational contraction since in this case their radii would be
Included: (REL)
Degenerate electrons + ions
radiation pressure & pressure ionization

\[
P_e = \frac{8\pi}{3\hbar^3} (2mkT)^{3/2} kT G_{\frac{3}{2}}(\phi, T), \quad G_{\frac{3}{2}} = \int_0^\infty \left(1 + \frac{kT}{2mc^2} u\right)^{3/2} \frac{u^{3/2}}{e^{-\Psi+u} + 1} du \tag{13}
\]

\[
N_e = \frac{\rho}{\mu_e H} = \frac{4\pi}{\hbar^3} (2mkT)^{3/2} G_{\frac{3}{2}}(\phi, T),
\]

\[
G_{\frac{3}{2}} = \int_0^\infty \left(1 + \frac{kT}{mc^2} u\right) \left(1 + \frac{kT}{2mc^2} u\right)^{1/2} \frac{u^{3/2}}{e^{-\Psi+u} + 1} du. \tag{14}
\]

and the pressure of the heavy particles is approximately

\[
P_N = \frac{\rho}{\mu_N H} kT \tag{15}
\]

where \(\mu_e\) and \(\mu_N\) are the mean molecular weights of electrons and ions respectively. The equations of the isothermal cores are reduced to
Integrated envelope solution inwards, subtracting nuclear energy generation until \( L(r) = 0 \) to obtain the core-mass

\[
\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon \tag{9}
\]

\[
\varepsilon = \varepsilon_0 \rho X_H X_{C+N^2} \varepsilon^{-\tau} \tag{10}
\]

\[
\tau = 3 \left( \frac{\pi^2 M e^4 Z^2}{2\pi^2 k T} \right)^{1/8}, \quad \log \varepsilon_0 = 23.55 \tag{11}
\]

where \( X_{C+N} \), combined abundance of \( C \) and \( N \), is assumed to be 1% of the Russell mixture. The right part of Table 2 shows the values where \( L(r) \) vanishes. At this point \( P, T \) and \( M(r) \) are continuous with those of the interior, but \( \rho \) must satisfy the following conditions owing to the disconti-
mass fraction of the core $M^*/M$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capella</td>
<td>0.047</td>
<td></td>
</tr>
<tr>
<td>Zeta Aur.</td>
<td>0.028</td>
<td></td>
</tr>
</tbody>
</table>

smaller than S-C limit (1942)

Table 1.

<table>
<thead>
<tr>
<th>star</th>
<th>spectral type</th>
<th>effective temperature</th>
<th>mass $M/M\odot$</th>
<th>radius $R/R\odot$</th>
<th>luminosity $L/L\odot$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap. A</td>
<td>GO</td>
<td>5200</td>
<td>4.18</td>
<td>15.9</td>
<td>120</td>
</tr>
<tr>
<td>ζ Aur. K</td>
<td>cK4</td>
<td>3200</td>
<td>14.8</td>
<td>200</td>
<td>6310</td>
</tr>
</tbody>
</table>

Table 3.

<table>
<thead>
<tr>
<th>star</th>
<th>$X^*$</th>
<th>$T^*$</th>
<th>$\log \rho^*$</th>
<th>$M^*/M\odot$</th>
<th>$\sigma^*/R\odot$</th>
<th>$\psi_C$</th>
<th>$\rho_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cap.</td>
<td>0.35</td>
<td>$42 \cdot 10^6$</td>
<td>1.50</td>
<td>0.198</td>
<td>0.0303</td>
<td>17</td>
<td>$2.9 \cdot 10^5$</td>
</tr>
<tr>
<td>ζ Aur.</td>
<td>0.37</td>
<td>$65 \cdot 10^6$</td>
<td>1.58</td>
<td>0.410</td>
<td>0.0186</td>
<td>31</td>
<td>$1.2 \cdot 10^6$</td>
</tr>
</tbody>
</table>

Hayashi C., 1947, PThPh, 2, 127
Proton-Neutron Concentration Ratio in the Expanding Universe at the Stages preceding the Formation of the Elements.

Chushiro Hayashi.

Department of Physics, Nagoya University.

(Received January 12, 1950)

§ 1. Introduction.

In the theory of the origin of the elements by Gamow, Alpher, and collaborators\(^1\), primordial matter (ylem) of the universe, which afterwards has been cooled down owing to the expansion of the universe and has formed the elements through nuclear reactions such as radiative capture and beta-decays, is assumed to consist solely of neutrons. At early stages, however, of high temperatures ($kT \gtrsim mc^2$, $m$ being the electron mass) in the expanding universe before the formation of the elements, induced beta-processes caused by energetic electrons, positrons, neutrinos and antineutrinos, in addition to the natural decay of neutrons, such as

\[
\begin{align*}
    n + e^+ & \longrightarrow p + \nu, \\
    n + \nu & \longrightarrow p + e^-, \\
    n & \longrightarrow p + e^- + \nu, \\
    h\nu + h\nu' & \longrightarrow e^+ + e^-.
\end{align*}
\]
Three types of Physics

Type A) Local Physics
- micro processes under given environment (Descartes)
- e.g., p/n-ratio in early Universe, origin of the elements
  though the environment changes in time as specified by other principle

Type B) Physics including (spatial) structure
- characteristic of Astronomy
- e.g., stellar spectra formed in stratified layers
  < structure given by other principle (incl perturbation method)

Type C) Global Physics for structure & its formation
- should be solved as a whole system (beyond Descartes)
  > behavior as a whole appears beyond the sum of local physics
el-deg core; effect of non-deg ion (1957)

Progress of Theoretical Physics, Vol. 17, No. 6, June 1957

Giant Stars with Shell Sources of C-N and p-p Reactions

Chushiro HAYASHI

Department of Physics, Kyoto University, Kyoto

(Received March 2, 1957)
Ion pressure makes the core mass smaller?

<table>
<thead>
<tr>
<th></th>
<th>without correction</th>
<th>with correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>log $L/L_{\odot}$</td>
<td>2.05</td>
<td>2.05</td>
</tr>
<tr>
<td>log $R/R_{\odot}$</td>
<td>1.15</td>
<td>1.16</td>
</tr>
<tr>
<td>$q_1$</td>
<td>0.33 &gt;</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Hayashi C., 1947, PThPh, 2, 127
Fig. 1. $U$–$V$ curves of the solutions of the partially degenerate isothermal cores. Full and dotted curves show the cases $\mu_n/\mu_e = 2$ and $\mu_n/\mu_e = \infty$, respectively. Values of $\psi$ are shown on the points on these curves.

Hayachi C., 1957: PThPh, 17, 727
contracting He core by Sandage & Schwarzschild 1952

similar figure for el-deg He core (no ion-pressure) by Schwarzschild Rabinowitz & Härm 1953

models with H-He intermediate zone (stability criterion) by Härm & Schwarzschild 1955
Fig. 7-6. Evolutionary track of a star of $15.6\, M_\odot$ superposed on the color-magnitude diagrams of \( h \) and \( \chi \) Persei. Segments of the track correspond to the phases: a--b, hydrogen depletion in the core; b--c, contracting helium core; c--d, helium depletion in the core; d--e, contracting carbon-oxygen core; and e--f carbon burning in the core.
later comp → **Red-giants** of 15.6 $M_\odot$ contains stars in He burning  
SN1978A → Star in the later phase **could be a yellow giant**,  
if its envelope mass was lost in the preceding phase
To extend the evolution computation through C-burning phase and beyond it was necessary to formulate the surface Boundary Conditions including the effects of convection with finite efficiency of heat transport co-existent with radiative transport, incomplete ionization, and opacity at low temperatures.
Boundary Condition from photosphere outwards

**photosphere:** optical depth (↓ with hydrostatic equil)

\[
\frac{2}{3} = \int_{\text{ph}}^{0} d\tau = \int_{\text{ph}}^{R} \kappa \rho dr = \int_{0}^{\text{ph}} \kappa \frac{r^2}{GM_r} dP \approx \frac{\langle \kappa \rangle R^2}{GM} P_{\text{ph}}
\]

**homology variables:**

\[ P = \frac{GM^2}{4\pi R^4}, \quad \rho = \frac{M}{4\pi R^3} \]

**effective polytropic index, and homology parameter:**

\[ N \quad \text{and} \quad Bp^N = f^{N+1} \]

**eq of state only at the photosph:**

\[ P = \left( \frac{k}{\mu H} \right) \rho T \]

**luminosity:**

\[ L = 4\pi R^2 \sigma T_{\text{eff}}^4 \]
HHS (1962)

\[ \kappa = \kappa_0 P^\alpha T^\gamma, \]

\[ s = \frac{Xk}{H} \left\{ (1+x) \left( \frac{5}{2} + \frac{x}{kT} \right) + 2 \ln \frac{x}{1-x} \right. \]
\[ + \frac{5}{2} \delta + \delta \ln \left( \frac{(8\pi H)^{3/2}}{h^3 P \delta} \right) \frac{(kT)^{5/2}}{(1+x+\delta)} \right\}, \]

\[ P_0 = (A/\kappa_0)^{1/(1+\alpha)}, \]

result: \( P_0 \) almost const

\[ E = 4\pi KG^{3/2} (\mu H/k)^{5/2} M^{1/2} R^{3/2}. \]

\[ K = \frac{P_4}{T_d^{5/2}}. \]

only when \( \Sigma_k \mu_k dN_k = 0 \)
i.e., when in equil.

E=45.46?
Success:
Hayashi lines and Hayashi phase, which opened a new paradigm for the origin of the solar system.

Hayashi lines
Hayashi & Hoshi (1961)
lower $T_e$ for larger $E$

Fig. 4-13. Curves of $E=10$ and 40 in the HR diagram for given stellar masses (population I composition: $X=0.61$, $Y=0.37$, $Z=0.02$).
comparison with observed giant branches of some clusters
HHS (1962)
Comparison with young cluster NGC 2264
Hayashi, PASJ vol.13, pp. 450-452, 1961
Hayashi phase

pre-main-seq contr;
initially
wholly conv case

\[ 1 \, \mathcal{M}_\odot \]

HHS (1962)

Dwarf-type solution

lower \( T_e \)

for

larger \( E \)
Hoyle & Schwarzschild 1955 for Globular Cluster
fitted to env sol
higher $T_e$ for higher $E$

Described also in detail in Schwarzschild’s book 1958
fitted also to core sol
(time sequence)

### §24. GROWTH OF ISOTHERMAL CORE

![Graph showing growth of isothermal core](image)

**Fig. 24.2.** Hertzsprung-Russell diagram representing two evolutionary phases with helium cores of moderate size, for various trial values of $E$ (data from Table 24.1).

### Fig. 24.3.** Approximate evolutionary tracks in Hertzsprung-Russell diagram for stars of $1.2 M_\odot$ in phases with large helium cores. The numbers give the mass fraction of the core, $q_1$, as it increases during the evolution. (Hoyle and Schwarzschild, Ap.J., Supplement No. 13, 1955)

Giant-type env solution
Hayashi lines for different $N_{\text{eff}}$

eliminate $R$, $P_{\text{ph}}$, $\rho_{\text{ph}}$, $f_{\text{ph}}$

result $L \sim T^{\beta}$:

$$
\left( \frac{L}{4\pi\sigma} \right)^{(N-1)/2} = \left( \frac{4\pi}{B\langle \kappa \rangle} \right) \left( \frac{\mu H G}{k} \right)^{N+1} T_{\text{eff}}^{N-3}
$$

relation to homology parameter:

<table>
<thead>
<tr>
<th>$N_{\text{eff}}$</th>
<th>$13/4$</th>
<th>$3$</th>
<th>$2$</th>
<th>$3/2$</th>
<th>$1$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>$+2/9$</td>
<td>$0$</td>
<td>$-2$</td>
<td>$-6$</td>
<td>$\infty$</td>
<td>$+6$</td>
</tr>
</tbody>
</table>

for $N = 3/2$ : $B = E$

for $N = 3$ and

$$
\frac{1}{n+1} = \frac{3}{16\pi acG T^4 M_r} \frac{P \kappa L_r}{M_r} : B = \frac{1}{4C_E}
$$
Difference on E dependence comes from:

**definition:**

\[ E = 4\pi K G^{3/2} (\mu H / k)^{5/2} M^{1/2} R^{3/2} . \]

\[ K = \frac{P_d}{T_d^{5/2}} \]

**fitted to core:**

\[ E = \exp(\sigma_c - \sigma_d) \left[ \phi_d^{1/2} \xi_d^{3/2} \left( \frac{M}{M_d} \right)^{1/2} \left( \frac{R}{R_d} \right)^{3/2} \right] \]

For dwarf-type sol: [...] ~45. Radiative core results in smaller E
For giant-type sol: \( \xi \) very large but compensated by large diff in \( \sigma ' s \)

**surface at pt \( d \) only:**

\[ E = \frac{2(2\pi)^{1/2}}{(h/2\pi)^3} \left( \frac{\mu}{\mu^{(el)}} \right)^{3/2} \exp(5/2 - \sigma^{(el)}) M^{1/2} R^{3/2} \]

↑ phtoshp cond

For \( R=\text{const} \): smaller E → larger \( \sigma \) → higher \( P_{rad} / P_{gas} \)
→ more photons escape → higher L (Hayashi)
Evolution in the forbidden region?

Not the dynamical evol, but thermal instability

HHS (1962)
Change of entropy distribution from birth through Hayashi phase

1: surface cooled*
2: convection inwards (Hayashi phase)
3: radiative core appears (core cooled**)
4: Henyey stage

* optically thin
** more transport ideal gas & Kramers’
Why Prof. Hayashi so great

- Constructed not only the theory of stellar evolution and protostar formation, but also a systematic theory for the origin of the solar system, Kyoto model.
- Awarded:
  - for evolution of the stars (& protostar 1961-)
    Eddington medal (1970); Imperial Prize of the Japanese Academy (1971)
  - for origin of the Solar System 1970-
    (promoted also cooperative research with geoscientists)
    Order of Culture (1986); Order of Sacred Treasure, First Class (1994); Kyoto Prize (1995)
  - for lifetime contribution, Bruce Medal (2004)
- Pioneering works (He began the fields before others can notice)
- Established the discipline of nuclear astrophysics in Japan
  (Nurtured so many disciples)
References

• Astronomy & Geophysics, 51: 3.36. 24 MAY 2010, by D.Sugimoto
• Hayashi C.,1947:Giant Stars Producing Energy by C-N Reactions,PThPh,2,127
• Hayachi C.,1957:Giant Stars with Shell Sources of C=N and p-p Reactions,PThPh,17,727