The force hierarchy and geodynamo regimes of Earth's core

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GFD seminar, Kyōto

Thanks to collaborators:
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University of Glasgow

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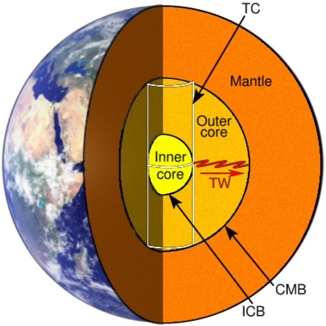
Reversals of the eld dipolarity occur (seemingly at random intervals and a reversal takes thousands of years to complete).

Structure of Earth



ICB = Inner core boundary
CMB = Core-mantle boundary
TC = Tangent cylinder

Fluid outer core is seat of dynamo giving rise to geomagnetic eld.

Convection arises from heat and light material released at inner core boundary.

Magnetic eld is continually replenished through induction (combining Faraday's law, Ampere's law, and Ohm's law)

Twisting and stretching of eld lines by chaotic convection generates electric current, in turn re-generating magnetic eld.

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Aim to match simulations to observations of the changing geomagnetic field thereby understanding dynamics in the core.

Geodynamo simulations - physical setup

Spherical polar coordinate system $(r; \ ; \ )$.

Spherical shell radially bounded above at $r = r_o$ by an electrically insulating mantle and below at $r = r_i$ by an electrically insulating (or conducting) inner core.

Rotates about the vertical (z-axis) with rotation rate and gravity acts radially inward, $g = gr$.

Boussinesq approximation used - density, treated as a constant except for the source of buoyancy

Fluid is assumed to have constant values of , , and , the outer core density, kinematic viscosity, thermal di usivity and magnetic di usivity respectively.

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Often use $\widetilde{Ra}' = Ra/Ra_c = \widetilde{Ra}/qRa_c$, as a measure of supercriticality. Ra_c is the critical Rayleigh number for the onset of (non-magnetic) convection.

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! Allows for the identi cation of suitable parameter regimes despite input parameters not close to Earth-like values.
Then preserve the force balance by moving all parameters towards Earth-li values in a systematic way.

Force balances

Forces acting in the non-dimensionalised system are:

$$F_I = E_m \left(\frac{\partial u}{\partial t} + u \cdot \nabla u \right) \qquad F_P = -\nabla p \qquad F_C = -2\hat{z} \times u$$

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CIA balance, $F_C \approx F_I \approx F_A$. At high convective driving and weak magnetic field.

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Some previous investigations of force balances:

- Rotvig & Jones, Phys Rev E, 2002
- Soderlund+, PEPS, 2015
- Yadav+, PNAS, 2016
- Schae er+, GJI, 2017
- Schwaiger+, GJI, 2019, 2021
- Teed & Dormy, JFM, 2023

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But that is not the case!

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Teed & Dormy (2023) proposed forming `solenoidal forces' by directly eliminating gradient parts of all forces to observe the important rst order balance (MAC, VAC, etc.) directly

F = r A + r ' ; eliminate r ' by:
- curling F. (Note: Taylor-Proudman constraint is formed this way!)
- projecting of forces onto their solenoidal part: r A

! Perform curl

Remove gradient parts

Teed & Dormy, 2023

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Potential bistability - di erent regimes at the same input parameters

Weak and strong field branches - simulations

Dormy, 2016; identification of strong field branch and bistability in DNS at $E = 3 \times 10^{-4}$ and $E_m = 1.7 \times 10^{-5}$

Requires E_m to be chosen within a `sweet-spot' range of values (dependent on E)...

Regime diagrams

Each plot is decreasing E

Decreasing E_m

Solutions become non-magnetic

Solutions become non-dipolar

Solutions become non-dipolar with increasing Ra^0

Bifurcation diagrams

Bifurcation diagrams for
$E = 3 \quad 10^{\ 4}$

Dormy, 2025

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Weak-strong branching is found at low enough E_m

Dormy, 2025

Bifurcation diagrams

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Tentative 3D bifurcation diagram for xed$E$

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Bifurcation diagrams

Dormy, 2025

- Tentative 3D bifurcation diagram for xed E
- Weak-strong branching occurs via a cusp singularity
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Distinguished limit

System of equations has four (main) control parameters $(E, E_m, q, \widehat{Ra})$...

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Need to study dependence of such solutions on E and E_m to help determine constants

Some questions to address

Now we'll look at some results on (solenoidal) forces (Teed & Dormy, 2023) and upcoming work on geodynamo branches as E is lowered (Teed & Dormy, 2025).

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Key questions to keep in mind:

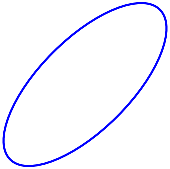
1. Do solenoidal forces help in identifying the leading order force balance and hierarchy in geodynamo simulations?
2. What regime produces the desired MAC balance relevant to Earth's core dynamics and where is it located in parameter space?
3. How does the branching between weak and strong regimes persist/scale as parameters are moved towards Earth-like values? I.e. lowering E and E_m.

Branches of dynamo action ($E = 10^{4}$)

Branches of dynamo action ($E = 10^{-4}$)

At higher E_m:

Branches of dynamo action ($E = 10^{4}$)



At higher E_m:

Dipolar regime...

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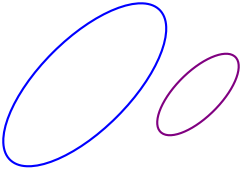

At higher E_m:

- Dipolar regime...
- ...transitions to multipolar regime at large enough $\widetilde{Ra}$

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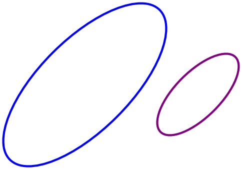

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At lower E_m:

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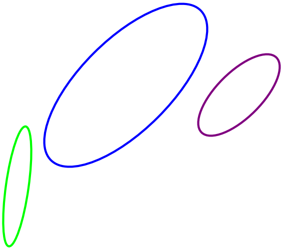

At higher E_m:

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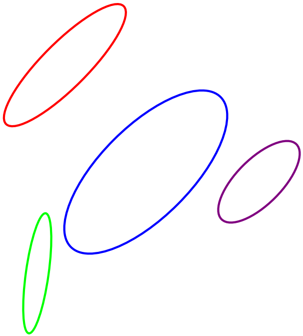
At lower E_m:

- Weak eld dipolar regime (^{0} 1)...

Branches of dynamo action ($E = 10^{\;4}$)



At higher E_m:

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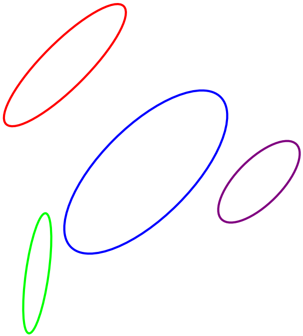
Branches of dynamo action ($E = 10^{\ 4}$)



At higher E_m:

- ~~Dipolar~~ `Strongish' dipolar regime...
- ...transitions to multipolar regime at large enough Ra

At lower E_m:

- Weak eld dipolar regime (^{0} 1)...
- ...transitions to strong eld dipolar regime (^{0} 1)
- Bistability between weak and strong branches in a region of Ra-space

Typical regimes (aE = 10 ^{4})

Strongish dipolar

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Fluctuating multipolar

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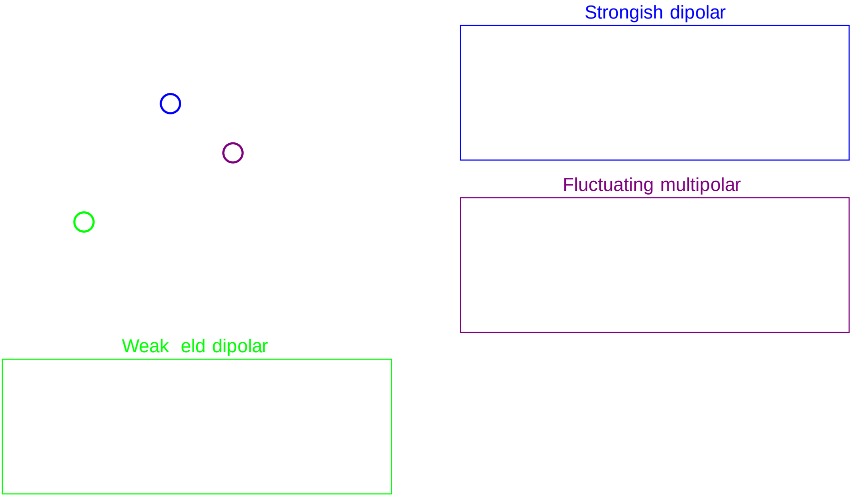

Typical regimes (aE = 10^{4})

Strongish dipolar

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Weak eld dipolar

Strong eld dipolar

Comparing forces and solenoidal forces

! Perform curl

Remove gradient parts

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Solenoidal forces reveal clear leading order CIA balance for multipolar regime

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Forces show `QG-MAC' balance with $\mathbf{F}_C^{ag}$ $\mathbf{F}_C$ at some scales. Strange!

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- Solenoidal forces reveal expected leading order VAC balance for weak eld regime

Strong eld dipolar

- Forces show `QG-MAC' balance with $\mathbf{F}_C^{ag}$ $\mathbf{F}_C$ at some scales. Strange!
- Solenoidal forces reveal clear leading order MAC balance at large scale for strong eld regime

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Strong eld dipolar

- Forces show `QG-MAC' balance with $\mathbf{F}_C^{ag}$ $\mathbf{F}_C$ at some scales. Strange!
- Solenoidal forces reveal clear leading order MAC balance at large scale for strong eld regime
- Usefulness of ageostrophic Coriolis force lost at lengthscales where balance is not QG

Solenoidal forces

Strongish *Multipolar*

Weak eld *Strong eld*