Stellar structure and evolution

Fundamental and Unified Understandings for Stellar Structure and Evolution

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Difference in approach

before computer age  
~ ca. 1970
• looked for generalized understandings
• processes idealized to know what are essential (idealized model)
• more easily foresee cases with different values of parameters
• can be extended to global physics as a system and structure formation

after computer age  
ca. 1970 ~
• Ask computer what will be the result
• every detail included (fine model) for local physics but their effects hard to know
• One can play more with computers (computer games?)
• global physics hard to know, but the results compared with observation in more detail
Solving for Stellar Structure

Multi-timescale problem

- mechanical (hydrodyn) eqn: \( \tau_{\text{sound}} \sim \text{days} \)
- heat transport to steady state: \( \tau_{\text{heat}} \sim \text{M yrs} \)
- secular change (nuclear): \( \tau_{\text{nuclear}} \sim \text{G yrs} \)

Strongly non-linear problem

Hydrostatic equilibrium – multiplications only

\[
\begin{align*}
\frac{dP}{dr} &= -\frac{GM_r \rho}{r^2} \\
\frac{dM_r}{dr} &= 4\pi r^2 \rho \\
1 + \frac{1}{N} &= \frac{d\log P}{d\log r} / \frac{d\log \rho}{d\log r}
\end{align*}
\]

not \( \frac{dy}{dt} = A + B \) according to main terms
approx 1: \( \frac{dy}{dt} = A \)
2: \( \frac{dy}{dt} = B \)
3 (steady state): \( A+B=0 \)

or eqs of heat transport & energy conservation
Pressure distribution in the core of 12 & 30 $M_\odot$ Nomoto et al (1979)

Extremely wide dynamic range due also to infinite range of grav force

very fine meshes req’ed

Pressure distribution in the core of 12 & 30 $M_\odot$
Nomoto et al (1979)

Results in many concepts out of common sense

Sugimoto D., Nomoto K., 1979, SSRv, 25, 155
3.1. HYDROSTATIC EQUILIBRIUM IN NON-DIMENSIONAL FORM

Let us define the non-dimensional variables by

\[ r = r_0 \xi, \quad M_r = M_0 \phi, \]
\[ P = P_c \hat{\omega}, \quad \rho = \rho_c \eta. \]

If we take

\[ r_0^2 = \frac{1}{4 \pi G} \frac{P_c}{\rho_c^2}, \]
\[ M_0^2 = \frac{1}{4 \pi G^3} \frac{P_c^3}{\rho_c^4}, \]

equations of hydrostatic equilibrium (2.1)–(2.3) are rewritten as

\[ U = \frac{d \ln \phi}{d \ln \xi} = \frac{\xi^3 \eta}{\phi}, \quad V = -\frac{d \ln \hat{\omega}}{d \ln \xi} = \frac{\phi \eta}{\xi \hat{\omega}}, \]
\[ \frac{d \ln \eta}{d \ln \hat{\omega}} = \frac{N}{N+1}. \]

The boundary conditions at the center are

\[ \phi = 0, \quad \hat{\omega} = 1, \quad \eta = 1, \quad \text{at} \quad \xi = 0. \]
Important!

Hydrostatic equilibrium described only with P & \( \rho \)

Complications arise in T thru Eq of States, Heat Transport etc

Find solutions only with P & \( \rho \) and then translate them to T

\[
P = \frac{\Lambda}{\mu \beta} \left( \frac{k}{m_{\text{amu}}} \right) \rho T
\]

eq of state

can absorb complications arising from electron degeneracy, radiation pressure, and changing chemical compositions
Gravitational Contraction

When interior of the star is cooled (entropy s decreased) the temperature rises (gravothermal)

Important Transformations

\[ M_1^2 = \frac{1}{4\pi G^3} \frac{P_c^3}{\rho_c^4} \varphi_1^2 \]
\[ r_1^2 = \frac{1}{4\pi G} \frac{P_c}{\rho_c^2} \xi_1^2 \]

(core) mass & radius \( \varphi_1 \) and \( \xi_1 \) depend weakly on \( N \)

& M1 determine everything

\( \varphi_1 \) almost determined by Nc / by
Lines with constant $T_c$

$M_1^2 = \frac{1}{4\pi G^3} \frac{P_c^3}{\rho_c^4} \varphi_1^2$

and Eq of state determine

$T$ increases until iron photodiss SN

$\leftarrow$ Chandrasekhar limit; REL, $\rightarrow$ electron capture SN

nucl inst (core) leading to He-flash, C-def SN

Min mass of H-burn

$T_c = 0$ white dwarf

brown dwarf

planet

Sugimoto D., Nomoto K., 1979, SSRv, 25, 155
Stable nuclear burning
(main sequence stars) and Luminosity

\[
\frac{M}{R} \sim \frac{P}{\rho} \sim T \text{ (gas),} \quad \sim \frac{T^4}{\rho} \text{ (rad)}
\]
\[
\tau \sim \kappa \rho R
\]
\[
L \sim \frac{\alpha T^4 V}{t_{\text{diff}}} \sim \frac{T^4 R^3}{\tau R/c} \sim \frac{T^4 R}{\kappa \rho} \sim M^3 \text{ (gas),}
\]
\[
\sim \frac{M}{\kappa} \text{ (rad : Eddington limit)}
\]

Wrong reasoning
Mass larger
→ gravity stronger
→ Temperature higher
→ Nuclear burning stronger
→ Luminosity higher

Right reasoning
Mass larger, but Temperature (almost) the same (nuclear E generation)
→ grav potential (almost) the same
→ Density lower
→ Optical depth smaller
→ Photons more easily diffuse out
→ Luminosity higher
→ slightly higher Temperature required
(evolution towards) Red Giant Stars consisting of core and envelope of different compositions

Core contracts – Envelope expands (to core-halo str)

H-rich envelope

H-exhausted He-core

H envelope

H-burning shell $T_H = \text{const}$
Denoting with subscript $c$: the center of the core
1: core edge just inside of the H-burning shell

<table>
<thead>
<tr>
<th>core contracts</th>
<th>but</th>
<th>T1 stays const</th>
</tr>
</thead>
</table>

$\rightarrow r_1$ and the volume of the core stays const

$\rightarrow \rho_c^{\uparrow}$ but $\rho_1^{\downarrow} \rightarrow P_1$ lowered

nevertheless

the same mass of the env must be sustained against gravity

for equilibrium solution (boundary value problem):
Place the bulk of the env-mass where the gravity weak, i.e., where the radius larger

**NB:** To realize it (initial value problem): raise $T_1$ slightly for a little while to supply add’l nucl energy
hydrostatic eqn described only with \( P & \rho \)

\[
\frac{dP}{dr} = -\frac{GM_r \rho}{r^2}
\]

\[
\frac{dM_r}{dr} = 4\pi r^2 \rho
\]

(local) polytropic index

singularity on \( 2U + V - 4 = 0 \)

homology invariants

\[
U = \frac{d \log M_r}{d \log r} = \frac{4\pi r^3 \rho}{M_r}
\]

\[
V = -\frac{d \log P}{d \log r} = \frac{GM_r \rho}{rP}
\]

\[
1 + \frac{1}{N} = \frac{d \log P / d \log r}{d \log \rho / d \log r}
\]

\[
d \ln M_r = -U \frac{d \ln U - d \ln V}{2U + V - 4}
\]
U-V plane

red giant comes here

dwarf region

giant region

2U + V - 4 = 0

center
polytropes (N=const)

Fig. 3-2. The $U$-$V$ curves for $N=1.5$.  

Fig. 3-3. The $U$-$V$ curves for $N=4$.  

Fig. 3-4. The $U$-$V$ curves for $N=\infty$. 

Hayashi C., Hoshi R., Sugimoto D., 1962, PThP, 22, 1
envelope solutions
for el scattering opacity

Hayashi C., Hoshi R., Sugimoto D., 1962, PThP, 22, 1
core solution
contracting core with $T$ gradient

\[ P = \frac{\Lambda}{\mu \beta} \left( \frac{k}{m_{amu}} \right) \rho T \]

Fig. 4-9. The $U$-$V$ curves of contracting cores ($\kappa L_r/M_r = \text{constant}$, $\beta = 1$).

Hayashi C., Hoshi R., Sugimoto D., 1962, PThP, 22, 1
electron degenerate core with $\Lambda$ (and $\mu$) gradient

$log U$ vs $log V$ plane

admired by Martin as Hayashi's invention

Hayachi C., 1957: PThPh, 17, 727

Fig. 1. $U$-$V$ curves of the solutions of the partially degenerate isothermal cores. Full and dotted curves show the cases $\mu_n/\mu_e = 2$ and $\mu_n/\mu_e = \infty$, respectively. Values of $\psi$ are shown on the points on these curves.
Fitting of envelope to core
Giant sol has a topology with loop

$\log U = 0.01$

Hayashi C., Hoshi R., Sugimoto D., 1962, PThP, 22, 1
Required for giant-type solution
- giant-type topology with loop in U-V plane

- Large concentration in the core,
  i.e., large ratio of \((P/\rho)_c \div (P/\rho)_1\)
  and Large mass fraction of the envelope

Not the case of giant-type solution

- Small envelope mass fraction:
  - much mass has been lost in the preceding phase
  - accreting WD (nova), X-ray burst of n-star
- Non-degenerate isothermal core smaller than S-C limit
- Chemically homogeneous star
  - the point \((P/\rho)_1\) comes
    not close to the singularity but to the surface
Comparison: Effect of ion pressure in electron degenerate core

Processes in competition - 1

\[ P = \frac{\Lambda}{\mu} \left( \frac{k}{m_{\text{amu}}} \right) \rho T \]

center: \( \Lambda_c \) large for el, 
\[ \frac{1}{\mu} \sim \frac{1}{\mu_{\text{el}}} \]; \( P_{\text{ion}} \ll P_{\text{el}} \)

core edge
ion neglected: \( \Lambda=1 \), \( \frac{1}{\mu} = \frac{1}{\mu_{\text{el}}} \)
ion taken into account:
\[ \Lambda=1, \frac{1}{\mu}=\frac{1}{\mu_{\text{el}}} + \frac{1}{\mu_{\text{ion}}} \]
\( \rightarrow \) higher \( \Lambda_c \) required for the same difference in \( P/\rho \)

For el + ion, \( \Lambda_c=30 \)

For el only, \( \Lambda_c=20 \)

Fig. 1. \( U-V \) curves of the solutions of the partially degenerate isothermal core. Full and dotted curves show the cases \( \mu_{\text{el}}/\mu_s=2 \) and \( \mu_{\text{el}}/\mu_s=\infty \), respectively. Values of \( \phi \) are shown on the points on these curves.
Processes in competition - 2

Helium-burning phase: competition between T and μ(center)

\[ P = \frac{\Lambda}{\mu \beta} \left( \frac{k}{m_{\text{amu}}} \right) \rho T \]

- onset, Y=1.0
- He flash, Y=1.0
- min R, Y~0.3

as He consumed, both T and μ increases.

Hayashi C., Hoshi R., Sugimoto D., 1962, PThP, 22, 1
Thermal pulses of helium shell-burning (AGB stars)

Processes in competition - 3

\[ P = \frac{\Lambda}{\mu \beta} \left( \frac{k}{m_{\text{amu}}} \right) \rho T \]

- H-burning shell
- He-burning shell
- C+O core
- growing He-burning

- He-zone

- Deposit of entropy \( \rightarrow \) He-zone expands
- \( T_{\text{He}} / T_{\text{H}} \) increases
- cooling down
- relaxation osc

- Instability ceases
- (spherical config = gravothermal, i.e., giant-type 2U+V-4~0)
- unstable
- (planer config = non-gravothermal, i.e., dwarf type)
Processes in competition – 4
Entropy aspects

\[ P = \frac{\Lambda}{\mu/\beta} \left( \frac{k}{m_{amu}} \right) \rho T \]

0-3: outer edge of C+O core
T~P/\rho stays const

1-3: bottom of He-zone
T~P/\rho increases
S increases in He-zone
He-zone expands

2-3: heat diffusion

3: top of He-zone
S becomes very large

When S at the core edge becomes as high as S in the surface convection zone (← it invades into the helium zone

more details later
Mass loss / accretion / flow in close binary / repeating thermal pulses/ etc

\[ s = s(M_r, t) = s(q, t); \quad M_r = M_1 q \]

\[ \frac{ds}{dt}_{M_r} = \left( \frac{\partial s}{\partial t} \right)_q - \frac{d \ln M_1}{dt} \left( \frac{\partial s}{\partial \ln q} \right)_t \]

eg, \( \sim 1000 \)

but computed easily (it has nothing to do with numerical stability, since \( t \) not contained)
**Accretion and Mass exchange in binary stars**

**Wrong Statement**
The star puffs up (since its radius increases)

**Right Statement**
Accreted matter just piling-up and compressed; radius for a Lagrange mass shell is shrinking

If $S$ is const (convective), no energy required to pushing-out the env mass: rapid mass exchange

\[ L_r = \int_0^r T \left( \frac{ds}{dt} \right) dM_r \]

\[ \left( \frac{ds}{dt} \right)_{M_r} = \left( \frac{\partial s}{\partial t} \right)_q - \frac{d \ln M_1}{dt} \left( \frac{\partial s}{\partial \ln q} \right)_t \]

Energy released / required for mass flow

**Wrong Statement**
The star puffs up (since its radius increases)

**Right Statement**
Accreted matter just piling-up and compressed; radius for a Lagrange mass shell is shrinking
Development and invasion of Surface Convection Zone

Wrong Statement
Radiation cannot transport such a high heat flux
↓
Energy generation in the interior is automatically adjusted down

Right Statement
Surface is relatively cooled down when the density is low (when the radius large)

↑
S in surface region becomes lower than S in the interior
or the latter becomes higher than the former

e.g., Hayashi phase

e.g., AGB stars
Invasion of surface conv into the core

to include mixing of diff compositions:

\[
\frac{ds^{(eq)}}{T} = \frac{dq}{T} = ds + \sum_k \frac{\mu_k}{T} dN_k
\]

- #1: fully convective
- #2: rad core developing

Surface cooled

Conv level lowered by cooling

Invasion of surface conv into the core

s increases at core edge

Core ≤⇒ Envelope

Convection level determined by radius (surf. cond)

\[ g^{(eq)}(q) + \text{const} = \ln \left( \frac{P^{3/2}}{\rho^{5/2}} \right) \]

Homogeneous; pre-main-seq contr
Thermal Pulses of He-shell burning in Asymptotic Giant Branch (AGB)

shell flash – non-linear oscillation

H-rich env

C+O core

He-zone

thin dwarf-type unstable

thick (exp’d) giant-type stable

Fujimoto & Sugimoto (1979)

approaches $2U + V - 4 = 0$
progress of the flash
and
non-linear oscillation

Sugimoto D., Fujimoto M. Y., PASJ, 20, 467
Unified & Similarity understanding for repeating nuclear shell-burning instabilities

<table>
<thead>
<tr>
<th>Thermal pulse</th>
<th>Nova Explosion</th>
<th>X-ray Burst</th>
</tr>
</thead>
<tbody>
<tr>
<td>- e.g., C+O core</td>
<td>- White dwarf</td>
<td>- Neutron star</td>
</tr>
<tr>
<td>- He-burning</td>
<td>- H-burning</td>
<td>- He-burning</td>
</tr>
<tr>
<td>- added by H-shell burning</td>
<td>- added by accretion</td>
<td>- Accreted H burned quietly into He</td>
</tr>
<tr>
<td>- thin-shell instability</td>
<td>- ← same</td>
<td>- ← same</td>
</tr>
<tr>
<td>- non-degenerate</td>
<td>- med. degeneracy</td>
<td>- strong degeneracy</td>
</tr>
<tr>
<td>- ceased by transit to spher geometry</td>
<td>- ← same or by explosion</td>
<td>- ceased by fuel He-exhaustion</td>
</tr>
<tr>
<td>- period controlled by $L_H$ and mixing</td>
<td>- period controlled by accretion rate</td>
<td>- ← same</td>
</tr>
<tr>
<td>- formation of C-star and dredging-up of s-process elements</td>
<td>- slower accr results stronger explosion due to higher deg</td>
<td>- If time should allow, its fate would be collapse into BH</td>
</tr>
<tr>
<td></td>
<td>- fate; SN Ia</td>
<td></td>
</tr>
</tbody>
</table>
Effect of rapid neutrino loss beyond C-burning

- \( \text{L}_{\text{ph}} \ll \text{L}_{\nu} \approx \text{L}_n \); Radiative heat transport negligible
- Time-scale for heat diffusion too long \( \tau_{\text{ph}} \gg \tau_{\nu} \approx \tau_{\text{nucl}} \)
- Entropy distribution \( s(M_r) \) is determined by
  \[
  \text{eq}(1): \quad T \frac{ds}{dt} = -\varepsilon_{\nu} + \varepsilon_n + \left[ \frac{dq}{dt} \right] \quad \leftarrow \text{negligible}
  \]
- Thermal process separable from hydrostatic equilibrium
  (decoupled; gravothermal but not in thermal equilibrium)
- Computation becomes easier because of the decoupling

\[
\begin{align*}
\text{give } s(M_r, t) & \quad \longrightarrow \quad \text{obtain h.s.eql} & \quad \longrightarrow \quad \text{obtain } T(M_r, t) \text{ fm } s(M_r) & \& P(M_r, t) \\
\text{replace } s(t+dt) \text{ with } s(t) & \quad \longrightarrow \quad \text{calculate } s(M_r,t+dt)=s(M_r, t) + ds \text{ using eq}(1)
\end{align*}
\]
If neutrino loss is not included in computation, strong invasion of the surface conv $60 \, M_\odot$(1974)

Sugimoto D., Nomoto K., IAUS, 66, 105
With neutrino loss included; \(30 \, M_\odot\)

surf conv stops invading (No time for redistributing s)

Fig. 3. Chemical evolution of the star of \(30 \, M_\odot\) computed by Sugimoto 1970b) and by Sugimoto and Nomoto (1974). Neutrino loss is taken into account. Shaded regions are in convective equilibrium. A part of the hydrogen-rich envelope is omitted from the top of the figure.

Conclusion

• Referring to global understanding for the structure of non-linear system and for resulting gravothermal nature,

• it is easy to explain stellar structure and evolution in various phases, systematically

Vote, please: agree / disagree / abstain

Another Vote, please:
Do you think a text book necessary which describes such a theory in a unified form?
yes / no / uncertain
References

- Sugimoto D., Nomoto K., 1979, Presupernova model and supernovae, SSRv, 25, 155
- Sugimoto D., 1971; Mixing between Stellar Envelope and Core in Advanced Phases of Evolution. III ---Stellar Core of Initial Mass 1.5M_ȯ---, PThPh, 45, 761
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