Gravity Wave Dynamics in Hot Extrasolar Planet Atmospheres

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1. Introduction

Stably-stratified atmospheres can support internal gravity waves, which arise from the buoyancy of the fluid. The waves are readily excited by heating and flow over topography and are a ubiquitous feature of the terrestrial atmosphere. They have been intensely studied over the past several decades. They transport momentum and heat within a fluid and the effects they have on the mean flow are routinely parametrized in GCMs of the Earth's atmosphere.

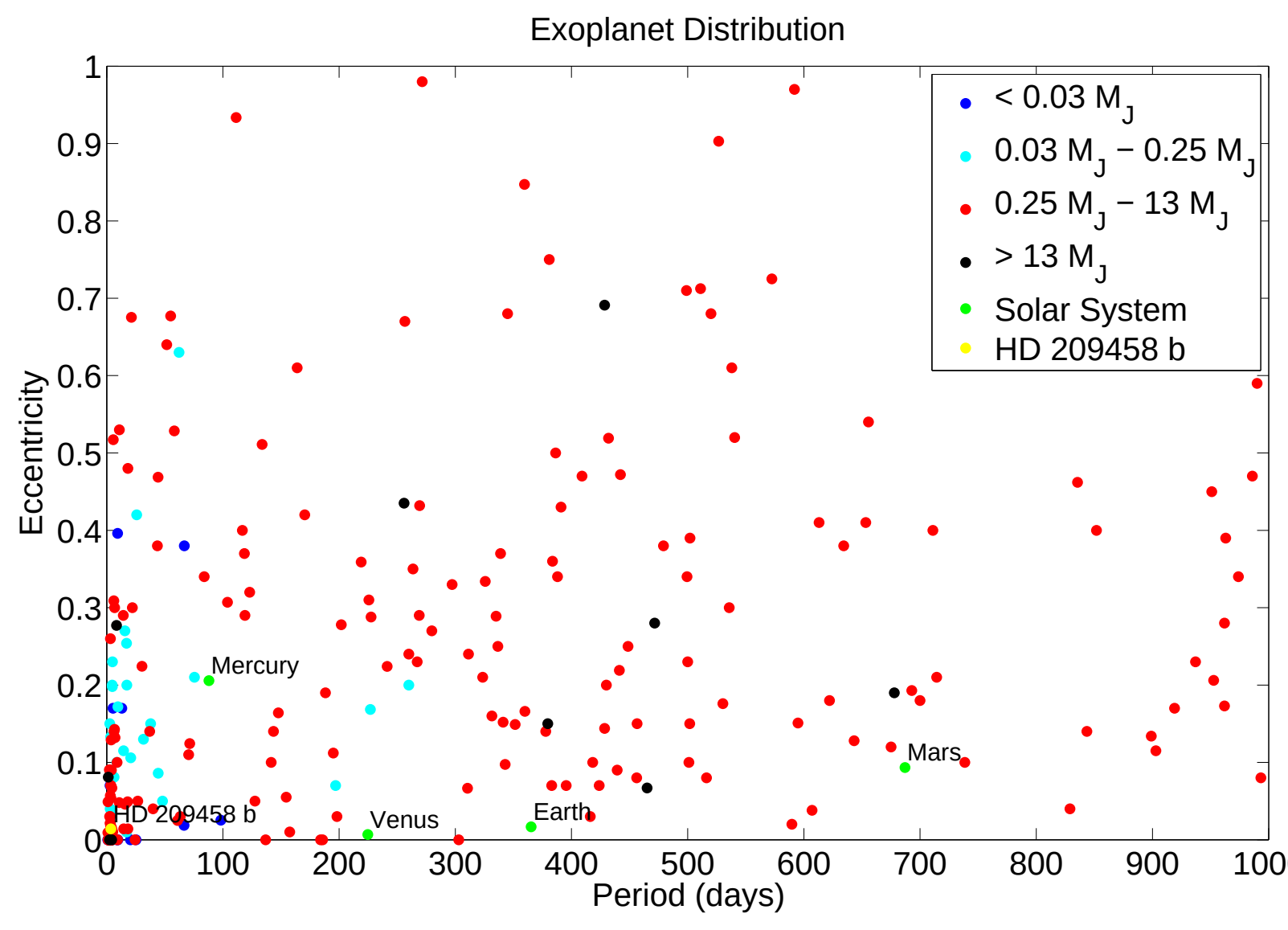

Fig. 1: Known extrasolar planets. Solar System bodies are shown for comparison and the hot-Jupiter HD 209458 b is highlighted. (Data from exoplanet.eu)

There are over 400 known extrasolar planets. Their orbits and masses vary far more than for the planets in the Solar System, as shown in Fig. 1. Many planets are very close to their parent star: nearly 70 have orbits of four days or less, they are assumed to be spin-orbit synchronised. They are exposed to a very large energy flux.

There is much interest in modelling the atmospheric circulations in these extreme conditions (e.g., Cho, 2008; Showman et al., 2009). We consider the effects gravity waves have on these planets by studying their behaviour in an atmosphere based on the hot-Jupiter planet HD 209458 b.

2. Gravity Waves

Internal gravity waves have large scale effects in the terrestrial atmosphere - for example, the Quasi-Biennial Oscillation (Baldwin et al., 2001). They have also been observed in many Solar Systems atmospheres beyond the Earth (e.g., Young et al., 1997). The atmospheres of extrasolar planets are expected to be stably stratified: so, gravity waves will be important for modelling their dynamics.

3. The Taylor-Goldstein Equation

The behaviour of a linear gravity wave is described by the *Taylor-Goldstein equation* which describes how the vertical velocity $\tilde{w}$ varies with altitude:

$$\frac{\mathrm{d}^2\hat{w}}{\mathrm{d}z^2} + \left[\frac{N^2}{(c-u_0)^2} + \frac{u_0''}{(c-u_0)} + \frac{u_0'}{H_\rho\,(c-u_0)} - \frac{1}{4H_\rho^2} - k^2\right]\hat{w} \;=\; \frac{\kappa\,\dot{Q}}{H_p\,(c-u_0)^2}\,e^{-z/2H_\rho},$$

$$\hat{w}(z) \;=\; \tilde{w}\,e^{-\chi(z)}, \qquad \chi(z) \;=\; \int_{z_b}^{z}\frac{\mathrm{d}\xi}{2H_\rho(\xi)}.$$

Here c is the horizontal phase speed and k is the horizontal wavenumber. The mean flow is u_0, primes are differentiation w.r.t. z, and $\dot{Q}$ is the heating. In the square brackets is the index of refraction.

The equation is solved numerically using Gaussian elimination with 3000 to 10000 equally spaced levels. As the equation is linear the wave can grow without bounds. To counter this a scheme is used to introduce saturation when the wave becomes convectively unstable.

4. HD 209458 b

The extrasolar planet HD 209458 b, is a hot-Jupiter planet that orbits its star at just 0.47 AU with a period of 3.52 days. It is expected to be tidally locked to its star. It has a radius of 1.32 R$_\mathrm{J}$.

Specific Gas Constant	R	3523 J kg^{-1} K^{-1}
Specific Heat at Constant Pressure	c_p	12300 J kg^{-1} K^{-1}
Acceleration Due to Gravity	g	10 m s^{-2}
Rotation Rate	Ω	2.08×10^{-5} s^{-1}

HD 2094568 b is a widely studied planet and it is assumed to be typical for the purpose of our study. The physical parameters we used are given in the table above.

The forcing in our exploration is centred at 1 scale height above the 1 bar level. It takes the form of a modified Gaussian that is zero beyond two half-widths ($\sim 0.3\ H_p$) from the forcing centre. The heating rate we use is modest at roughly 300 K per rotation (i.e. $\dot{Q}/c_p = 10^{-3}$ K s^{-1}).

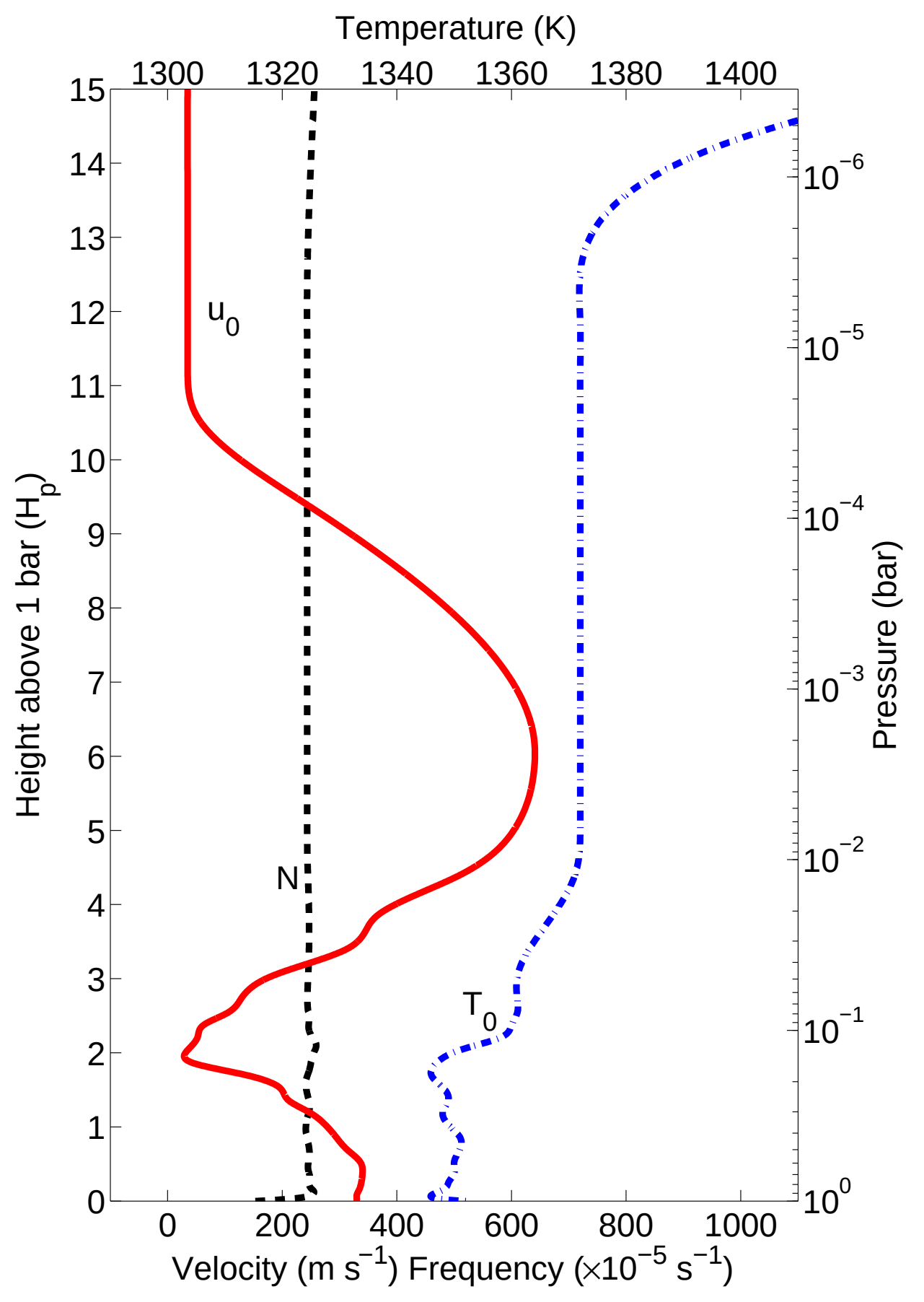

Fig. 3: Flow and temperature profiles.

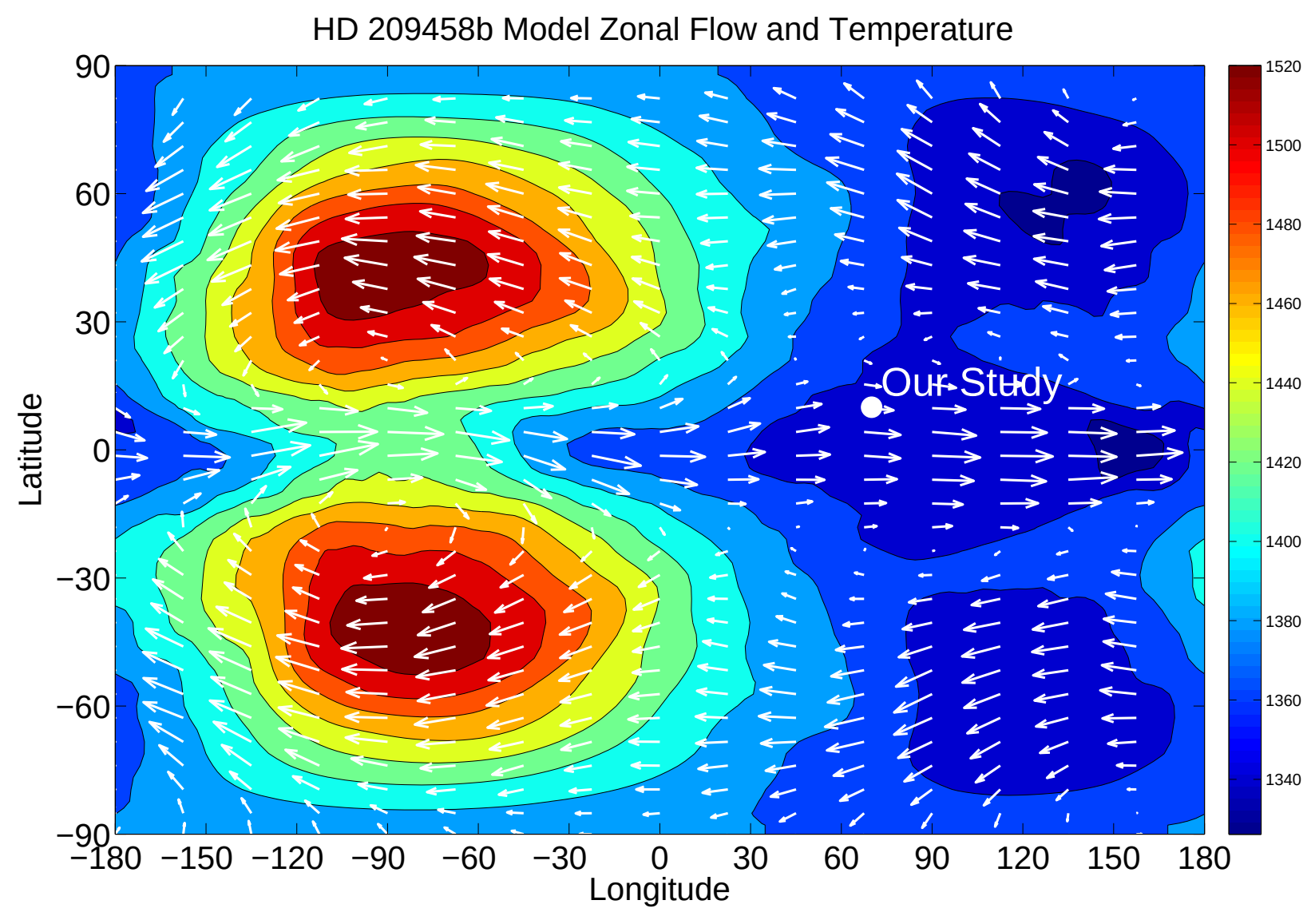

Fig. 2: Modelled flow and temperature on HD 209458 b at about 1 H_p above the 1 bar level. The longest zonal flow vectors are 533 m s^{-1}. The sub-stellar point is at the centre (0° N, 0° E).

The model atmosphere used is based on simulations generously shared by Thrastarson & Cho (2009), part of which is shown in Fig. 2. Our study is conducted at 10° N, 70° E. This point was chosen as it is in the equatorial jet and has a small Coriolis parameter, treated as negligible in our study. The profiles in Fig. 3 are based on the data, which extends to 6 H_p. We expanded it to 15 H_p, based on profiles from Flasar et al. (2004) and Koskinen et al. (2007). The resulting temperature profile is similar to that observed for Jupiter by the Galileo probe (Young et al., 1997).

Also in Fig. 3 is the profile of the Brunt–Väisälä frequency N. It varies little over the domain. The main contributor to variation in the index of refraction is the variation in $c - u_0$. This is different to the terrestrial situation.

5. Effect on the Background

As can be seen in Fig. 3 the wave propagates in a sheared environment. So, $c - u_0 = 0$ is possible in some layer known as a critical layer. A wave encountering a critical layer is shown in Fig. 4. The wave is attenuated by the encounter and the momentum it transports is deposited, causing the flow speed to change. In this case the acceleration, ∼250 m s^{-1} per rotation, is enough to double the flow speed at this layer in about 2 planetary rotations. In reality a spectrum of waves will encounter critical layers over a range of altitudes, causing accelerations over the range. Thus the critical layers filter out waves, preventing them from propagating to high altitudes.

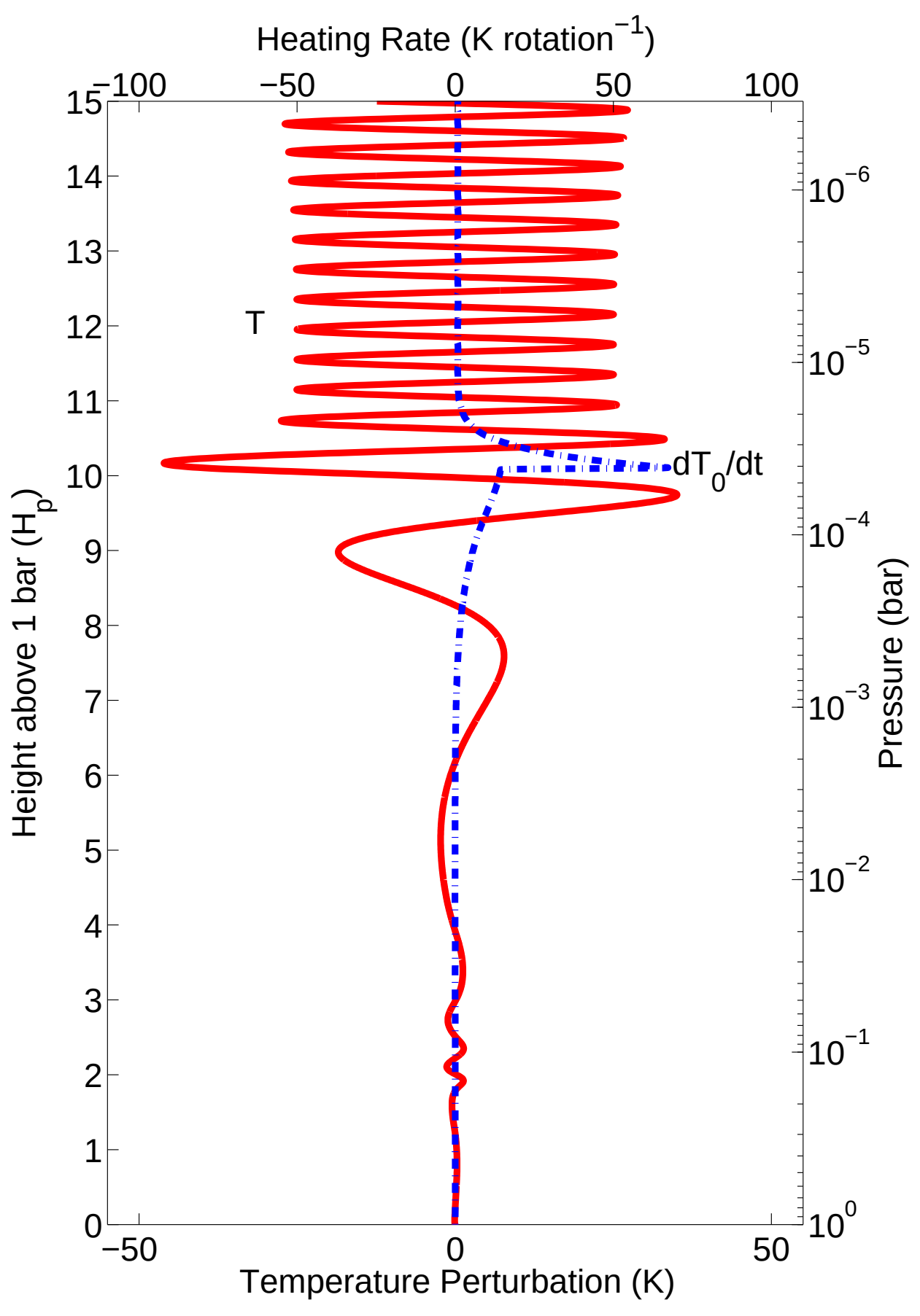

Fig. 5: A wave with $c = -40$ m s^{-1} and $2\pi/k = 2500$ km saturates above 10 H_p.

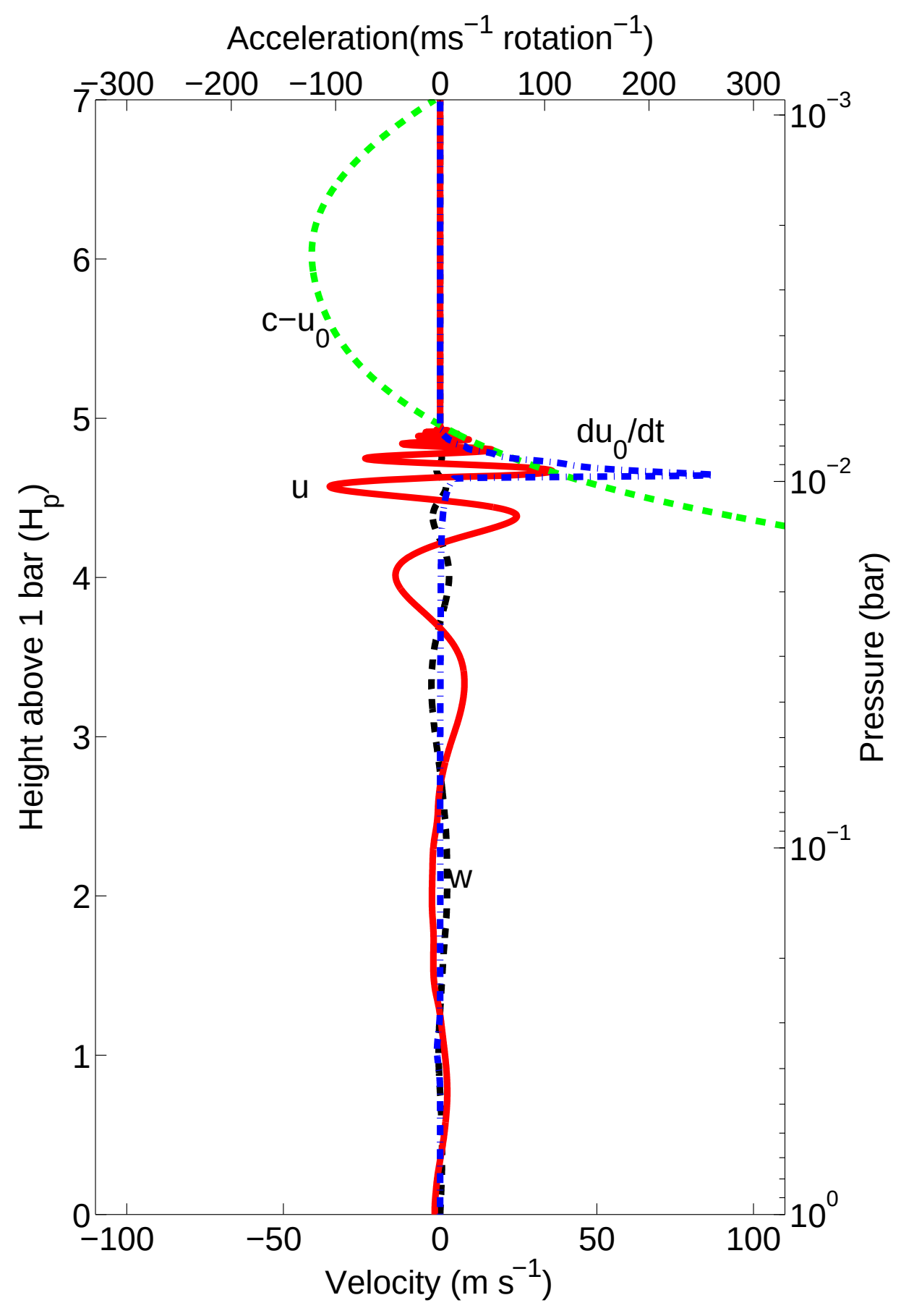

Fig. 4: A wave with c =600 m s^{-1} and $2\pi/k$ =2500 km saturates in an encounter with a critical layer near 5 H_p.

Gravity waves saturate when the temperature perturbations cause the atmosphere to become locally convectively unstable and the wave stops growing. This is shown in Fig. 5. Here we show the heating the wave induces on the background, roughly 75 K per rotation. Such heating could trigger new gravity waves and will induce additional dynamical effects. Waves originating from deeper levels in the atmosphere have larger effects. For example, excitation at the 30 bar level leads to heating of nearly 3000 K per rotation.

6. Horizontal Transport

For shorter waves it is possible for k^2 to dominate the index of refraction, which becomes imaginary (indicating that the wave is evanescent). A wave is reflected at a layer where it becomes evanescent, as can be seen in Fig. 6; the vertical group velocity vanishes at around the 10 H_p level, where $Re(m)$ vanishes. Where there are layers that support propagation between evanescent layers, the wave is ducted and energy is transported horizontally. The wave in Fig. 6 is an example of this. Waves are often ducted in jets and the energy is transported at roughly the speed of the jet.

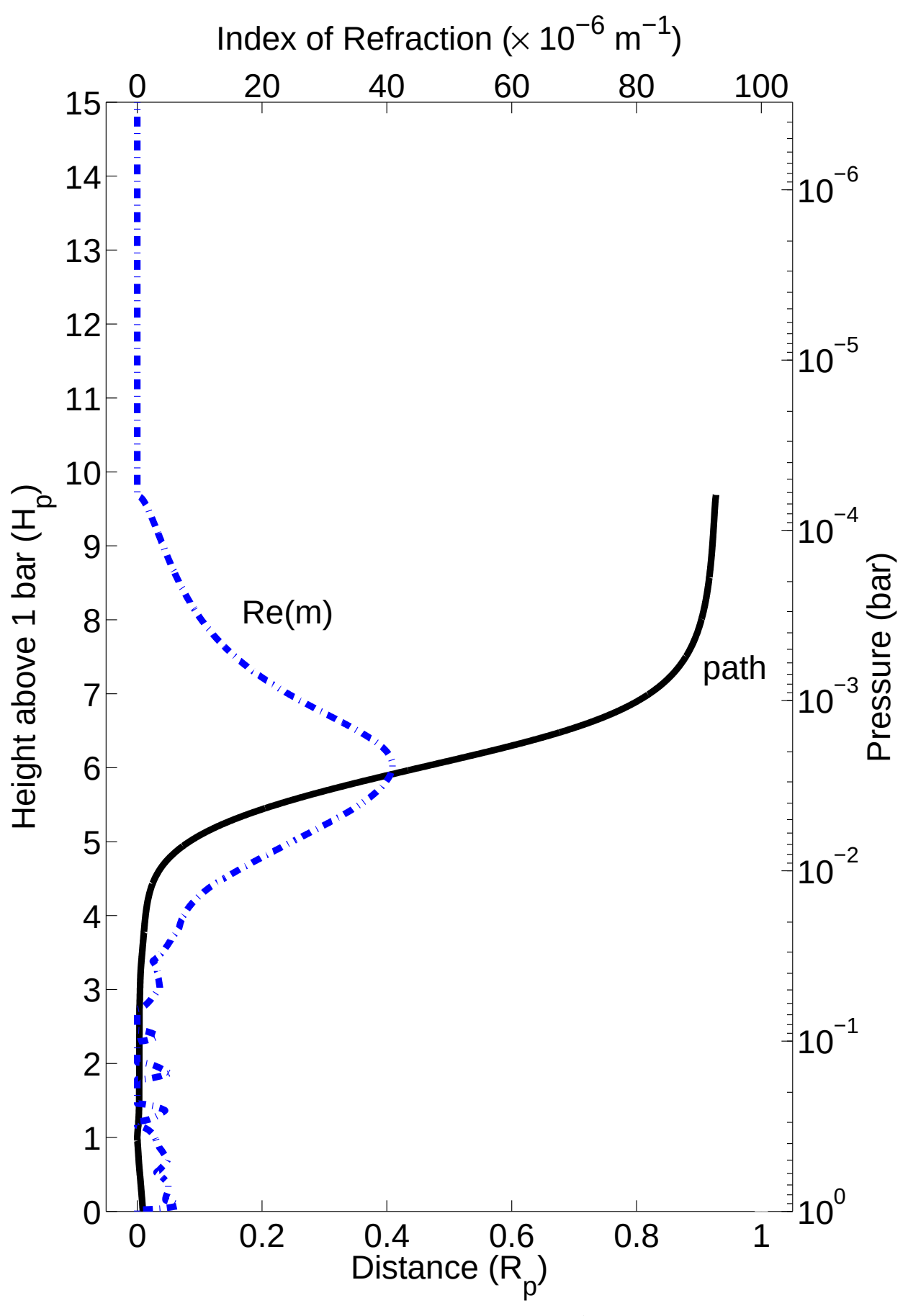

Fig. 7: The path of propagation of the wave in Fig. 6.

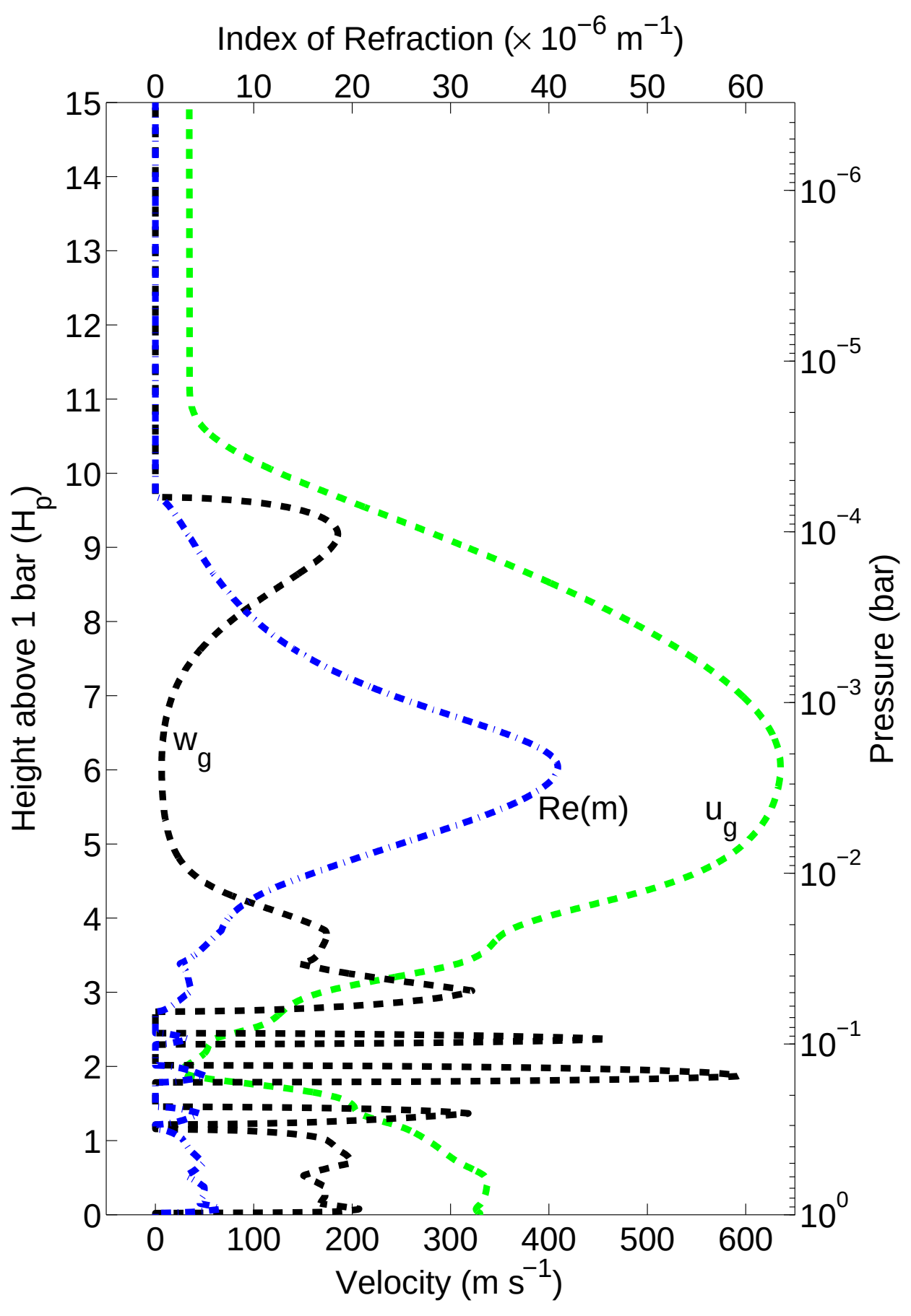

Fig. 6: A wave with c =700 m s^{-1} and $2\pi/k$ = 1410 km. The vertical and horizontal group velocities and the real part of the index of refraction are shown.

Energy can be transported across vast distances in ducts as Fig. 7 shows. With just one reflection the wave is carried about two planetary radii across the surface. This will enable energy to be carried across the terminator from the day-side to the night-side. The properties of the jet will change around the planet. In places it will become leaky and the wave will escape, via this mechanism heat and momentum can be transported between hemispheres thus contributing to the homogenisation of the atmospheric temperature.

7. Conclusions

The effect of gravity waves on the dynamics of extrasolar planet atmospheres is significant. However, their inclusion in general circulation models is computationally prohibitive due to the high resolution required. Accurate parametrisations of their effects need to be developed to mitigate this. Currently, such parametrisations for the Earth's atmosphere are in the vertical only. But, as we have shown, horizontal transport is also important on tidally locked planets. Therefore, parametrizations for modelling the circulation of such planets need to include the horizontal as well as the vertical transport of momentum and heat.

8. Acknowledgements

The authors thank Heidar Thrastarson for sharing information from his simulation work on extrasolar planet atmospheric circulation. We also thank Orkan Umurhan for helpful discussions. C.W. is supported by the Science and Technology Facilities Council (STFC). J.Y-K.C. is supported by NASA NNG04GN82G and STFC PP/E001858/1 grants.

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