Structure and Evolution of Stars
Anatomy of Universe fundamental Bricks

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Lecture plan

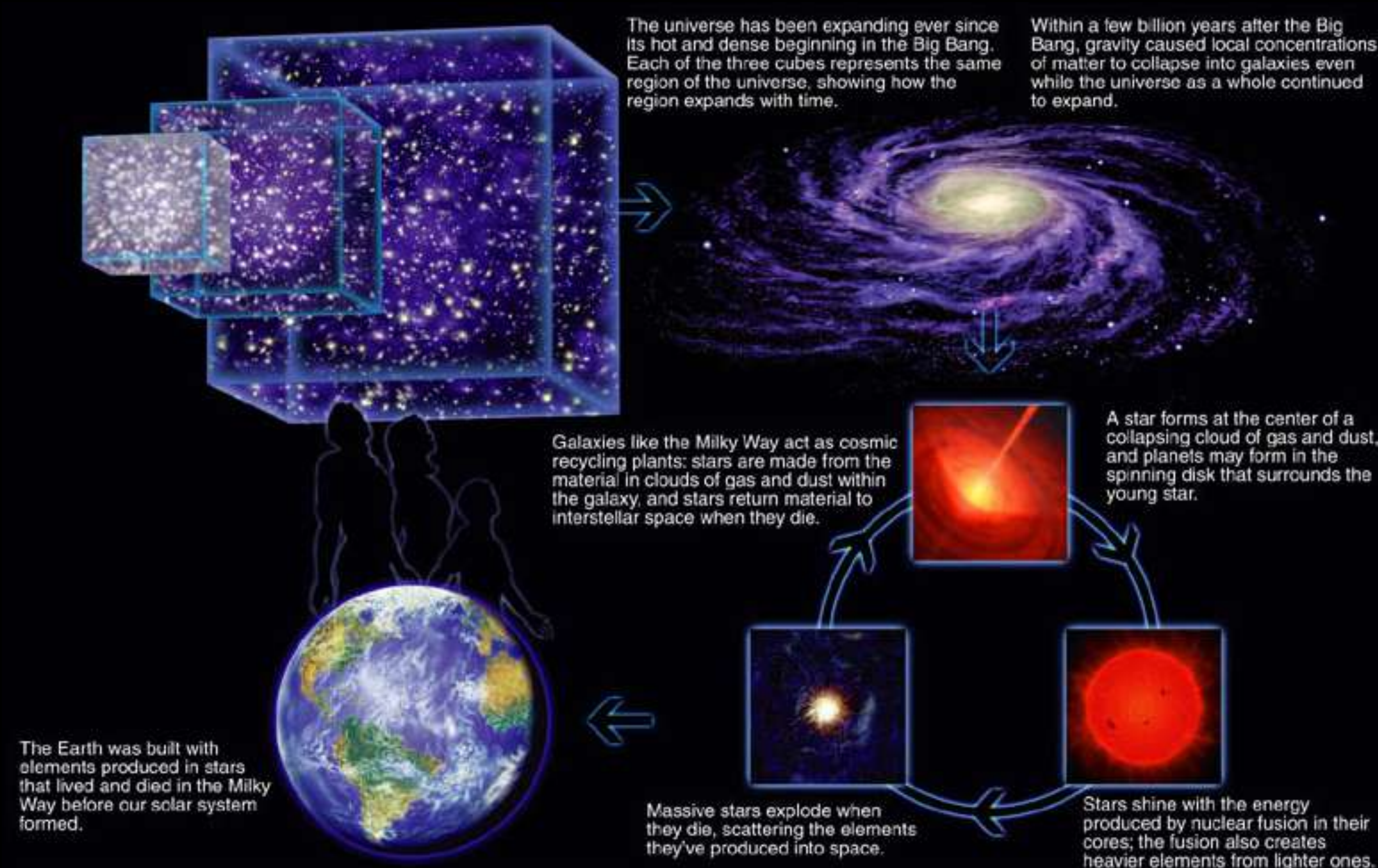
I. **What is a star?**

II. **Stars: main concepts**
- How do you stabilize a star?
- Un simplified model of stars: Lane Emden equation
- Stellar mass and luminosity: what limits?

III. **Life and death of stars**

Book on stellar evolution/structure: Cox & Giuli 1968 « Principles of Stellar structure »
Hansen & Kawaler 1994 « Stellar interiors »
Kippenhahn, Weigert & Weiss 2012 « Stellar structure and evolution »

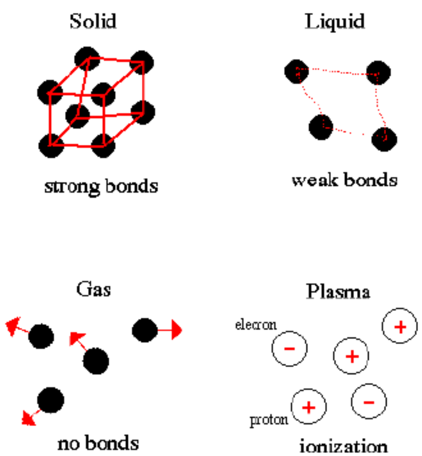
Stars: Universe elementary blocks

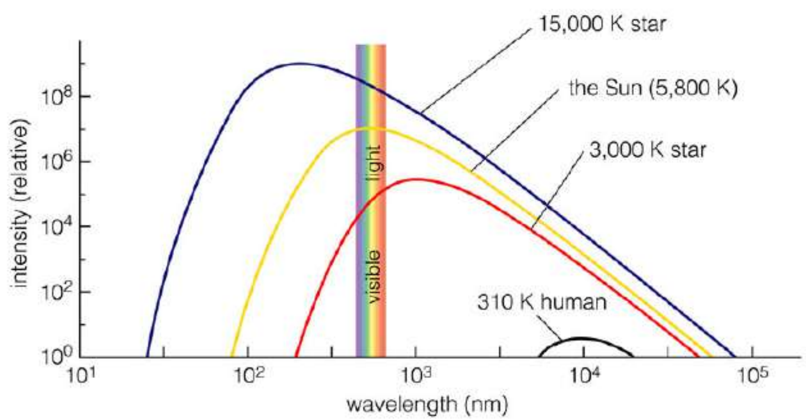


What is a star?

It is a self-gravitating ball of hot gas:
the pressure of the gas balances gravity.

This gas is very hot (and therefore ionized): it is a plasma that emits light.

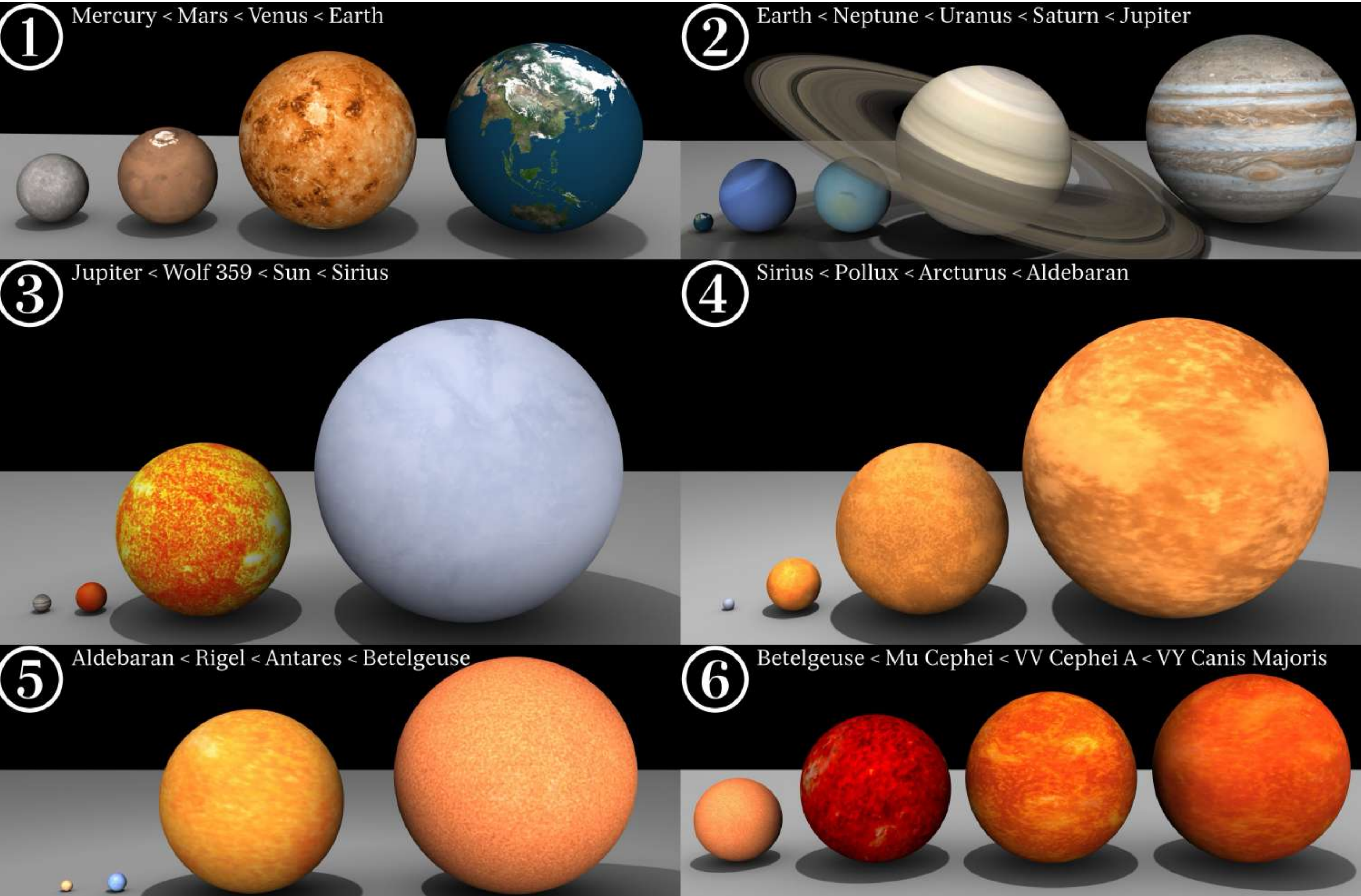




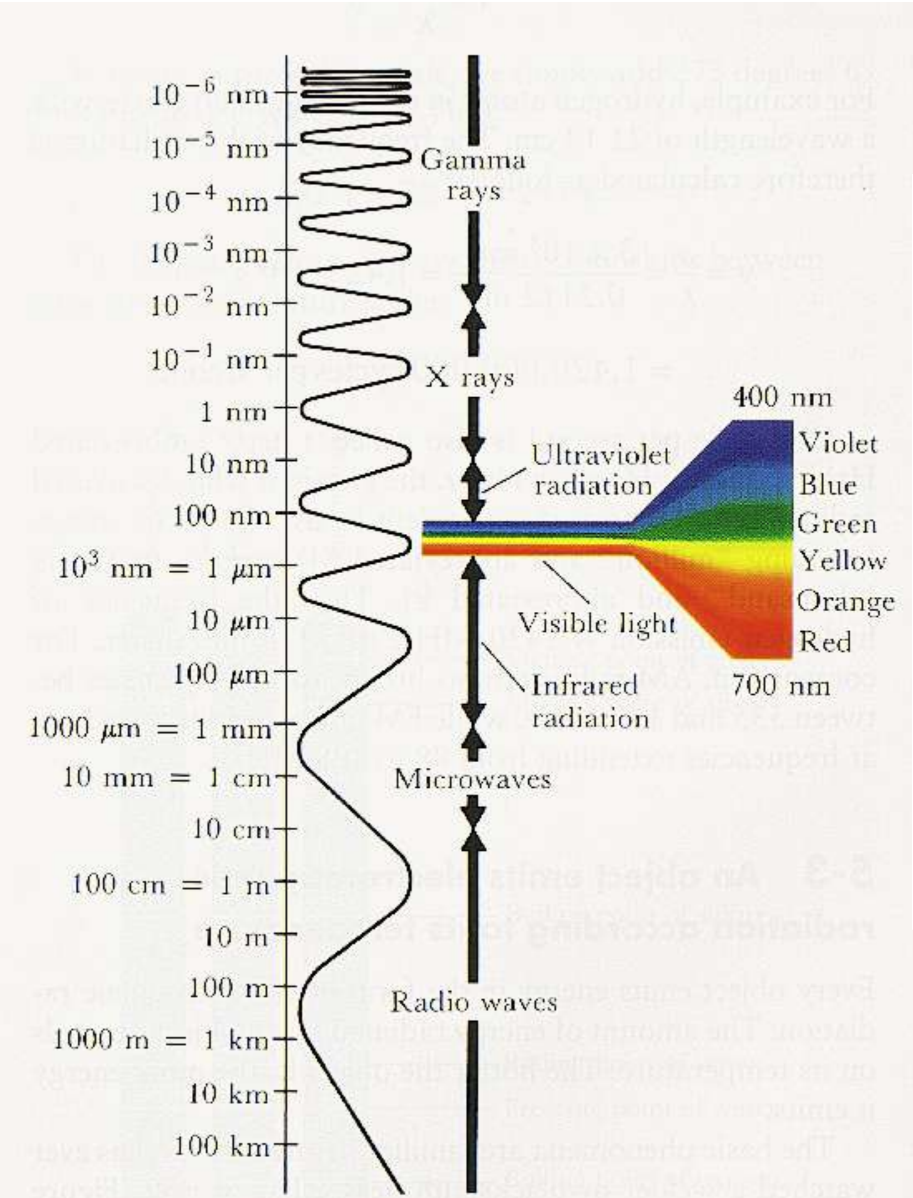
lishing as Addison Wesley.

This energy flow must last for a long time > millions of years
An efficient energy source to ensure this: nuclear reactions (see slide 27)
Only massive objects >~0.08 solar masses have a core temperature high enough to trigger these reactions (see slide 55)

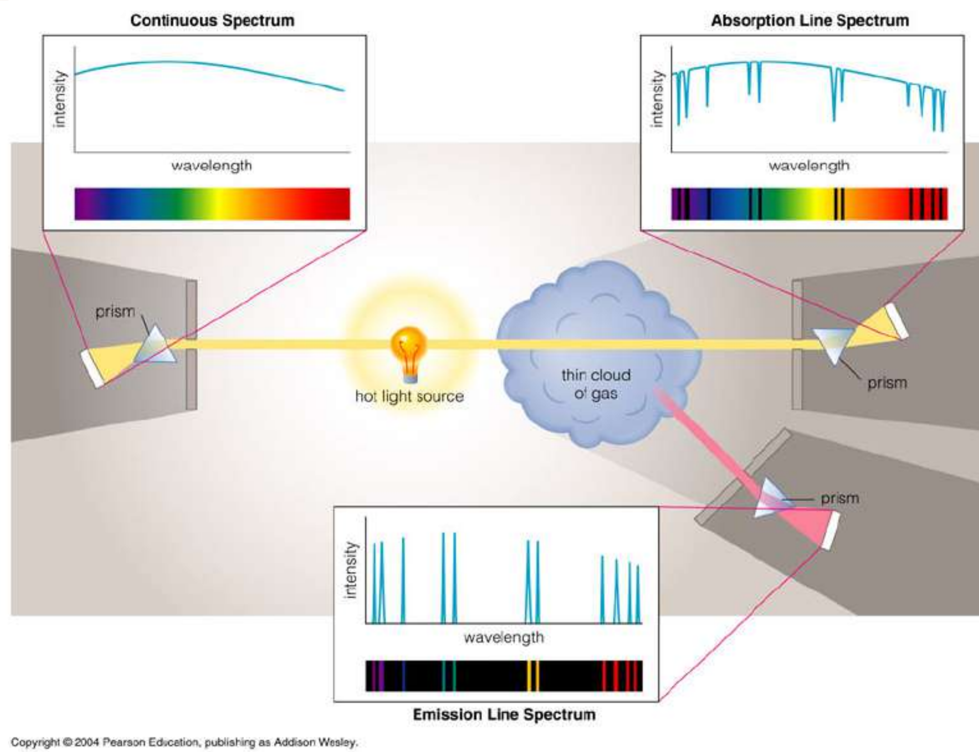
Relative size: from planets to stars



Light, messenger of stars



Kirchhoff Law

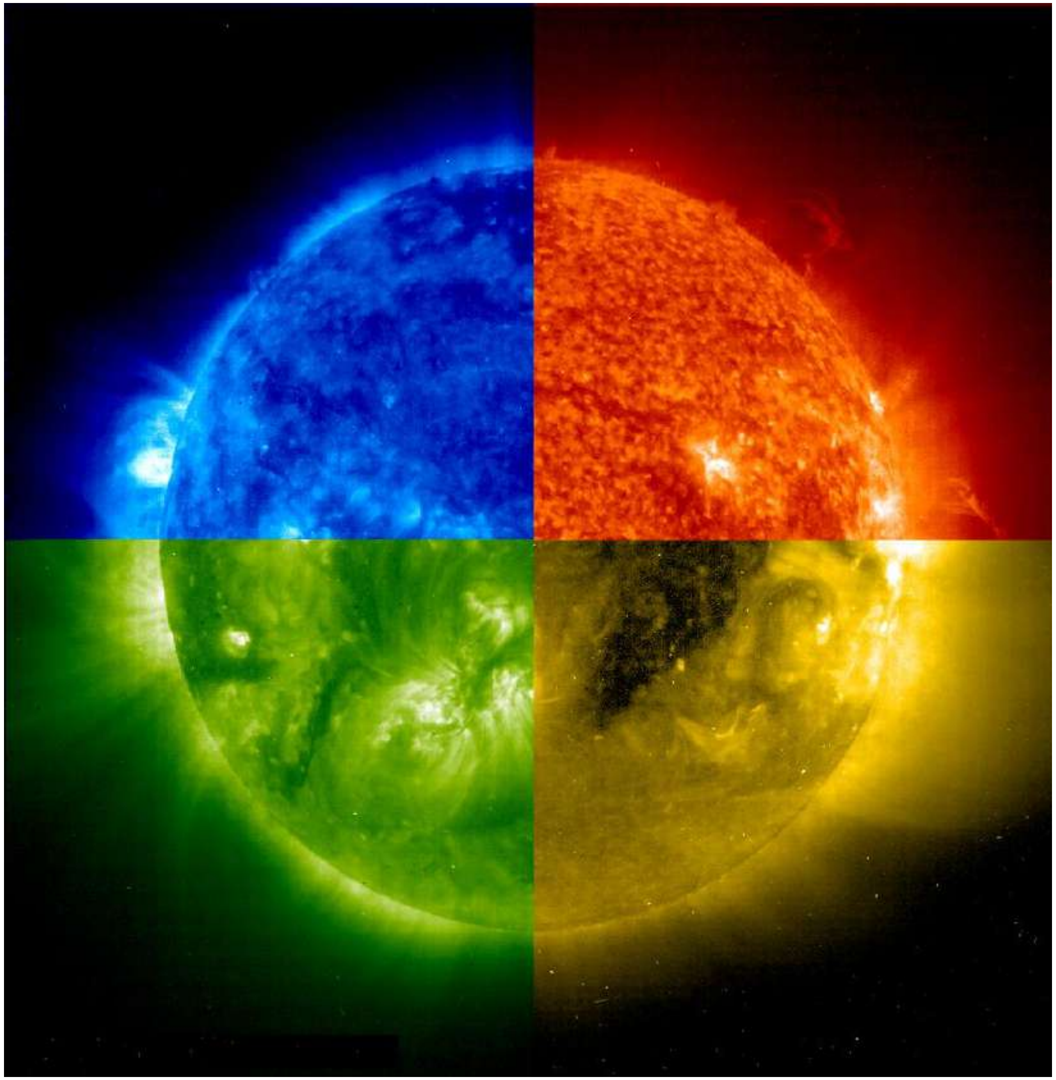


The Sun seen at 4 different frequencies/temperatures

UV Light

Iron
8/9 times ionized
17,1 nm
0.8 Millions K

Helium
1 times ionized
30,4 nm
60000 K



Iron
11 times ionized
19,5 nm
1.5 Millions K

Iron
14 times ionized
28,4 nm
2 Millions K

Black body radiation

Body with a rich energy spectra able to excite all light frequencies

Photons spectral density

$$n(\nu)\,\mathrm{d}\nu = 2\,\frac{4\pi\nu^2\,\mathrm{d}\nu}{c^3}\,\frac{1}{\exp(h\nu/kT)-1}$$

Energy density

$$u = \int_0^\infty h\nu \times n(\nu)\,\mathrm{d}\nu = \frac{8\pi^5 k^4}{15(hc)^3}\,T^4$$

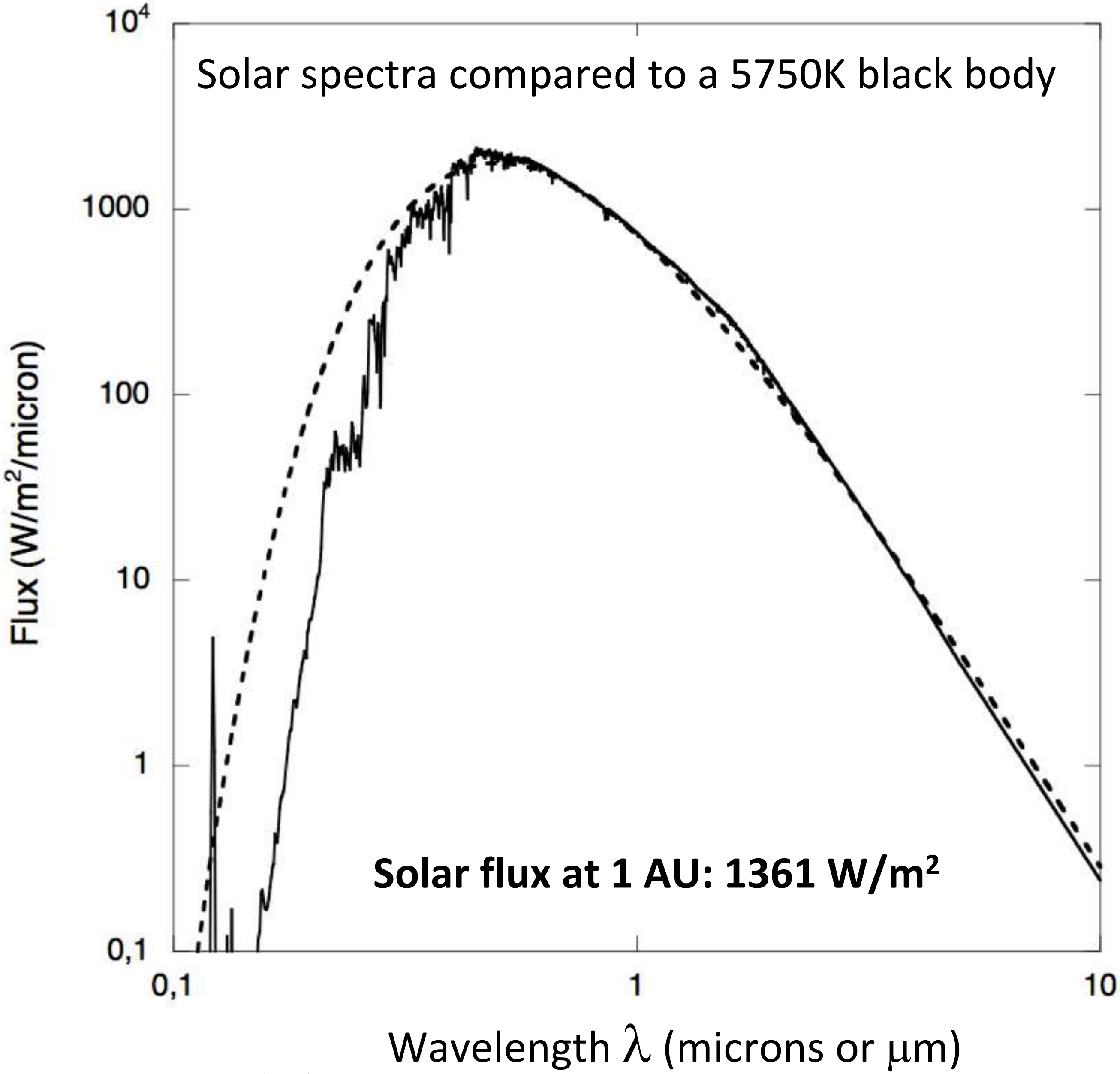
Energy flux

σ

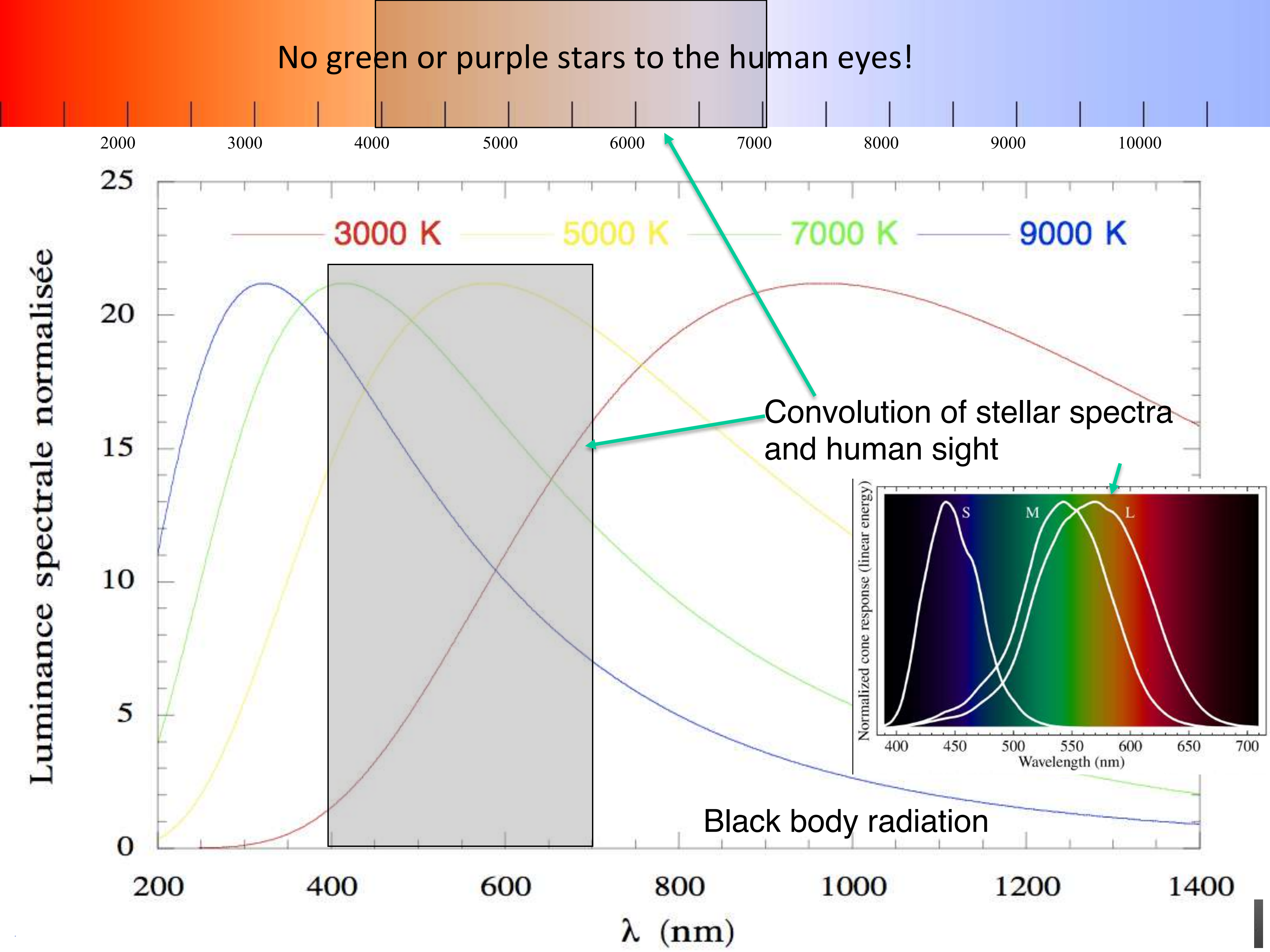
$$\phi = \int_{\mathrm{espace}} \int_0^\infty c\cos\theta \times h\nu \times n(\nu)\,\mathrm{d}\nu\,\frac{\mathrm{d}\Omega}{4\pi} = \boxed{\frac{2\pi^5 k^4}{15h^3c^2}}\,T^4$$

h Planck constant, k boltzmann constant

Stars ressemble black bodies !

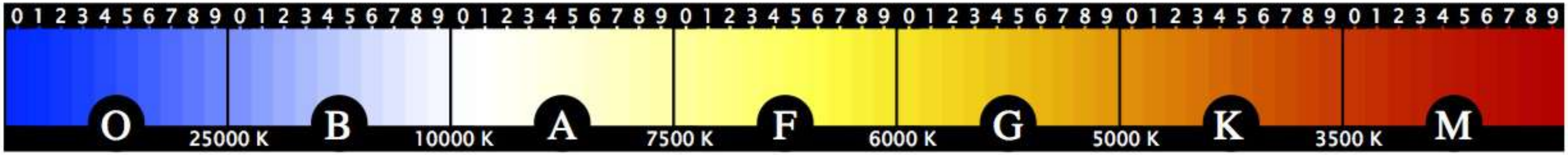


Wien's Law
$\lambda = 2900/T_{eff}$ (μm)



No green or purple stars to the human eyes!
2000
3000
4000
5000
6000
7000
8000
9000
10000
3000 K
5000 K
7000 K
9000 K
Luminance spectrale normalisée
25
20
15
10
5
0
200
400
600
800
1000
1200
1400
λ (nm)
Convolution of stellar spectra
and human sight
Normalized cone response (linear energy)
S
M
L
400
450
500
550
600
650
700
Wavelength (nm)
Black body radiation

Harvard classification of stars



Class	Temperature	Absorption lines
O	> 25 000 K	Azote, carbone, hélium, oxygène
B	10 000 – 25 000 K	Hélium, hydrogène
A	7 500 – 10 000 K	Hydrogène
F	6 000 – 7 500 K	Fer, titane, calcium, strontium
G	5 000 – 6 000 K	Calcium, hélium, hydrogène, métaux
K	3 500 – 5 000 K	Métaux, TiO
M	< 3 500 K	Métaux, TiO

Spectral types and color of Stars

A classification of stars based solely on spectral type is not sufficient to highlight their characteristics, since two stars of the same spectral type can have different sizes and luminosities. To remedy this, in 1942, astronomers Morgan, Keenan, and Kelman adopted a more detailed classification of spectral characteristics (the MKK classification). Stars are classified within each spectral type by decreasing luminosity into five classes, denoted by Roman numerals from I to V.

Table 16.2 Stellar Luminosity Classes

Class	*Description*
I	Supergiants
II	Bright giants
III	Giants
IV	Subgiants
V	Main sequence

The Sun is a G2V star

Luminosity: $L=4\pi R^2\sigma T^4$

Brightness = $1 / D^2$

D (parsec) = 1/p (angle arcsecondes)

RGB= red giant branch

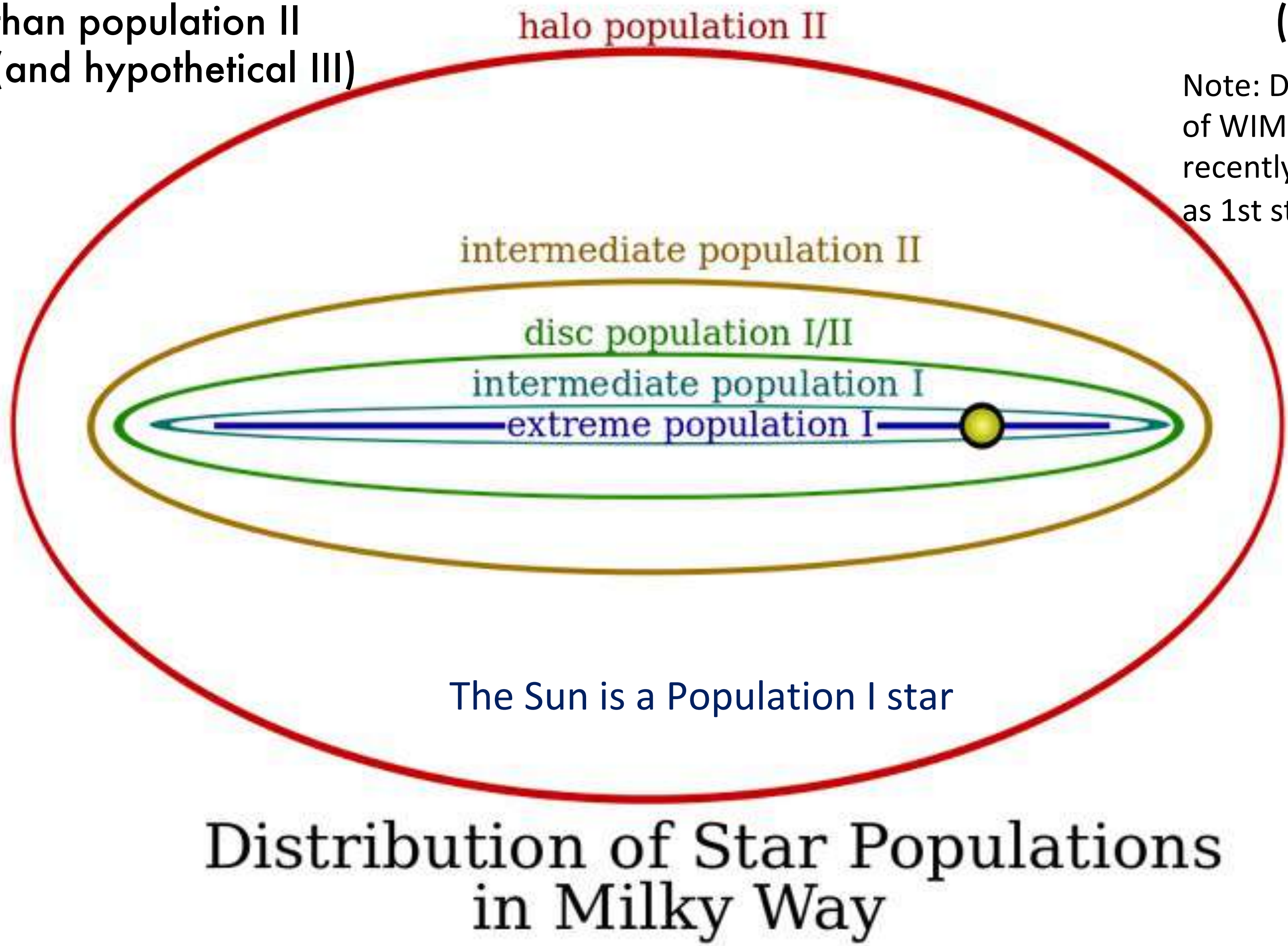
AGB=asymptotic giant branch

Population of Stars in our Galaxy:

Population I are younger and more metallic stars than population II (and hypothetical III)

Population III
First stars
(no metals)

Note: Dark stars made of WIMPS* have also recently been proposed as 1st star generation



*WIMPS: weakly interacting massive particles a dark matter candidate

Hertzsprung-Russell diagram (1910)

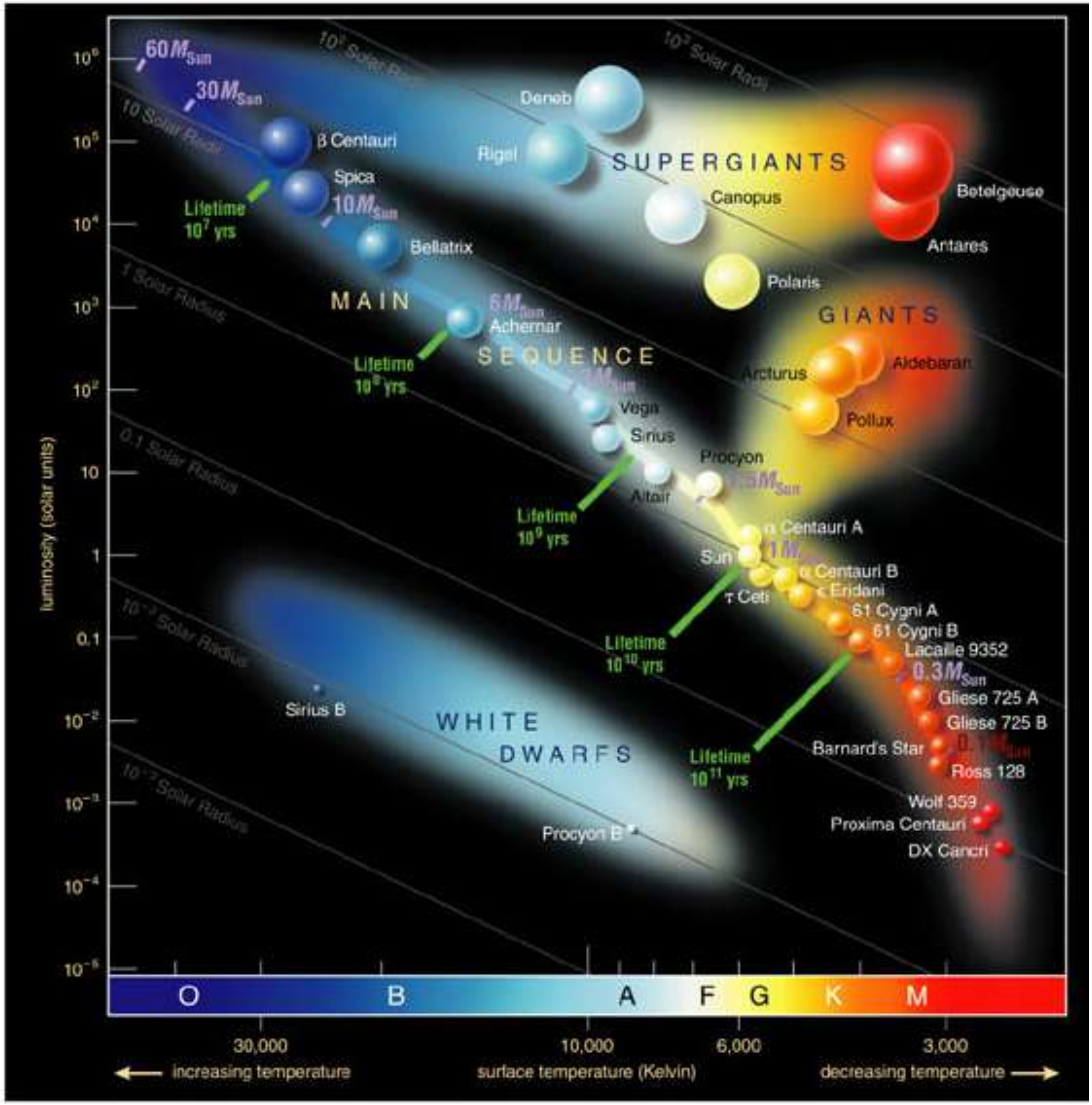
$$L = 4\pi R^2 \sigma T^4$$

« Michelin » maps of stars

Stefan-Boltzmann constant

$$\sigma = \frac{2\pi^5}{15}\frac{k^4}{h^3 c^2}$$

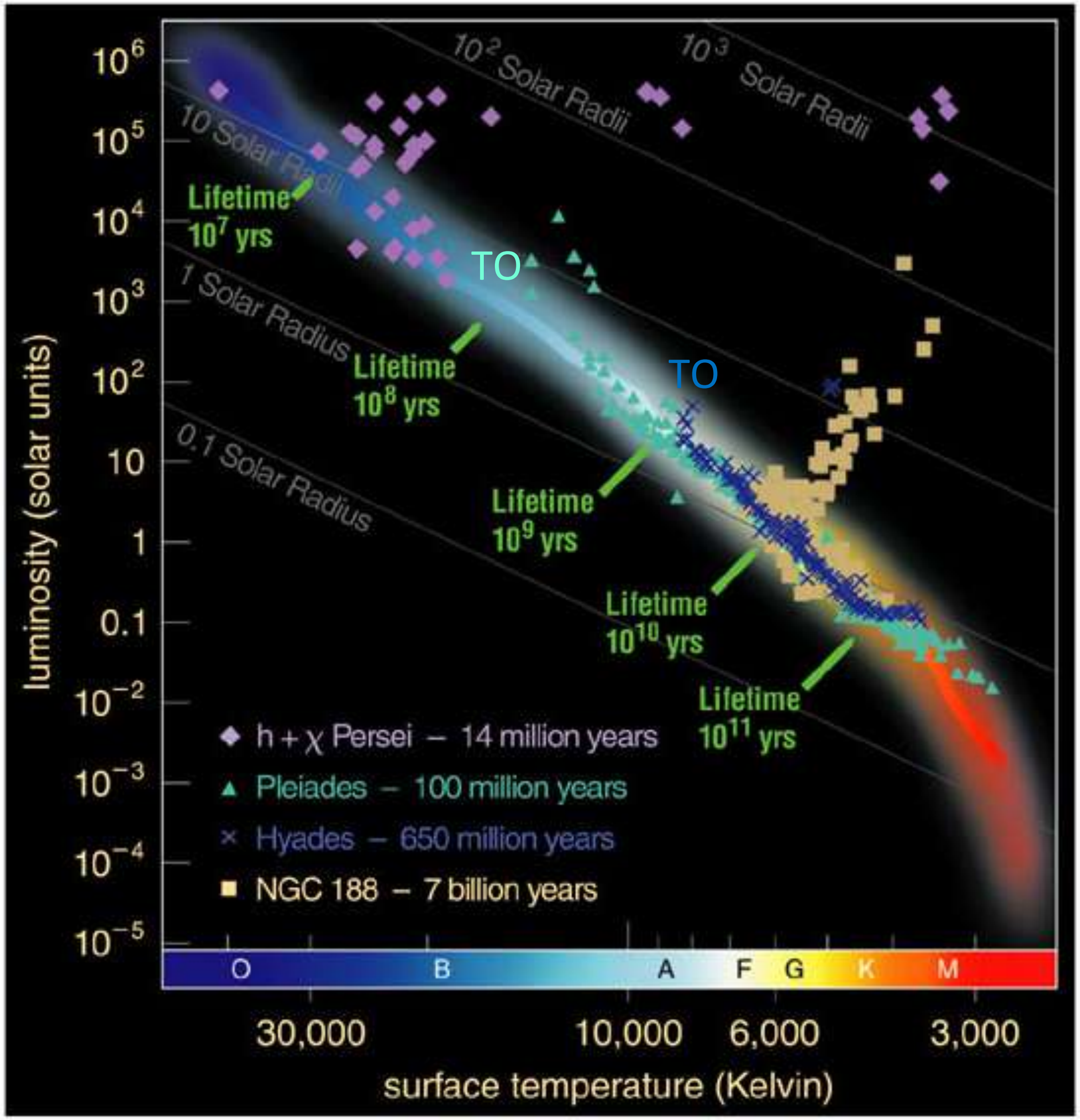
$$\sigma = 5,67\,10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$$



Observational Hertzsprung-Russell Diagram

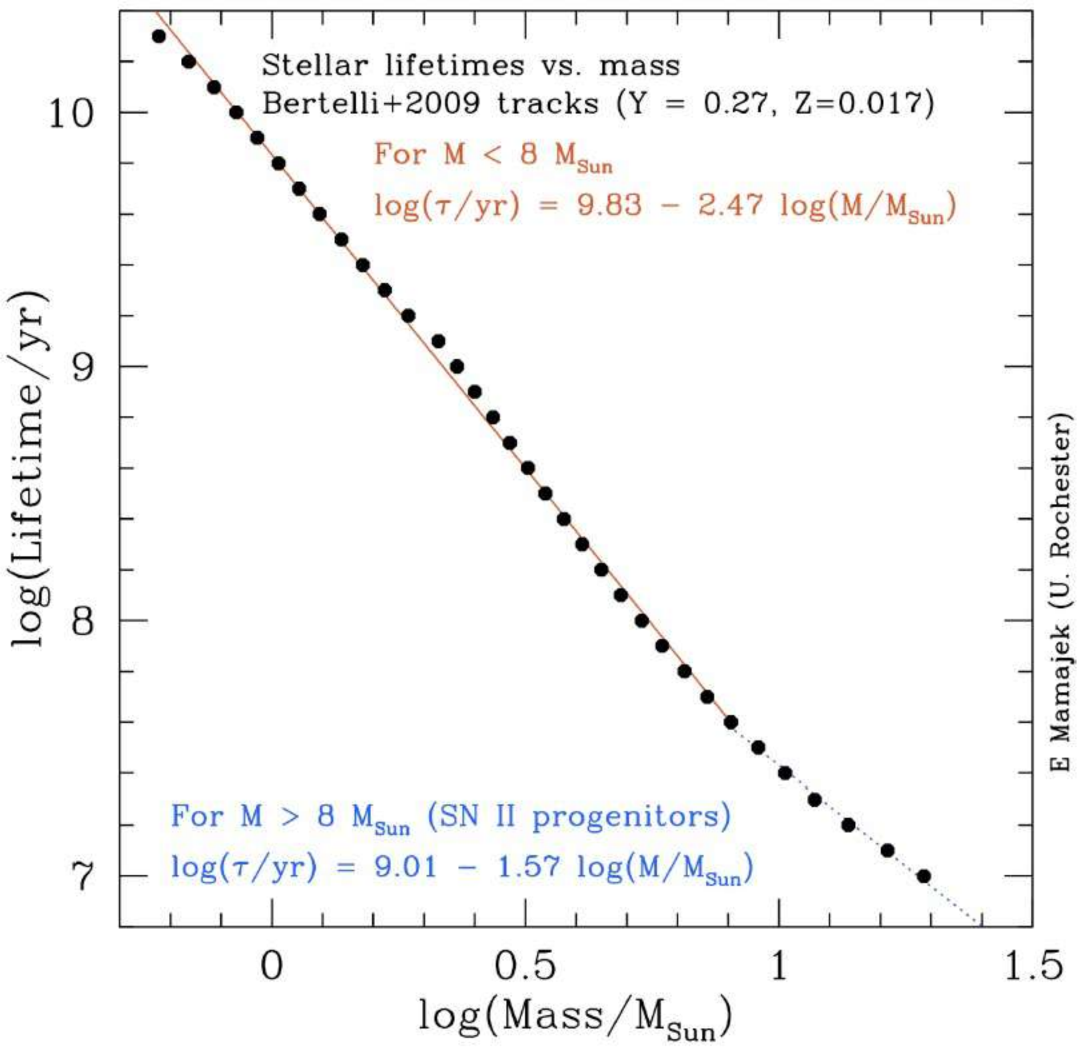
Note the difference in the shape of the HR diagram observed depending on the age of the star cluster considered.

We see stars (masses) disappear and others appear depending on their respective time to form or leave the main sequence



* The inflection point when the star leaves the main sequence (MS) is called Turn-off (TO) point

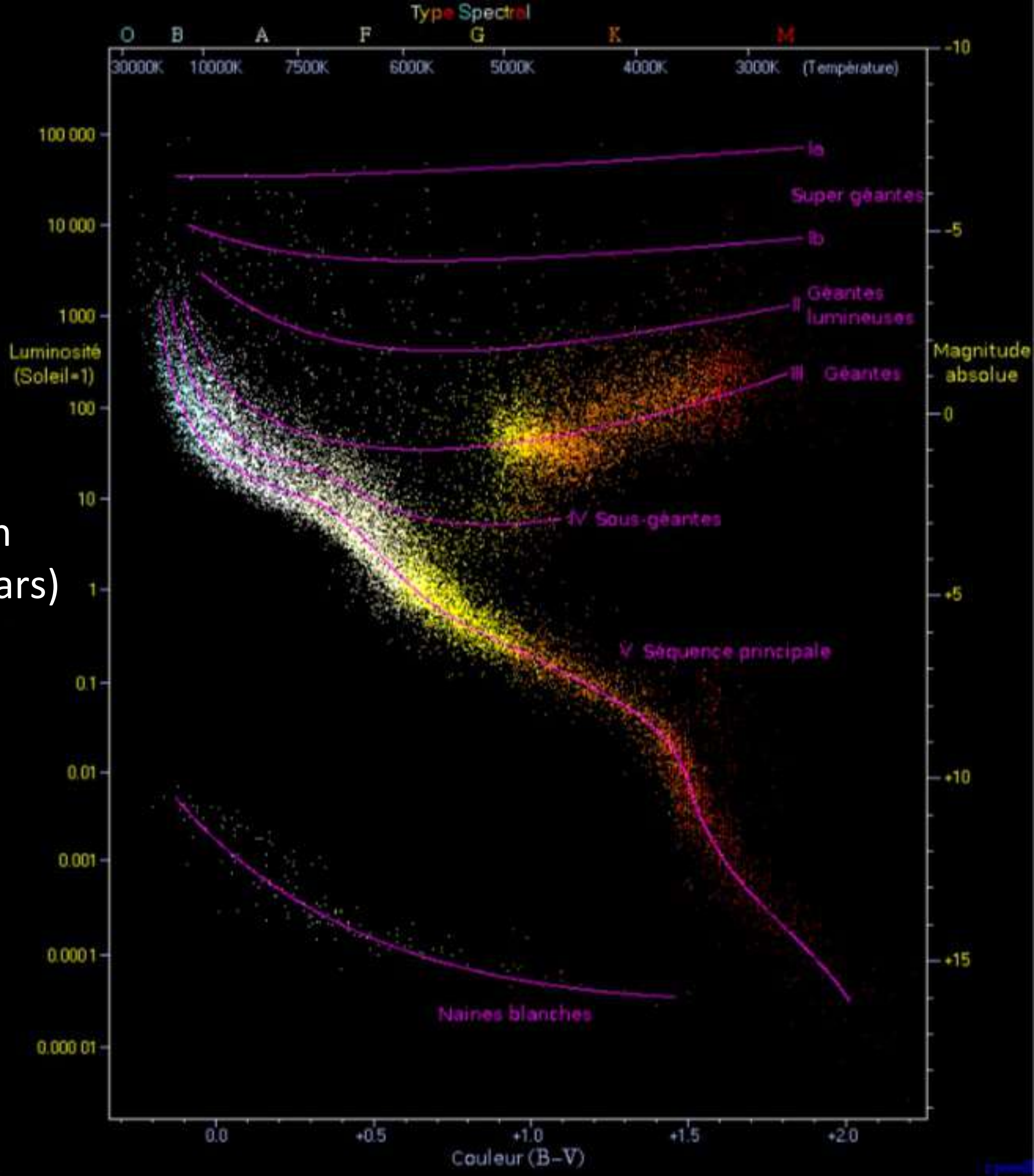
Stellar lifetime (I)



If one considers that $L_* \sim M_*^{-3.5}$ (slide 41)

Then one can show that star lifetime on the MS is: $t_{MS} \sim M_*^{-2.5}$

HR diagram realized with Hipparcos data (118 000 stars)



GAIA DR3 HR Diagram

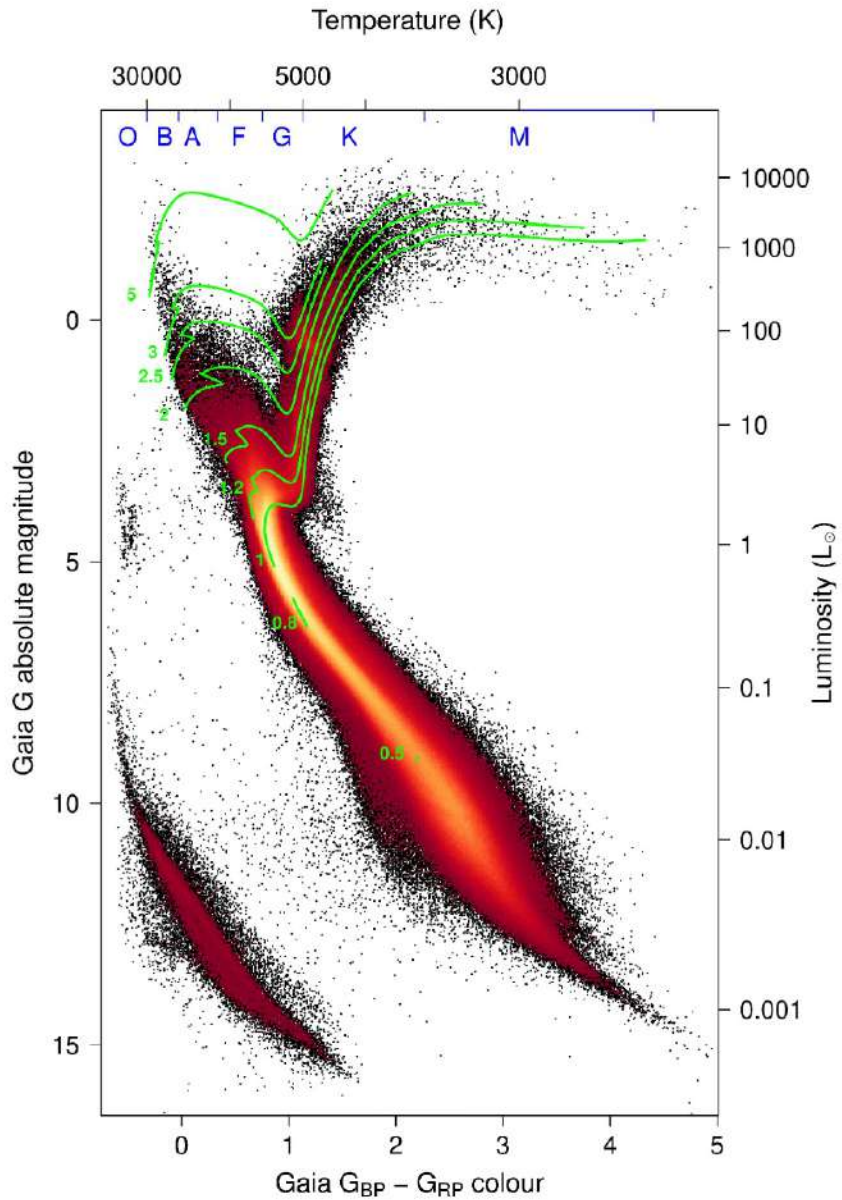
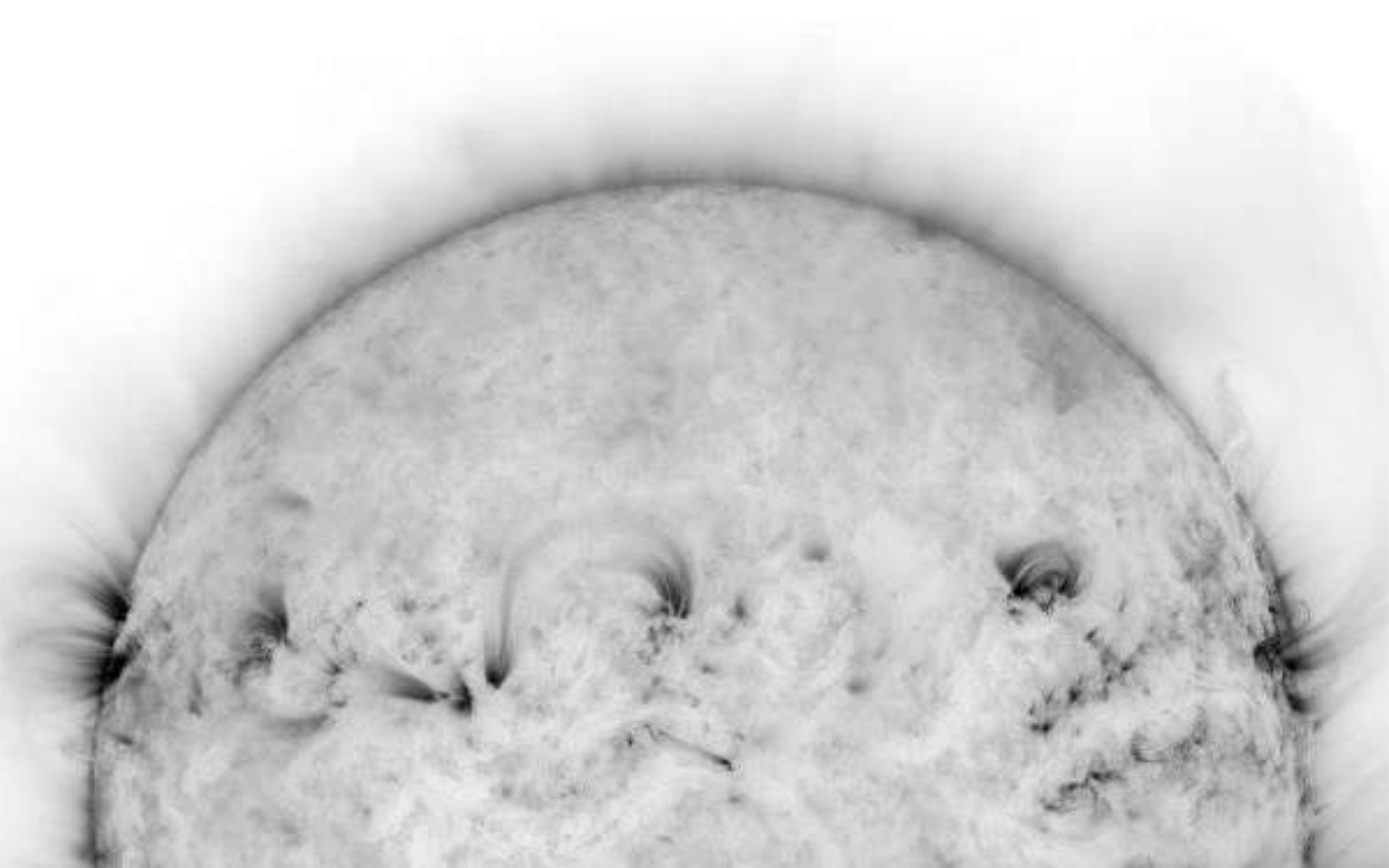


Figure 8.1 – Hertzsprung-Russell diagram from Gaia DR3 adapted from (Babusiaux et al., 2023) on which evolutionary tracks (green lines) from the Main Sequence to the RGB phase (limited to the age of the Universe, hence the very short track for very low-mass stars) for several stellar masses have been superposed. Copyright Carine Babusiaux.

Stars: main concepts

How does a star stabilize?



First considerations for the internal structure of a star

Hypothesis: a star possesses a spherical symmetry

Intensity gravitational field

$$g(r) = \frac{GM(r)}{r^2}$$

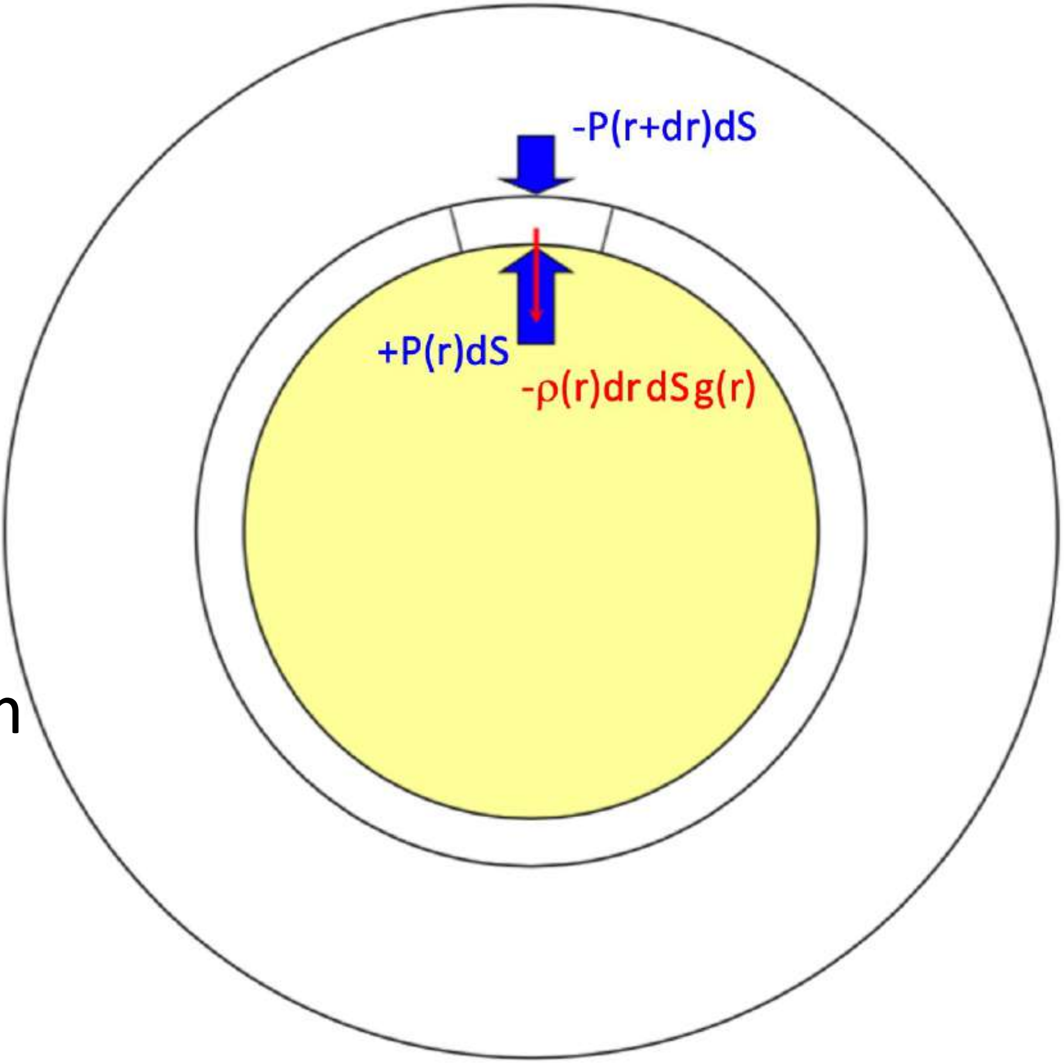
Mass of a stellar spherical shell

$$dM = 4\pi r^2 \rho(r) \mathrm{d}r$$

Mechanical equilibrium condition for a star

$$-P(r+dr)\mathrm{d}S + P(r)\mathrm{d}S - \rho(r)\mathrm{d}S\mathrm{d}r g(r) = 0$$

Pressure variations compensate gravity



Hydrostatic equilibrium

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\, g(r)$$

And if there was no pressure?

$G = 6.67\ 10^{-8}\ cm^3/g/s^2$
Cas Solaire
$R \sim 7\ 10^{10}$ cm; $M \sim 2\ 10^{33}$ g

Simple dimensional analysis
$R/t^2 \sim GM/R^2$
$t_{ff} \sim \text{sqrt}(R^3/GM) \sim 1600$ s

The star would collapse on itself !

How long would that take ?

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} \longrightarrow \tau_{ff} = \sqrt{\frac{3\pi}{32G\bar{\rho}}}$$

free fall timescale

And for the Sun?

Mean solar density is1.4 g/cm^3

The Sun would collapse on itself in ~ 1800 s (1/2 h)!

Jeans Mass

The characteristic time associated with the pressure gradient is the "sound" time associated with the speed of sound in the star.

$$\tau_s = \frac{R}{c_s} \qquad c_s = \sqrt{\gamma P/\rho}$$

Hence a ISM cloud collapse if $\tau_{ff} \ll \tau_s$

This means $R^2 \gg \frac{c_s^2}{G\bar{\rho}} = \lambda_J^2$ (Jeans lenght)

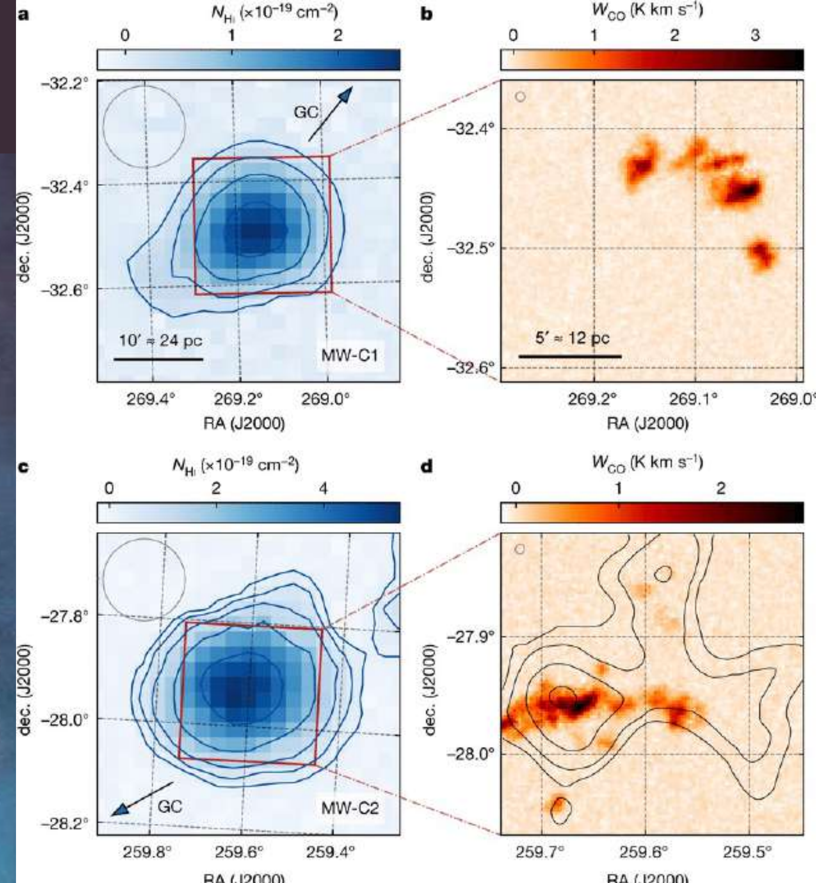
Which results into the famous Jeans mass: $M_J = \frac{4}{3}\pi\lambda_J^3\bar{\rho}$

$$= 20 \cdot \left(\frac{T}{\mathrm{K}}\right)^{1.5} \left(\frac{n}{\mathrm{cm}^{-3}}\right)^{-0.5} \mathrm{M}_\odot$$

Hydrogen cloud: $M_J = 10^{3\text{-}4} M_\odot$ T~100K

Molecular cloud: $M_J = 10^{1\text{-}2} M_\odot$ denser and colder (10 K)

The “pillars of creation”

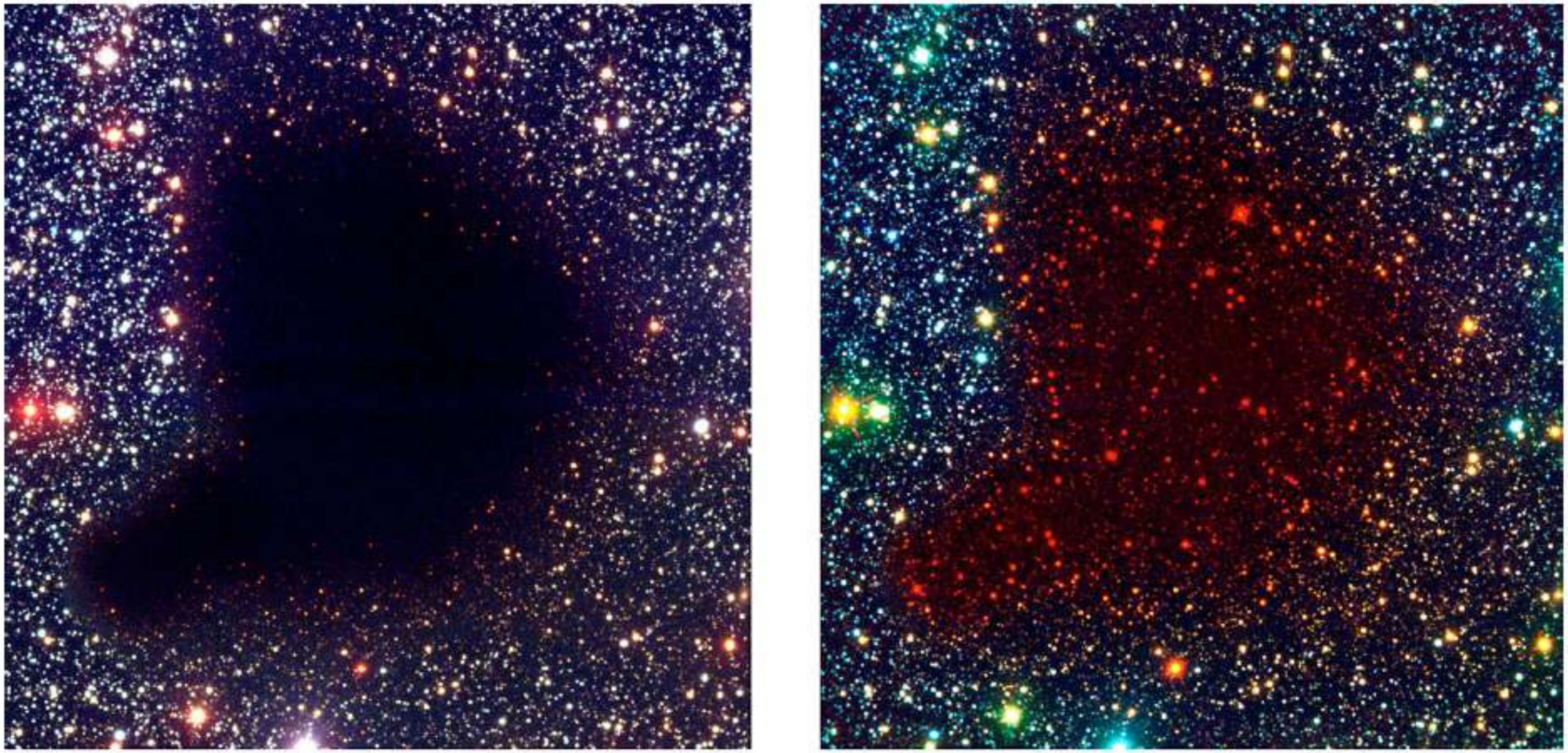


Atomic and molecular clouds
(Theodoro et al. 2020)

Phase		n [cm^{-3}]	T[K]	M_{tot}[M$_\odot$]
atomic (HI)	cold	$\backsim 25$	$\backsim 100$	$1.5 \cdot 10^9$
	warm	$\backsim 0.25$	$\backsim 8000$	$1.5 \cdot 10^9$
molecular (H_2)		$\gtrsim 10^3$	10-50	10^6
ionized	HII	$\backsim 1 \cdots 10^4$	$\backsim 10000$	$5 \cdot 10^7$
	diffuse	$\backsim 0.03$	$\backsim 8000$	10^9
	hot	$\backsim 6 \cdot 10^{-3}$	$\backsim 5 \cdot 10^5$	10^8

Table 1.1: Components of the ISM (phases)

Exemple of a cold dark cloud in Visble revealed by IR observations



B, V, I

B, I, K

Figure 5.3: Colour composites of ESO images of the Dark Cloud Barnard 68 taken through different optical and near-IR filters. Dust in the cloud hides stars located behind it; because dust scatters infrared light less efficiently than optical light, the opacity of the cloud is reduced in the colour composite that includes the near-IR K-band filter. The cloud is located at a distance of 160 pc and is seen against the background of a rich star field in the Milky Way. Curiously, there are no foreground stars. Barnard 68 seems to be a molecular cloud in the earliest phase of collapse to form new stars; for this reason it is the subject of many studies at a variety of wavelengths.

Stellar lifetime (II)

If the star radiates away its gravitational potential energy then:

$$\tau = \frac{U_G}{L_\star}$$

$$U_G = \beta \frac{GM_\star^2}{R_\star}$$

β=3/5 for an homogeneous sphere

In the Solar case this yields

$$\left\{ \begin{array}{l} R_\odot = 6.96\,10^5 \text{ km} \\ M_\odot = 1.99\,10^{30} \text{ kg} \\ L_\odot = 3.83\,10^{26} \text{ W} \end{array} \right. \rightarrow \left\{ \begin{array}{l} U_G \approx 2.3\,10^{41} \text{ J} \\ \tau \approx 19\,10^6 \text{ years} \end{array} \right.$$

Age du système solaire ≈ 4.56 10^9 ans !

Then how did the Sun succeed to radiate for such a long time?

Where does the radiatie energy come from in that case?

The Sun has radiated 3.83 10^{26} W during 4.56 10^{9} ans.
(simplified assumption as $L_{Sun}(t)$ varies, see slides 47)

It contains $M_{Soleil}/m_p \sim 10^{57}$ protons

If we convert that to energy per protons, its luminosity corresponds to ~ **0.38 MeV/proton.**

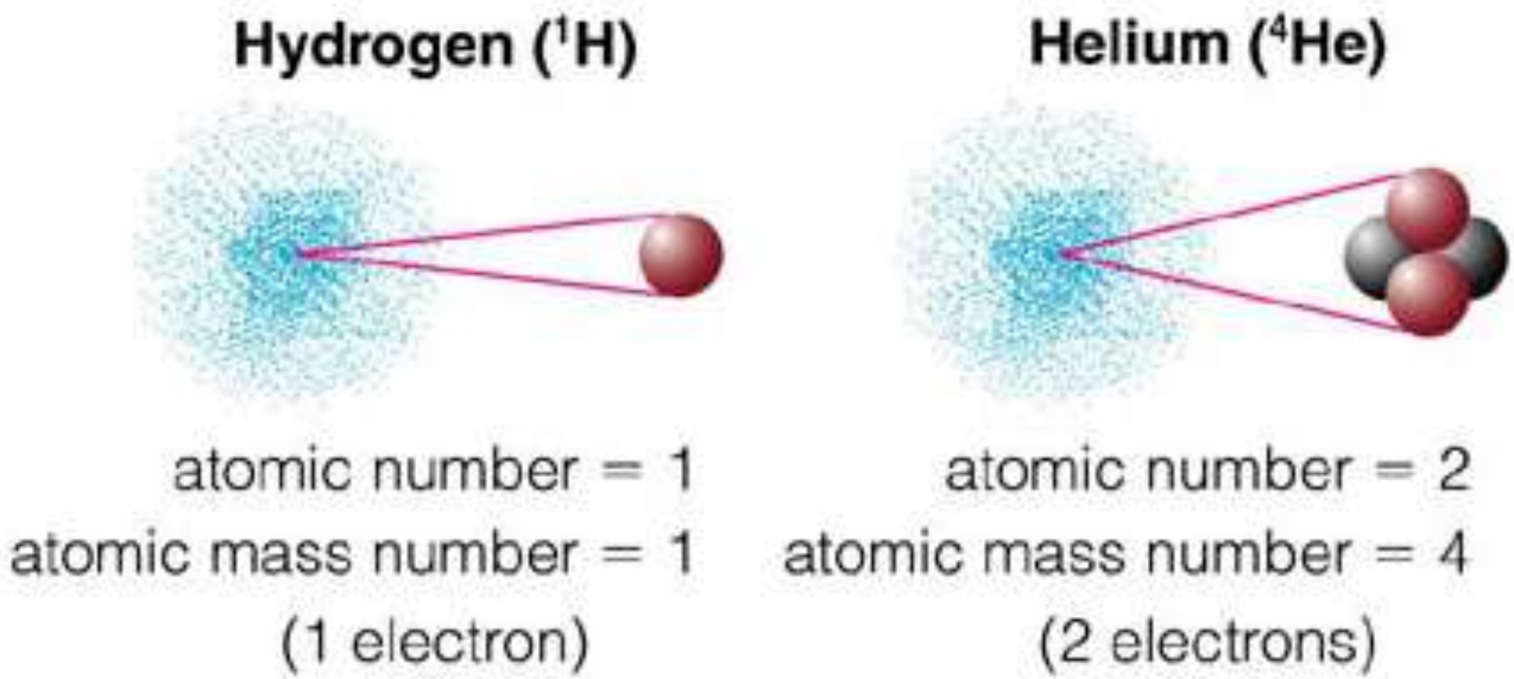
Chemical reactions could provide 1 eV/proton

Gravity provides 2 keV/proton (cf previous derivation)

Nuclear fusion of 2 protons provides up to **1MeV/proton**

So there is not alternative to nuclear energy to explain the long lasting intense brightness of star over secular ages!

Hydrogen atom fusion



Fusion of 4 Hydrogene atomes in one Helium ('pp' chain or CNO cycle):

$$4\,p \rightarrow {}^{4}He + 2e^{+} + 2\nu_e + E_\gamma$$

PP chain nuclear rate: $\mathcal{E}_{pp} \propto X^2 \rho T^4 \quad \mathrm{erg\,s^{-1}\,g^{-1}}$

Note on nuclear reaction rate

It is convenient to put the stellar reaction rates in a power-law form:

$$r_{it} \simeq r_0 X_i X_t \, \rho^\gamma \, T^\beta \tag{7.23}$$

where r_0 is a constant, $X_{i,t}$ are the mass fractions of the two particles, the exponent of the density dependence is normally $\gamma = 2$ for two-body collisions, whereas the power-law dependence on the temperature β can range from ~ 1 to $\gtrsim 40$. If $\mathcal{E}_0$ is the energy released per reaction, then the rate of energy release per unit mass of nuclear fuel is:

$$\mathcal{E}_{it} = \left(\frac{\mathcal{E}_0}{\rho}\right) r_{it} \qquad \frac{\mathrm{erg\,s^{-1}\,cm^{-3}}}{\mathrm{g\,cm^{-3}}} \Rightarrow \mathrm{erg\,s^{-1}\,g^{-1}} \tag{7.24}$$

or

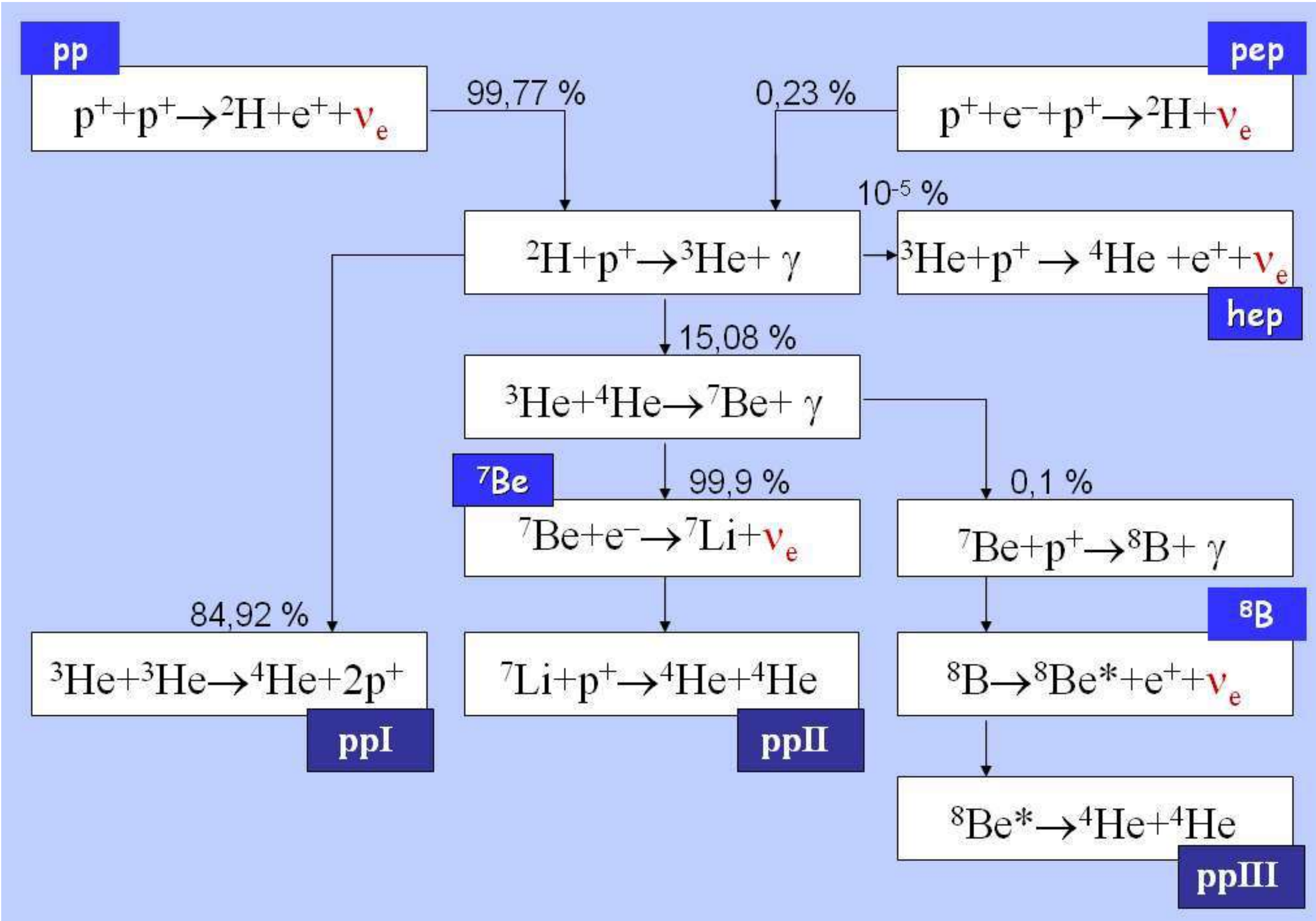
$$\mathcal{E}_{it} = \mathcal{E}_0 \, r_0 \, X_i X_t \, \rho^\alpha \, T^\beta \tag{7.25}$$

where $\alpha = \gamma - 1$.

From Prof. M. Pettini, IoA, University of Cambridge

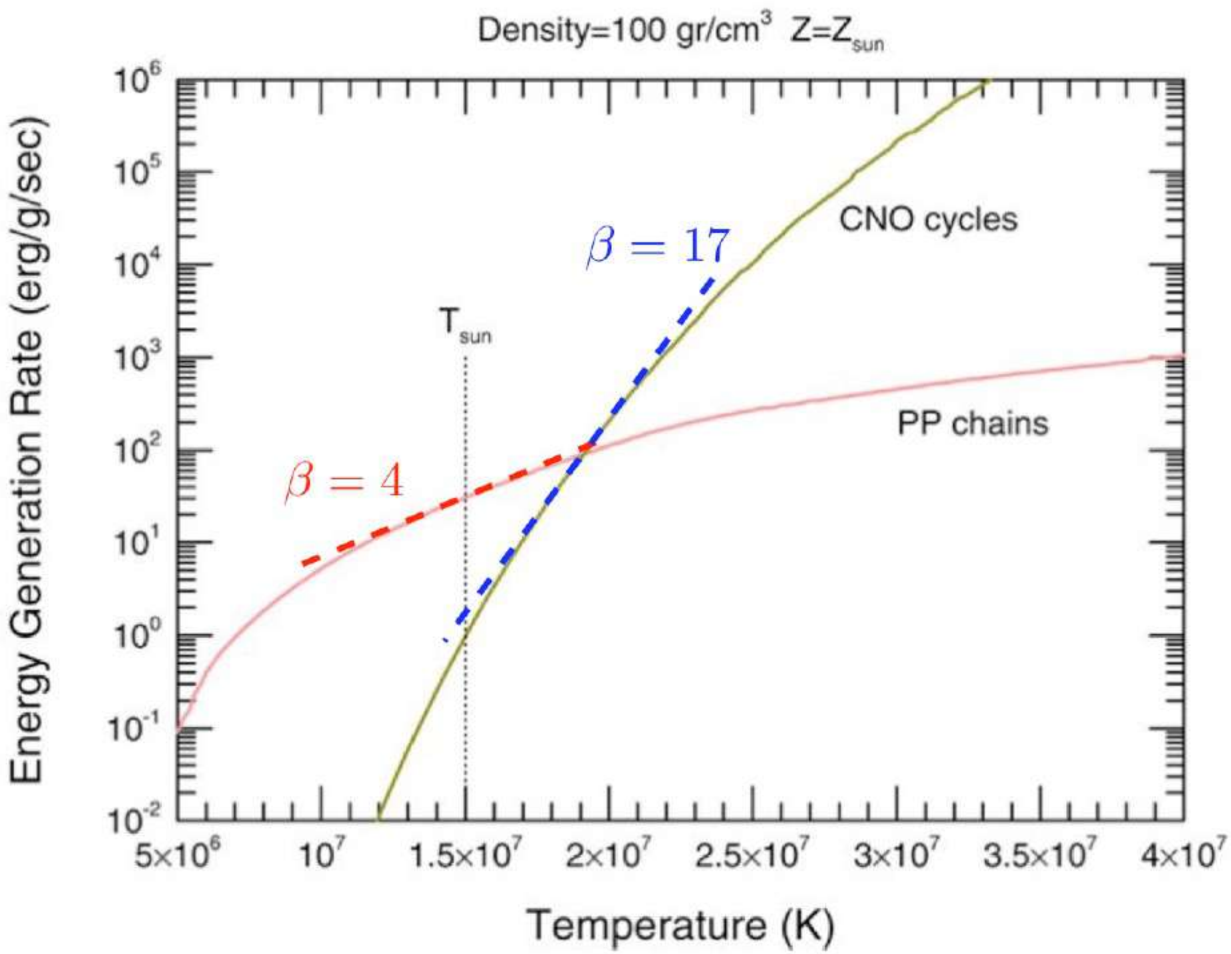
Hence for pp: the exponent α =2-1=1 and for triple alpha reaction α =3-1=2 (cf slide 70)

P-P chains vs cycle CNO



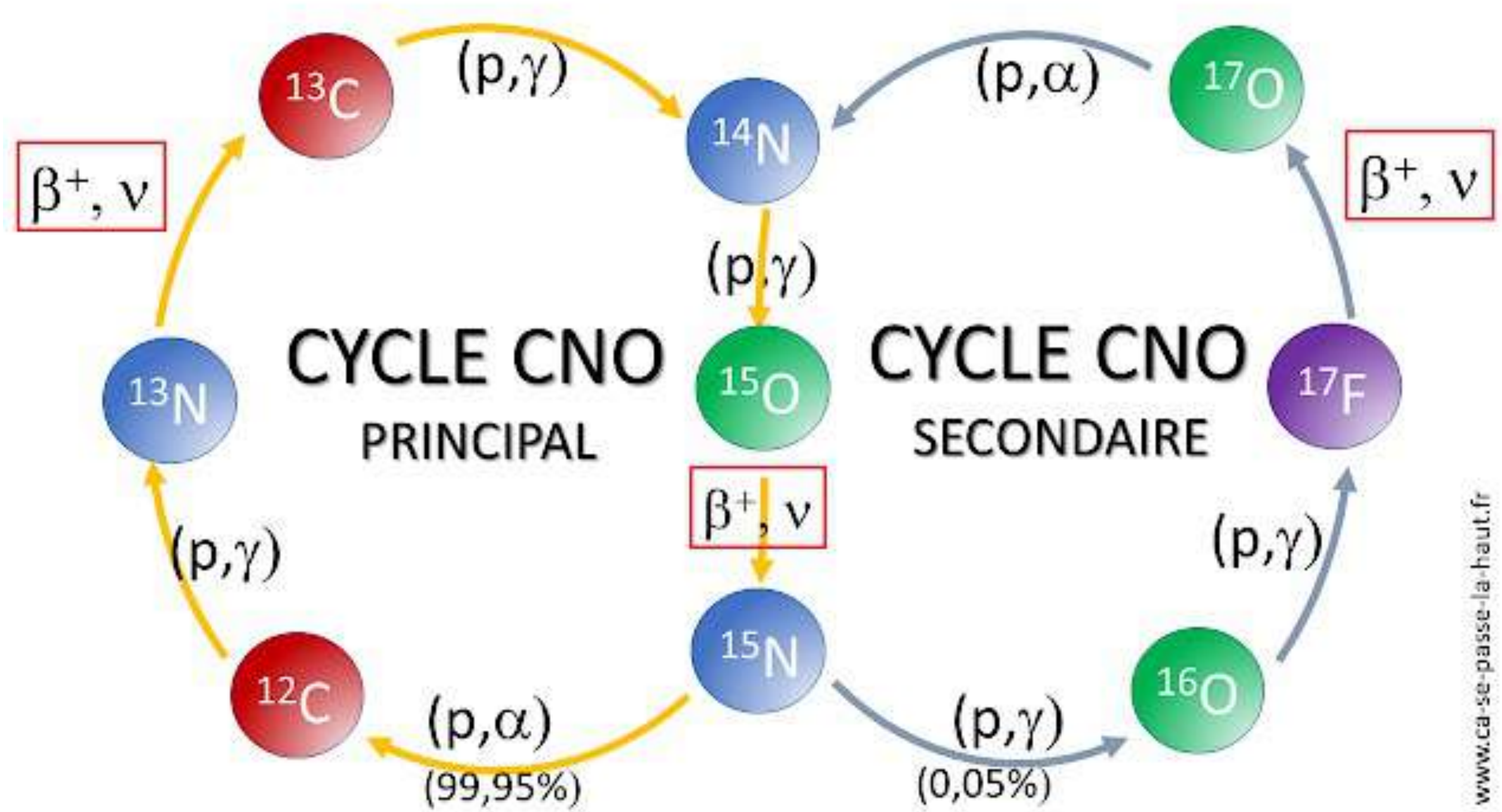
Dominate Helium4 creation in stars less massive than 1.3 Msol

PP-chain vs CNO bi-cycle energy generation vs stellar core Temperature



see Clayton 1968, « Principles of stellar evolution and nucleosynthesis » book

P-P chains vs cycle CNO

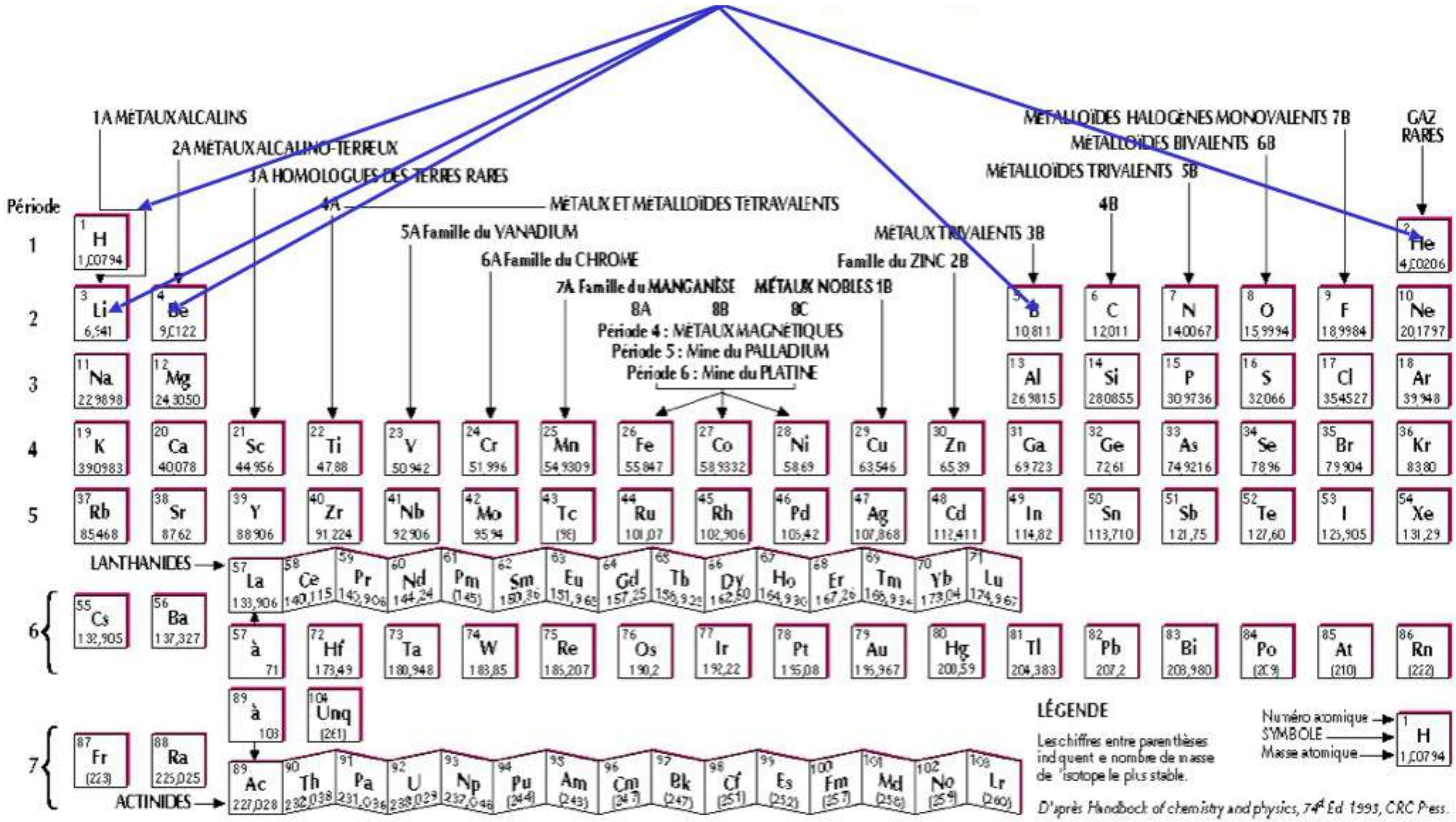


Dominate Helium4 creation in stars more massive than 1.3 Msol

CNO bi-cycle nuclear rate: $\mathcal{E}_{\rm CNO} \propto X\, X_{\rm CNO}\, \rho\, T^{17} \qquad {\rm erg\, s^{-1}\, g^{-1}}$.

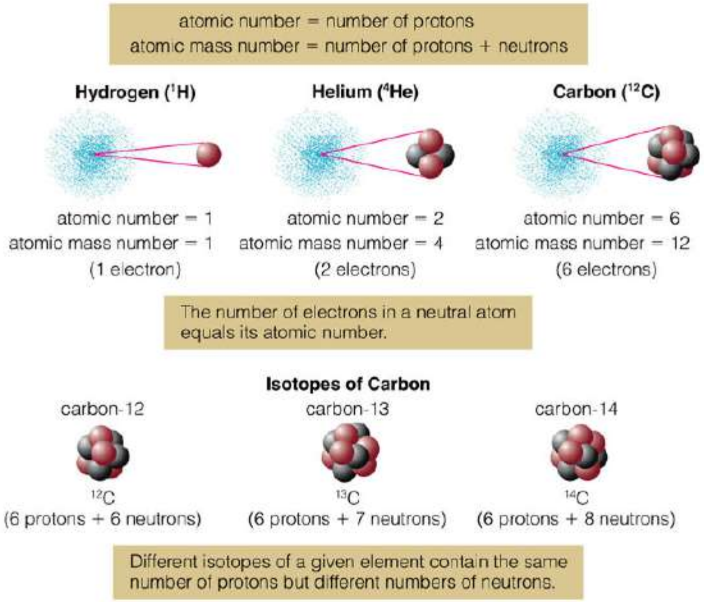
Mendeleïev periodic element table

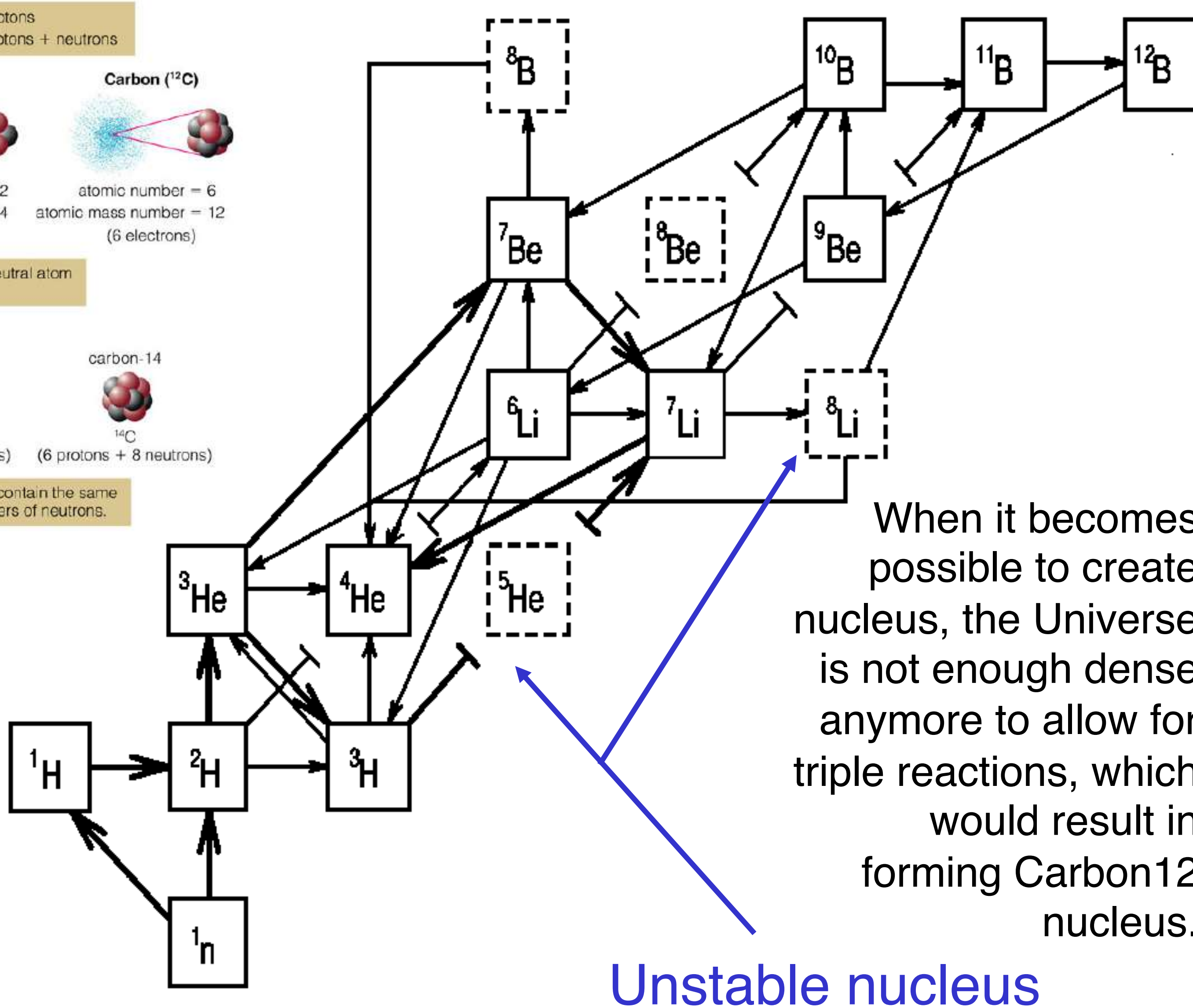
Created during the Big Bang (B and Be only as side trace products)



We are all « star dust »

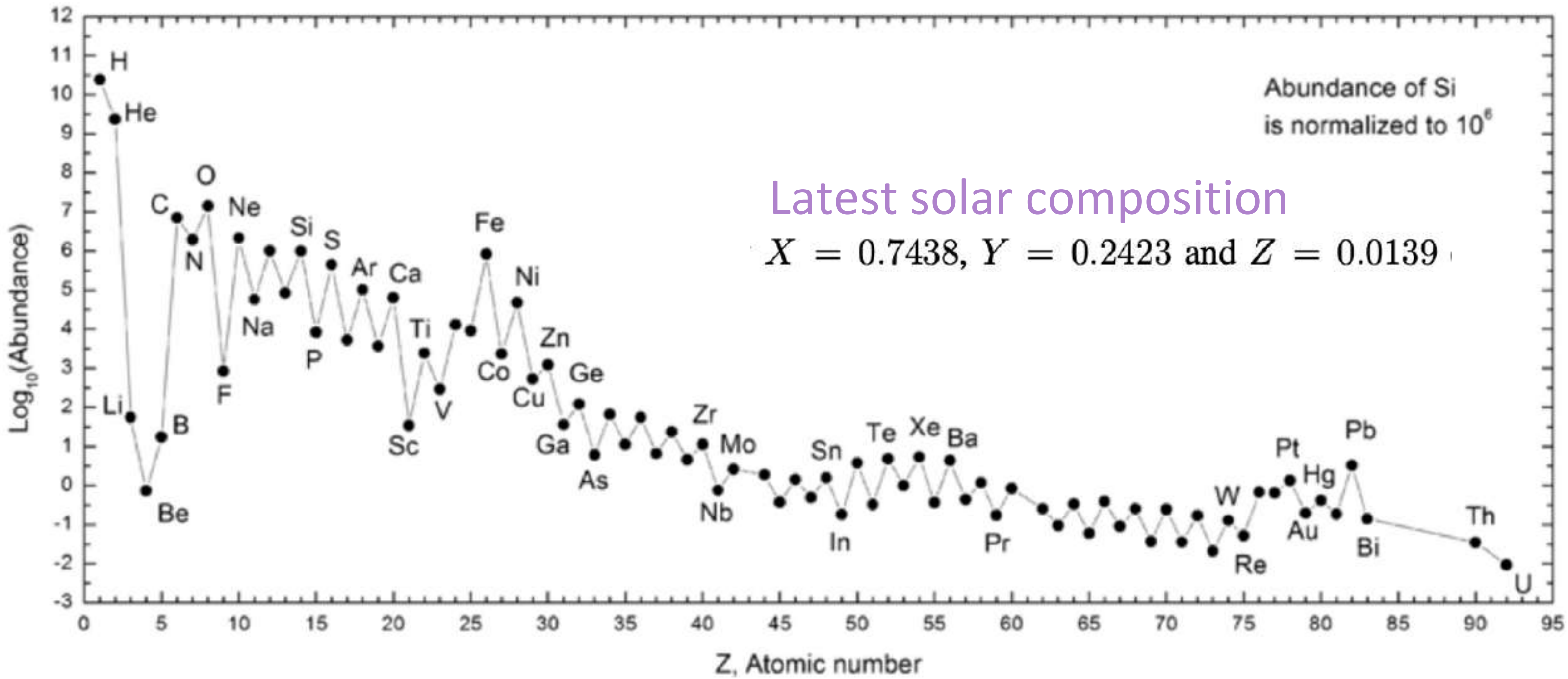
Hole in nuclear stability at A=5 and A=8





When it becomes possible to create nucleus, the Universe is not enough dense anymore to allow for triple reactions, which would result in forming Carbon12 nucleus.

Cosmic chemical abundances & mean molecular weight μ_{mol}



Latest solar composition

$X = 0.7438,\ Y = 0.2423$ and $Z = 0.0139$

A useful quantity to take into account abundances in stellar plasmas is the mean molecular weight μ_{mol}. It is related to the number density n and the density ρ by: $\mu_{mol} = \frac{\rho}{m_H n}$. The fluid plasma density ρ is by definition the sum of all the individual species densities, e.g. $\rho = \sum_i \rho_i$, such the mean molecular weight:

$$\mu_{mol} = \sum_i \frac{\rho_i}{m_H n_i},$$

with $n_i = \frac{\rho_i}{\mu_i m_H}$, and μ_i the species atomic weight. In stellar plasma it is usual to define relative abundances $X_i = \frac{\rho_i}{\rho}$. It is standard to define the Hydrogen abundances as X, the helium abundance as Y and all the other elements (also called "metals" or "heavy elements") by Z, such that $X + Y + Z = 1$. With such definitions $n_i = \frac{\rho}{m_H}\frac{X_i}{\mu_i}$. The *metallicity* of a star is the amount of metals in it, which is often quoted as $[Z/X]$ ratio (or [Fe/H]). It is indeed standard astronomy practice to define the abundance of an element Z in a cosmic body with respect to that of the Sun by $[Z/H] \equiv \log(N_Z/N_H) - \log(N_Z/N_H)_\odot$, with N_i the number of atom i (see Figure ↑).

$$n = n_e + \sum_i n_i = \sum_i (1 + Z_i) n_i.$$

$$P = nk_BT = (n_e + \sum_i n_i)k_BT$$

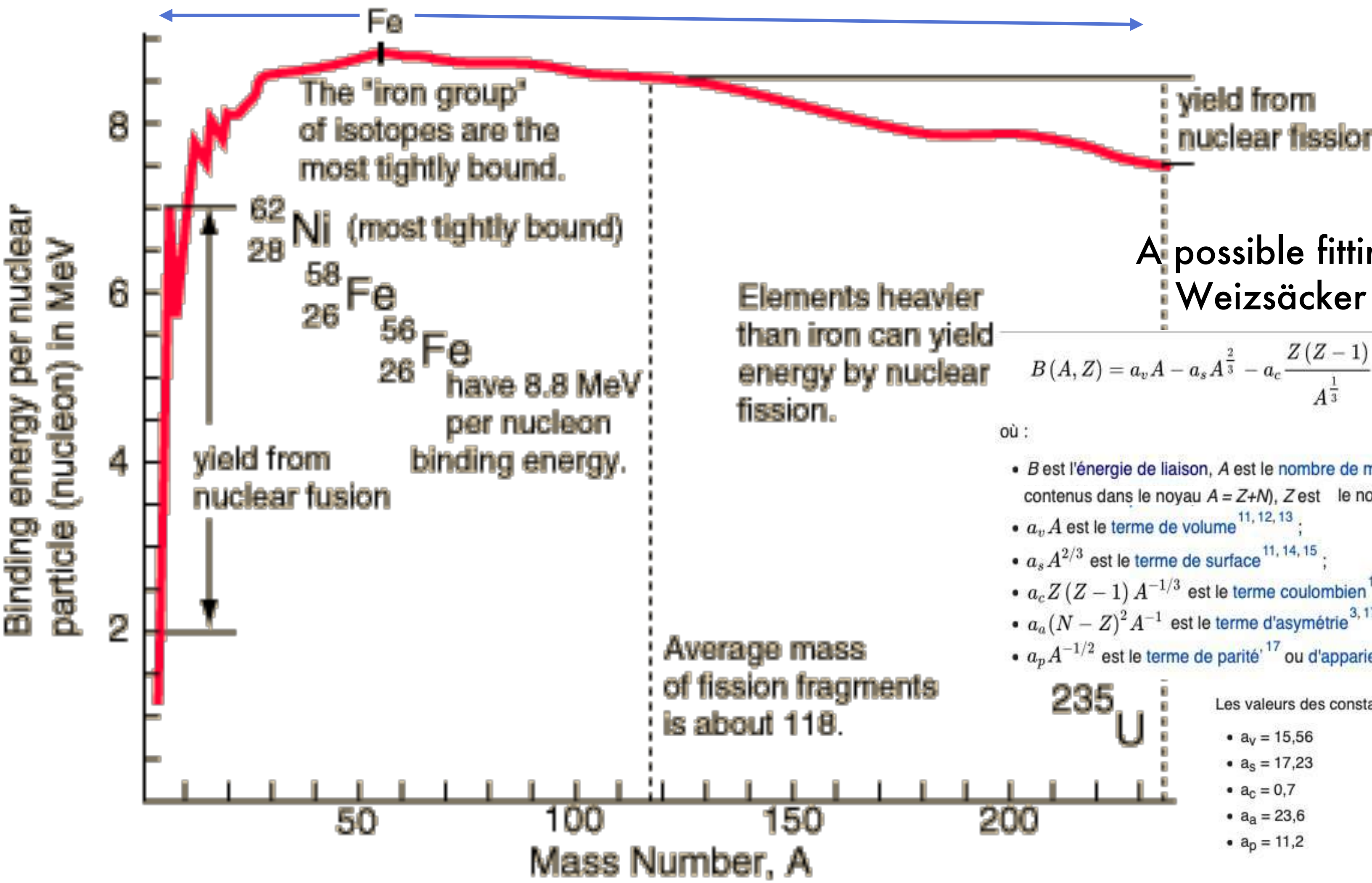
$$= k_BT \sum_i (1+Z_i)\frac{\rho}{m_H}\frac{X_i}{\mu_i} = \Re\rho T \sum_i \frac{(1+Z_i)X_i}{\mu_i} = \frac{\Re}{\mu_{mol}}\rho T,$$

which yields: $\mu_{mol}^{-1} = \sum_i \frac{(1+Z_i)X_i}{\mu_i}$. Please note that there is an ongoing discussion on solar abundances as discussed in (Asplund et al., 2021; Christensen-Dalsgaard, 2021).

Nucleus binding energy

(mass difference the nucleus and the sum of its constituant nucleons (p, n))

Fusion <-> Fission



A possible fitting formula
Weizsäcker formula

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_a \frac{(A-2Z)^2}{A} \pm a_p A^{-\frac{1}{2}}$$

où :

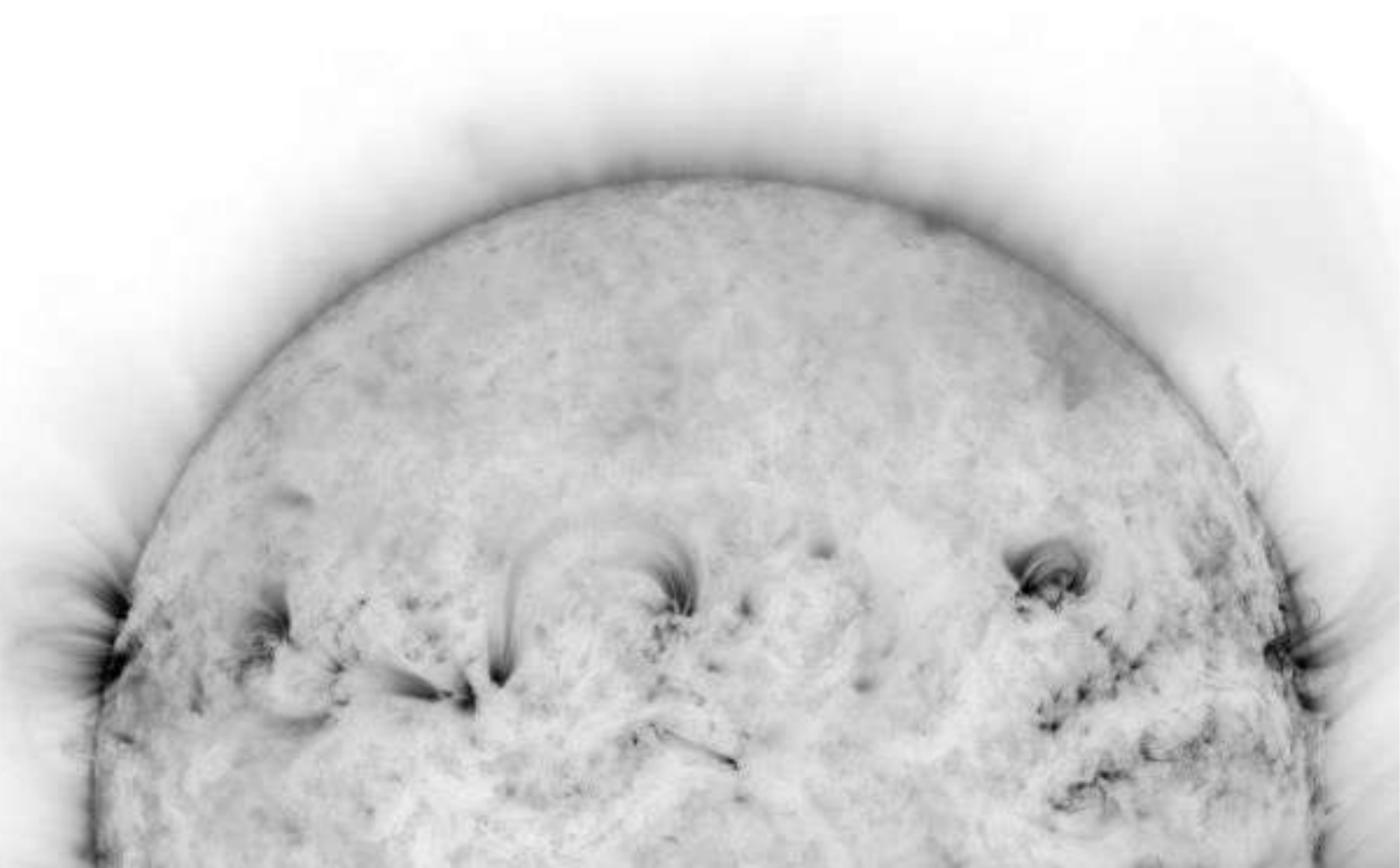
- B est l'énergie de liaison, A est le nombre de masse (ou nombre de nucléons contenus dans le noyau $A = Z+N$), Z est le nombre de protons ;
- $a_v A$ est le terme de volume[11, 12, 13] ;
- $a_s A^{2/3}$ est le terme de surface[11, 14, 15] ;
- $a_c Z(Z-1) A^{-1/3}$ est le terme coulombien[11, 16, 17] ;
- $a_a (N-Z)^2 A^{-1}$ est le terme d'asymétrie[3, 17, 18, 19] ;
- $a_p A^{-1/2}$ est le terme de parité[17] ou d'appariement[3, 20].

Les valeurs des constantes utilisées sont (en MeV) :

- $a_V = 15{,}56$
- $a_S = 17{,}23$
- $a_C = 0{,}7$
- $a_A = 23{,}6$
- $a_P = 11{,}2$

Stars: main concepts

A first model of stellar interior



Basic equilibria of spherically symmetric star

Up to now, we have ontained **3 equations** but with **4 unknowns !**

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)g(r)$$

$$g(r) = \frac{GM(r)}{r^2}$$

$$dM = 4\pi r^2 \mathrm{d}r\rho(r)$$

Hence we must connect P and ρ in order to go forward

Polytropic equation

$$P = K\rho^{\frac{n+1}{n}}$$

We call n the **polytropic index** and usually note $\gamma = (n+1)/n$

if n=0, The density is constant (incompressible gas)
if n=3/2 The gaz is said to be adiabatic (perfect mono-atomic ideal gas)
it is also the case for a fully degenrate electron gas
if n=3 It is a mixture of perfect ideal gas and photon gas
it is also the case for a fully degenrate relativist electron gas
if n=∞ The gas is isothermal

To simplify notations we will make the following change of variable

$$\rho = \rho_c \phi^n \rightarrow P = K\rho_c^{1+1/n}\phi^{n+1}$$

Lane-Emden equation

So we have 4 equations with 4 unknown that we can reduce to

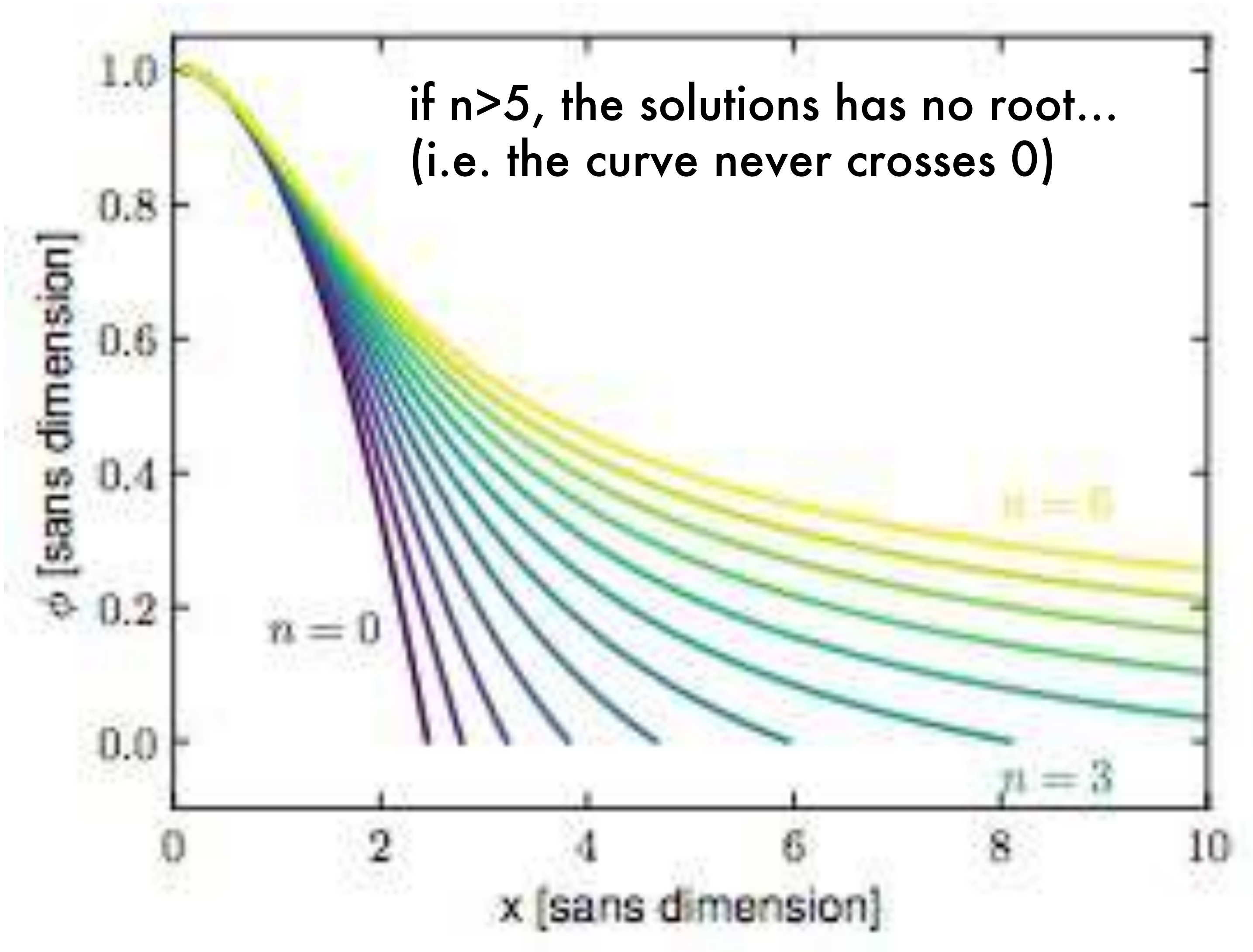
$$\frac{1}{x^2}\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\frac{\mathrm{d}\phi}{\mathrm{d}x}\right) = -\phi^n$$

where $x = \frac{r}{\lambda}$ and $\lambda^2 = \left(\frac{(n+1)K\rho_c^{1/n-1}}{4\pi G}\right)$

with conditions such that Φ =1 and Φ' =0 at the star's center.

Analytical solutions exist for n=0,1 and 5. Solutions for other n values must be found numerically.

Lane-Emden solutions



Stellar structure obtained with Lane-Emden equations

Radius: Obtained with the first 0 of function Φ

$$R_\star = x(\phi = 0)\lambda$$

Mass:

$$M_\star = \int_0^r 4\pi r^2 \rho(r) \mathrm{d}r$$

No physical solutions if n ≥ 5!

$$U_G = -\int_b^t \frac{Gm}{r} dm = -\frac{GM_*^2}{2R_*} - \frac{1}{2}\int_b^t \frac{Gm^2}{r^2} dr$$

$$U_G = -\left(\frac{3}{5-n}\right)\frac{GM_*^2}{R_*}.$$

if n ≥ 5, the star is not gravitationally bound anymore

Solar case for n=3, M=2 .10^{30} kg **and** R = 7. 10^{8} m:

Lane-Emden: ρ_c = 76.4 g/cm^3 ; T_c = 12.1 10^6 K

Calcul ab-initio: ρ_c = 158 g/cm^3 ; T_c = 15.7 10^6 K

Equations of stellar internal structure and evolution (I)

They can be deduced from the equations of motion and conservation of energy, considering hydrostatic and thermal equilibrium:

Stellar structure equations

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

$$\frac{\partial L}{\partial m} = \epsilon_N - \epsilon_\nu - T\frac{\partial S}{\partial t}$$

$$\frac{\partial T}{\partial m} = -\frac{Gm\,T}{4\pi r^4\,P}\nabla$$

eq 1.X

It is necessary to have a precise microscopic description of the plasma under consideration (equation of state P(r,T,Xi), nuclear reaction networks enabling the production of energy ε_N(r,T,Xi) and opacity κ(r,T,Xi)).

Note: X_i chemical composition of element i
X= H, Y= ^{4}He, Z gather all other heavier elements

Schwarzshild Criteria

(see next lecture for derivation)

In general, the temperature gradient is determined by heat diffusion and by convection, if the particular layer inside a star is convectively unstable. The standard, Schwarzschild, criterion for stability is:

$$\nabla_{rad} < \nabla_{ad} \qquad \text{stable,}$$

$$\nabla_{rad} > \nabla_{ad} \qquad \text{unstable.}$$

where

$$\nabla_{rad} \equiv \frac{\kappa L_r}{16\pi c G M_r} \frac{3P}{aT^4},$$

$$\nabla_{ad} \equiv \left(\frac{\partial \ln T}{\partial \ln P} \right)_S$$

The temperature gradient may be calculated as

$$\nabla_T = \nabla_{rad} \quad \text{when} \quad \nabla_{rad} < \nabla_{ad},$$

$$\nabla_T = \nabla_{conv} \quad \text{when} \quad \nabla_{rad} > \nabla_{ad},$$

$$\nabla_{rad} > \nabla_{conv} > \nabla_{ad}.$$

Equations of stellar internal structure and evolution (II)

Schwarzshild criteria

Equation 1.X does not specify yet the source of energy transport inside the star and the value taken by ∇. As seen in §1.3.2, the criterion most often used to identify the transport mode is the *Schwarzschild* criterion characterizing convective instability, e.g. $\nabla_{rad} > \nabla_{ad}$, where ∇_{rad} and ∇_{ad} are respectively the radiative and adiabatic gradients introduced above (see also Cox and Giuli (1968); Kippenhahn et al. (2013)). In the case of the Sun, it is the sharp increase in plasma opacity due to the change in the ionization state of carbon, nitrogen and oxygen that causes convective instability to start around 71% of the solar radius away from the center. Then, the temperature gradient ∇ in the case of radiative or convective transport will take either of these expressions depending on the outcome of the *Schwarzschild* criterion:

$$\frac{\partial T}{\partial m} = -\frac{1}{4\pi\rho r^2}\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}\text{, if } \nabla_{rad} < \nabla_{ad} \text{ the transport is radiative } \nabla = \nabla_{rad}$$

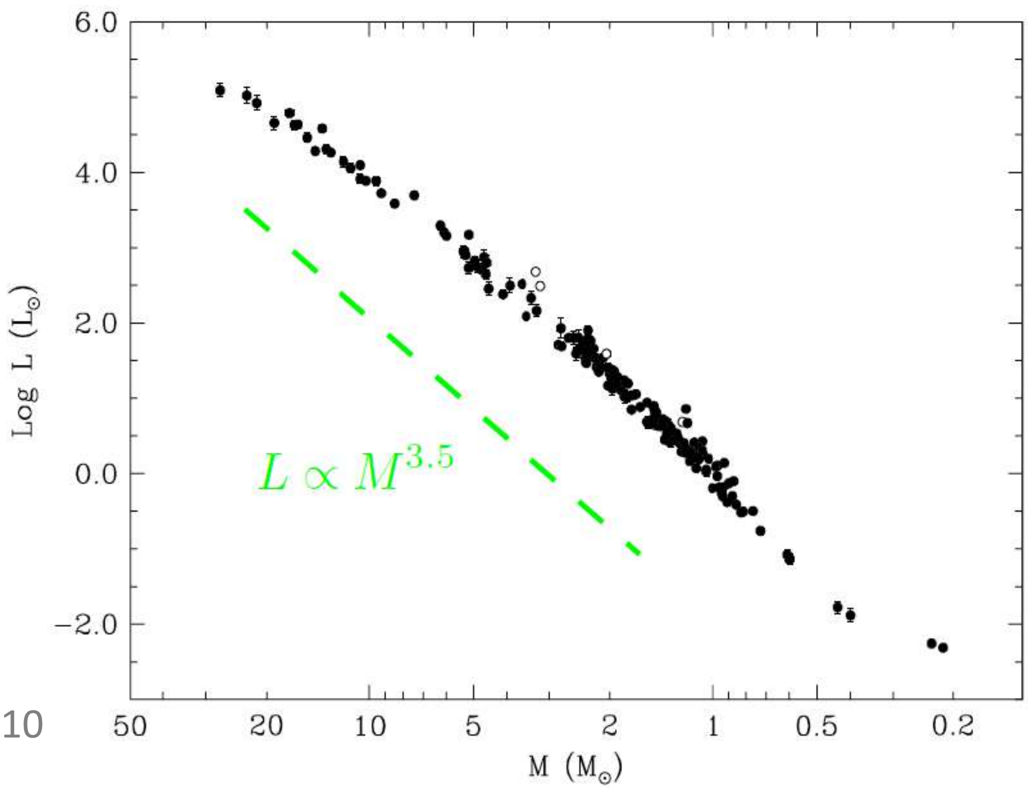
$$= \frac{\Gamma_2 - 1}{\Gamma_2}\frac{T}{P}\frac{\partial P}{\partial m}\text{, if } \nabla_{rad} > \nabla_{ad} \text{ the transport is by adiabatic convection, } \nabla = \nabla_{ad}$$

κ is opacity, and Γ_2 adiabatic exponent.

In the Sun, the strong increase of opacity near 0.7 R_{Sun} triggers convection.

See next Lecture for an improved traitement of convection

Homology relations, Mass-Luminosity

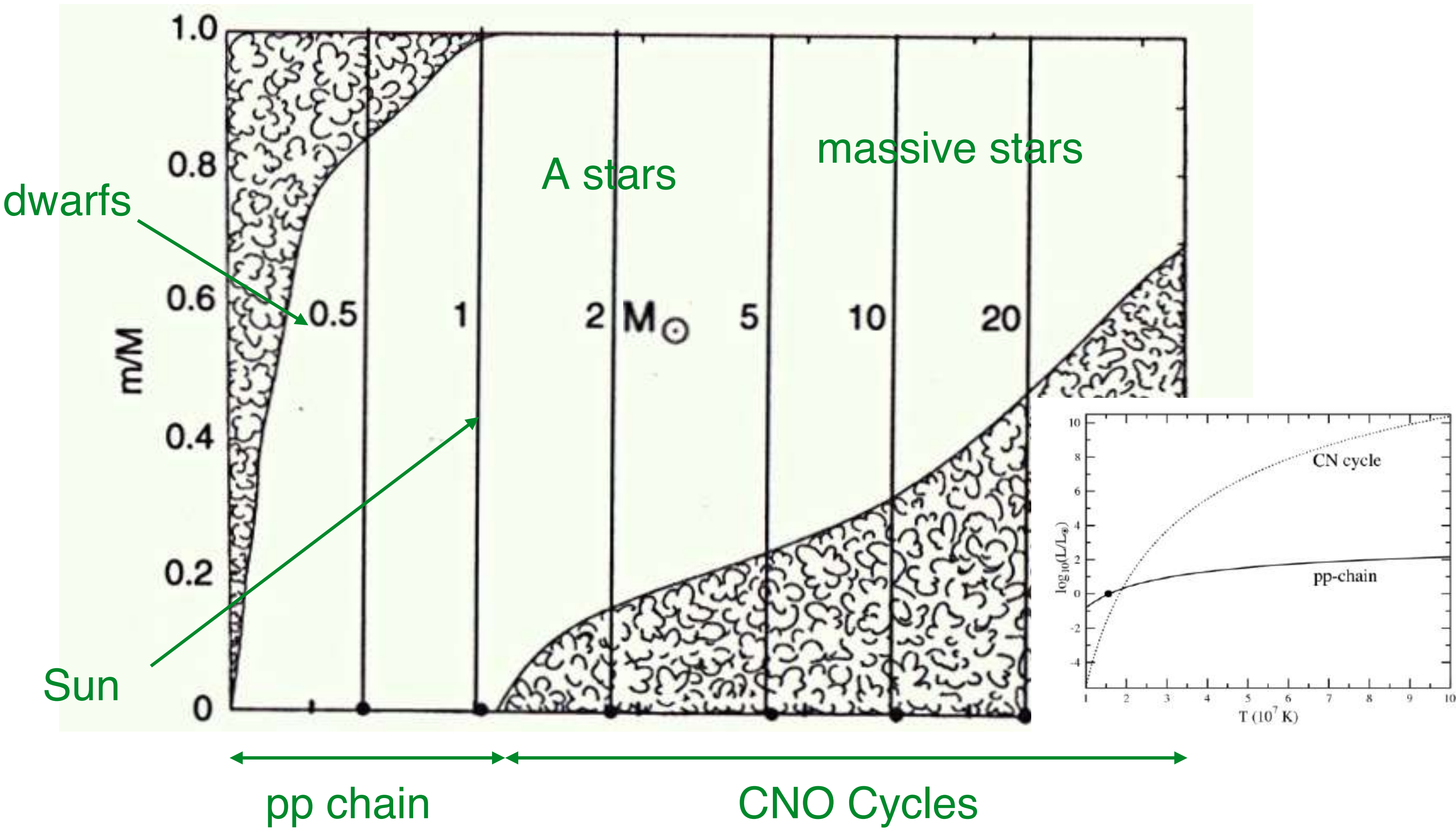


Torres et al. 2010

If the star maintains its luminosity via nuclear reaction rates then we can write:
$dM_*/dt = k\ L_*$ => $t \sim M_*/L_* = M_*/M_*^{3.5} = M_*^{-2.5}$ (cf. slide 16)

Stellar convection zones vs M*

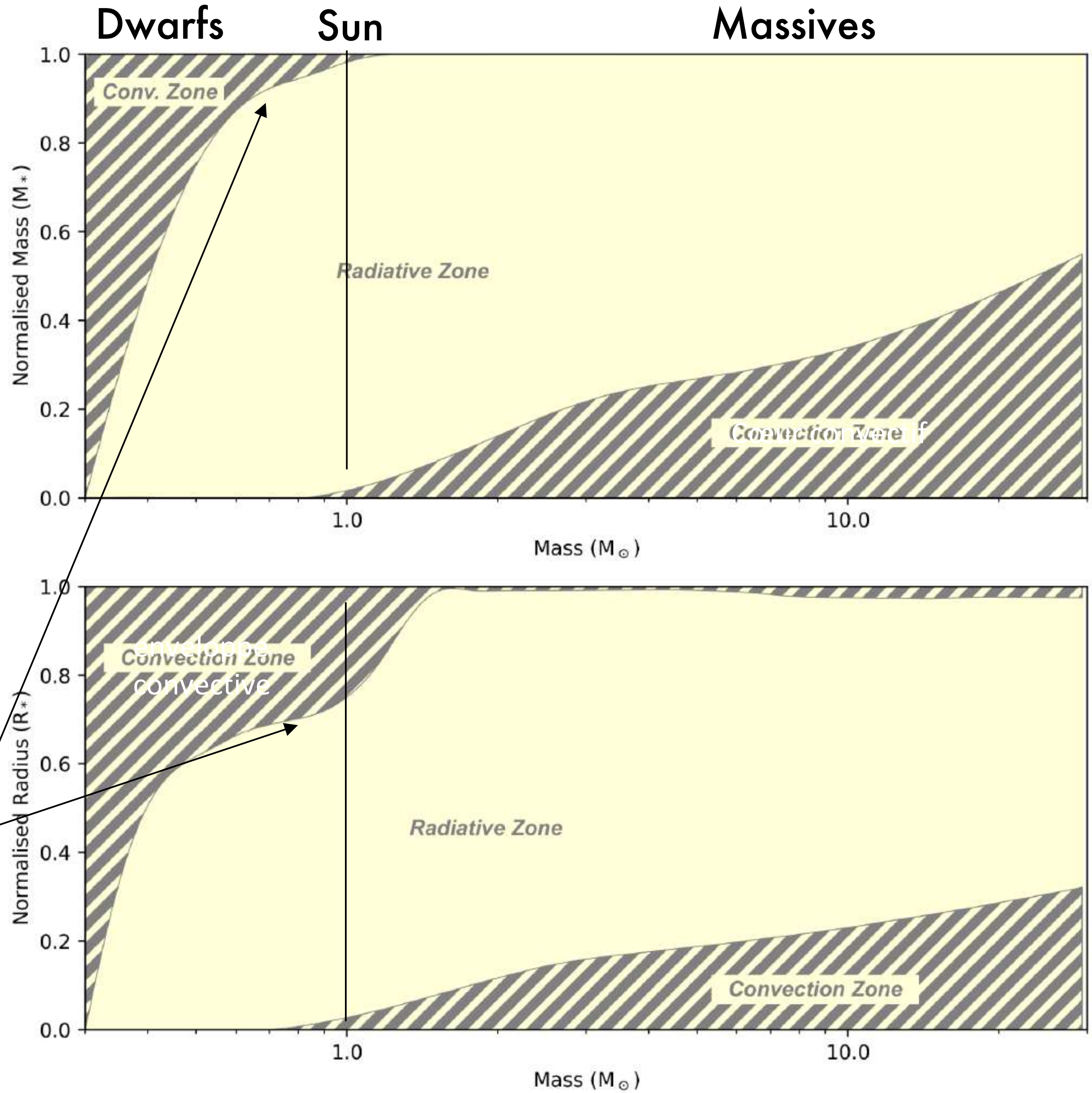
Transition between envelope and core convection: ~1.3 Msol



Stellar convection zones vs M_*

In mass
(cf previous slide)

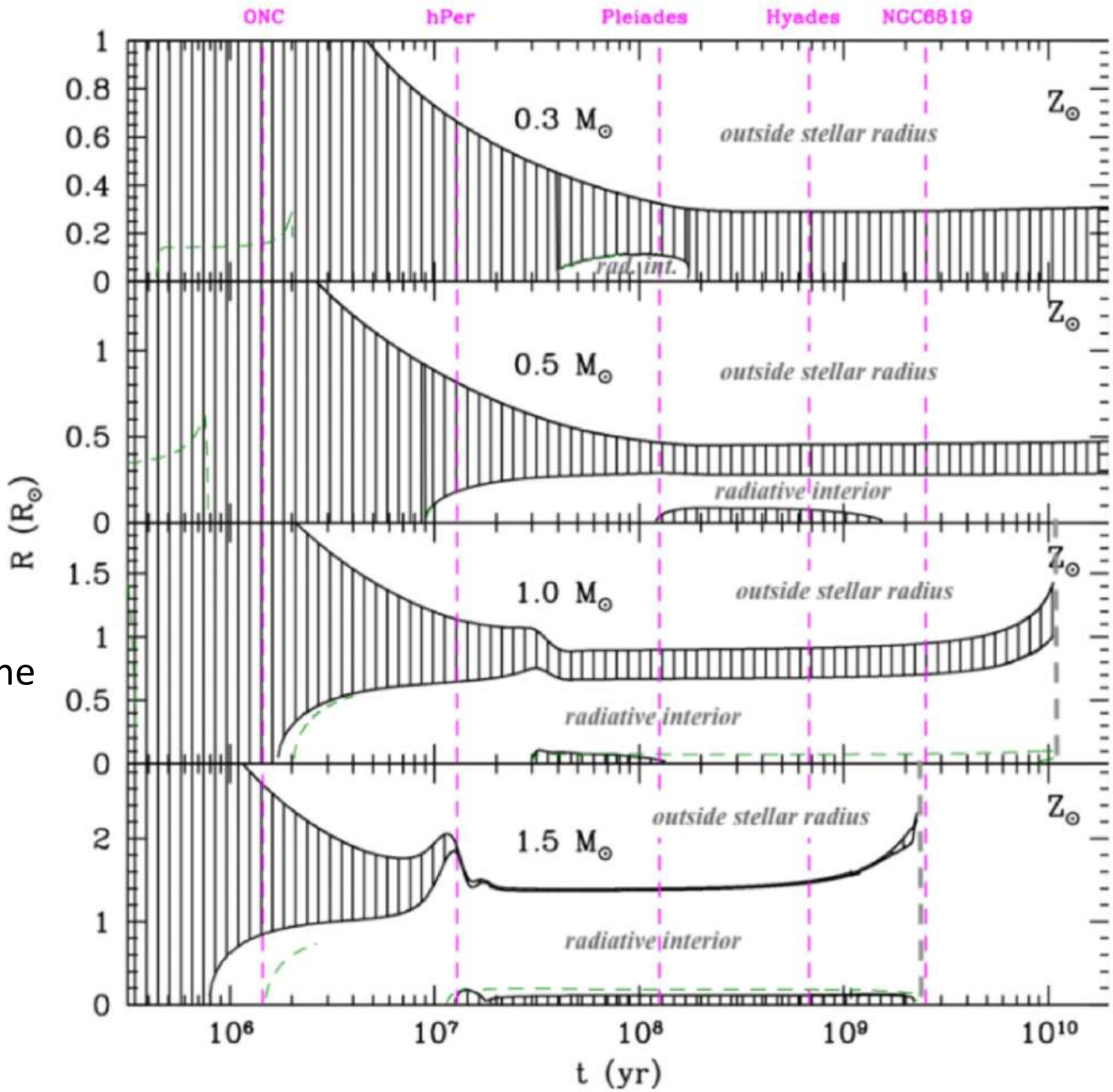
Solar case:
2% in mass
30% in radius



in radius
(notice
the difference
between
0.5 & 1 M_{Sun})

Kippenhahn Diagram

Evolution of the location of the convective zone for Different stars of solar or low mass (convective zone (CZ) =hatched areas).

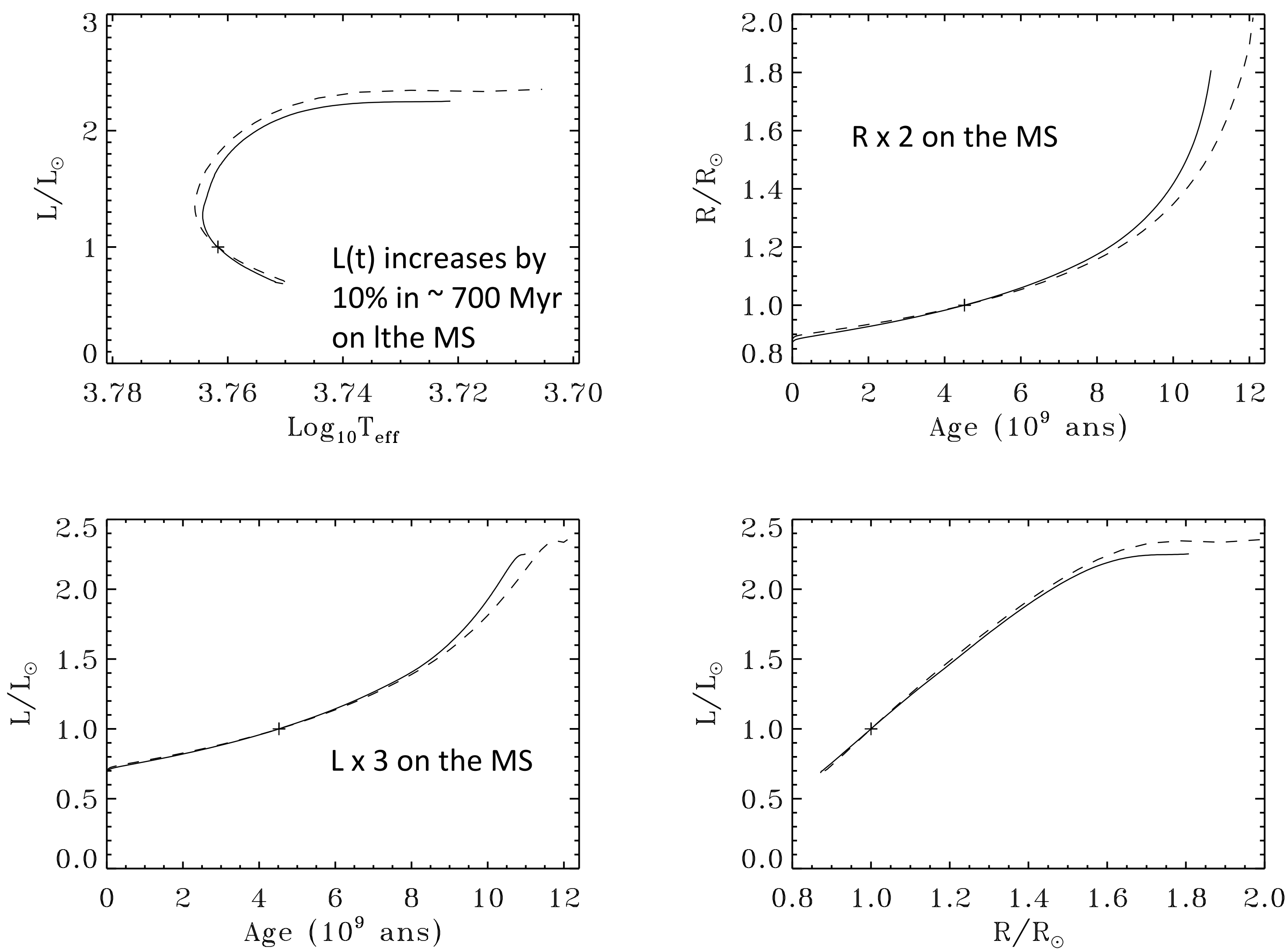


Names of famous stellar clusters

(solar metallicity Z)

Gallet et al .2015

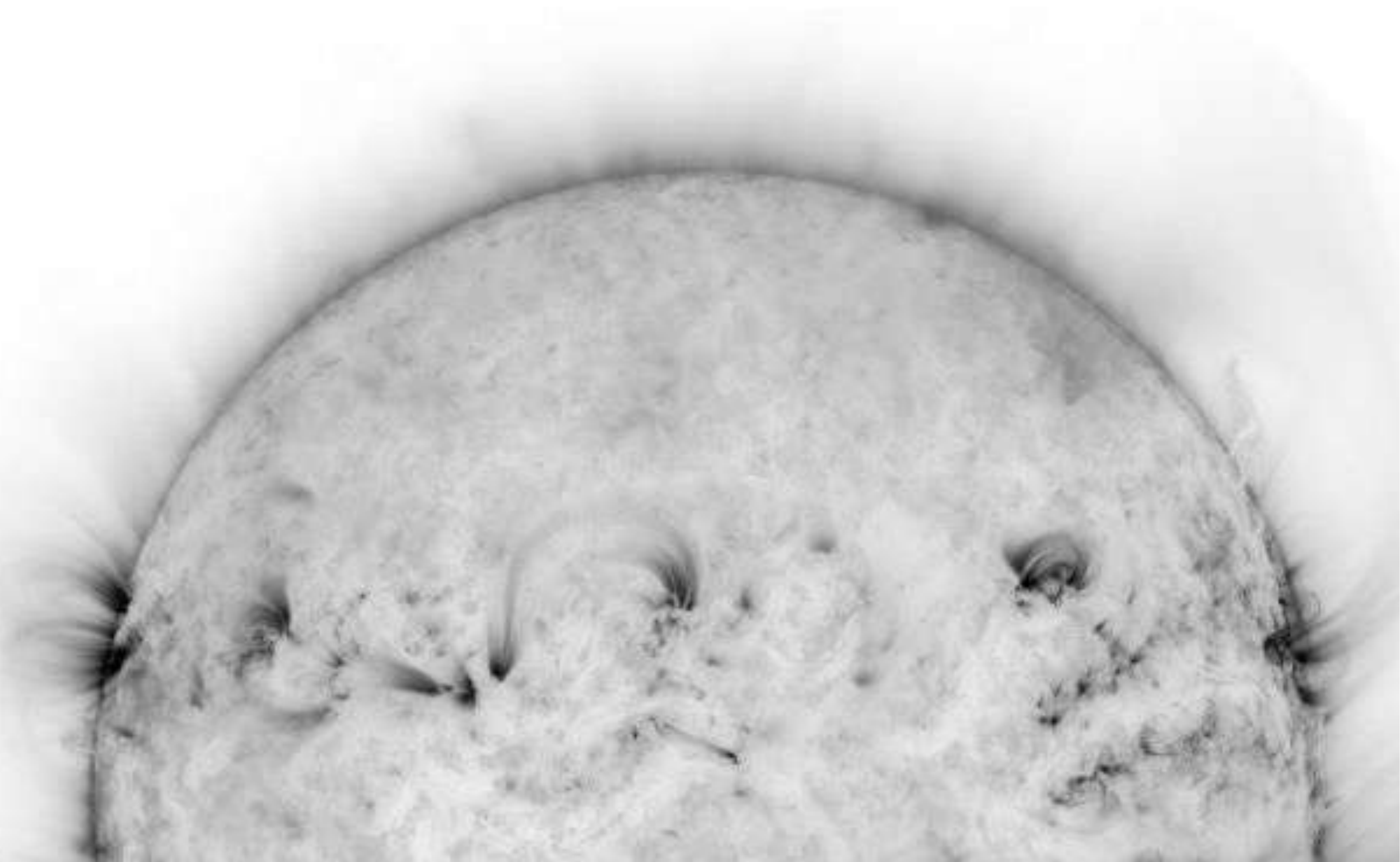
Evolution of the Sun global properties



The variations will be even greater in the RGB and AGB phases (see slide 63)

Stars: Main concepts

Mass and luminosity of stars: what bounds?



Eddington limit

Suppose the star mechanical equilibrium is only due to radiation pressure

$$\frac{dP_{rad}}{dr} = \frac{4aT^3}{3c}\frac{dT}{dr}$$

If this region transport heat by radiative diffusion, then we use the expression for temperature gradient

$$\frac{dP_{rad}}{dr} = -\frac{\kappa_{op}\,\rho}{4\pi c}\frac{L_*}{r^2}\,.$$

This gradient cannot exceed the hydrostatic gradient otherwise the star will not hold, i.e.

$$\left|\frac{dP_{rad}}{dr}\right| < \left|\frac{dP}{dr}\right| = \frac{Gm\rho}{r^2}\,.$$

Equating the two expressions, one obtains the maximum luminosity achievable, also known as the Eddington luminosity limit $L_* < L_{Edd}$:

$$L_{Edd} = \frac{4\pi c\,Gm}{\kappa_{op}} \simeq 3.8\times 10^4\left(\frac{M}{M_\odot}\right)\left(\frac{0.34}{\kappa_{op}}\right)L_\odot\,.$$

Mass-Luminosity relation

In reality the radiative pressure writes $P_{\mathrm{rad}} = \frac{4\sigma}{3c} T^4$

Virial theorem writes $2E_T = -E_G$

$$3\frac{M_\star}{m_p}kTV \qquad \beta\frac{GM_\star^2}{R_\star}$$

β=3/5 for an homogeneous sphere

Which yields

$$L_\star = \left(\frac{4\pi}{3}\right)^2 \frac{\sigma}{\kappa}\left(\frac{\beta G m_p}{\sqrt{2}\, 3k}\right)^4 M_\star^3$$

Maximum stellar mass

We just obtained the maximum luminosity of a star and a relationship between its mass and brightness.

We can deduce the maximum **stellar mass has being**

$$L(M) = L_{max}$$

$$M_{max} \sim 60 N_0 m_p \sim 110 M_\odot$$

$$N_0 = \left(\frac{\hbar c}{G m_p^2} \right)^{3/2} \sim 2.2 10^{57}$$

In reality it is more complex than this simple calculation, as some stars exceed this limit... This is an active area of research!

Minimum mass for becoming a star

A star is defined by the existence of nuclear fusion at its core. To be a star, the core temperature must therefore exceed the ignition temperature Tig~10^6 K.

The maximum core temperature is given when the thermal energy equals the Fermi energy. We can then find

$$kT = \frac{\beta^2}{18}\left(\frac{4}{9\pi}\right)^{2/3}\left(\frac{N}{N_0}\right)^{4/3} m_e\, c^2$$

Number of particules/unit volume

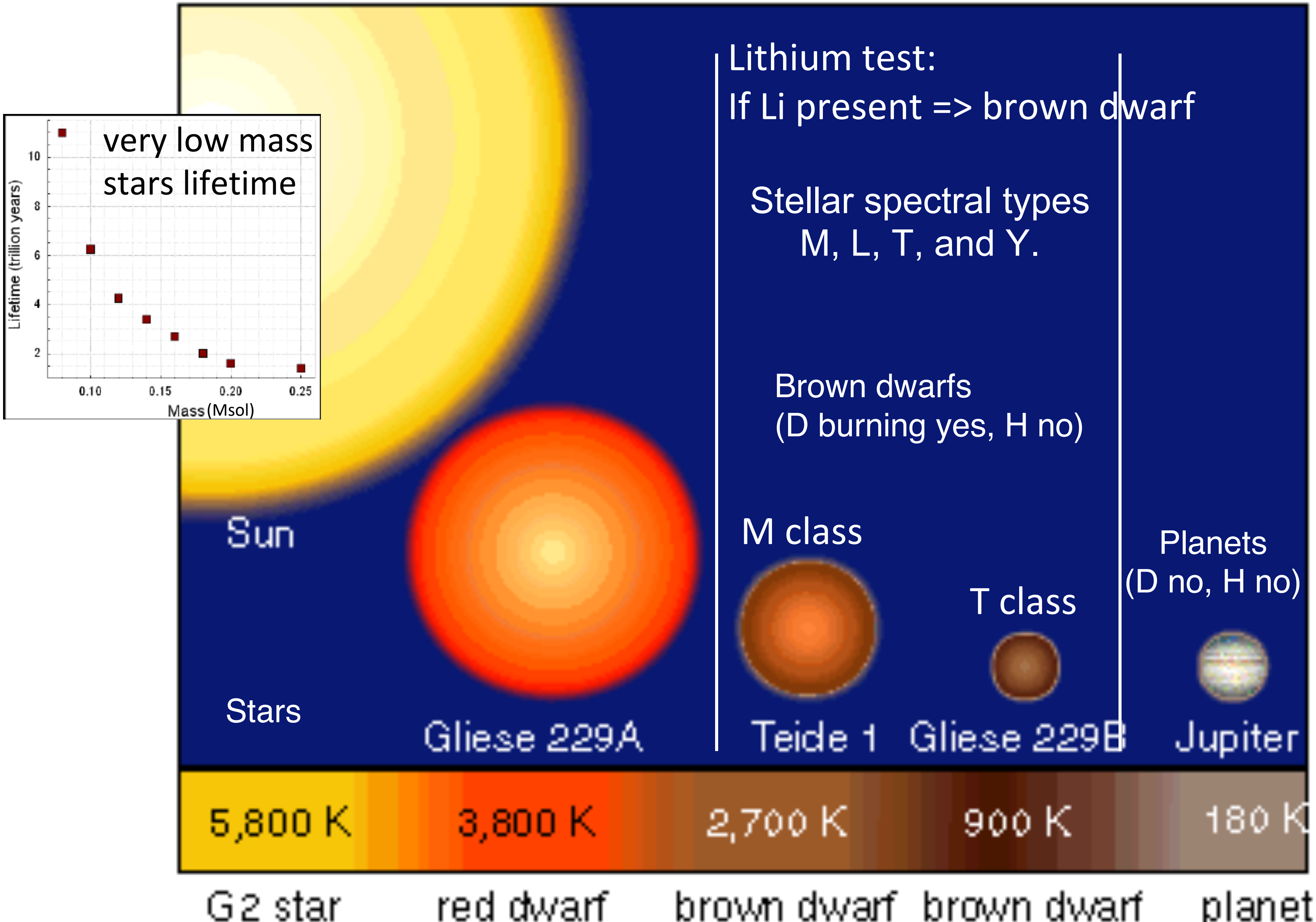
$$N_0 = \left(\frac{\hbar\, c}{G m_p^2}\right)^{3/2}$$

This translates into a minimum mass (via T=T_{ig}, we determine N_{min}/N_0~ 0.038)

$$M_{min} = N_{min} m_p \sim 0.07 M_\odot$$

More complete computation yield 0.084 Msol

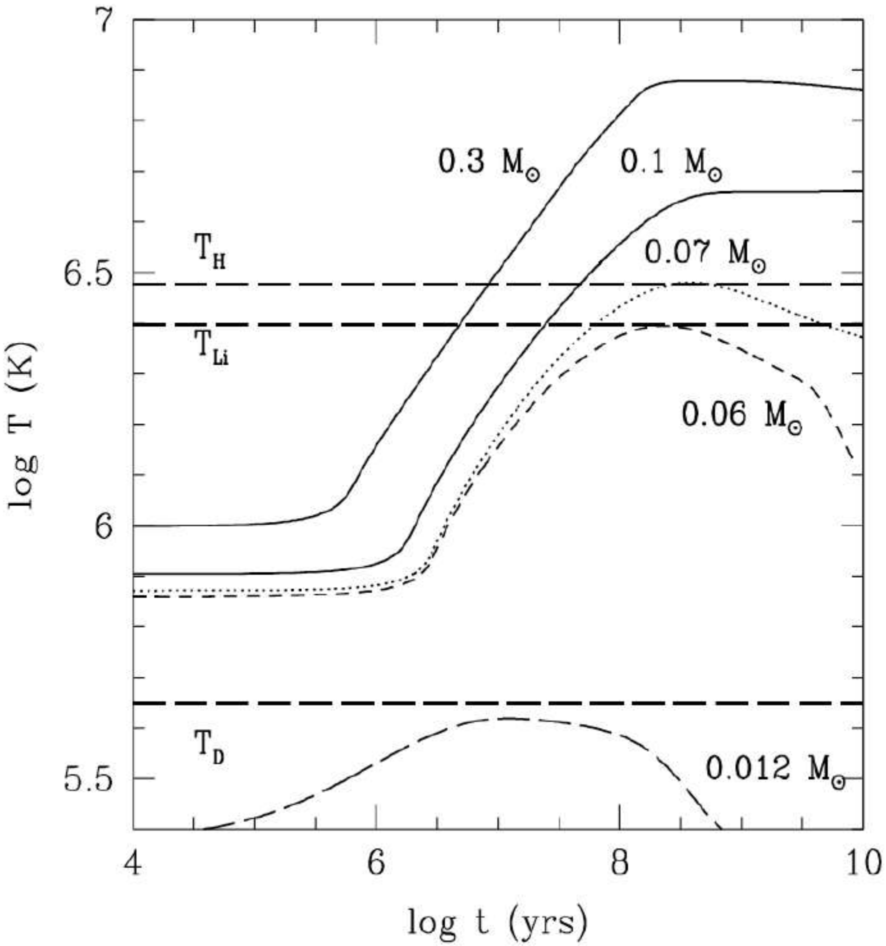
Low mass stars and giant planets

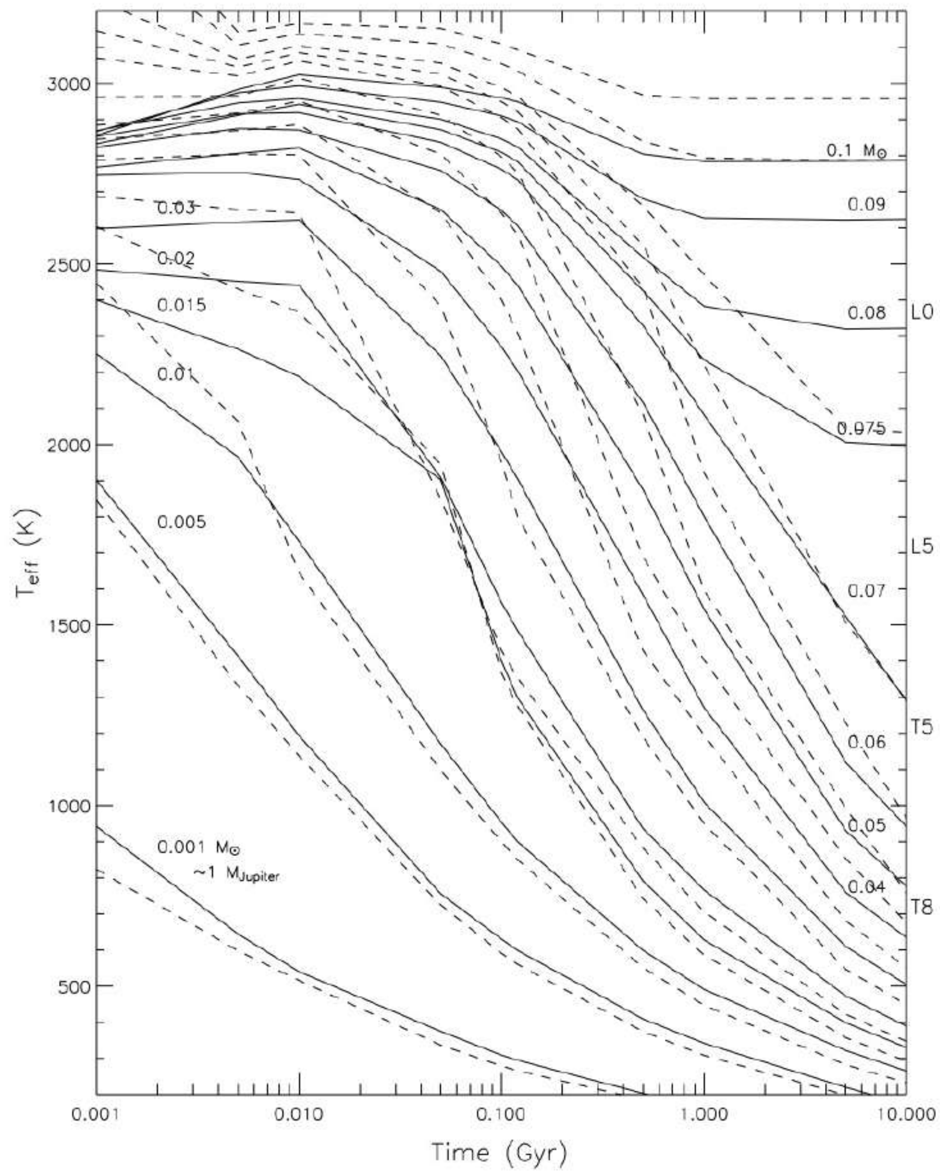


M must be greater than 0.084 M_{Sun} to become a star (H burning)

Note: between 0.065-0.084 Msol, some Li can be burnt

Low mass stars and giant planets

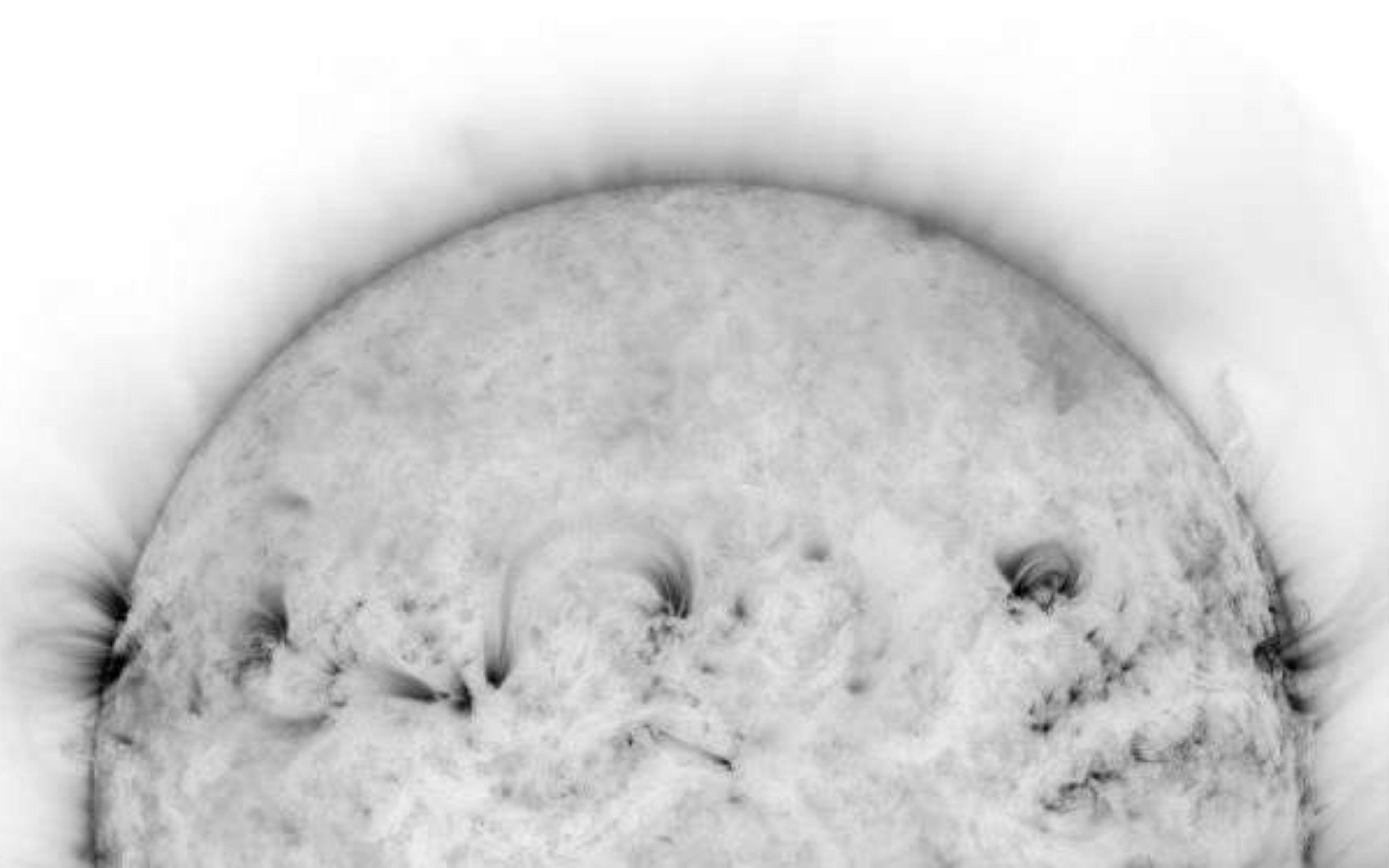




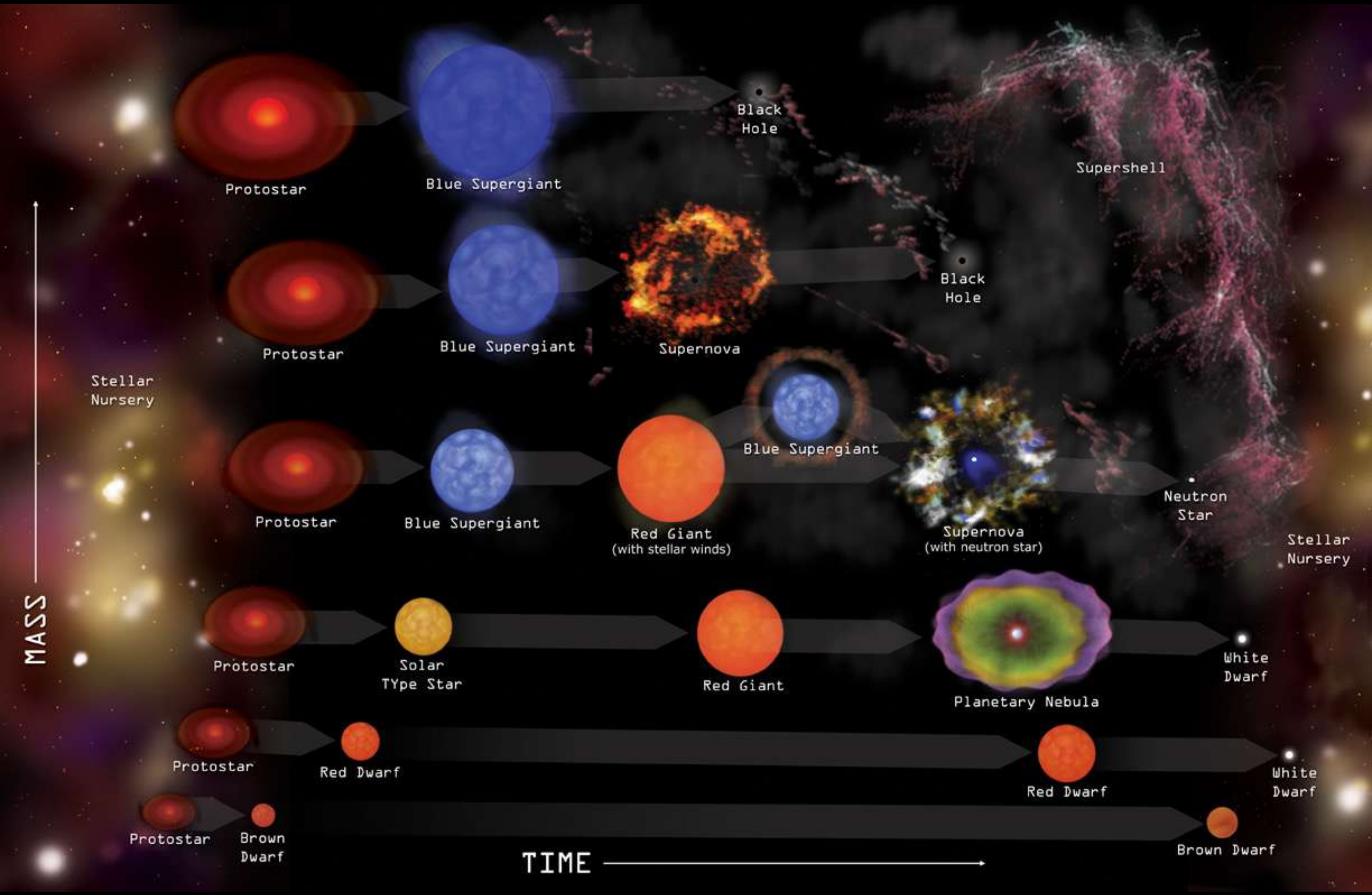
Evolution of T_{eff} and of the core T_c in low mass objects ($M_* < 0.3\ M_{Sun}$). Minimum temperature for H, D or Li burning (dashed horizontal lines)

Burgasser 2004

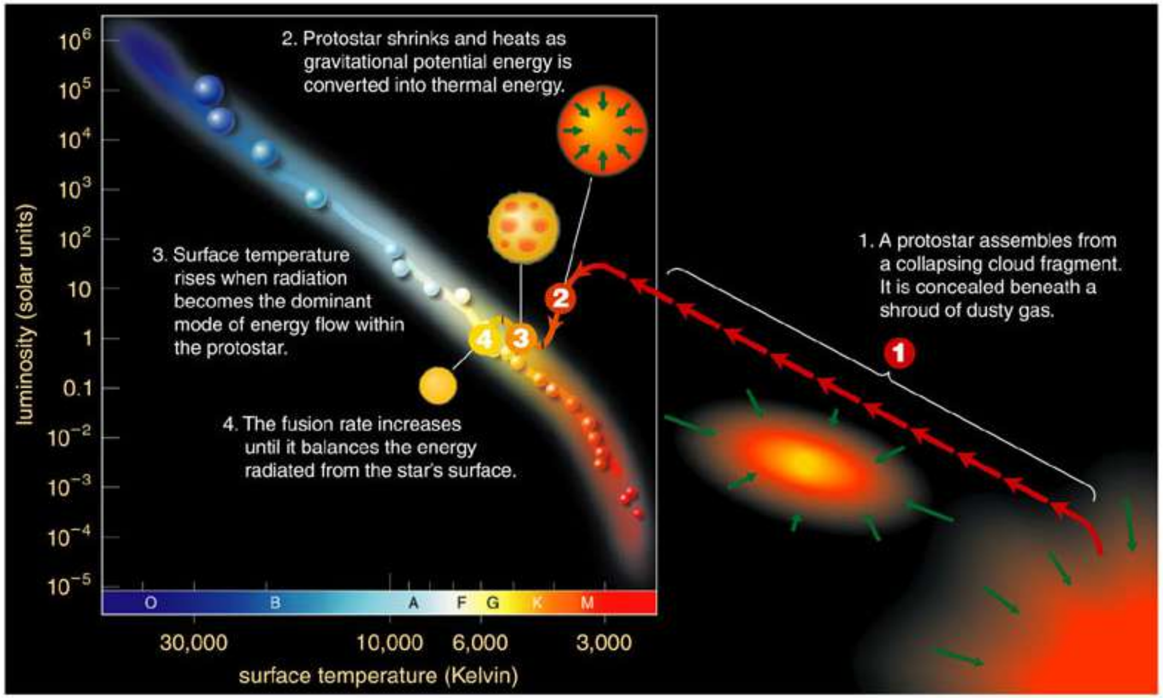
Life and death of stars

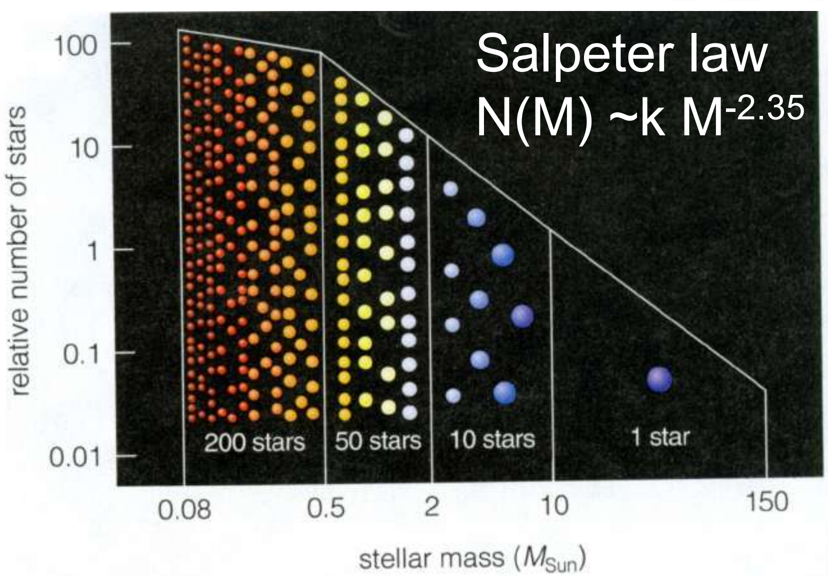


Stellar evolution: summary

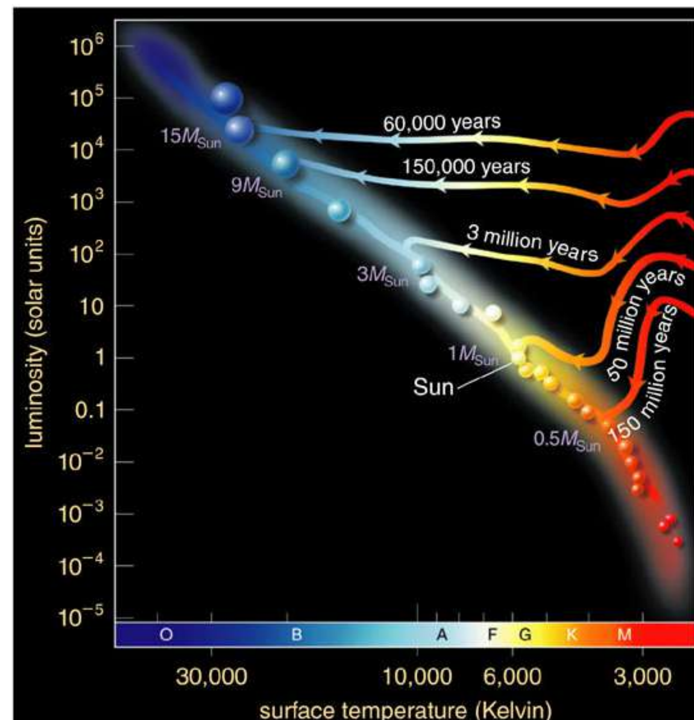


Stellar evolution: formation, young stars



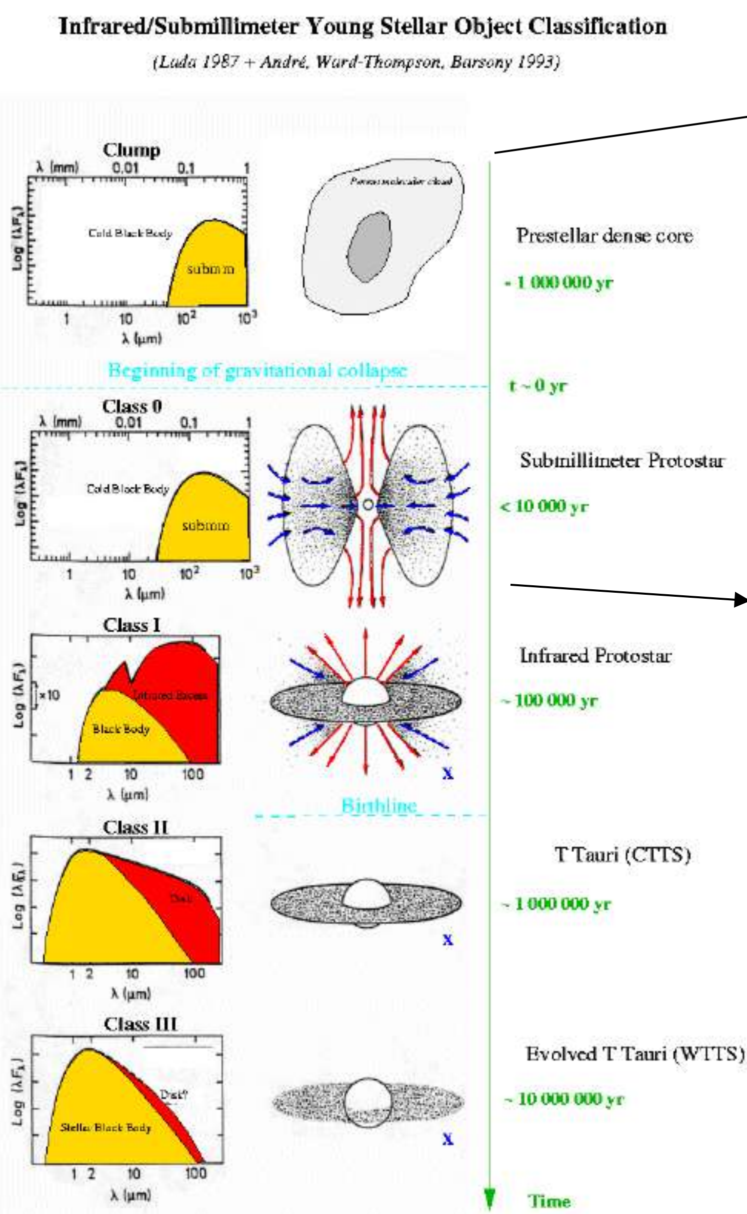


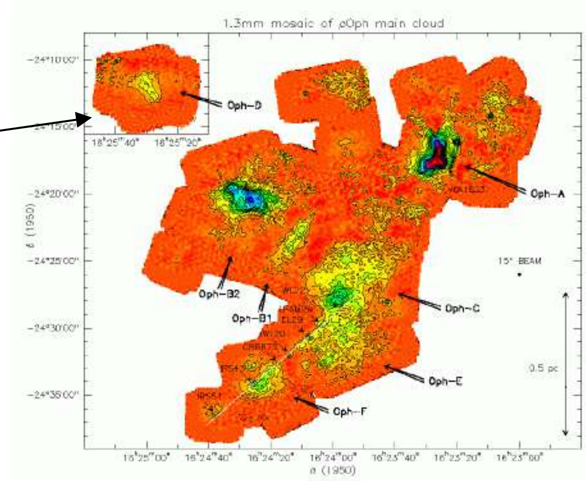
H-R Diagram

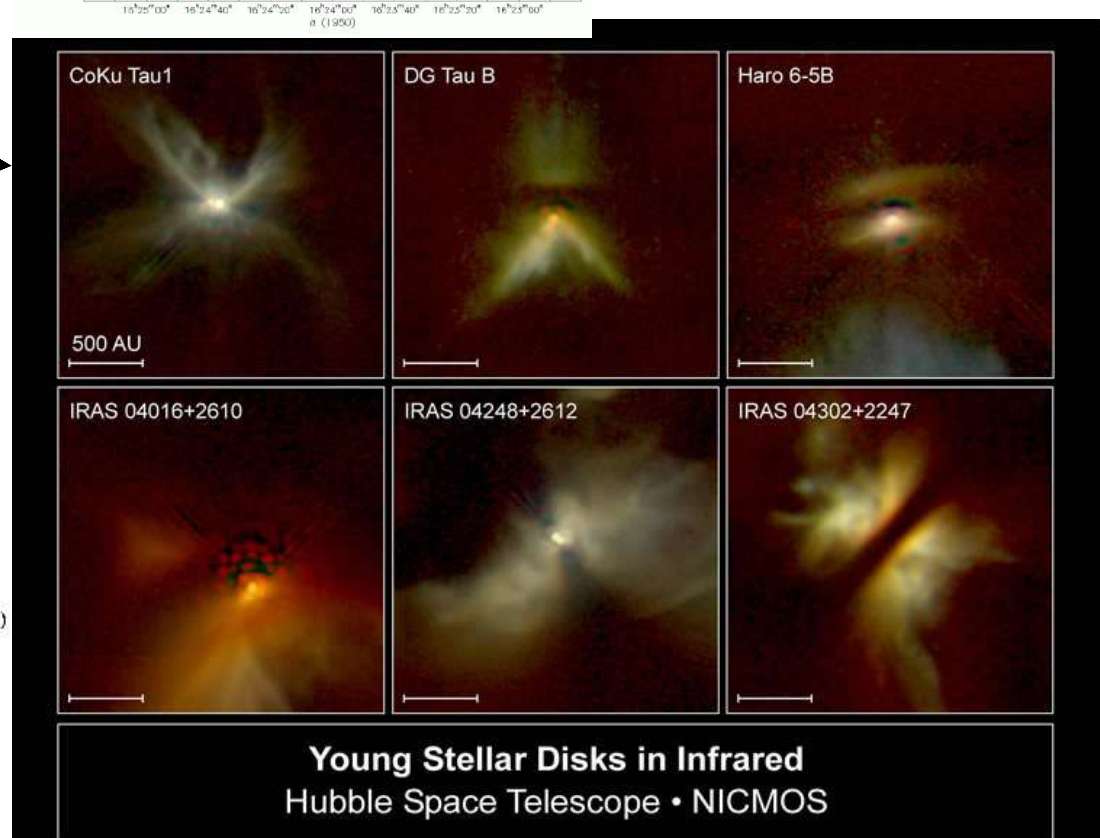


ˁcation, publishing as Addison Wesley.

Stellar evolution: T-Tauri phase

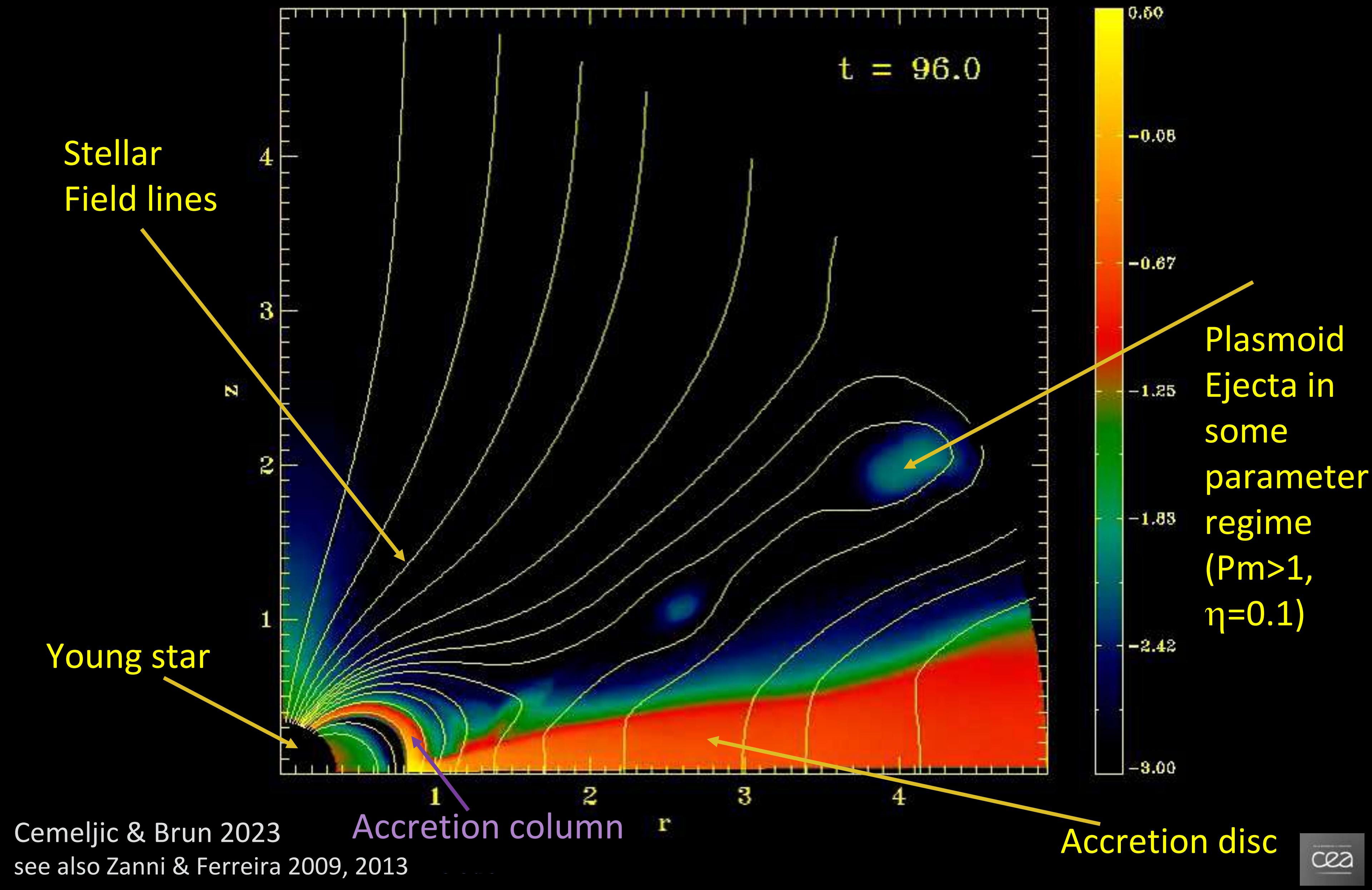






Ph. André (DAp)

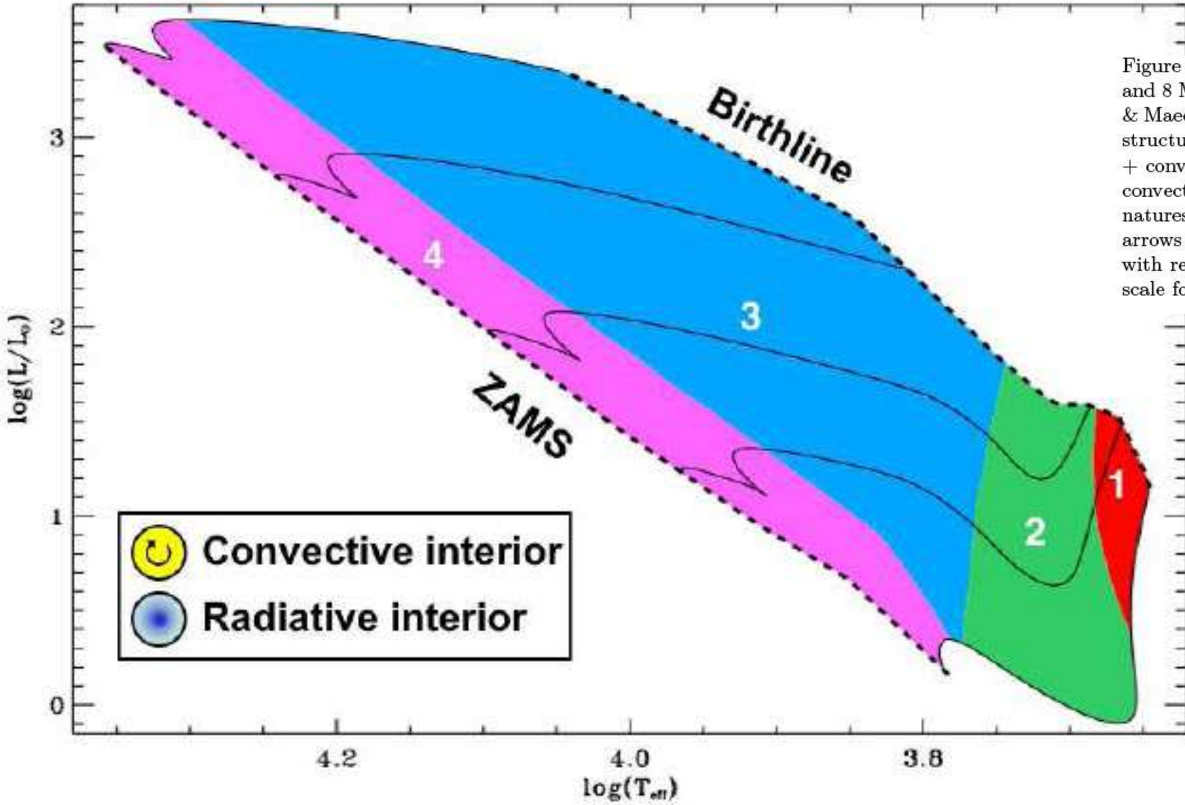
Young magnetic star-accretion disc interaction



Cemeljic & Brun 2023

see also Zanni & Ferreira 2009, 2013

Stellar structure along Pre Main Sequence for various Stellar Masses



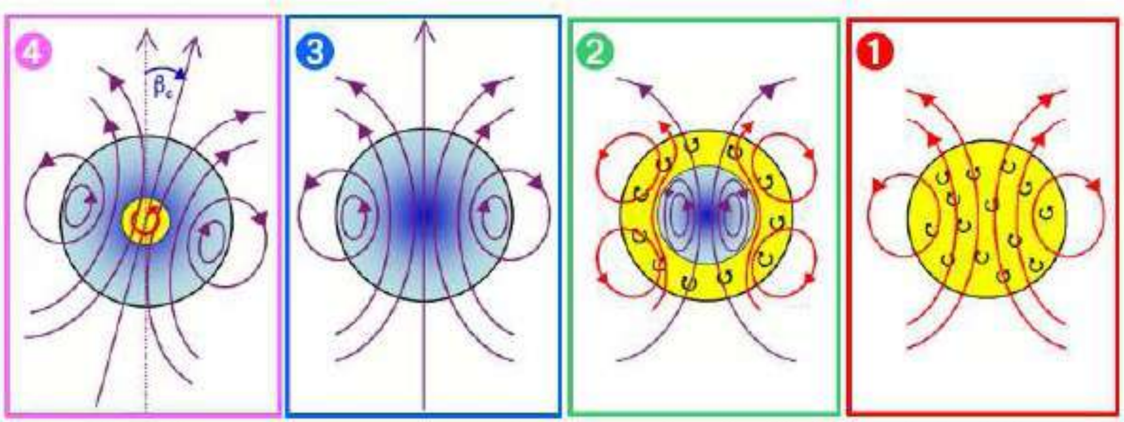
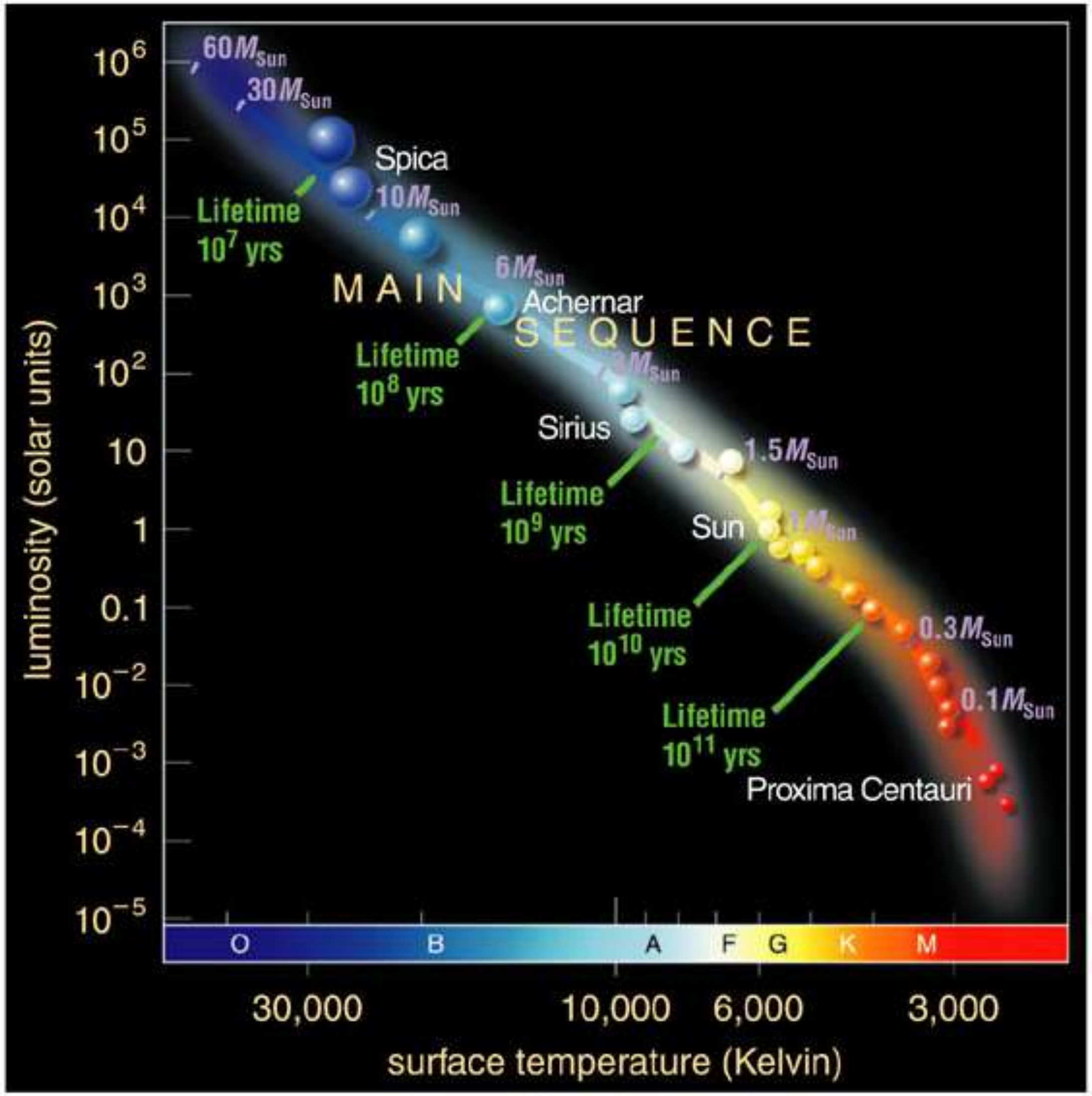


Figure 1. **Top:** Pre-main sequence evolutionary tracks (full lines) for 1.2, 2, 3, 5, and 8 $M_\odot$, computed with CESAM (Morel 1997). The birthline is from Behrend & Maeder (2001). The shaded area separate regions with different stellar internal structures: phase 1 (red), fully convective interior ; phase 2 (green) radiative core + convective envelope ; phase 3 (blue), fully radiative interior ; phase 4 (pink), convective core + radiative envelope. **Bottom:** Schematic view of the different natures of the magnetic fields for the four different phases. Yellow with circular arrows regions represent convective interior creating dynamo fields (represented with red field lines). Blue regions represent radiative interiors containing large-scale fossil fields (represented with purple field lines).

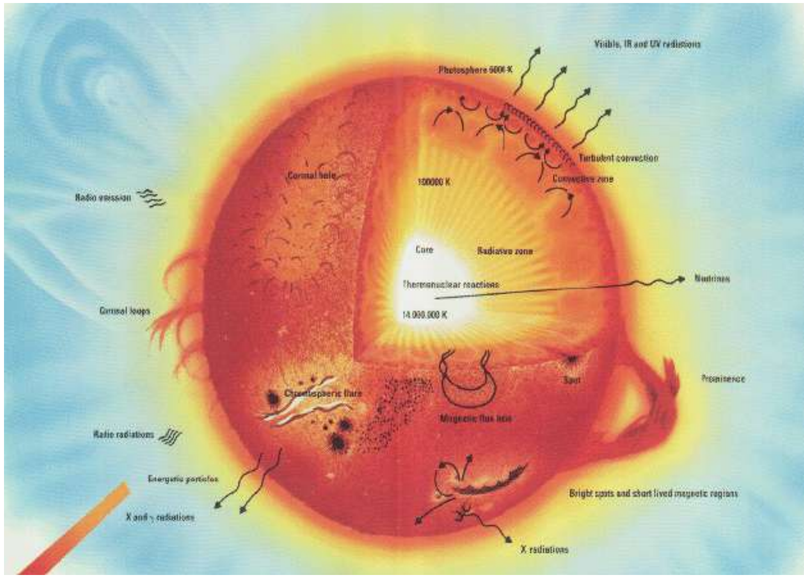
Courtesy of Alecian et al. 2014

Stellar evolution: "Main sequence"

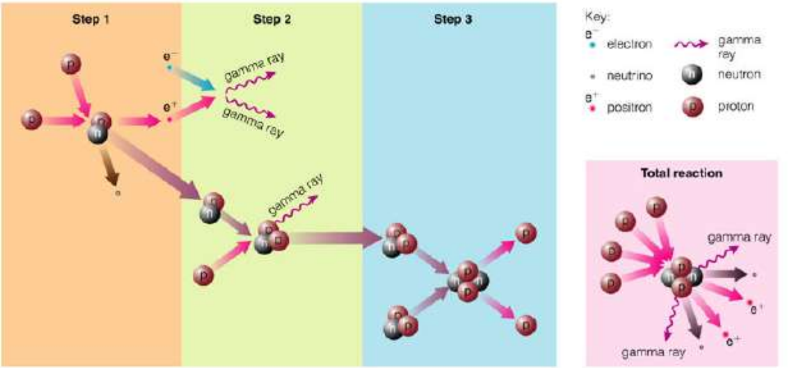


on Education, publishing as Addison Wesley.

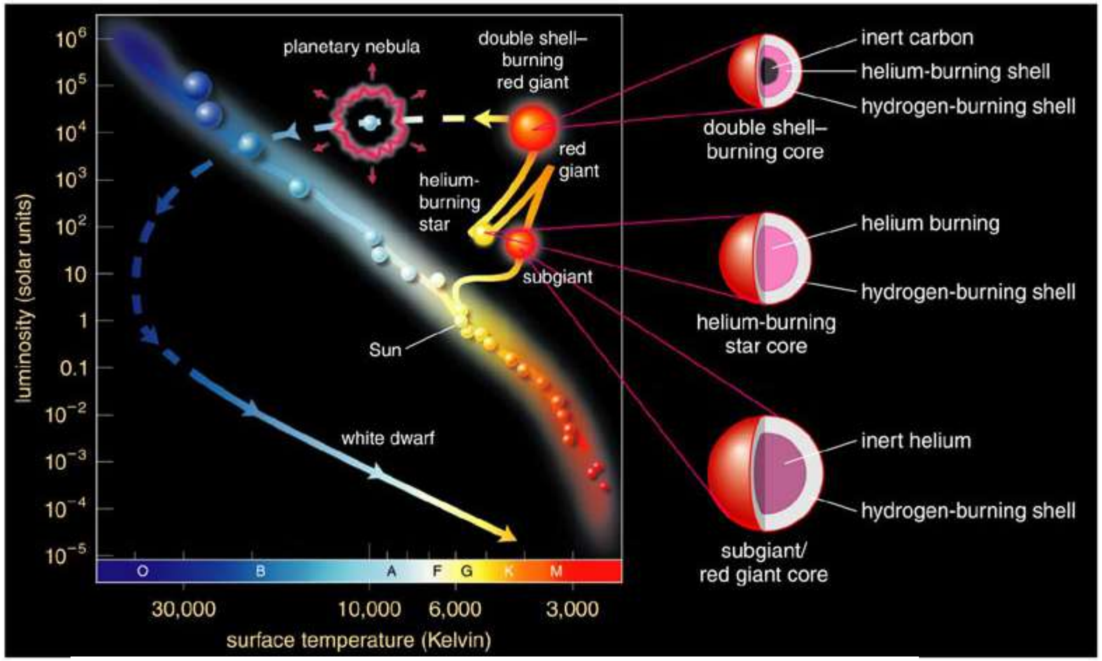
Solar Case

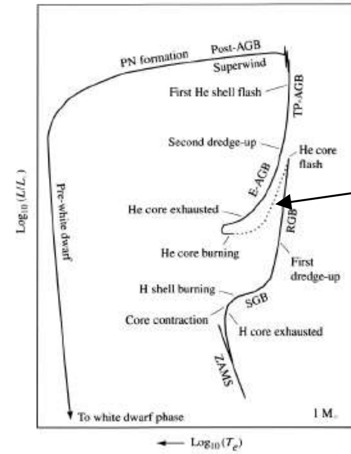


Hydrogen burning

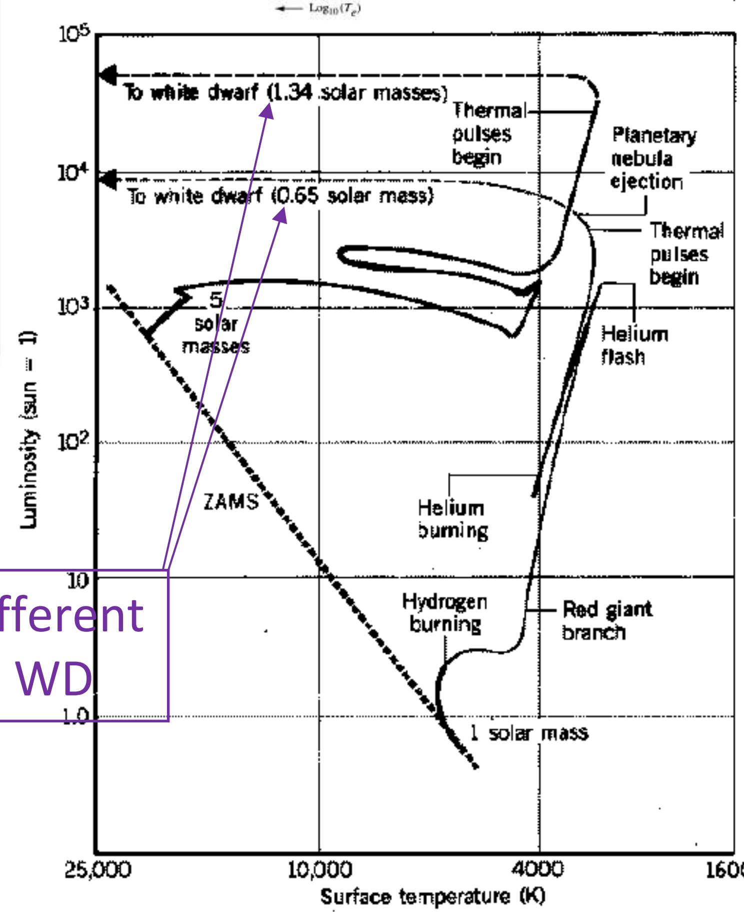


Stellar evolution: between 1and 5 M_{Sun}



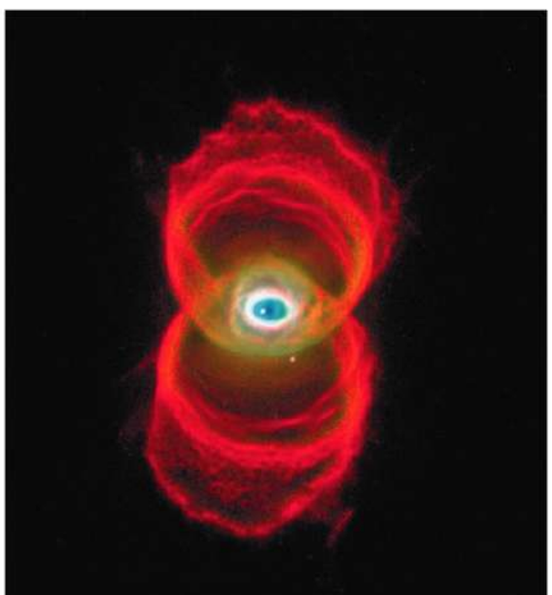


Fast transit in HR diag after He flash



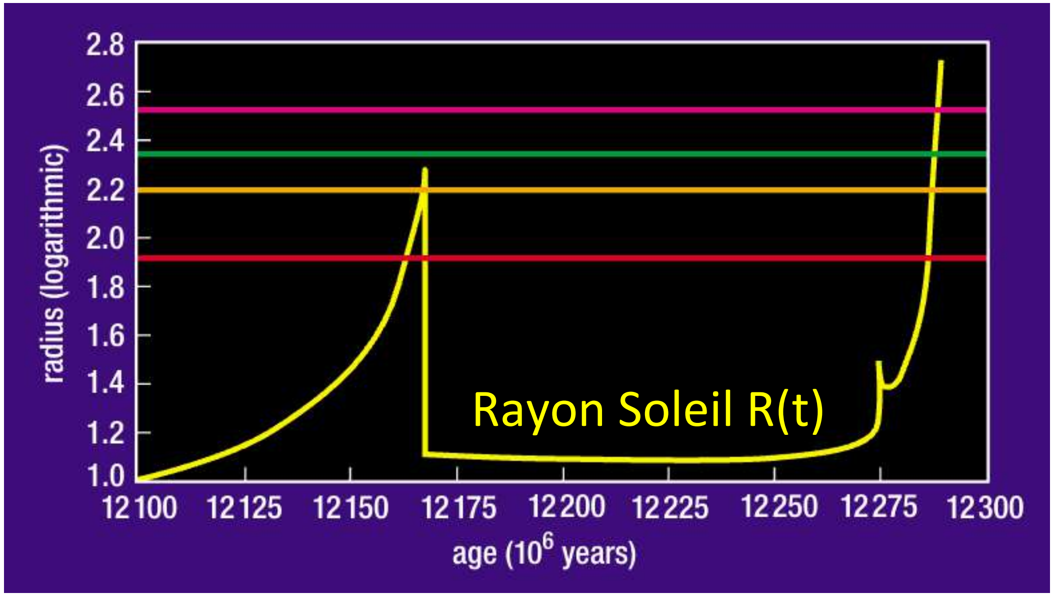
Note the different mass of the WD

Theoretical evolutionary tracks off the ZAMS for the stars of 1 and 5 solar masses.



Hourglass « Planetary » nebula

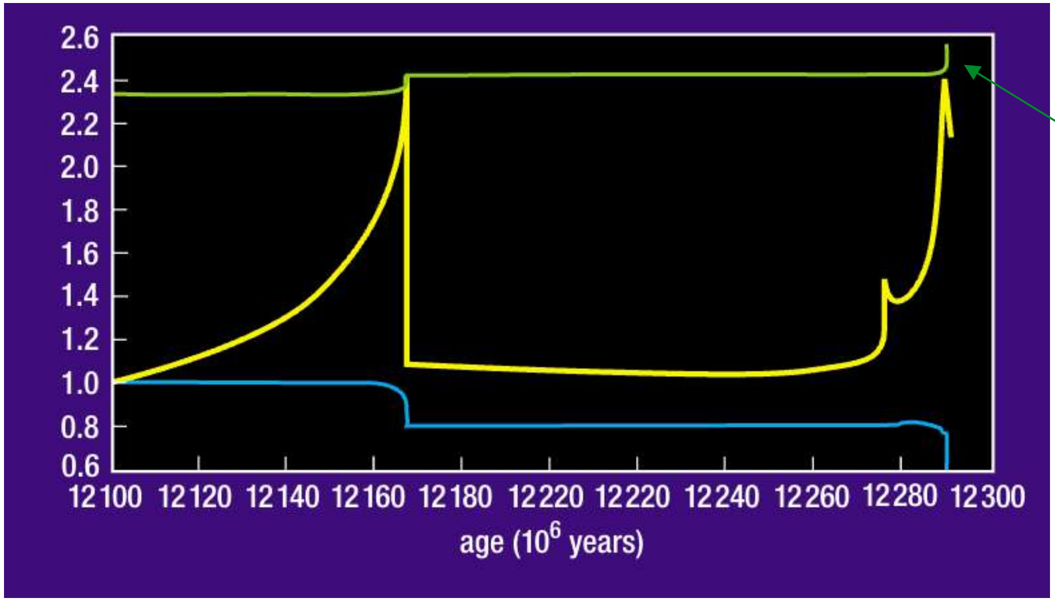
Zoom on future solar RGB & AGB stages



Orbites:
Mars
Terre
Venus
Mercure

Without mass loss:

Earth and Mars swallowed.



With mass loss:

Earth survives soalr expansion as a AGB, but will still end up swallowed since tidal effect makes them spiral into the Sun! ($J_* > 3\ J_p$)

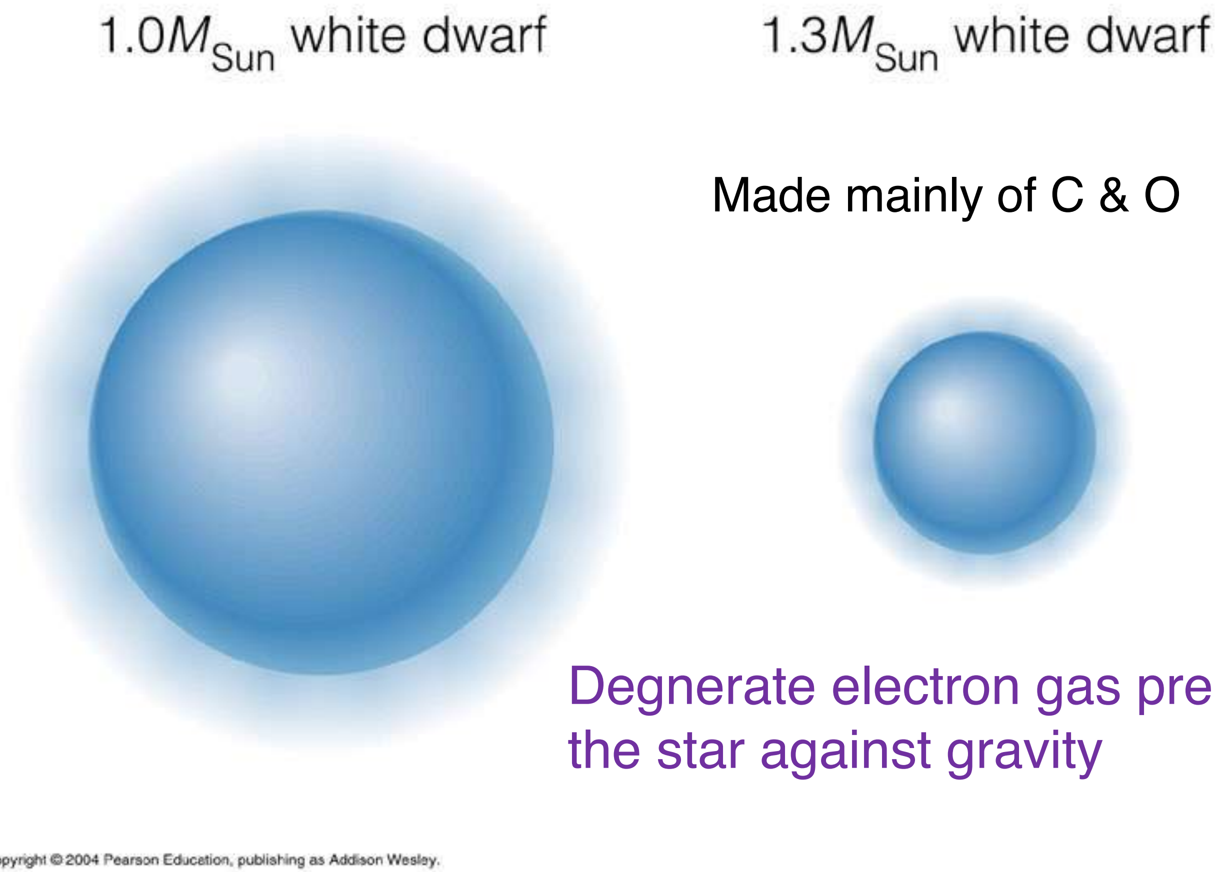
We note (blue line) that the Sun will Loose twice about 20% of its mass => un residual of 60% de M_{Sun} (initial) Cf. white dwarf mass in next slide.

Stellar evolution: white dwarfs

These stars cannot go beyond Chandrasekhar mass: 1.4 M_{Sun}

(Rayon plus petit)

R~10^4 km



Made mainly of C & O

Degnerate electron gas pressure sustains the star against gravity

Low mass stars (M<3 M_{Sun}) => 0.6 M_{sun} white dwarfs (as for the future Sun)
Intermediate mass stars (3<M<9 M_{Soleil}) => 1 M_{sun} white dwarfs
(good candidates for SNIa in binary systems)

Stellar evolution: white dwarfs

These stars cannot go beyond Chandrasekhar mass: 1.4 M_{Sun}

Why?

Equilibrium between potential and kinetic energies $E_g \sim E_k$

$$\frac{GM_\star m}{R_\star} \quad \text{— unit mass —} \quad m\frac{Np^2}{m_e}$$

Degenerated Electrons: $p \sim \Delta p \sim \hbar n^{1/3}$ (Heisenberg)

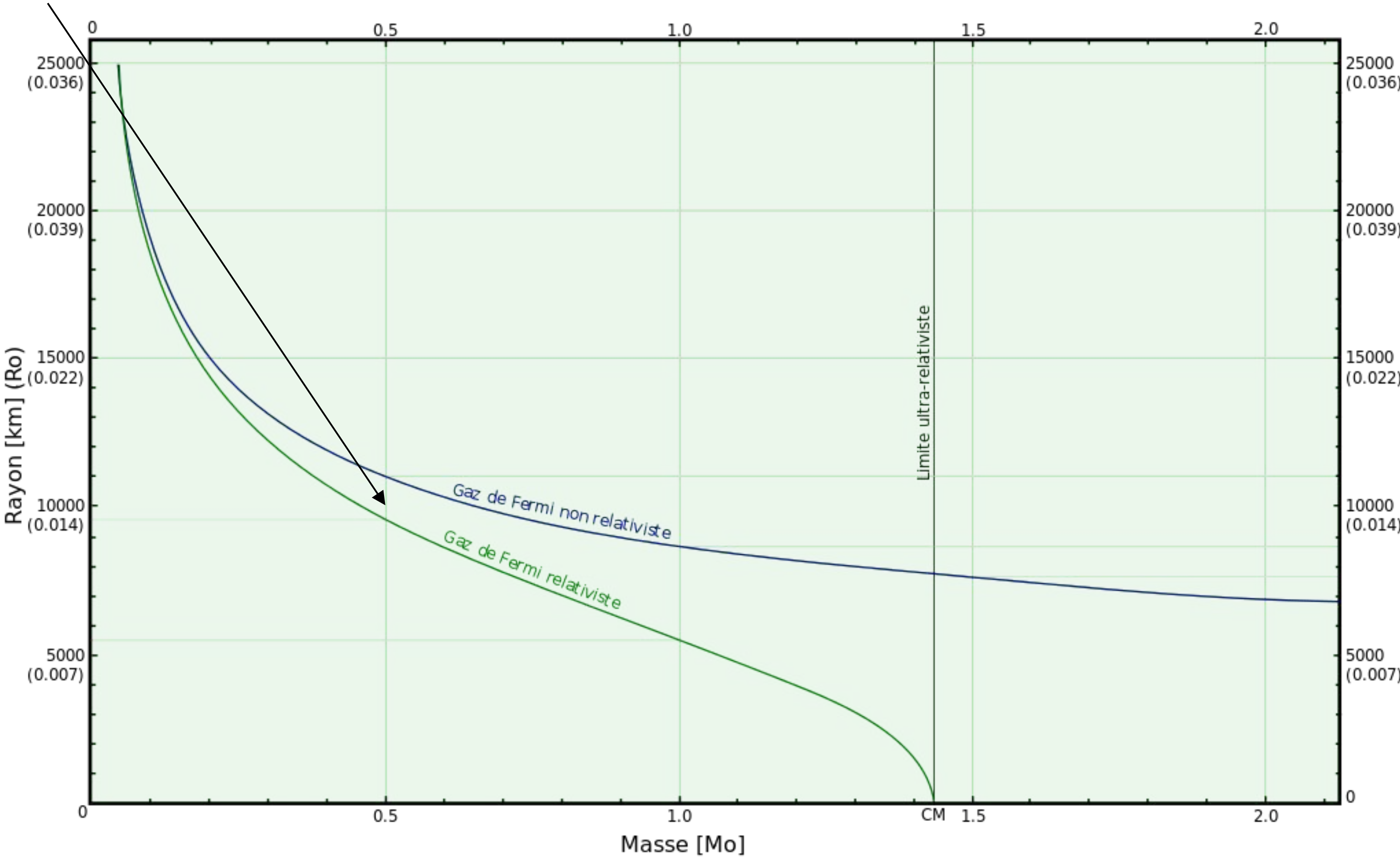
Then $$R_\star \sim \frac{N^{5/3}\hbar^2}{2m_e G M_\star^{1/3}} \sim M_\star^{-1/3}$$

But if the mass increases, then the electron degenerate pressure increases as well and they become relativistic!

Stellar evolution: white dwarfs

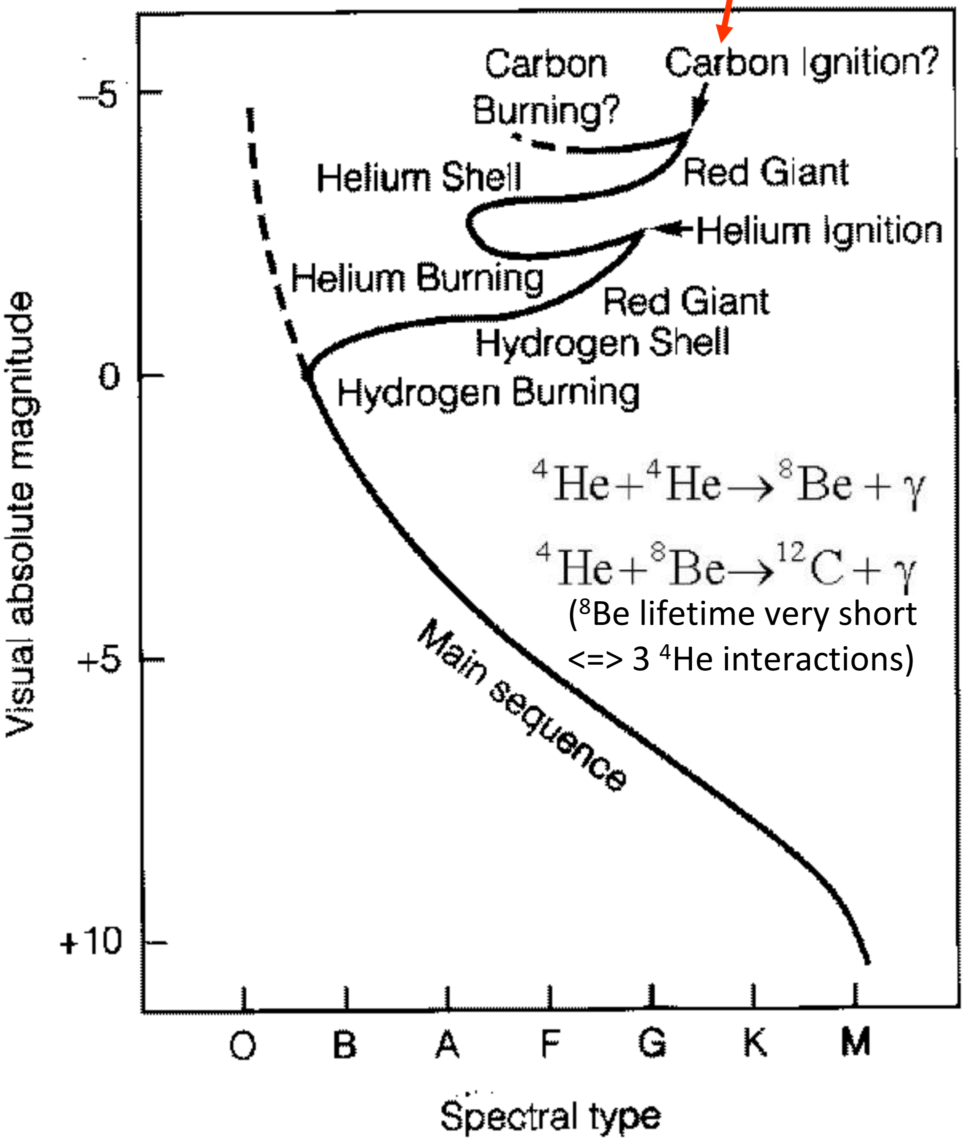
These stars cannot go beyond Chandrasekhar mass: 1.4 M_{Sun}

Why?



Stellar evolution: stars > 8-9 M_{Sun}

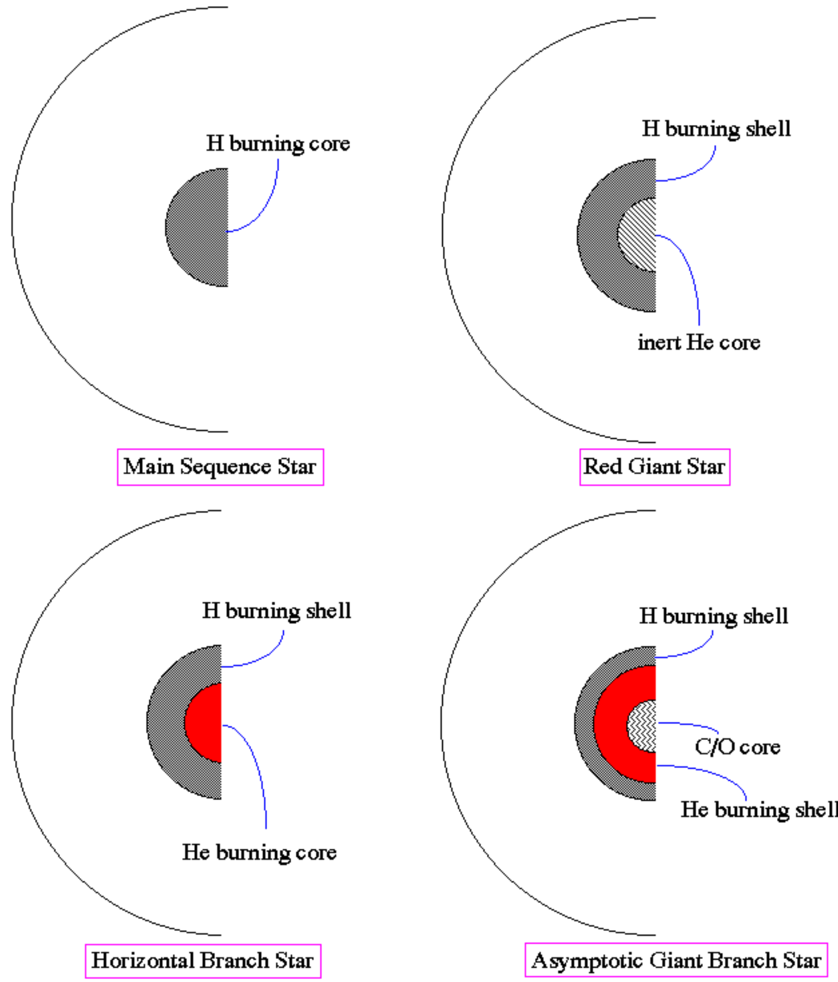
depends on stellar mass (yes if > 8-9 M_{Sun})



$$^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^8\mathrm{Be} + \gamma$$

$$^4\mathrm{He} + {}^8\mathrm{Be} \rightarrow {}^{12}\mathrm{C} + \gamma$$

(^{8}Be lifetime very short <=> 3 ^{4}He interactions)



Triple-alpha nuclear rate:

$$\mathcal{E}_{3\alpha} \propto Y^3 \ \rho^2 \ T^{40} \qquad \mathrm{erg\,s^{-1}\,g^{-1}}$$

Advanced phase in the nuclear core of a star ($M_* > 8\text{-}9M_{sol}$)

When enough carbon has been formed via 3-alpha reactions, then other heavy Elements can be created by Helium4 capture

$$ {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma \qquad {}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{20}_{10}\mathrm{Ne} + \gamma $$

Once the carbone has been created, if core temperature T_c is greater than ~ 6 10^8 K, Then carbon fusion can start:

$$ \begin{aligned} {}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{16}\mathrm{O} + 2\,{}^{4}\mathrm{He} \\ &\rightarrow {}^{20}\mathrm{Ne} + {}^{4}\mathrm{He} \\ &\rightarrow {}^{23}\mathrm{Na} + p^{+} \\ &\rightarrow {}^{23}\mathrm{Mg} + n \\ &\rightarrow {}^{24}\mathrm{Mg} + \gamma \end{aligned} $$

If T_c > 10^9 K, alors l'oxygène peut fusionner

$$ \begin{aligned} {}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{24}\mathrm{Mg} + 2\,{}^{4}\mathrm{He} \\ &\rightarrow {}^{28}\mathrm{Si} + {}^{4}\mathrm{He} \\ &\rightarrow {}^{31}\mathrm{P} + p^{+} \\ &\rightarrow {}^{31}\mathrm{S} + n \\ &\rightarrow {}^{32}\mathrm{S} + \gamma \end{aligned} $$

Beyond T_c = 1.5 10^9 K, then the photons energy is enough to photo-dissociate the nucleus, helping the burning of Silicium to create a stable nucleus, with very high binding energy near the iron peak.

see Clayton 1968, « Principles of stellar evolution and nucleosynthesis » book

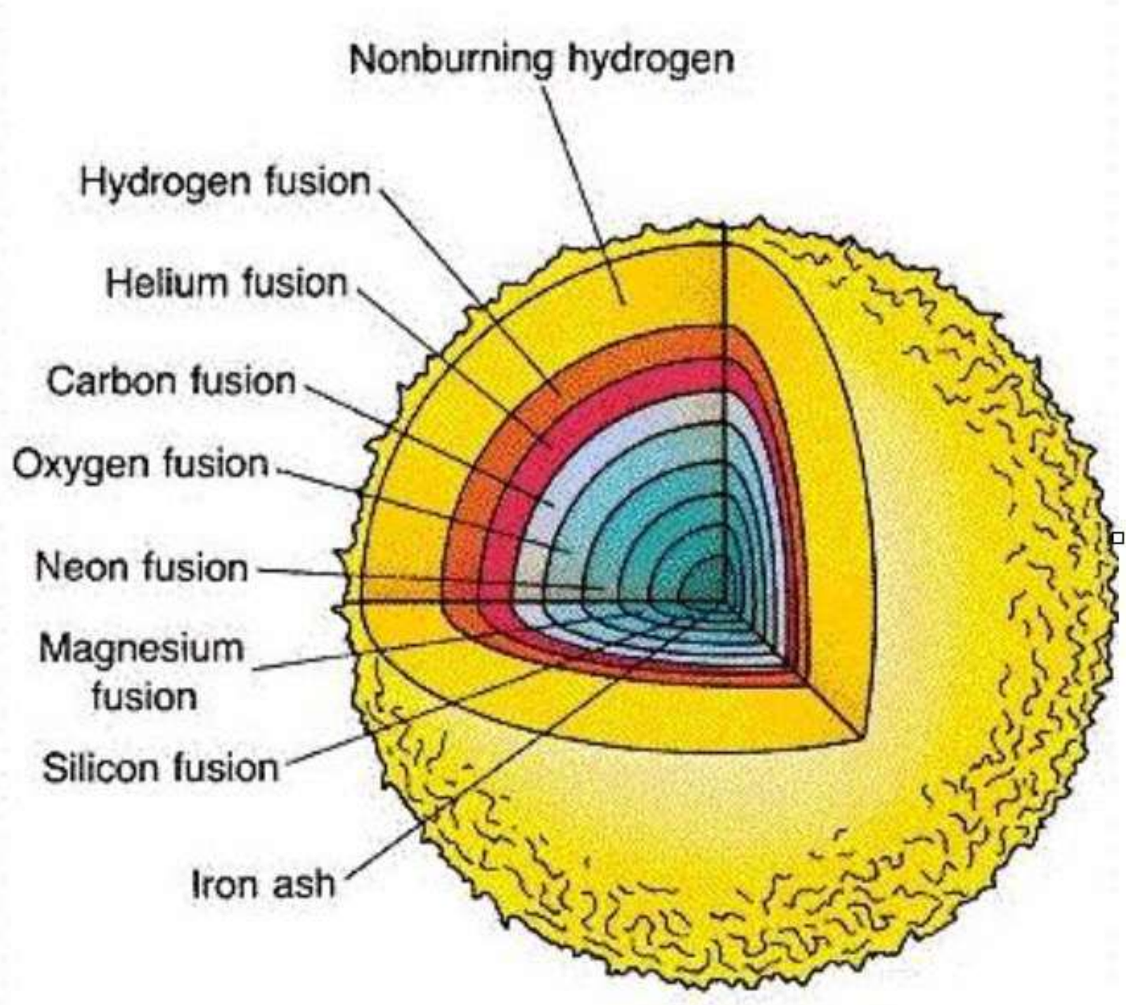
Stellar evolution: stars > 10-12 M_{Sun}

20 M_{Sun} star

Combustion	T 10^6 K	ρ kg/cm³	Durée années
H	37	0.0045	8.1 millions
He	188	0.97	1.2 millions
C	870	170	976
O	1980	5550	1.25
Si	3340	33400	0.03

These succesive nuclear burning phase necessarly stops at **Iron** that is **energetically sterile**

The remnant celestial objects mass is greater than1.4 M_{Sun}. These stars then either become **neutron star** or **black holes**!



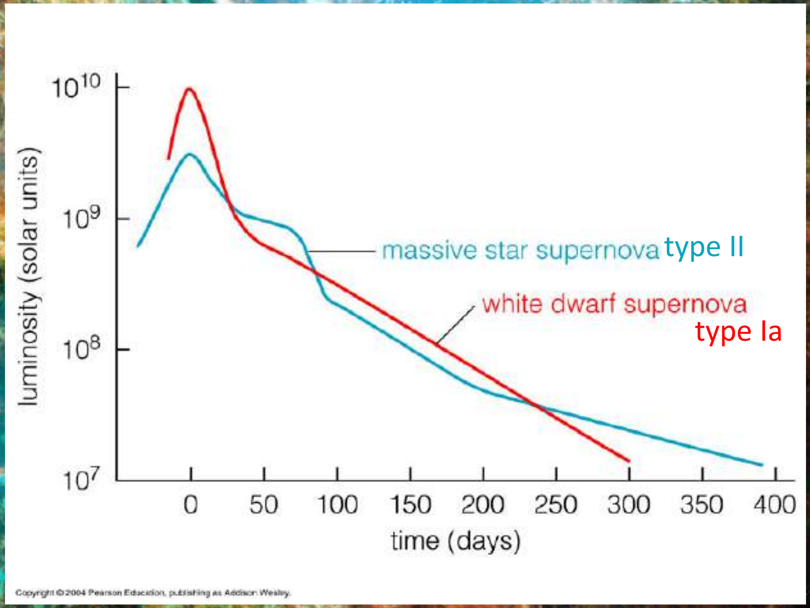
Note: between 8-9 and 10-12 Msun only carbon is consumed and a « runaway » instability leads to a supernova

Stellar evolution: Supernova type II

Gravity leads the star into a formidable implosion: Supernova de **type II**.

The outer parts of the star are free falling, collapsing and crashing onto the neutron star rigid surface. A violent compression reverse the motion and create a shock wave that make the star explodes.

Heavier atoms are hence feeding the interstellar medium!

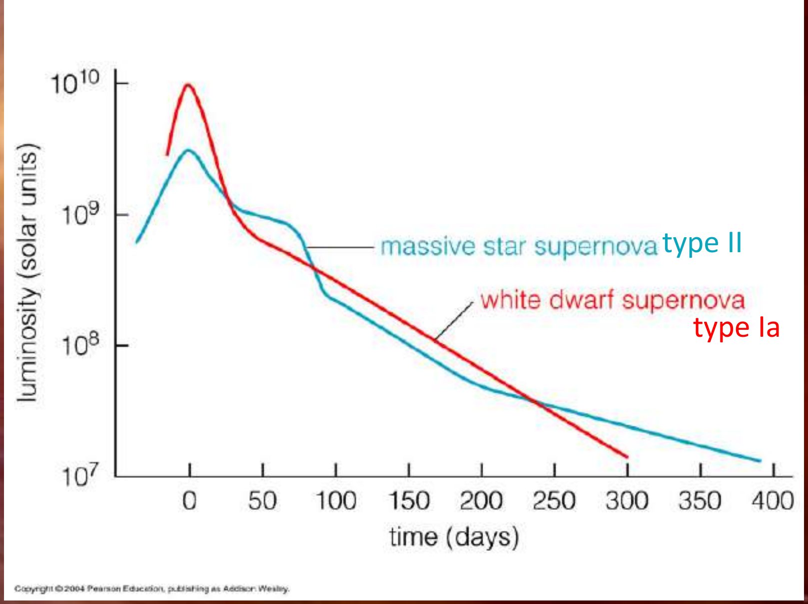


Note: supernova type II, Ib/Ic are due to massive star collapse

Crabe nebulae

Stellar evolution: Supernova Ia

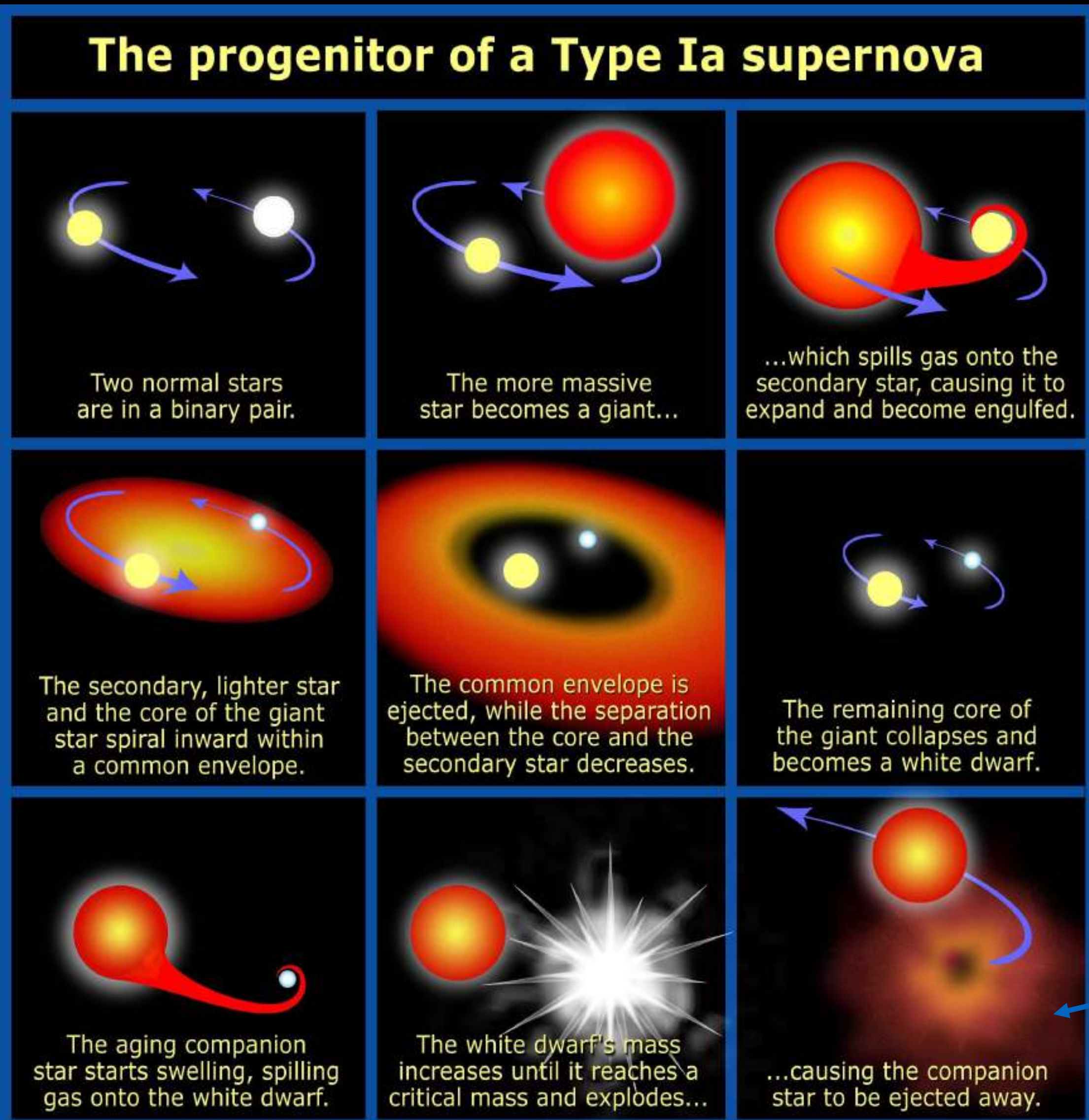
Type II vs type I supernova: Observational signature differs by the presence or lack of Hydrogen



The accretion of a star on a white dwarf (WD) in a binary system can lead the WD to go over Chandrasekhar mass: supernova de **type Ia**

Artist view

Supernova Ia: What future outcome for the stellar pair?



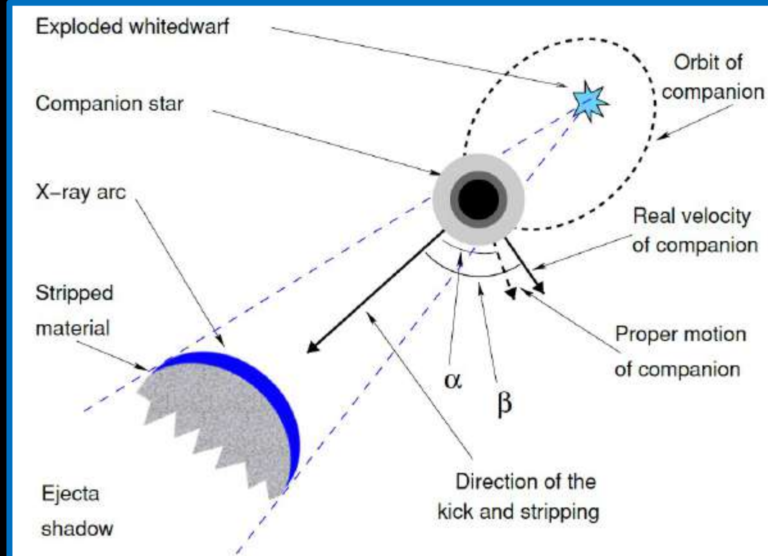


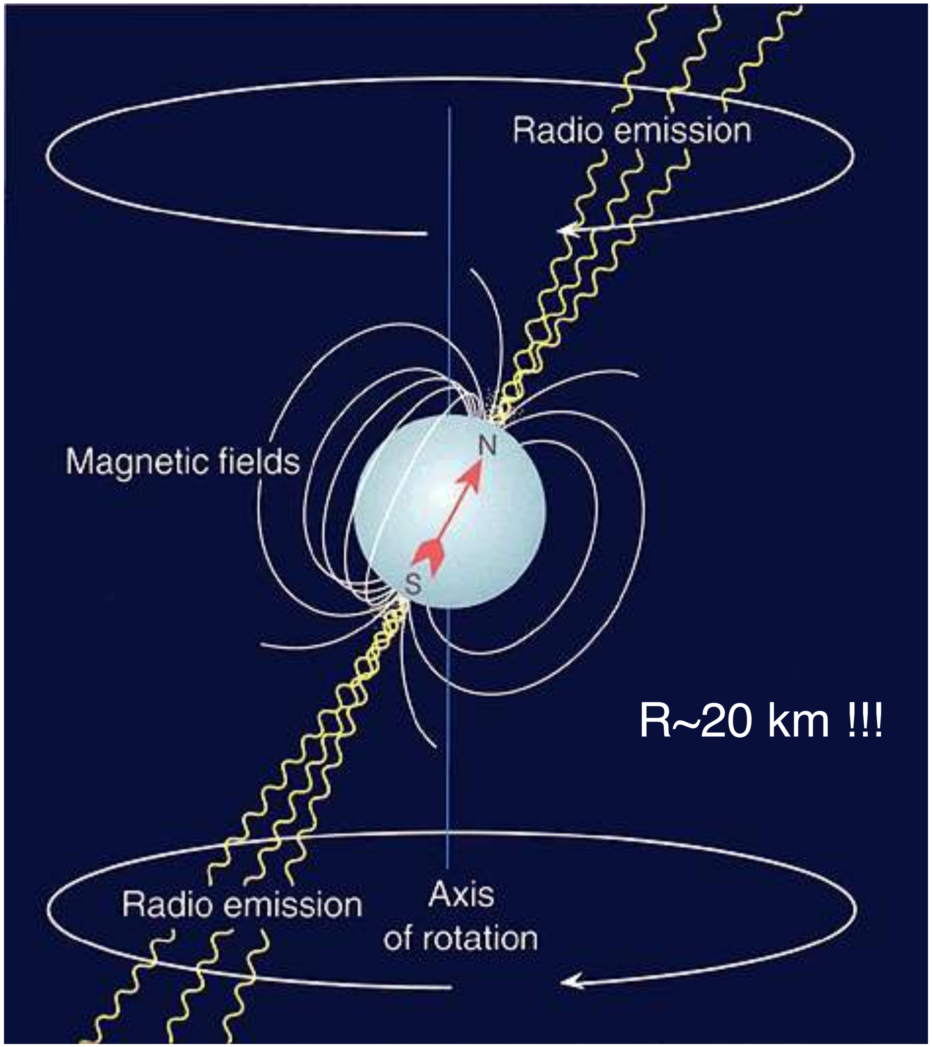
Figure 4. Illustration of the progenitor binary and the impact of the SN on the companion star. The companion star gained a kick in addition to the original orbital motion. Therefore, the angle between the kick direction and the companion star velocity has a projected (observed) value $\alpha = 63^\circ$, but is $\beta (= 67^\circ)$ in real space. The outer envelope of the companion star is stripped by the fast expanding SN ejecta in the same direction as the kick. The observed X-ray arc represents the shock wave generated by the interaction between the stripped envelope and the ejecta, which are behind the X-ray arc. The blocking of the stripped envelope produces a cone that is SN ejecta deficient.

Expulsion of a part of the RGB envelope and impact on its motion.

Lu, Wang et al. 2011, ApJ

Stellar Evolution: Neutron stars

These (degenerate) stars cannot go over Oppenheimer-Volkoff limit mass: ~3 M_{Soleil}



Somes become pulsars

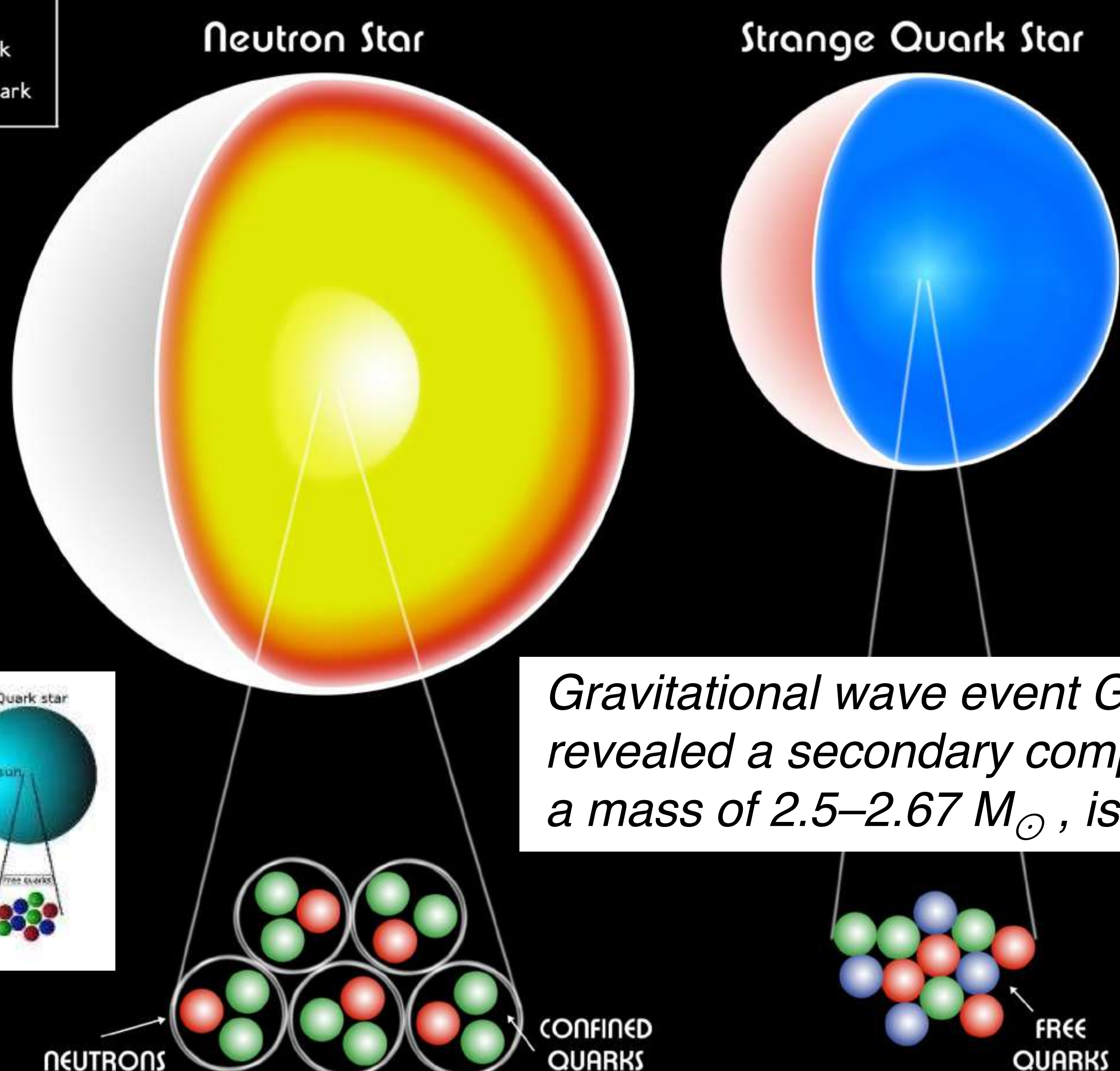
Depends on EoS of neutron matter in super dense conditions ($\rho \sim 10^{14-15}$ g/cm^3)

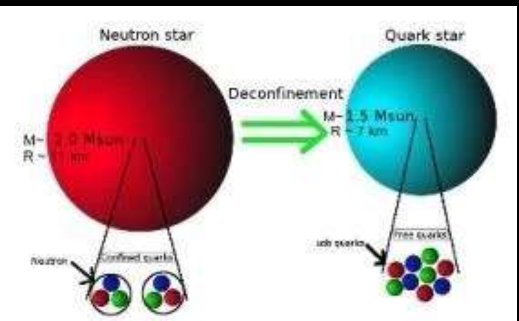
Its the degenerate neutron gas pressure that compensates gravity and maintains the star equilibrium

PSR J0740+6620 Mass = 2.14 +/- 0.1 $M_\odot$

Crab Constellation Pulsar

Quark and Strange Quark Stars





Gravitational wave event GW190814 revealed a secondary component with a mass of 2.5–2.67 $M_{\odot}$, is it a SQS?

Bhattacharjee & Chattopadhyay 2025

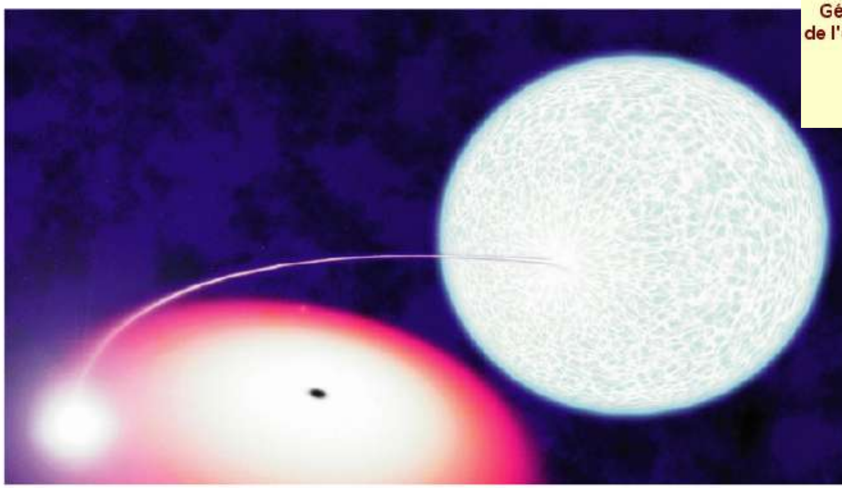
Stellar evolution: Black Holes

There is no real minimum mass for a Black Hole (the mass contained in the horizon at the Planck scale could set such a lower limit), more a minimum density. However for collapsing stars, the stellar core progenitor must be more massive (denser than a neutron (or even strange quark) star) that is given by the Openheimer-Volkoff mass: ~3 M_{Sun}

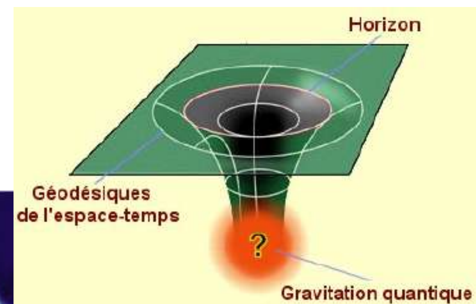
Attraction is such that even light cannot escape... (horizon R_s)
We can see a BH only in an indirect way, via the accretion disk of material falling into it (for instance coming from close orbiting stars via X-ray radiation)

$$R_s = 2GM/c^2 \sim 3\ M/M_{Sun}$$

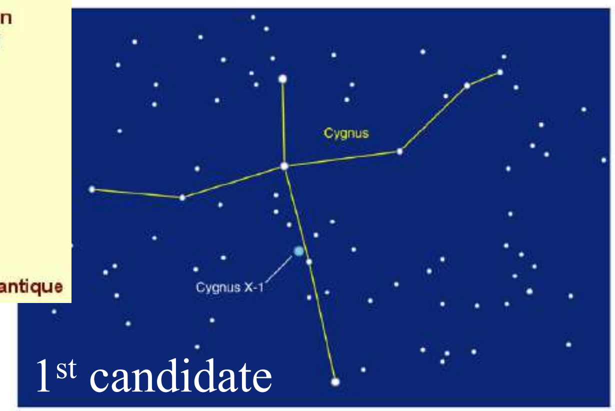
Schwarzschild radius (in km)



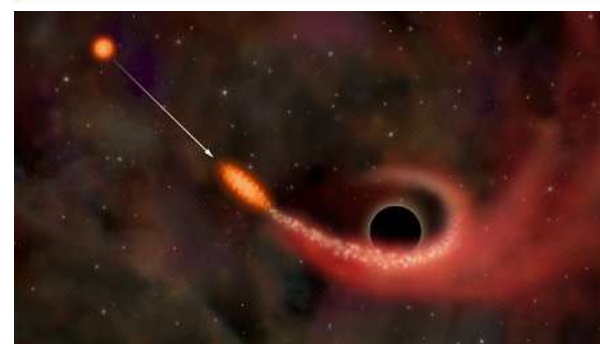
Accretion disk around a BH (Artist view)



Cartoon view of a BH, in which space is reduced to 2 dimensionsThe 3rd dimension then allows to see the space-time curvature



1st candidate



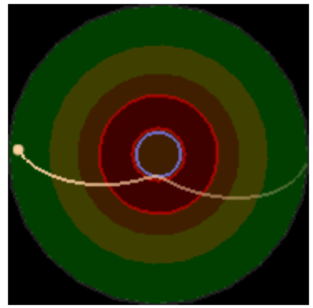
Stellar evolutionary stage: Black Holes

There are several type of Black Holes (BH): Schwartzschild, Kerr (spin), Reissner-Nordstrom (charged) and Kerr-Newman (spin and charged). Their mass M, angular momentum J and charge Q (depending on the case studied) are enough to characterize them outside (not inside) !

A rotation and/or a non-zero charge complexifies the BH metric. A 2nd horizon appears (we then speak of internal and external horizons) and the notions of worm hole and white hole emerge. However it seems that an instability of the internal horizon usually destroy the tunnel.

Color	Zone
Green	Stable circular orbits
Yellow	Unstable circular orbits
Orange	No circular orbits
Red line	Horizon
Red	Inside the horizon

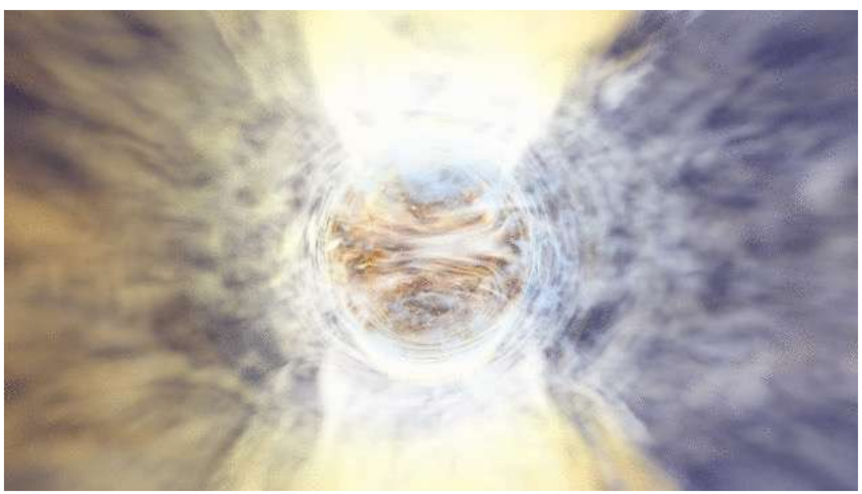
Schwartzschild



Color	Zone
Green	Stable circular orbits
Yellow	Unstable circular orbits
Orange	No circular orbits
Red lines	Horizons
Red	Between horizons

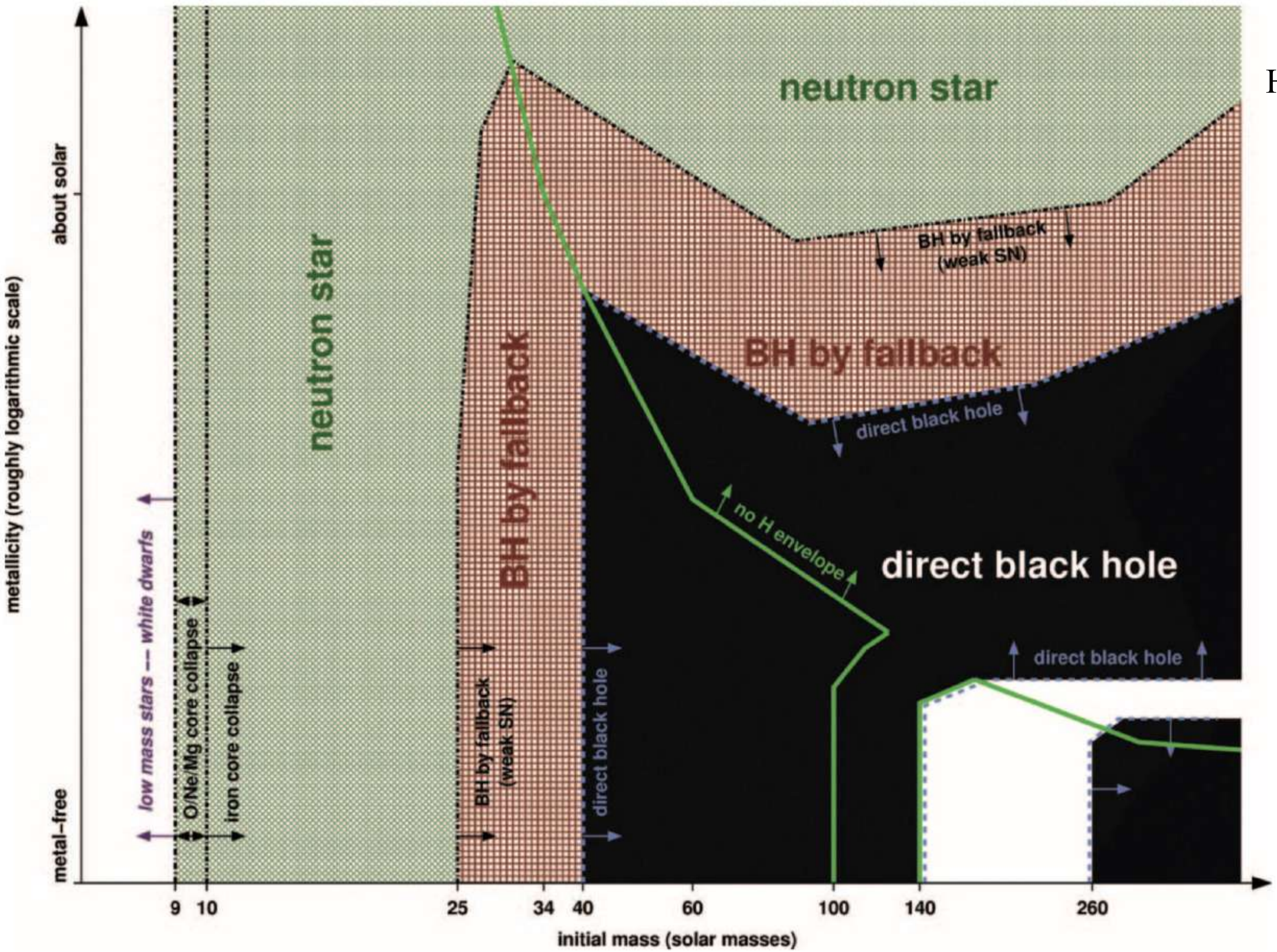
Reissner-Nordstrom

Hypothetical travel through a Reissner-Nordstrom BH



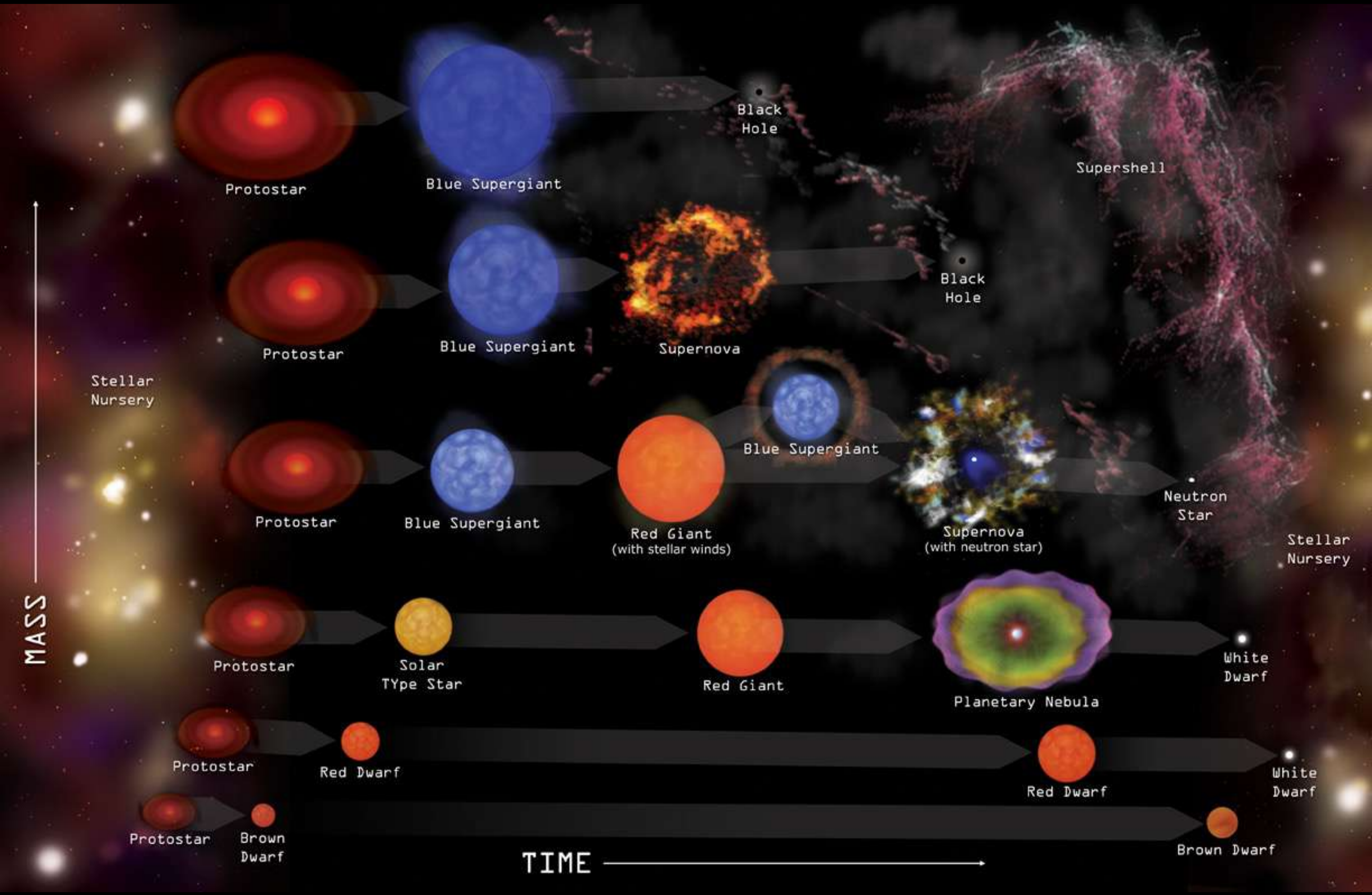
https://jila.colorado.edu/~ajsh/insidebh/schw.html

Stellar evolution: summary for massive stars outcome



Heger et al. 2003

Stellar evolution: summary



Next Lecture:

Heat transport in stars, stellar magnetism and dynamo action