Green’s function for a generalized two-dimensional fluid
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Introduction

Generalized two-dimensional (2D) fluid

The Governing equation of this system is the nonlinear advection equation for a scalar $u$, including a real parameter $\alpha$.

\[
\frac{\partial u}{\partial t} + J(\psi, q) = D + \mathcal{F}
\]

where $\psi$ is the stream function, $q$ is the vorticity, $D$ is the stream function, and $\mathcal{F}$ is the forcing term.

We calculate the above integral using the Fourier transform and the Gamma function.

\[
G(k) = \int_{-\infty}^{\infty} e^{-ikx} dx
\]

where $k$ is the wave number.

We calculate the inverse Fourier transform of $G(k)$.

\[
\hat{u}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(k) e^{ikx} dk
\]

Results and Discussion

I. Green’s function

The functional form of the Green’s function depends on $\alpha$.

1. Except for particular values of $\alpha$, where $\alpha \neq \pm 2n$ and $n$ being the integers,

\[
G(r) = \psi_{\alpha}^{(2n-2)}(\ln r + \frac{C}{r})
\]

where $C$ is the natural numbers.

2. For $\alpha$ being the positive even numbers, $\alpha = 2m$ where $m$ is the natural numbers,

\[
G(r) = \psi_{\alpha}^{(2n-2)}(\ln r + \frac{C}{r})
\]

3. For $\alpha$ being the negative even numbers, $\alpha = -2n$.

\[
G(r) = \psi_{\alpha}^{(2n-2)}(\ln r - \frac{C}{r})
\]

II. Azimuthal velocity around the vortex

The azimuthal velocity around the vortex can be calculated by differentiating the Green’s function.

\[
u = \frac{\partial G(r)}{\partial \alpha} = (\alpha - 2)\psi_{\alpha}^{(2n-3)}
\]

III. Existence of physically reasonable 2D fluids

• The azimuthal velocity around the vortex is a increasing function of $r$ for $\alpha > 2$.
  – It is physically reasonable that the azimuthal velocity decreases as the distance from the velocity source becomes farther.

• We conclude that physically reasonable 2D fluids exist only for $\alpha < 2$.

Conclusion

We have discussed the Green’s function for the generalized two-dimensional fluid.

The functional form of the Green’s function is

\[
G(r) = \psi_{\alpha}^{(2n-2)}(\ln r, (\alpha = 2n))
\]

– The functional form of the Green’s function is discontinuous at $\alpha = \pm 2n$.

– In contrast, the azimuthal velocity is continuous at $\alpha = 2n$.

– Physically reasonable two-dimensional fluids exist only for $\alpha < 2$.

Bibliography