FDEPS 20年と zonal flows

March 17, 2024
ＧＦＤセミナー特別編（支笏湖畔）

山田道夫
京都大学数理解析研究所

Joint work with K.Obuse and S.Takehiro

Congraturations!

- 林さん

御定年退職，おめでとうございます．

FDEPS Story

FDEPS started in 1999.

- Fluid Dynamics in Earth and Planetary Sciences (FDEPS)

- 連続講義形式の workshop を始めよう(1999年)
  - 年１回，１週間
  - 場所は駒場：東京大学数理科学研究科（当時）
- 開催費用は自前（科研費）で
  - 紐付きのお金は避ける（制約なしでやりたい）
  - 大きな workshop にはしない（参加者40人程度以下にする）
- 世界の leaders の講義を聴きたい
  - 講師リストを華やかにする（依頼しやすくなる．最初が肝心）
  - 講師も楽しめるように（東京では難しかった）

FDEPS 1999 (1st)

第一回 地球惑星科学における流体力学 ワークショップ

- 日程：
  1999年12月6日(月)～10日(金)
- 場所：
  東京大学大学院 数理科学研究科
  (東京都目黒区駒場3-8-1、京王井ノ頭線駒場東大前下車)
  地図 (地図上の建物 "j")

- アドバイザー（所属は当時）
  木田重雄（京大・工）
  M.McIntyre (DAMTP, Cambridge)
  三村昌泰（明治大・理工）
  中澤清（東工大・地球惑星）

1999.12.06(月）~07（火）講演会

M.McIntyre (DAMTP, Cambridge)
Atmosphere-Ocean Dynamics: Some Fundamentals
P.H.Haynes (DAMTP, Cambridge)
Stirring and mixing in the stratosphere
岩崎俊樹（東北大・理）
高解像度気象モデルとダイナミクス
木田重雄（核融合研）
ダイナモ問題と流体力学
小屋口剛博（東大・新領域創成）
火山現象のモデリングと流体力学
星野真弘（東大・理）
宇宙空間での磁気リコネクション

12.08 (水)~10 (金) 連続講義

P.H.Haynes (DAMTP, Cambridge)
Waves, turbulence and mean-flow generation in geophysical flows

The total mean force exerted on the critical layer is

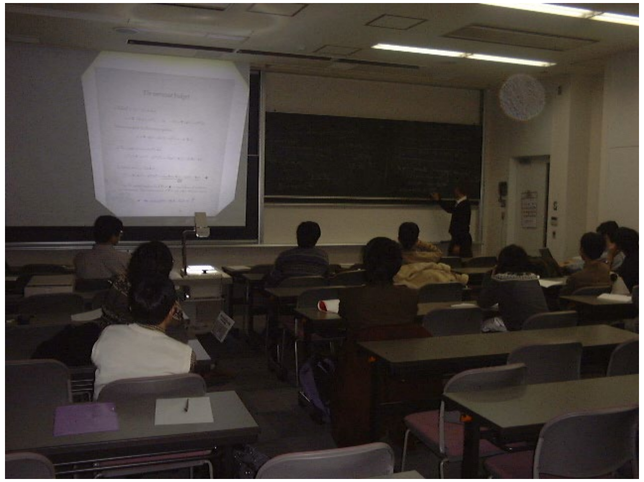
$$\int_{-}^{+} (-\overline{u'v'})_y dy = -[\overline{u'v'}]_{-}^{+} = \int_{-}^{+} \overline{\mathcal{D}} - \overline{\mathcal{A}}_t dy < 0$$

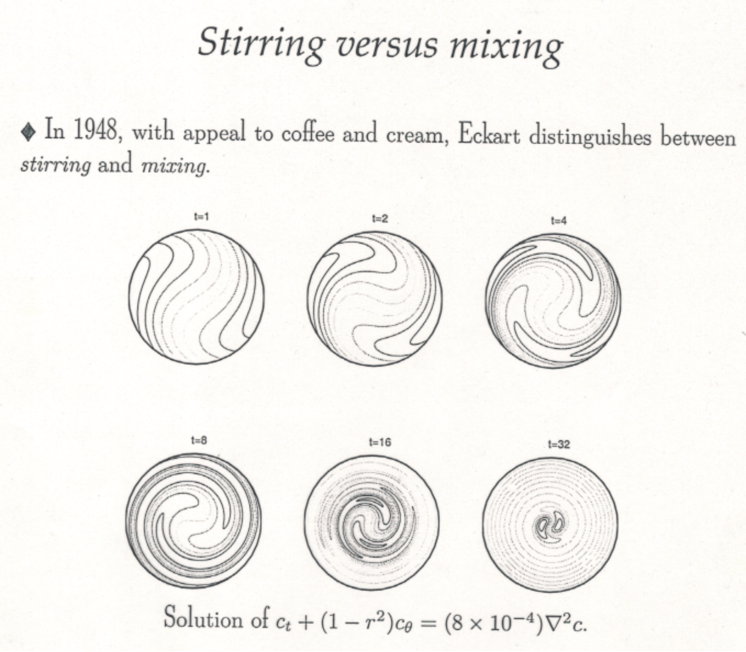
The negative (westward) force exerted on the mean flow in the critical layer region corresponds to a convergence of pseudomomentum flux into that region.

FDEPS 2000

- 第２回　2000.12.04(月）~08（金）
  - W.R.Young, (Scripps Inst. Oceanography)
    Biological tracers
  - P.Cessi（Scripps Inst. Oceanography)
    1. Simple models of ocean/atmosphere low-frequency variability
    2. Dissipation selection of low-frequency basin modes
  - 羽角博康（東大・気候システム）
    海洋大循環モデリングにおける「渦」パラメタリゼーションの様々な側面
  - 吉田茂生（名大・理）
    コア・マントル間熱的相互作用
  - 草野完也（広大・先端物質科学研究科）
    太陽プラズマにおける電磁流体力学
    ＜対流，ダイナモ，リコネクション＞
  - 榎本　剛（東大・理）
    8月に日本上空にできる等価順圧構造を持つ亜熱帯高気圧の成因

- 第３回　2001.12.03-07
  - R. Lindzen　（同時多発テロ事件により中止）





(from lecture by W.R.Young)

気象海洋分野

FDEPS 2002

- 第４回　2002. 11. 11-15

  I.M.Held (Princeton GFDL/NOAA)

  Atmospheric poleward energy fluxes, turbulent cascades, and the efficiency of the atmospheric heat engine.

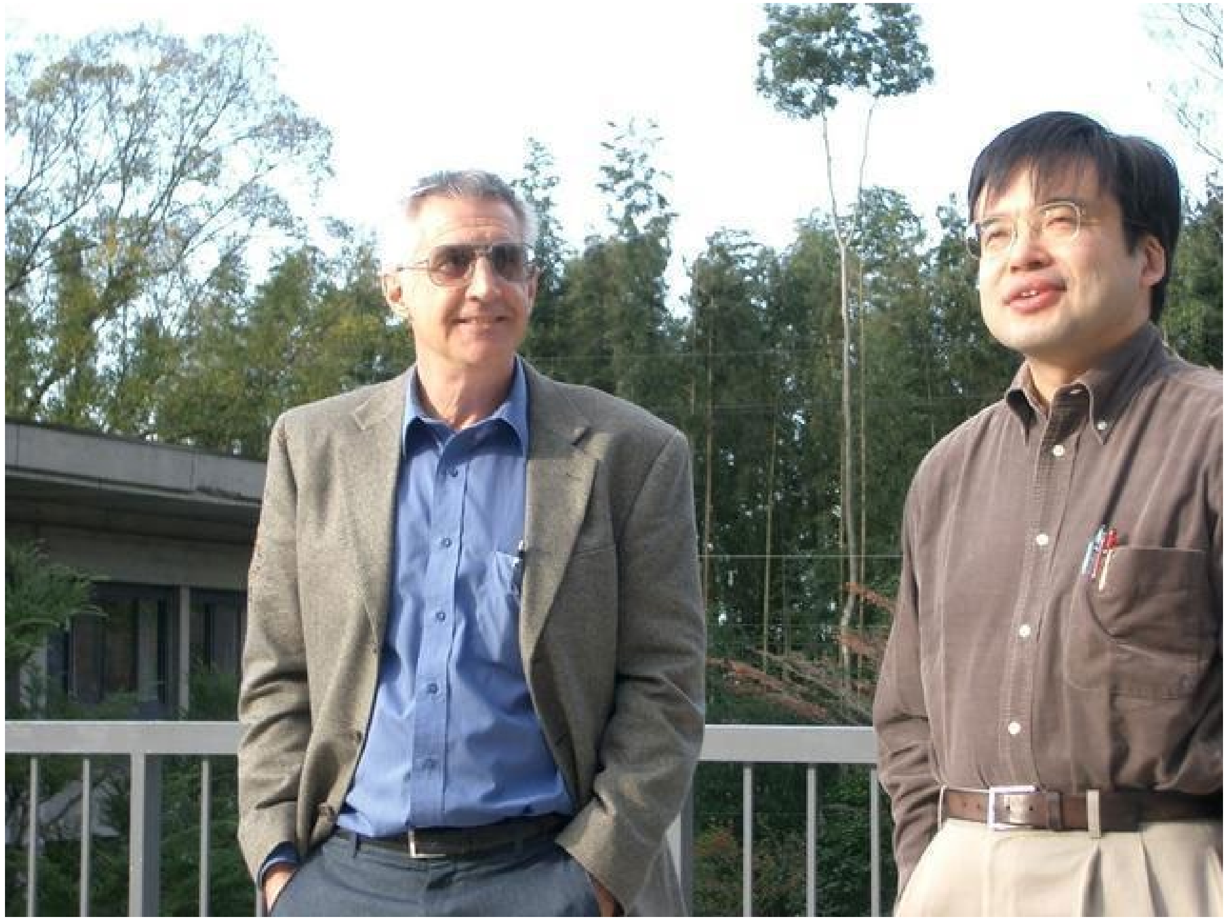
The Zonal Mean Climate of the Troposphere

Isaac M. Held

(from lecture by I.M.Held)

FDEPS 2003

第５回　2003. 11.17-21
（この年から京都関西セミナーハウスで開催）
R. Salmon (UCSD)
Introduction to Hamiltonian Fluid Dynamics



Mean flows and disturbances

Lagrangian for a one-dimensional fluid:

$$L[x(a,\tau)] = \iint d\tau\, da \left\{ \frac{1}{2}\left(\frac{\partial x}{\partial \tau}\right)^2 - E\left(\frac{\partial x}{\partial a}\right) \right\}$$

Hamilton's principle:

$$\int d\tau \;\; L[x(a,\tau)] = 0 \qquad \text{for arbitrary} \qquad \delta x(a,\tau)$$

(from lecture by R. Salmon)

第７回　2005. 11.08-11

G.K.Vallis (GFDL/Princeton Univ.)

Geostrophic Turbulence and the General Circulation of the Ocean and Atmosphere

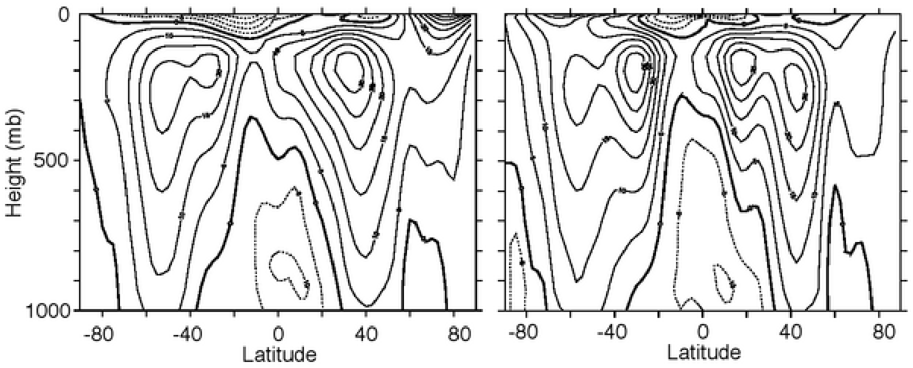


Fig. 3.1 The time-averaged zonal wind at 150°W (in the mid Pacific) in December-January February (DJF, left), March-April-May (MAM, right). The contour interval is 5 m s^{-1}. Note the double jet in each hemisphere, one in the subtropics and one in midlatitudes. The subtropical jets is associated with strong meridional temperature gradient, whereas the midlatitude jets have a stronger barotropic component and are associated with westerly winds at the surface.

(from lecture by G.K.Vallis)

FDEPS 2006 Spring Special

- 第８回　2006.03.27-31

  （この回だけ小樽朝里川温泉，FDEPS2001 のリベンジ）

  R.S. Lindzen（MIT）

  Is there really a basis for global warming alarm?

Wave Overreflection and Shear Instability

R. S. Lindzen and K. K. Tung

Center for Earth and Planetary Physics, Harvard University, Cambridge, MA 02138

(Manuscript received 2 February 1978, in final form 25 May 1978)

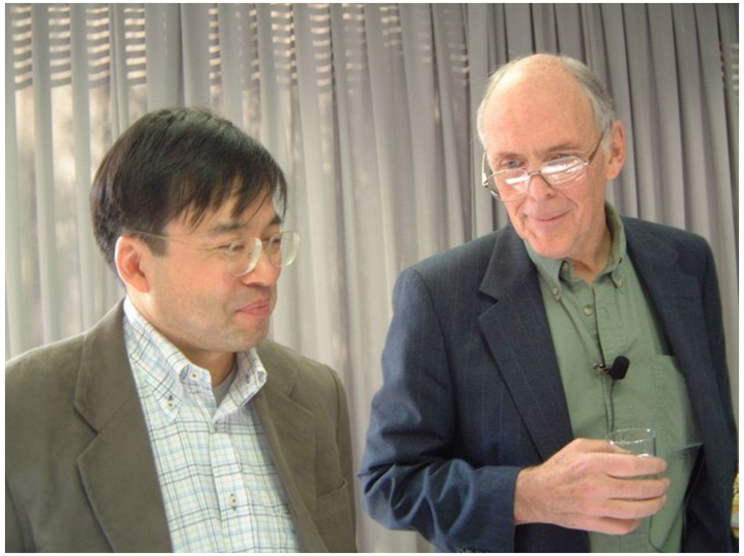
(from lecture by R.S.Lindzen)

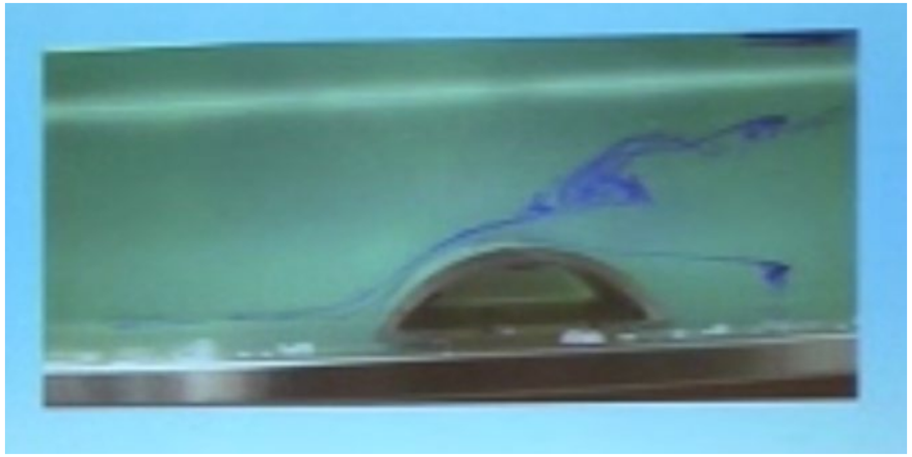
FDEPS 2007

- 第１０回　2007.11.5-8

  Peter B. Rhines (Univ. of Washington)

  “How do waves and eddies shape the general circulation, gyres and jet streams?” and …





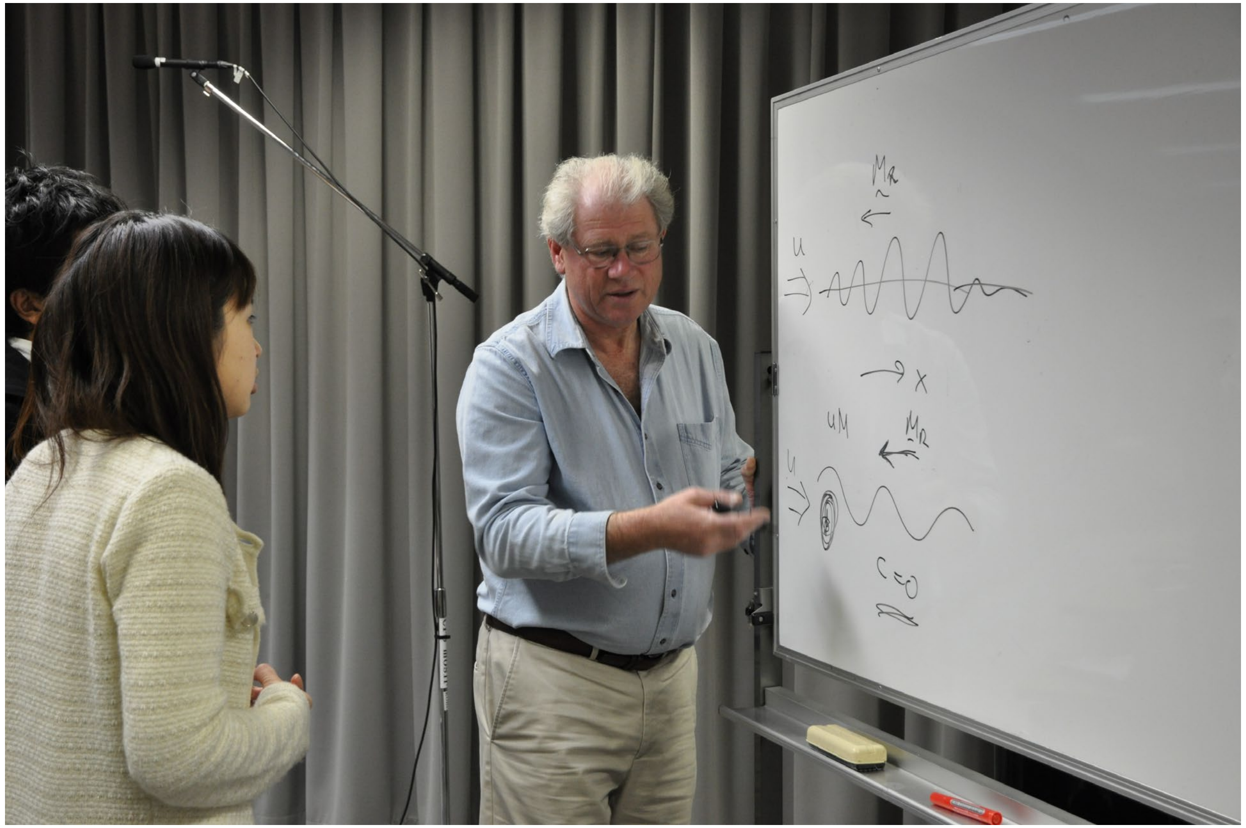
(from lecture by P.B.Rhines)

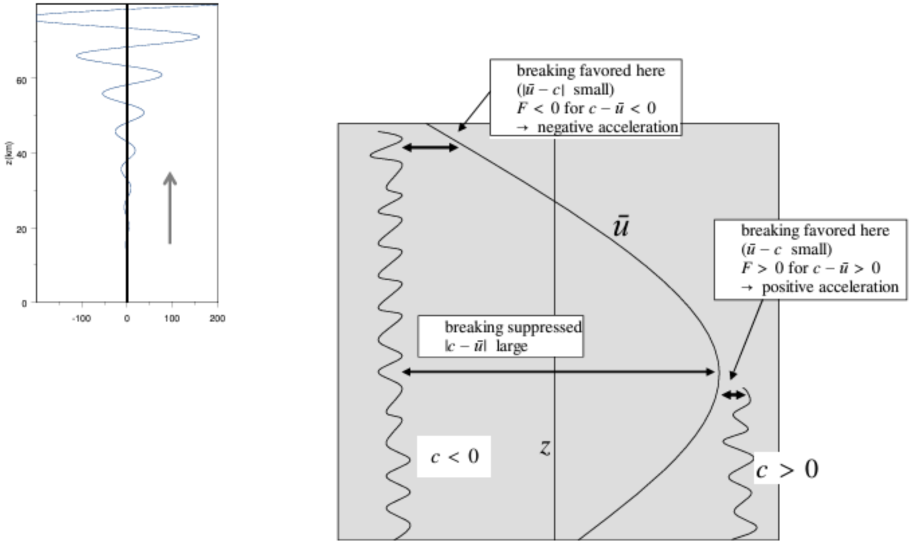
FDEPS 2010

- 第１３回　2010.11.16-19

  Raymond Alan Plumb (MIT)

  Waves and their role in the general circulation of the atmosphere





→ Internal gravity wave breaking can *reinforce* zonal flow
(we'll see importance of this later)

(from lecture by R.A.Plumb)

FDEPS 2012

- 第１５回　2012.11.18-21

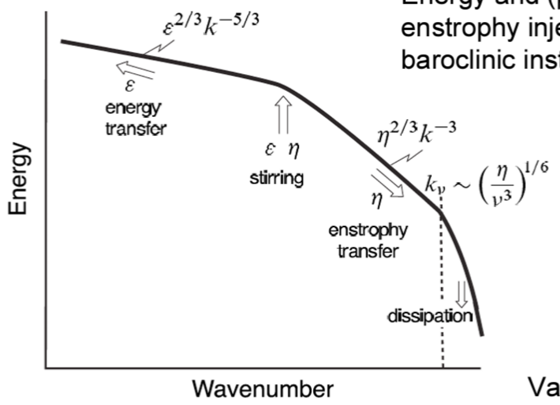
  Theodore G. Shepherd (Univ. of Reading)

  Fluid dynamics of atmospheric circulation

The classical picture of two-dimensional turbulence (after Kraichnan 1967 Phys. Fluids)

– Argued to be relevant to the atmosphere by Charney (1971 JAS)

Energy and (potential) enstrophy injection is by baroclinic instability



Vallis (2006)

(from FDEPS lecture by T.G.Shepherd)

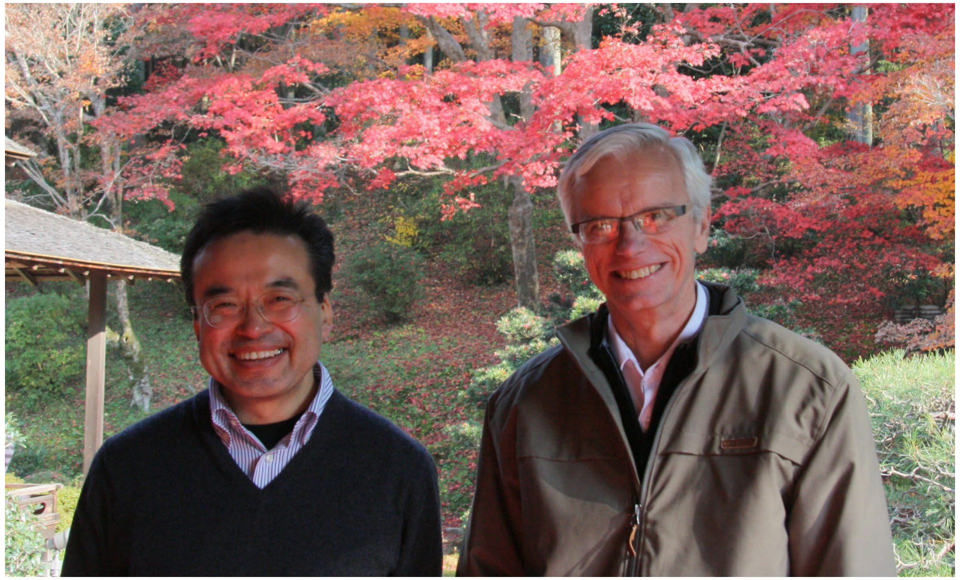
FDEPS 2013

- 第１６回　2013.12.03-06

  Adam Sobel (Columbia Univ.)

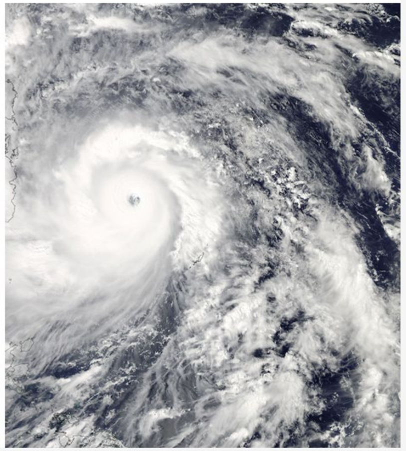
  Moist convection and tropical atmosphere dynamics

(Prof. Sobel and Prof. Tatsumi)



(Prof. Durran)

Super Typhoon Haiyan
Image: NASA



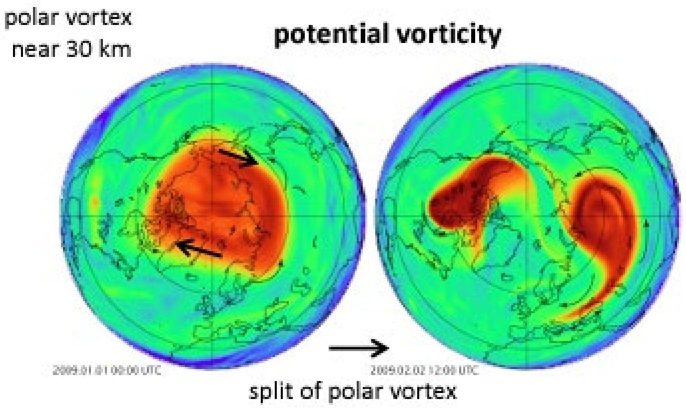
(from FDEPS lecture by A.Sobel)

FDEPS 2014

- 第１７回　2014.12.02-05

  William Randel (NCAR)

  Circulation, transport and chemical variability derived from satellite observations

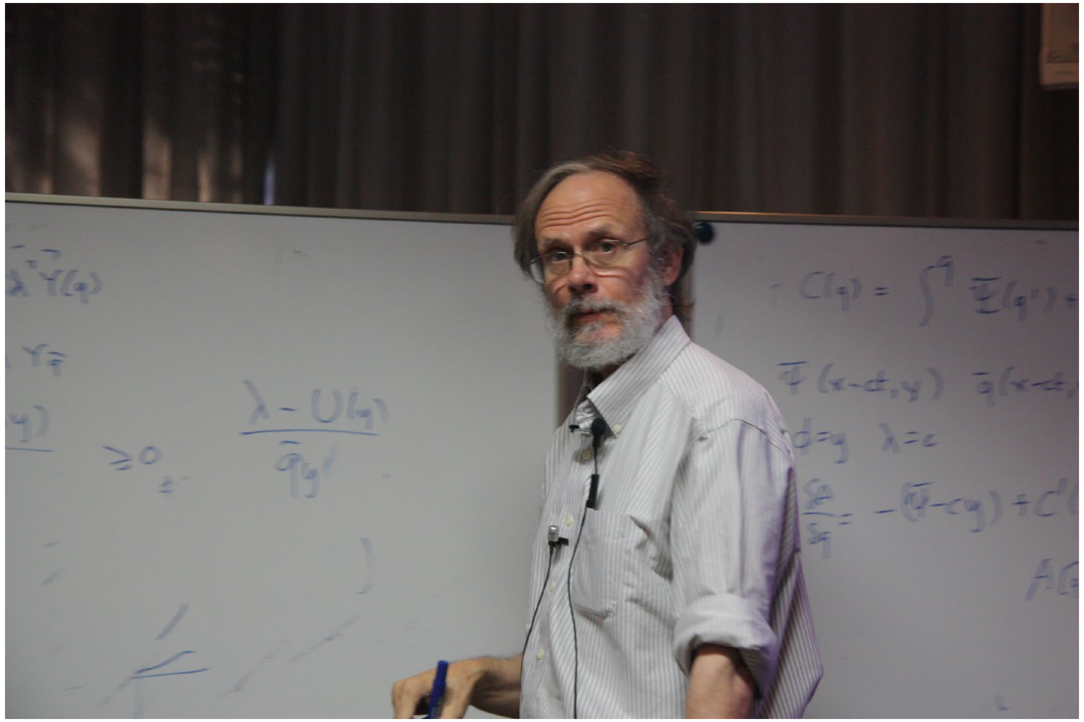


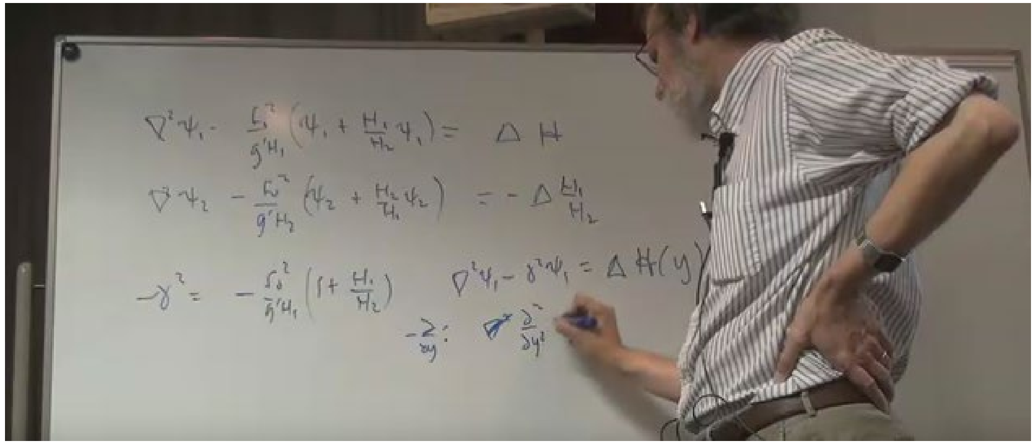
(from FDEPS lecture by R.Randel)

- 第１８回　2015.12.01-04

  Glenn Flierl（MIT）

  Dynamics of geophysical vortices and jets





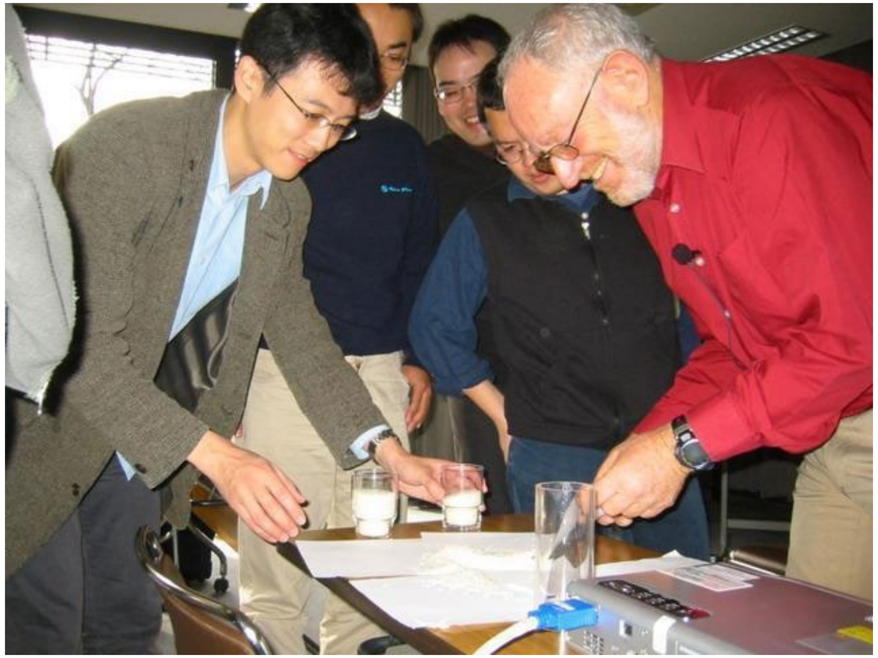
(from FDEPS lecture by G.Flierl)

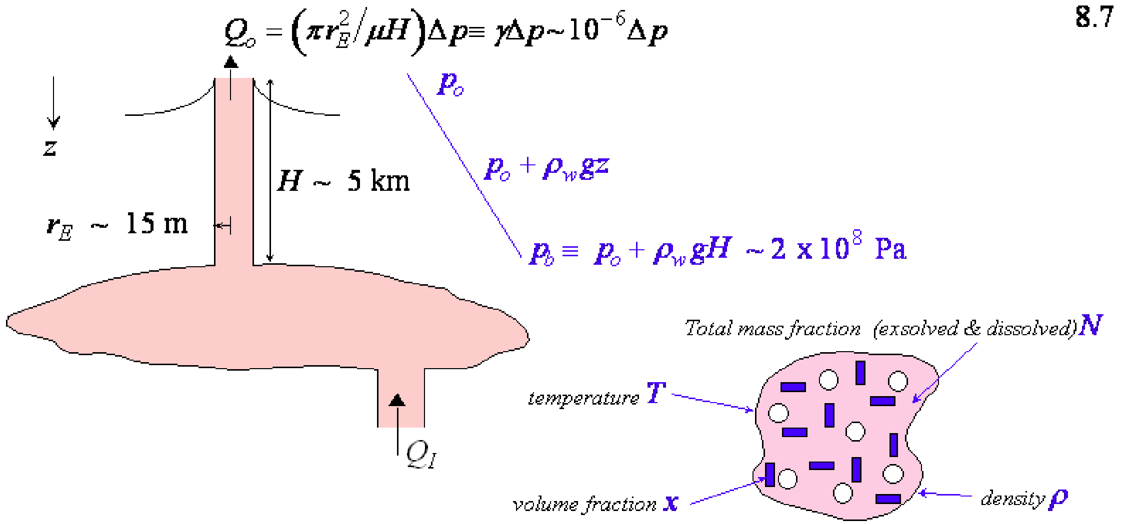
固体地球・宇宙惑星分野

- 第６回　2004. 12.06-09

  H.E.Huppert (Cambridge)

  Geophysical Fluid Dynamics





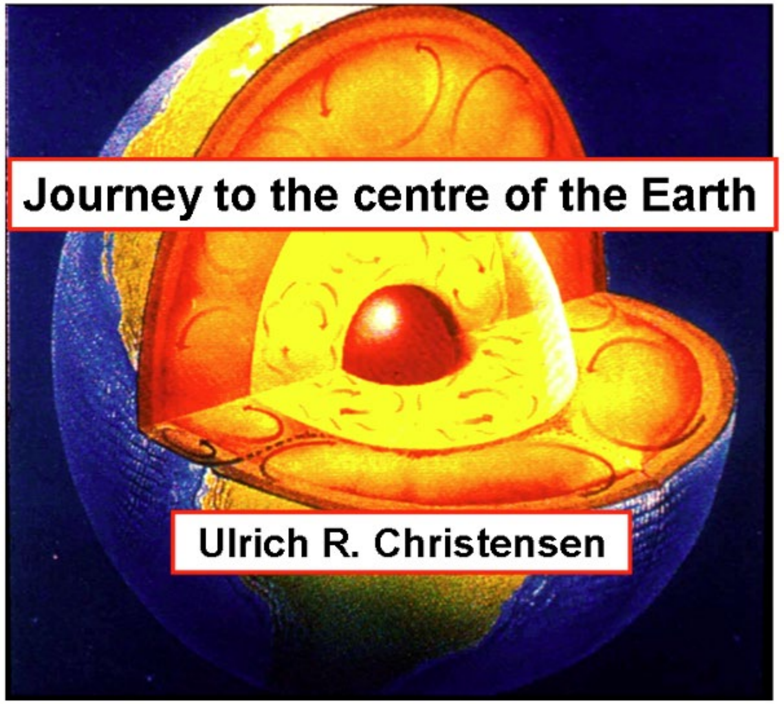
(from FDEPS lecture by H.E.Huppert)

FDEPS 2006

- 第９回　2006.11.28-12.1

  Ulrich Christensen (Max Planck Institut)

  Fluid dynamics of earth and planetary interiors



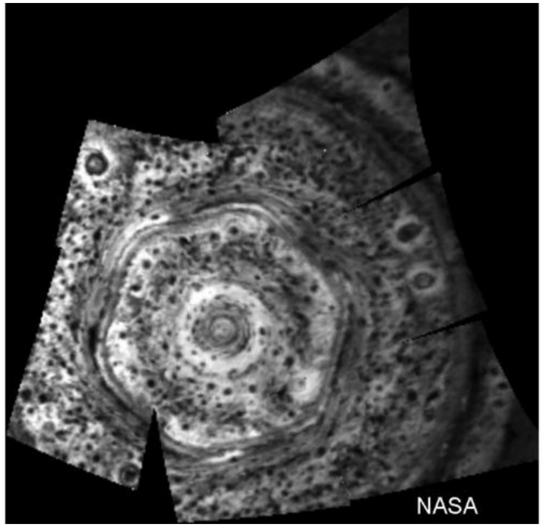
(From FDEPS lecture by U.Christensen)

FDEPS 2008

- 第１1回　2008.11.4-7

  Andrew Ingersol (Caltech)

  “Venus, Mars, Io, Triton, Titan, Jupiter, Saturn
  and other giant planets”



(from FDEPS lecture by A.Ingersol)

FDEPS 2009

- 第１２回　2009.11.3-6

  Peter Goldreich (IAS, Princeton)

  A tour through astrophysical fluid dynamics

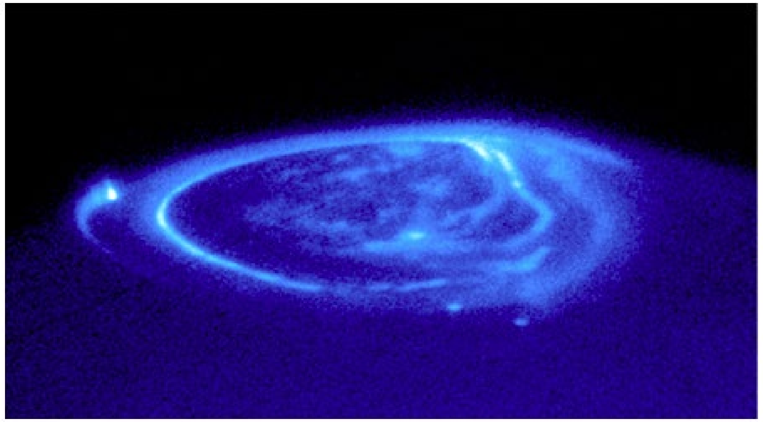


Figure 4.8: Aurora borealis on Jupiter. Three bright dots are created by magnetic flux tubes that connect to the Jovian moons Io (on the left), Ganymede (on the bottom) and Europa (also on the bottom). In addition, the very bright almost circular region, called the main oval, and the fainter polar aurora can be seen. Credit: John T. Clarke (U. Michigan), NASA image in UV, Hubble Space Telescope

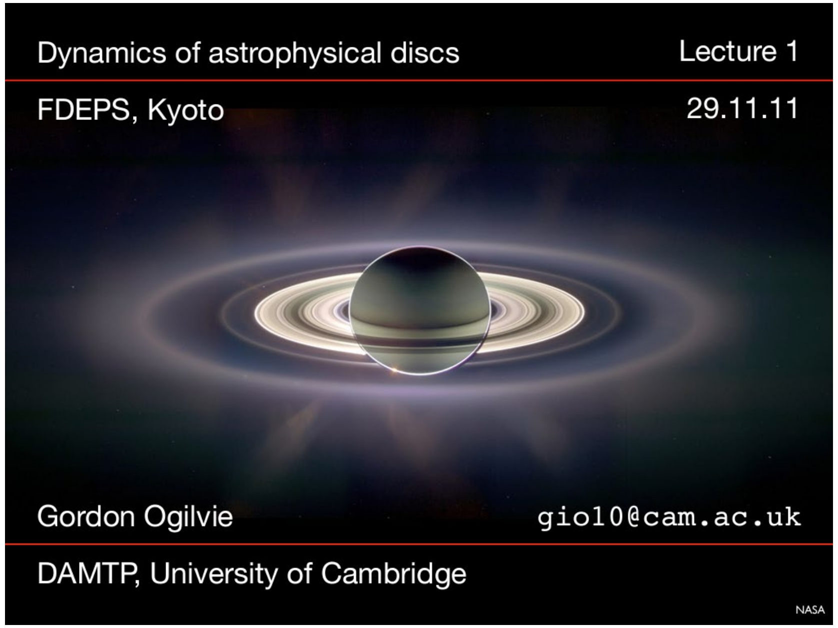
(from FDEPS lecture by P.Goldreich)

FDEPS 2011

- 第１４回　2011.11.29-12.02

  Gordon Ogilvie (Cambridge Univ.)

  Dynamics of astrophysical discs



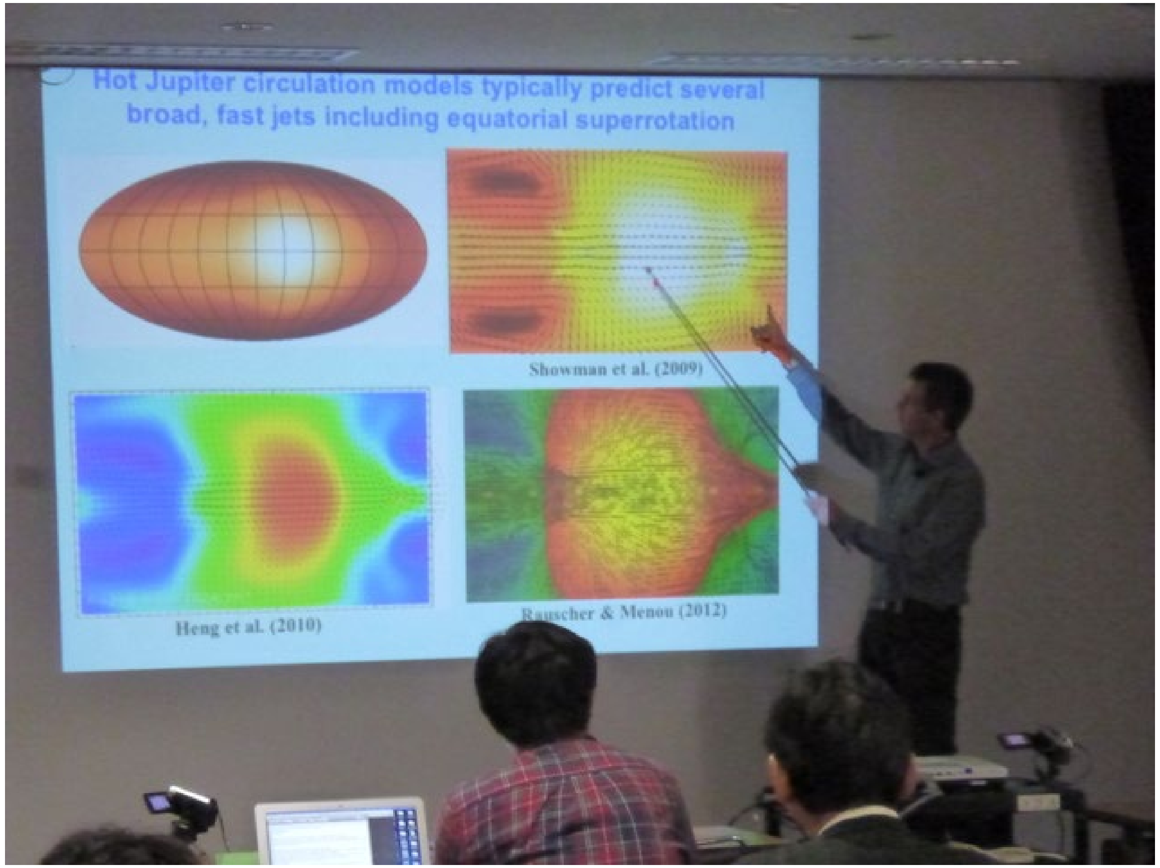
(from FDEPS lecture by G.Ogilvie)

FDEPS 2016

- 第１９回　2016.11.29-12.02

  Adam Showman (Univ. of Arizona)

  Large-scale Dynamics of Planetary and Exoplanetary Atmospheres



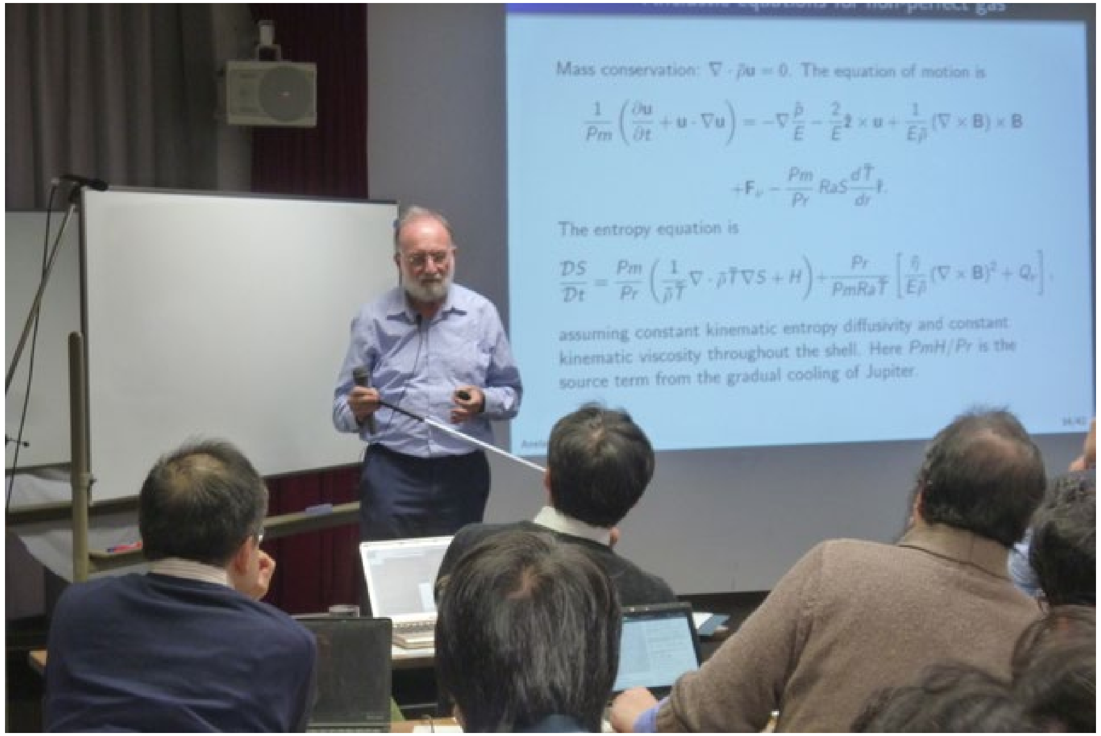
(from FDEPS lecture by A.Showman)

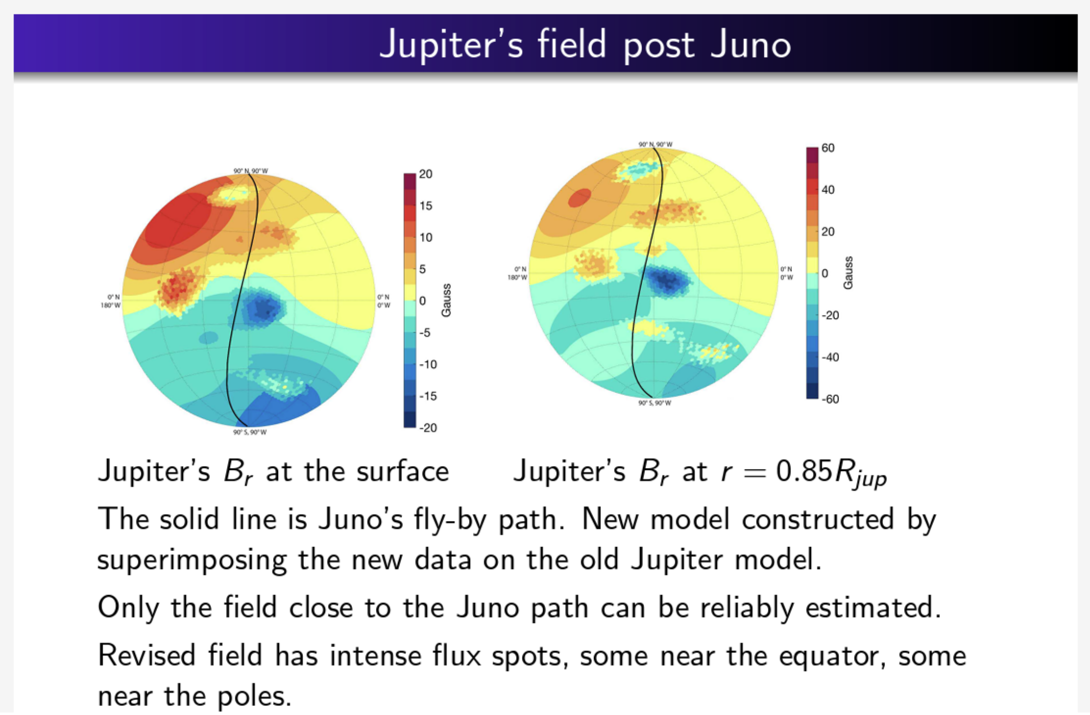
FDEPS 2017

- 第２０回　2017.11.28-12.02

  Chris A. Jones (Univ. of Leeds)

  Planetary interiors: Magnetic fields, convection and dynamo theory





(from FDEPS lecture by C.A.Jones)

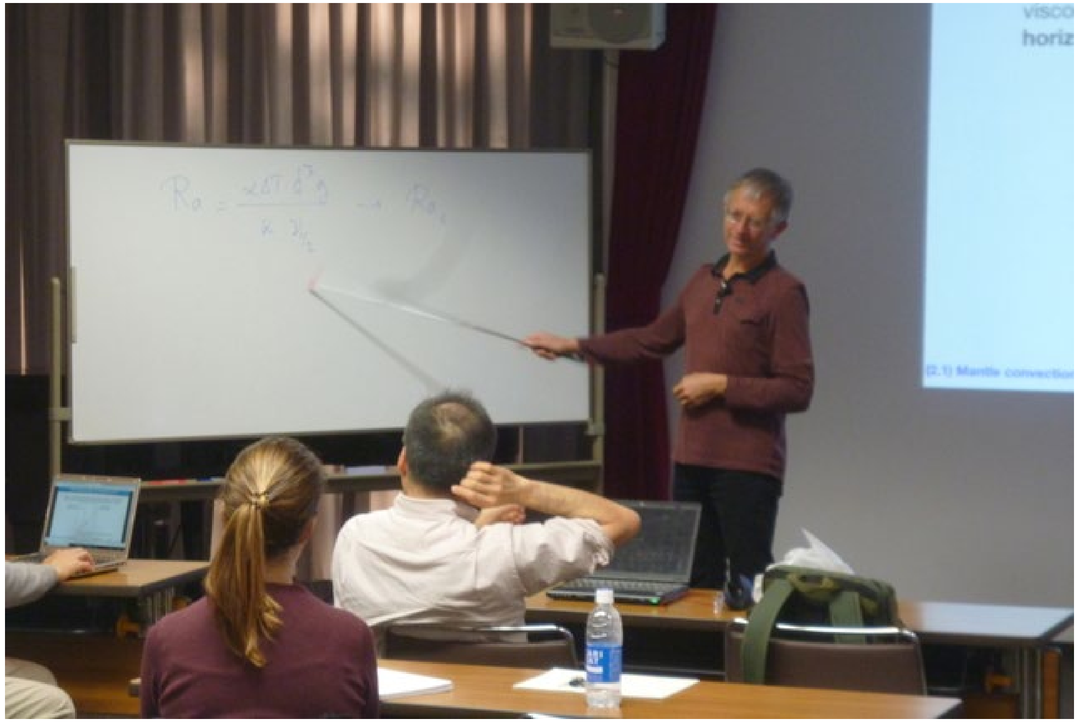
FDEPS 2018

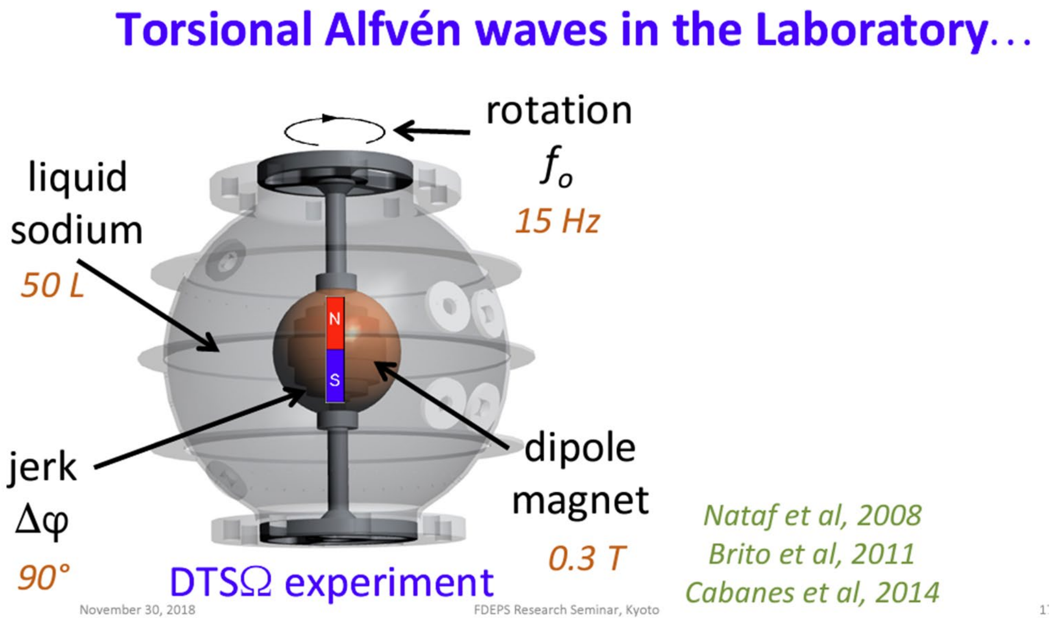
- 第２１回　2018.11.27-11.30

  Henri-Claude Nataf (Univ. Grenoble Alpes)

  Geophysical Fluid Dynamics:

  from the Lab, up and down!





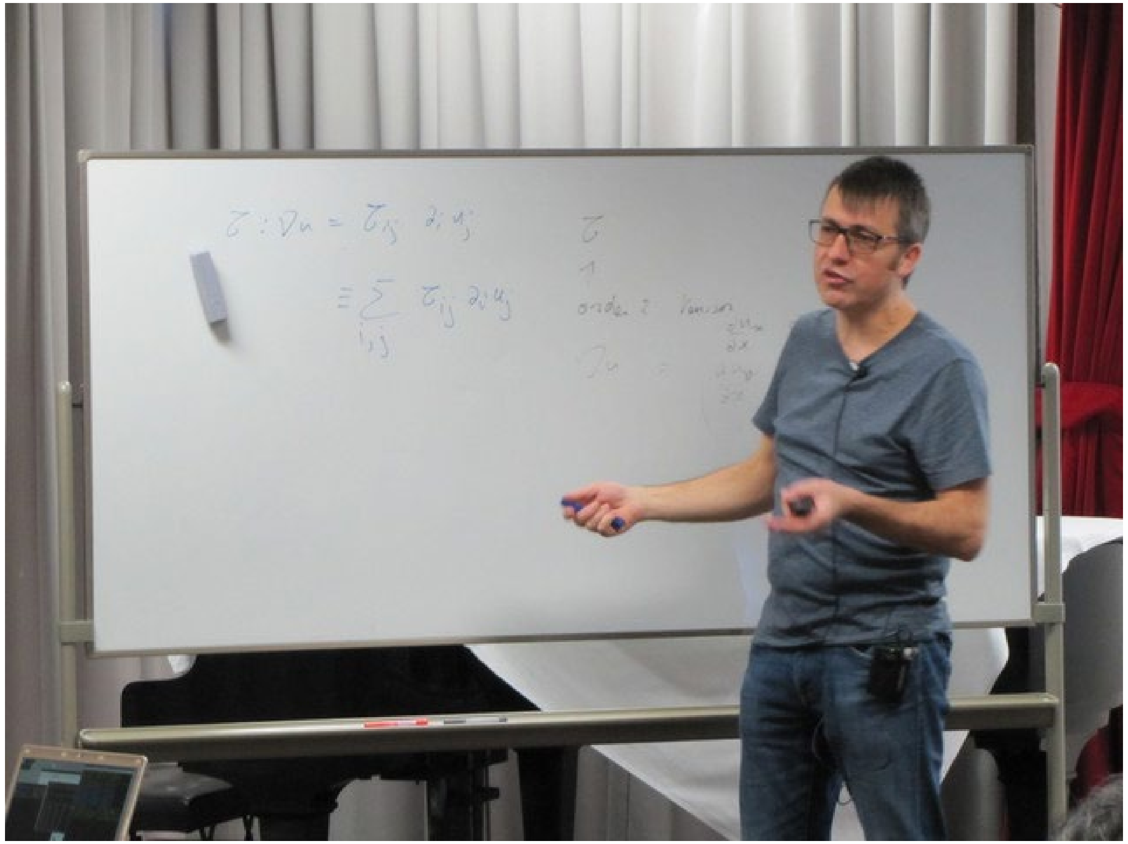
(from FDEPS lecuture by H.C.Nataf)

FDEPS 2019

- 第２２回　2019.11.26-11.29

  Stephane Labrosse (ENS Lyon)

  Convection in planetary interiors and implications for their evolution



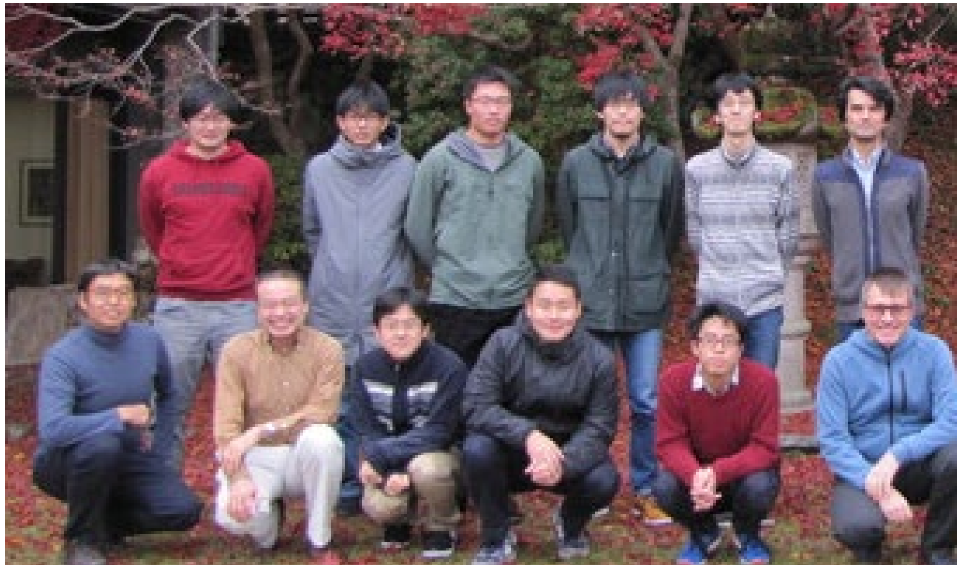
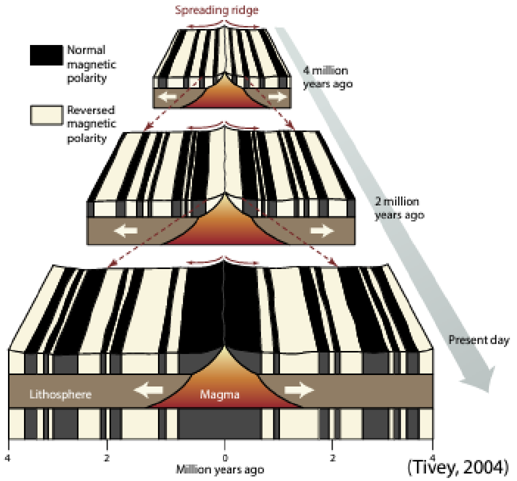


Plate tectonics



(Tivey, 2004)

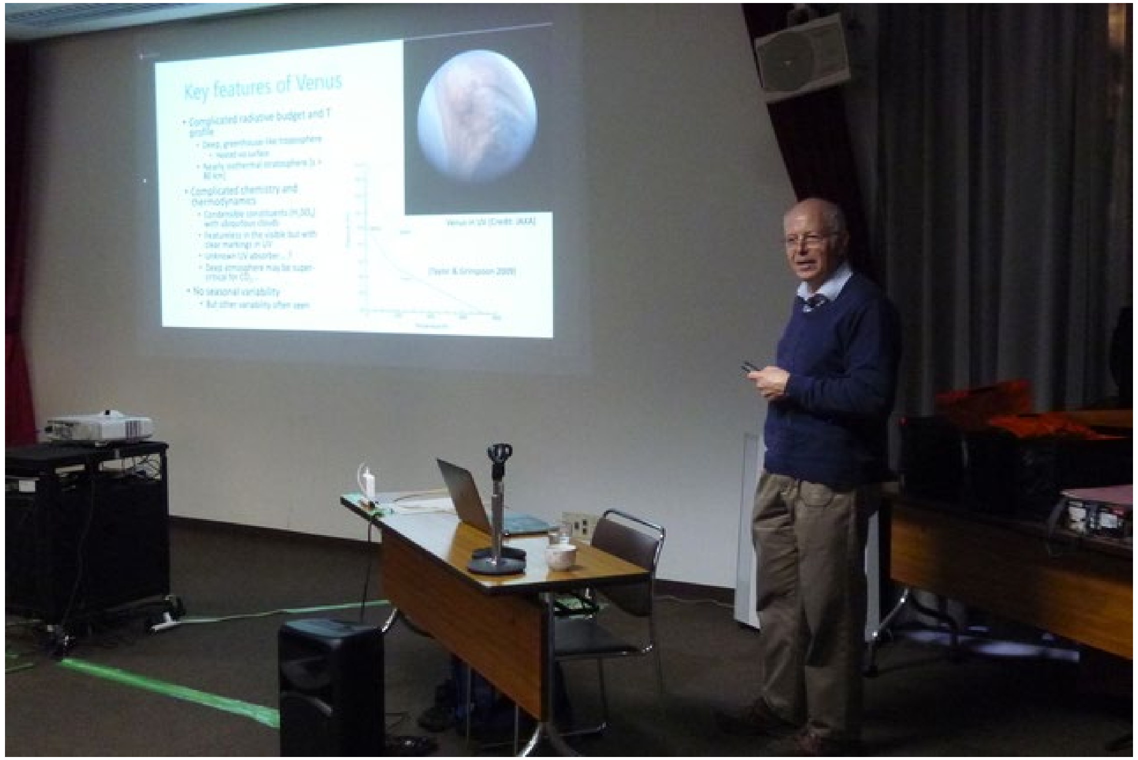
(from FDEPS lecture by S.Labrosse)

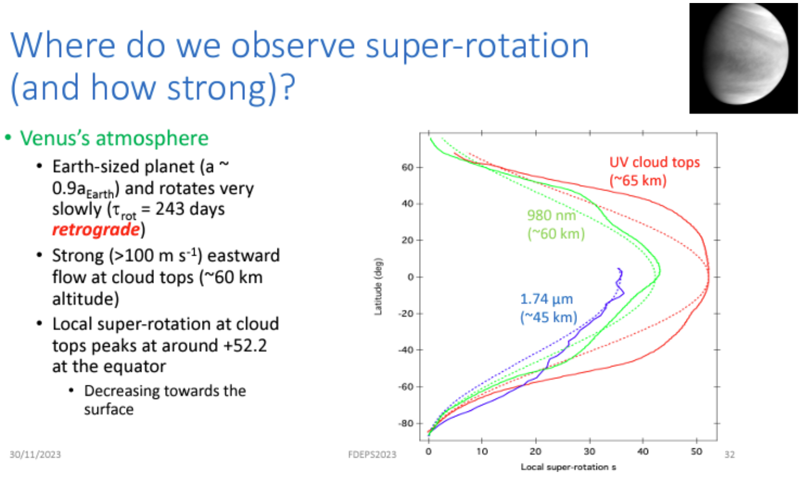
- 第２３回　2023.11.28-12.01

  Peter L. Read (Univ. of Oxford)

  Fast and Slow: Super-rotation Phenomena

  in Palnetary Atmosphere





(from FDEPS lecture by P.L.Read)

FDEPS 運営

- 裏方
    - 運営委員(Alphabet順):
        - 林 祥介 (神戸大・理) <幹事>
        - 石岡 圭一 (京大・理)
        - 石渡 正樹 (北大・理)
        - 竹広 真一 (京大・数理研)
        - 山田 道夫 (京大・数理研) <幹事>
    - Web 資料作成統括
        - 杉山 耕一朗 (松江高専)
    - 実行委員会
        - fdeps-staff at gfd-dennou.org
    - 運営資金
- Many thanks to colleagues, students, JSPS and RIMS.

Academic time

共鳴相互作用の効果

Vallis: "Atmospheric and Oceanic Fluid Dynamics" (2nd ed., Cambridge)

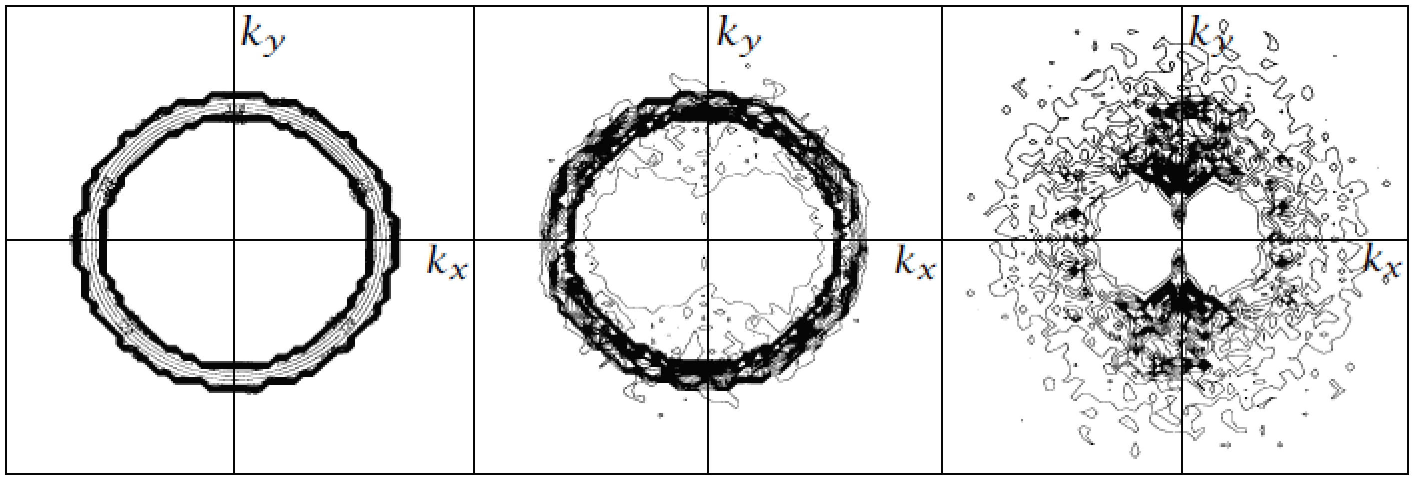


Fig. 12.3 Evolution of the energy spectrum in a freely evolving two-dimensional simulation on the β-plane. The panels show contours of energy in wavenumber (k_x, k_y) space at successive times. The initial spectrum is isotropic. The energy 'implodes', but its passage to large scales is impeded by the β-effect, and the second and third panels show the spectrum at later times, illustrating the dumbbell predicted by (12.14) and Fig. 12.2.[2]

$$\omega = -\frac{\beta k^x}{k^{x2} + k^{y2}}. \qquad \varepsilon^{1/3} k^{2/3} = \frac{\beta k^x}{k^2},$$

$$k_\varepsilon^x = \left(\frac{\beta^3}{\varepsilon}\right)^{1/5} \cos^{8/5}\theta, \qquad k_\varepsilon^y = \left(\frac{\beta^3}{\varepsilon}\right)^{1/5} \sin\theta \cos^{3/5}\theta,$$

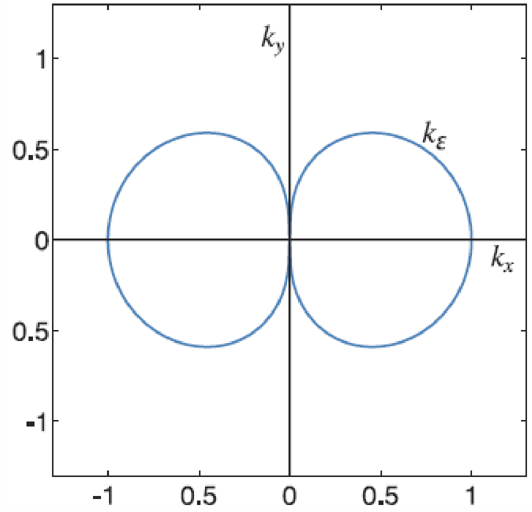
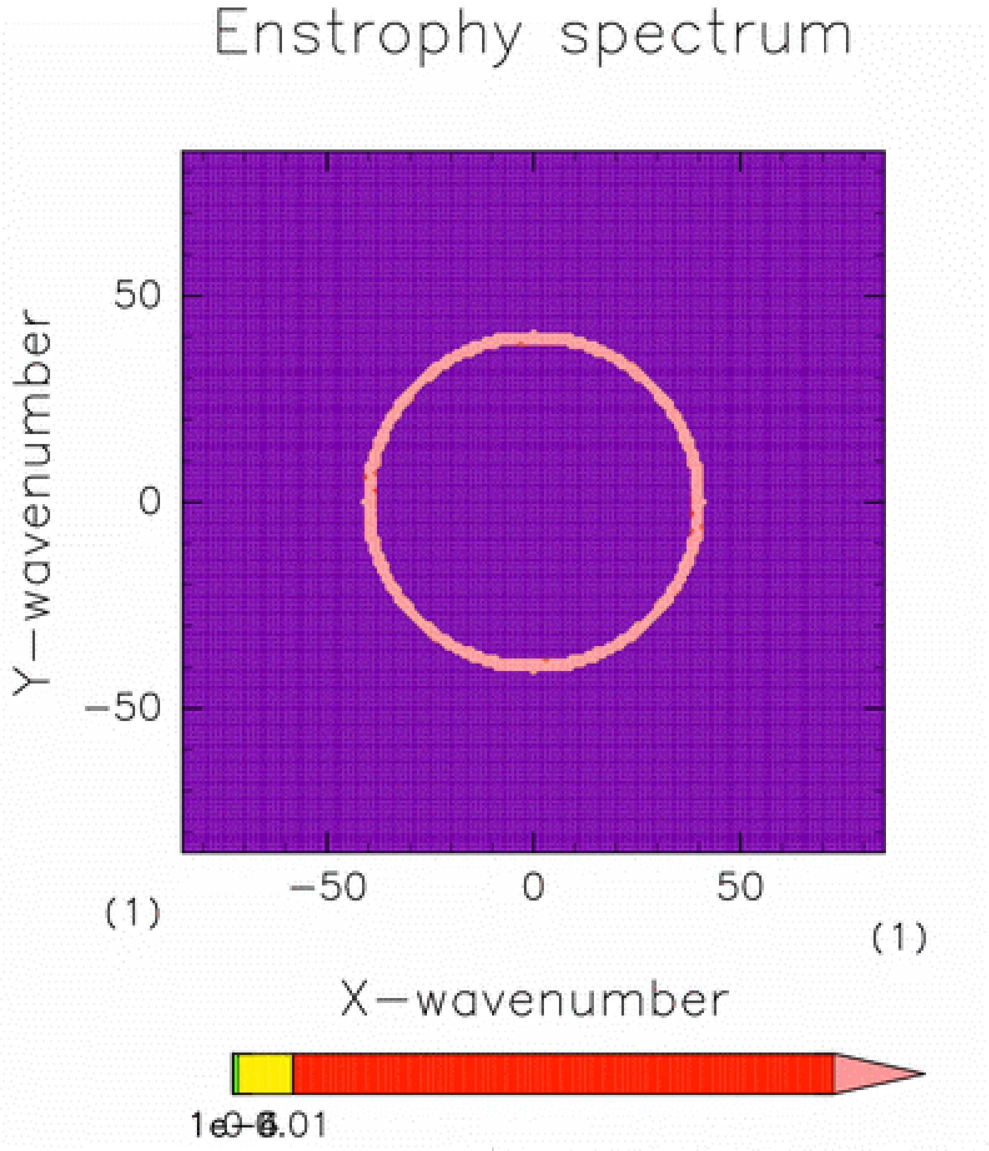
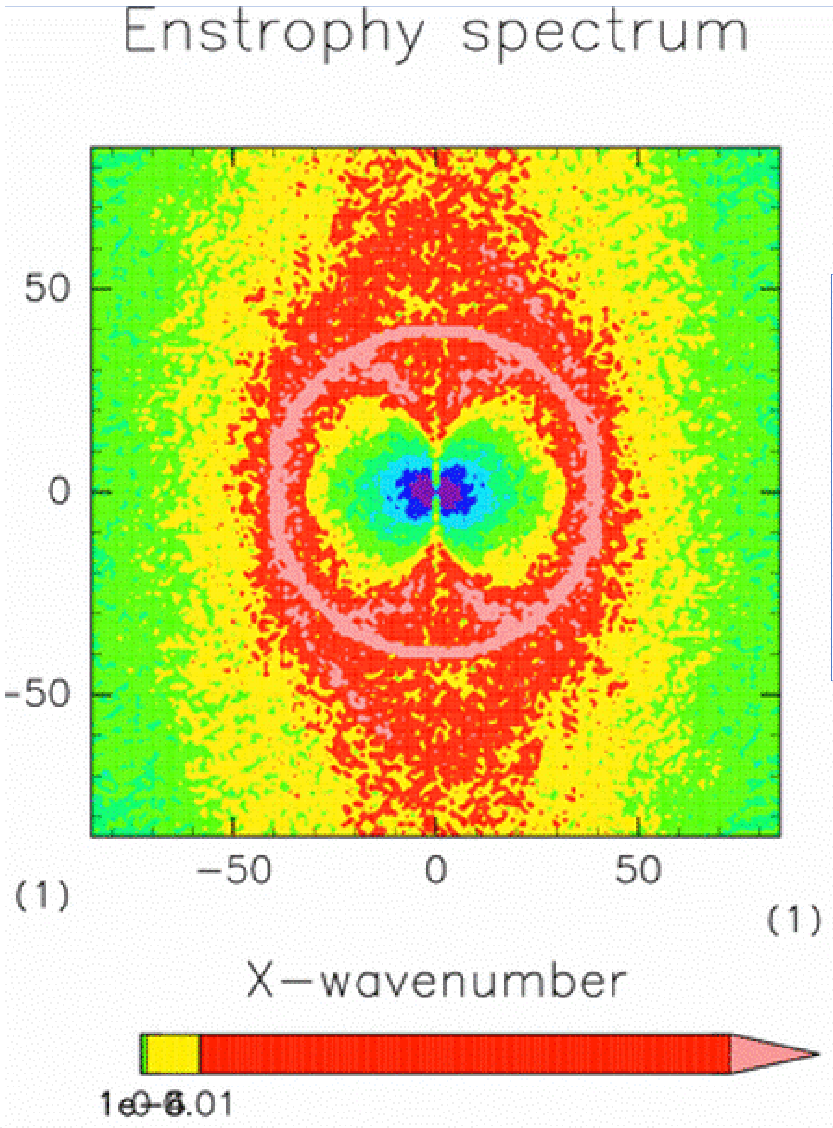


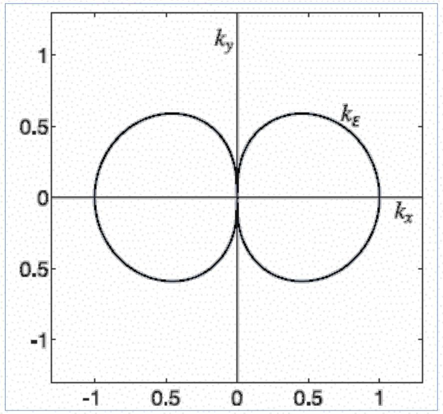
Fig. 12.2 The anisotropic wave–turbulence boundary k_ε, in wave-vector space calculated by equating the turbulent eddy transfer rate, proportional to $k^{2/3}$ in a $k^{-5/3}$ spectrum, to the Rossby wave frequency $\beta k^x/k^2$, as in (12.14).The wavenumbers are scaled such that $\beta^3/\varepsilon = 1$.

Within the dumbbell Rossby waves dominate and energy transfer is inhibited. The inverse cascade plus Rossby waves thus leads to a generation of zonal flow.

β 面乱流の dumbbell スペクトル







t=0.44 1

β 面上の非粘性 2 次元流

β 面上の流れ関数と運動方程式

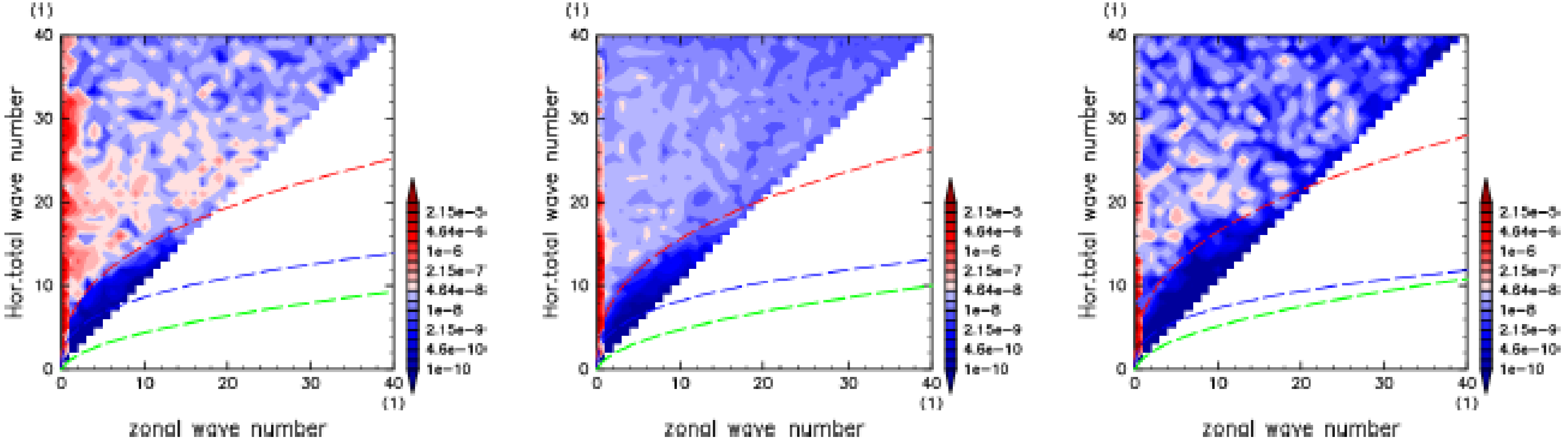
$$
\psi = \psi(x, y), \quad (u, v) = (-\partial_y \psi, \partial_x \psi),
$$
$$
\partial_t \omega + J(\psi, \omega) + \beta \partial_x \psi = 0, \quad (\omega = \Delta \psi).
$$

Fourier 級数 (Rossby 波) 展開

$$
\psi(x, y) = \sum_{m \in \mathbb{Z}} \sum_{n \in \mathbb{Z}} a_{m,n} \exp[i(mx + ny - \omega_{m,n} t)],
$$
$$
\omega_{m,n} = -\frac{\beta m}{m^2 + n^2}.
$$

回転球面上の 2 次元流 （thanks to 竹広さん）

2D kinetic energy spectrum



$t = 10^3$ $t = 10^5$ $t = 5 \times 10^6$

red :m_E, blue :m_R, green:m_H

- Inhibit of fluid motion in “dumbbell shape region”
- Anisotropic inverse cascade occurs around n axis:
  $\Rightarrow$ transition to larger scale structures in latitudinal direction

非線形相互作用の特徴

Zonal flow 生成の弱非線形理論

平均流の加速

２次の方程式

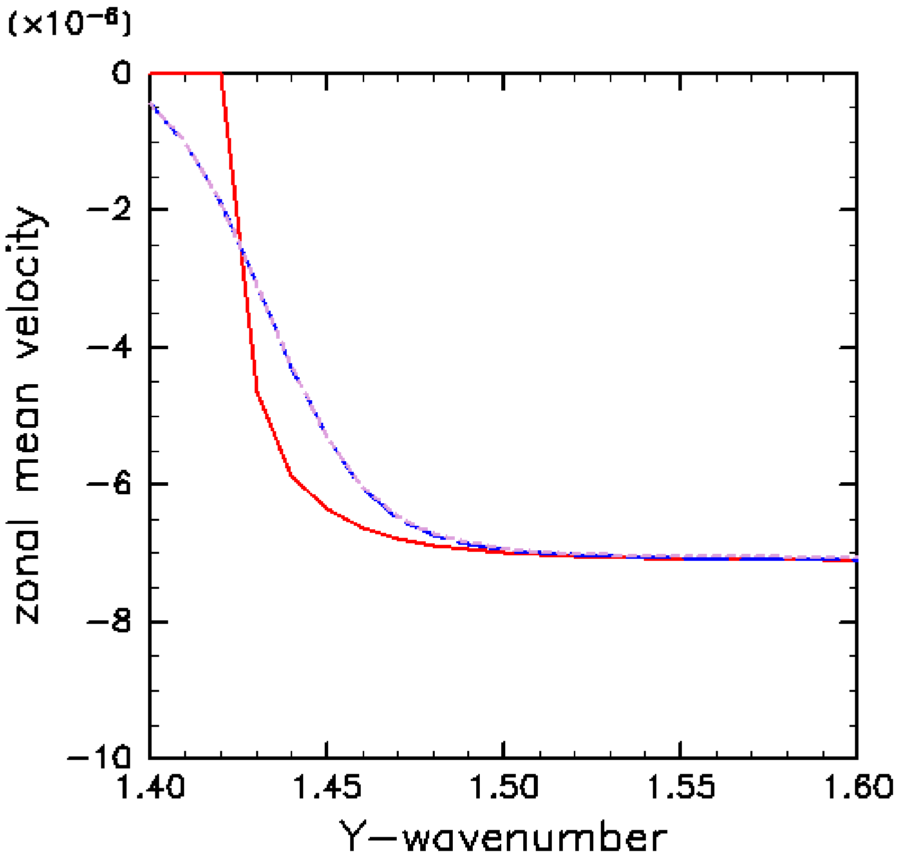
$$\frac{\partial}{\partial t}\overline{u}_2 = -\frac{\partial}{\partial y}\overline{u_1 v_1} + \frac{1}{R}\frac{\partial^2}{\partial y^2}\overline{u}_2.,$$

を y 積分および t 積分して平均流の全加速量を評価する．

$$\int_{y_1}^{y_2} \overline{u}_2(y, t_2)\, dy = -\left[\int_{t_1}^{t_2} \overline{u_1 v_1}(y_2, t)\, dt - \int_{t_1}^{t_2} \overline{u_1 v_1}(y_1, t)\, dt\right].$$

平均流加速の理論値と実験値

- 理論値と計算値の比較
  - Violet line: CHM 方程式の full simulation
  - Blue line: packet の数値計算による |R|, |T| を用いた理論値
  - Red line: 固有関数計算による |R|, |T| を用いた理論値



Beyond weakly nonlinear theory …

- Dynamics of two-dimensional fluid motion on rotating sphere.
  - Zonal flow formation
  - Theories based on
    - Energy inverse cascade
    - Critical layer of Rossby waves
    - Wave-mean flow interaction
      - Weakly nonlinear theory
    - (quasi) conserved quantities
    - …

- How is the nonlinear interaction?

Rotating sphere

- 2D non-rotating system (2D N-S eqn.)

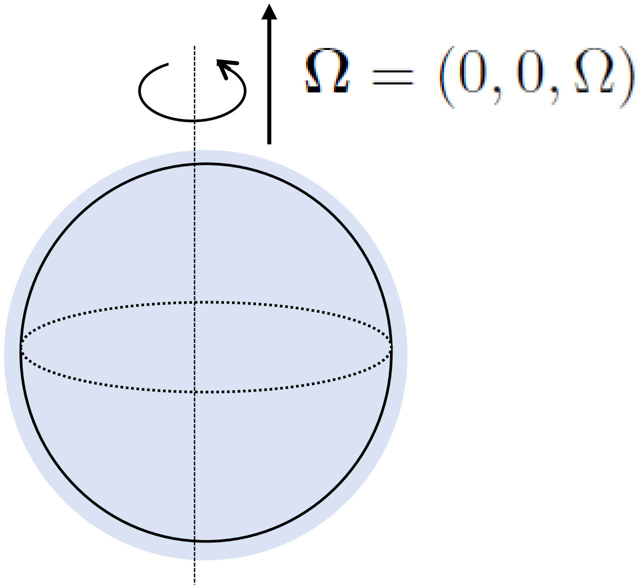
  ⇨ large-scale homogeneous and isotropic vortex

- 2D rotating system

  - **rotating sphere**

  ⇨ large-scale zonal flow formation

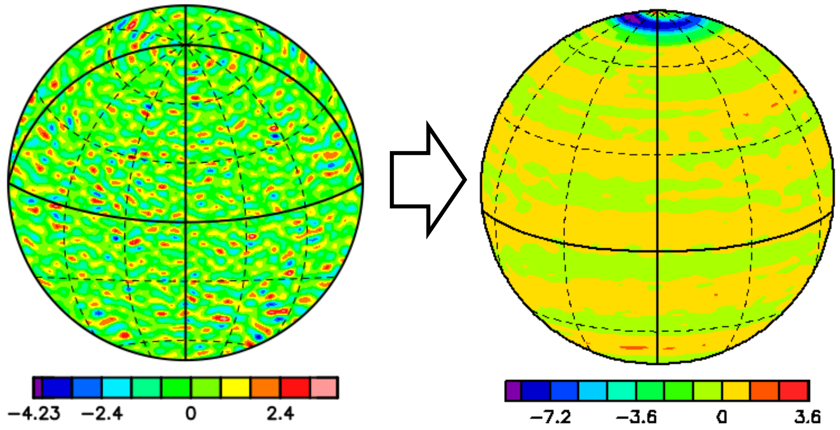
  (**inhomogeneous and anisotropic flow**)



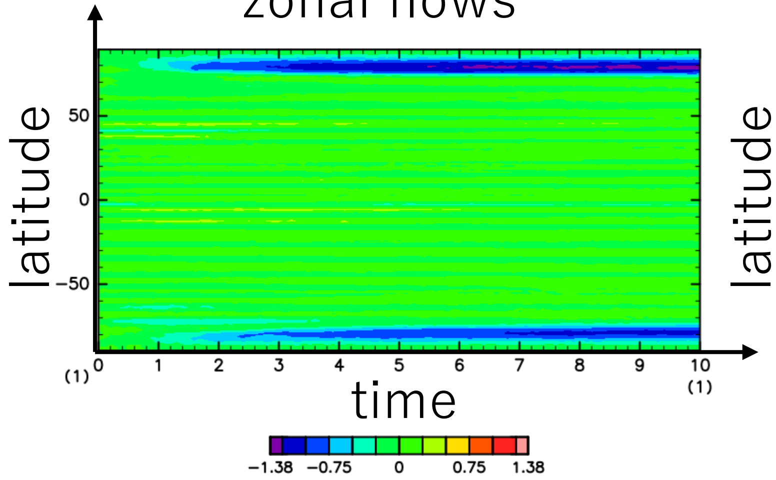
Zonal flow formation

- Unforced: westward circumpolar jets (Yoden and MY 1993)

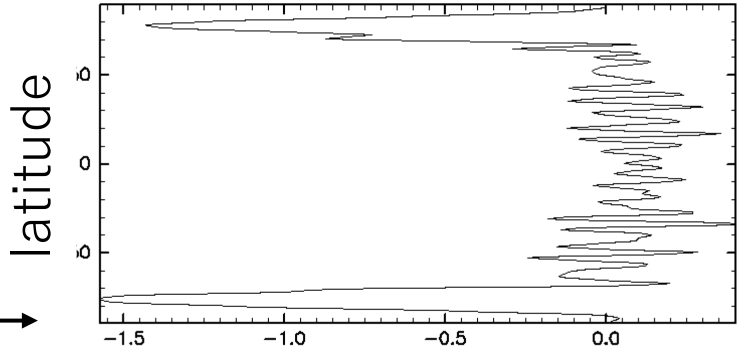
longitudinal-velocity



temporal development of zonal flows



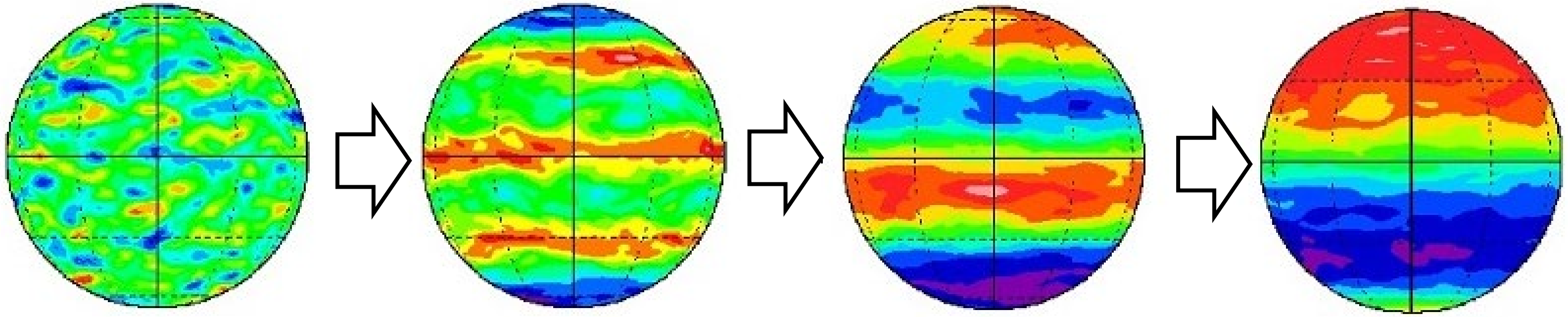
westward circumpolar jets



- Forced: multiple zonal band structure → a few large zonal flows

(Nozawa and Yoden 1997 , Obuse et al. 2010)

longitudinal-velocity



Inviscid dynamics

$$\frac{\partial \zeta}{\partial t} + J(\psi, \zeta) + 2\Omega \frac{\partial \psi}{\partial \lambda} = \cancel{\nu(\nabla^2 + 2)\zeta}$$

$$J(f, g) = \partial_\lambda f \partial_\mu g - \partial_\lambda g \partial_\mu f$$

λ : longititude, φ : latitude, μ : sin(latitude),

$\psi(\lambda, \mu, t)$: stream function,

$$u_\lambda = -\frac{\sqrt{(1-\lambda^2)}}{a}\frac{\partial \psi}{\partial \mu}, \quad u_\varphi = \frac{1}{a\sqrt{(1-\lambda^2)}}\frac{\partial \psi}{\partial \lambda},$$

$\zeta(\lambda, \mu, t) \equiv \nabla^2 \psi(\lambda, \mu, t)$: vorticity,

Ω=10000 : rotation rate of the sphere,

Ω

Nonlinear interaction coefficients

Time derivative of stream function spectrum:

$$\psi(t,\lambda,\mu) = \sum_{n=0}^{N}\sum_{m=-n}^{n} \psi_n^m(t)\, Y_n^m(\lambda,\mu)$$

$$\frac{\partial \psi_n^m(t)}{\partial t} = \frac{2im\psi_n^m(t)}{n(n+1)} + \frac{i}{2}\sum_{s=0}^{N}\sum_{r=-s}^{s}\sum_{k=0}^{N}\sum_{j=-k}^{k} \psi_k^j(t)\psi_s^r(t) H_{kns}^{jmr}$$

from linear term   from nonlinear term

$$H_{kns}^{j0-j} = \frac{s(s+1)-k(k+1)}{n(n+1)} L_{kns}^{j0-j}$$

$$L_{kns}^{j0-j} = (-1)^j L_{nks}^{0jj}$$

$$= (-1)^j \{E_{nks}^{0jj} - E_{skn}^{jj0}\}$$

$$= (-1)^j E_{nks}^{0jj}$$

$$E_{nks}^{0jj} = j\sqrt{2n+1}\sum_{q}\sqrt{2q+1}\int_0^{\pi} P_q^0 P_k^j P_s^j \sin\theta d\theta$$

$$\int_0^{\pi} P_a^b P_c^d P_e^f \sin\theta d\theta$$

$$= \frac{(e+a-c-1)!![(2c+1)(2a+1)(2e+1)]^{\frac{1}{2}}}{(e+c-a)!!(a+c-e)!!(c+a+e+1)!!}$$

$$\times \left[\frac{(c+d)!(c-d)!(a-b)!(e-f)!}{2(a+b)!(e+f)!}\right]^{\frac{1}{2}}$$

$$\times \sum_{h=0}^{c-d} \frac{(-1)^{\frac{1}{2}(e-a+c)+f+h}(e+f+h)!(a+c-f-h)!}{(c-d-h)!h!(e-f-h)!(a-c+f+h)!}$$

(Hull 1951)

necessary conditions for nonzero H_{kns}^{j0-j}:

- $q = n-1, n-3, n-5, \cdots, 1 \text{ or } 0$
- $n+k+s =$ odd integer
- $|n-s| < k < n+s$ (Silberman, 1954)

Nonzero triad interaction of Rossby waves

Triad interaction of three Rossby waves $Y_k^q \times Y_m^r \to Y_n^p$
has to satisfy

Necessary conditions for **three-wave interaction**

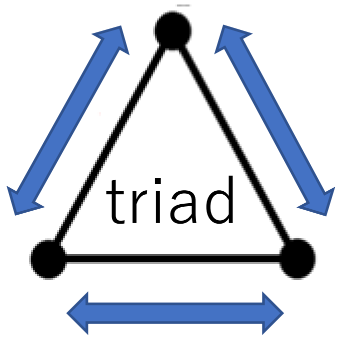
$$\begin{cases} p = q + r, \\ |k - m| \le n \le k + m, \\ n + k + m = \text{odd}, \\ n, k, m > 0,\ k \ne m,\ n \ne k,\ n \ne m, \end{cases}$$

Additional condition for **resonant interaction**

$$\frac{p}{n(n+1)} = \frac{q}{k(k+1)} + \frac{r}{m(m+1)}$$

Rossby waves

$$\begin{cases} Y_n^m(\lambda, \mu) \exp(i\omega t), \\ \omega = \dfrac{-2m\Omega}{n(n+1)} \end{cases}$$



three-wave nonlinear interaction

Parity

球面調和関数

$$Y_\ell^m(\mu,\varphi) = (-1)^{(m+|m|)/2}\sqrt{\frac{2\ell+1}{2}\frac{(\ell-|m|)!}{(\ell+|m|)!}}P_\ell^{|m|}(\mu)\sqrt{\frac{1}{2\pi}}e^{im\varphi},$$

$$P_\ell^{|m|}(\mu) = (1-\mu^2)^{|m|/2}\frac{d^{|m|}}{d\mu^{|m|}}P_\ell(\mu),$$

$$P_\ell(\mu) = \frac{1}{2^\ell \ell!}\frac{d^\ell}{d\mu^\ell}(\mu^2-1)^\ell.$$

(参照：https://dora.bk.tsukuba.ac.jp/~takeuchi/ の量子力学１の球面調和関数)

対称性 ($\mu \to -\mu$ についてのパリティ)

$$P_\ell(-\mu) = (-1)^\ell P_\ell(\mu),$$

$$P_\ell^{|m|}(-\mu) = (-1)^{\ell-m}P_\ell^{|m|}(\mu),$$

$$Y_\ell^m(-\mu,\varphi) = (-1)^{\ell-m}Y_\ell^m(\mu,\varphi).$$

Subspaces

$$Y_\ell^m(-\mu,\varphi) = (-1)^{\ell-m} Y_\ell^m(\mu,\varphi).$$

- 分類
  - ℓ の偶奇　E，O
  - mの偶奇　e，o
  - ex:　E_o = [ℓ が偶数，mが奇数の Y_ℓ^m 全体が張る線形空間]

- 非線形項の入る空間

  ex: J(E_o, O_o) ϵ E_e

	E_e	E_o	O_e	O_o
E_e	O_e	O_o	E_e	O_o
E_o	O_o	O_e	E_o	E_e
O_e	E_e	E_o	O_e	O_o
O_o	E_o	E_e	O_o	O_e

Invariant spaces

- 5 invariant spaces

南北対称流の一部　　　　　南北対称流

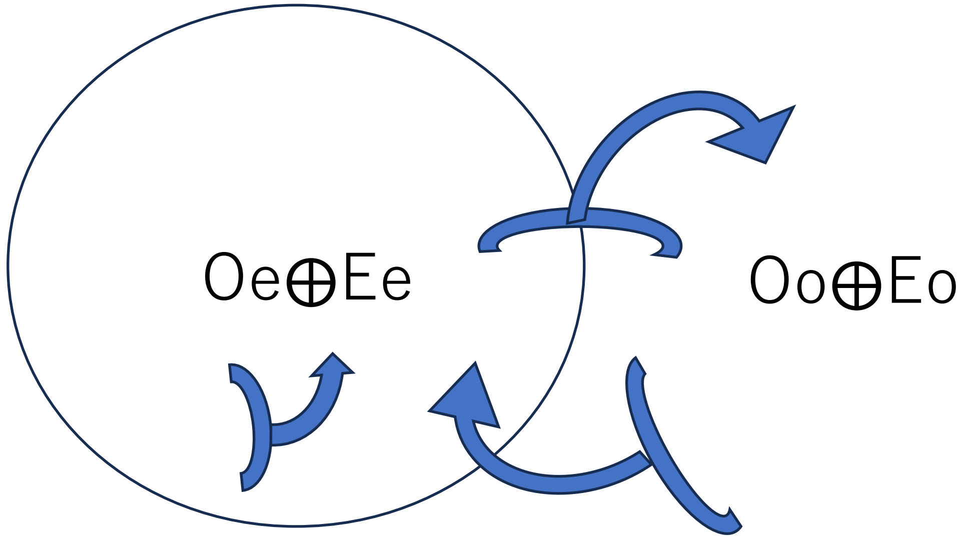
Oe ⊂ Oe⊕Ee, Oe⊕Eo, Oe⊕Oo

⊂ Oe⊕Ee⊕Eo⊕Oo

すべての流れ

	E_e	E_o	O_e	O_o
E_e	O_e	O_o	E_e	O_o
E_o	O_o	O_e	E_o	E_e
O_e	E_e	E_o	O_e	O_o
O_o	E_o	E_e	O_o	O_e

Klein の 4 元群
（Oe : 単位元）



Energy goes to Oe.

Sum of energy of zonal modes $\sum_{\ell:odd} E_\ell^0(\lambda,\mu)$, $\sum_{\ell:even} E_\ell^0(\lambda,\mu)$

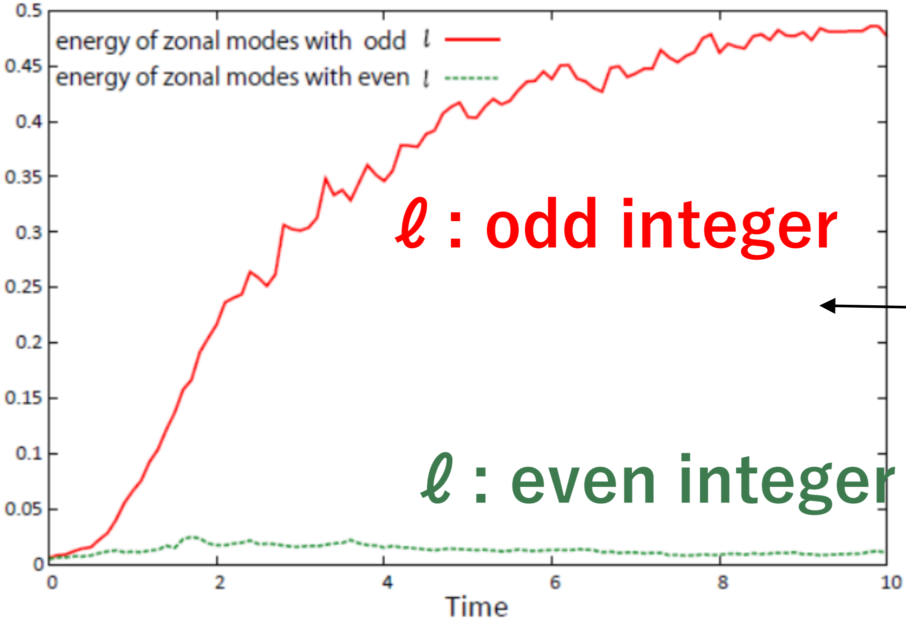
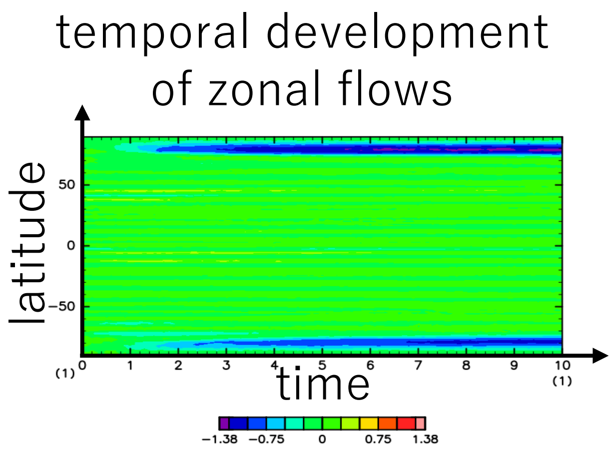


Fig. 3. (Color online) Temporal variation of the $Y^0_{l=odd}$ and $Y^0_{l=even}$ modes from $t = 0$ to 10 in $\nu_{2p} = 0.0$ case.



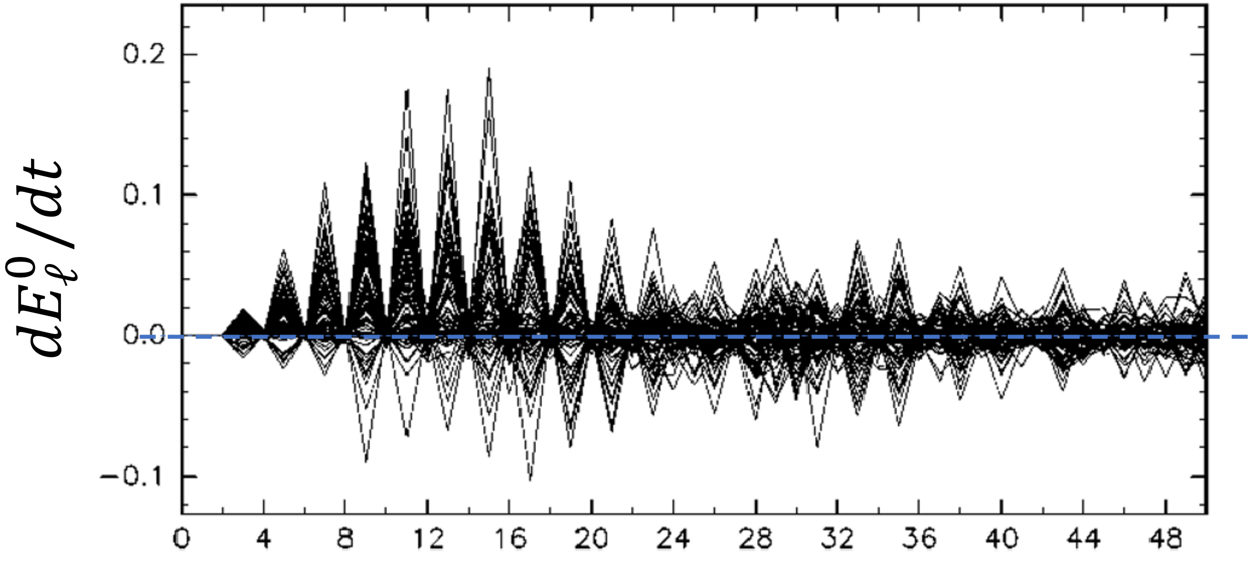
Energy is only accumulated to zonal modes with odd ℓ , $Y^0_{\ell=odd}$.

(Obuse and MY 2020)

Time variation of dE_ℓ^0/dt by nonlinear interaction

t=0-3.0

dE_ℓ^0/dt(overplotted)

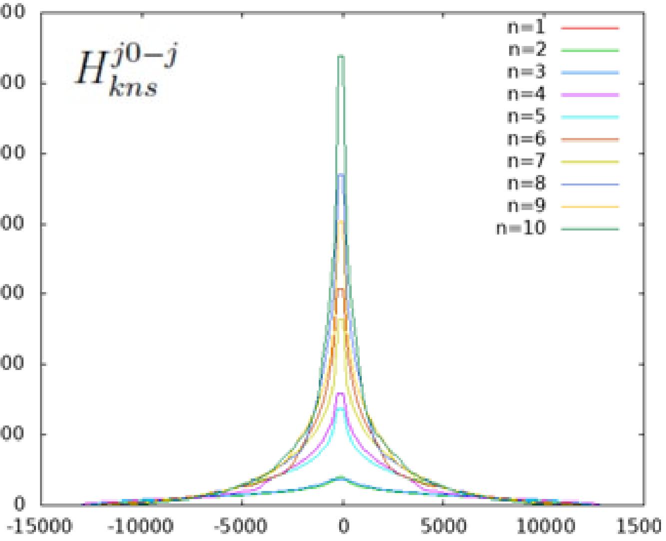


Total wave number(ℓ)

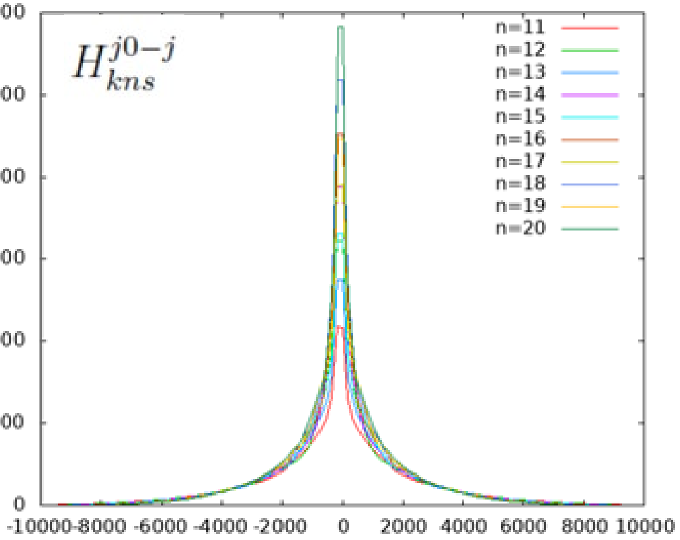
At low ℓ ,

- $\frac{dE_{\ell=odd}^0}{dt}$ is almost always positive
- $\frac{dE_{\ell=even}^0}{dt}$ is small but nonzero

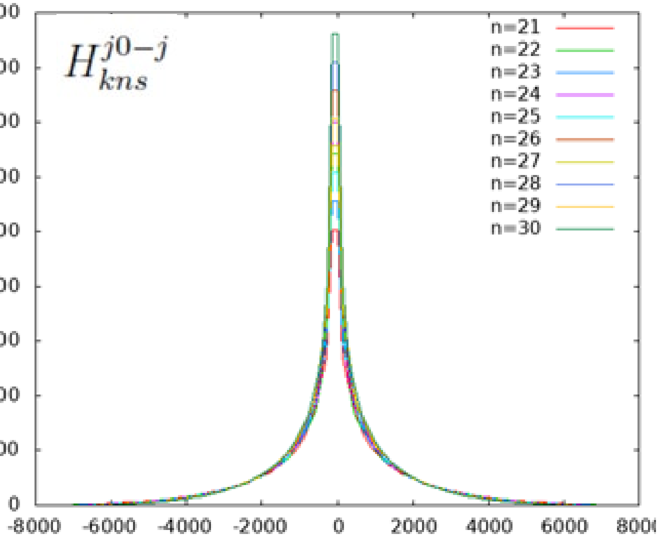
Coefficients of nonlinear interactions H_{kns}^{j0-j} $(Y_k^j \times Y_s^{-j} \rightarrow Y_n^0)$



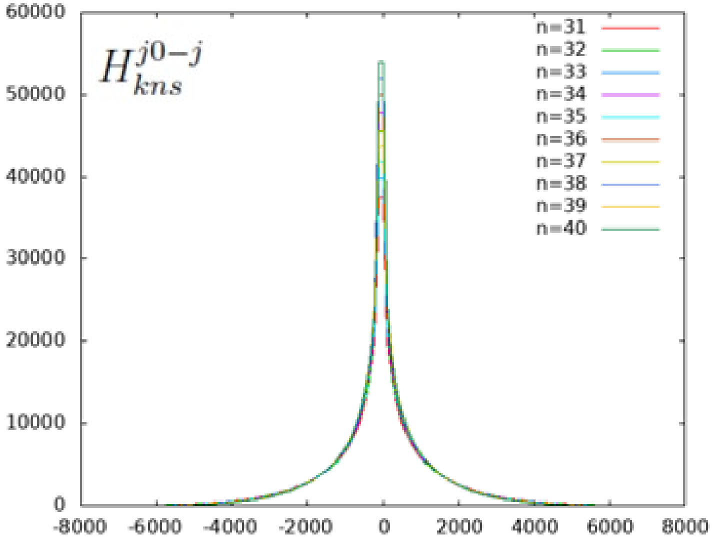
n=1~10



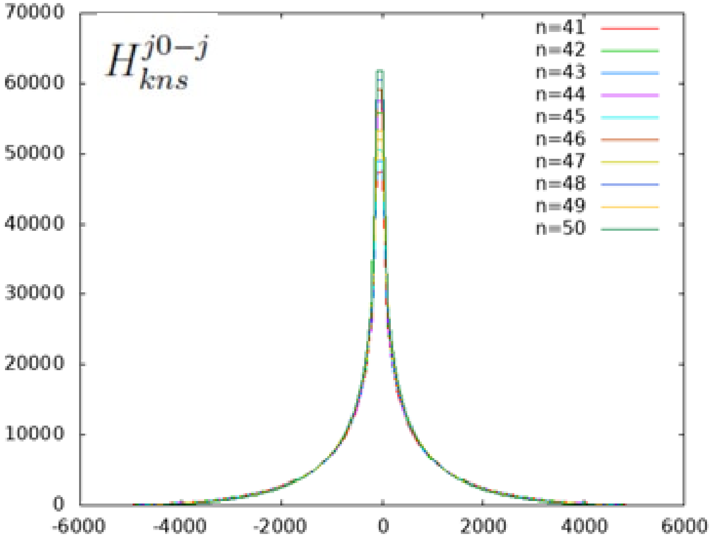
n=11~20



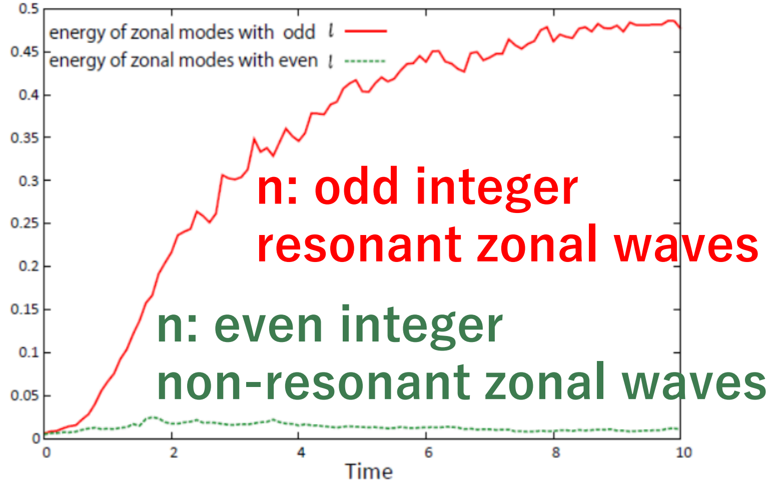
n=21~30



n=31~40



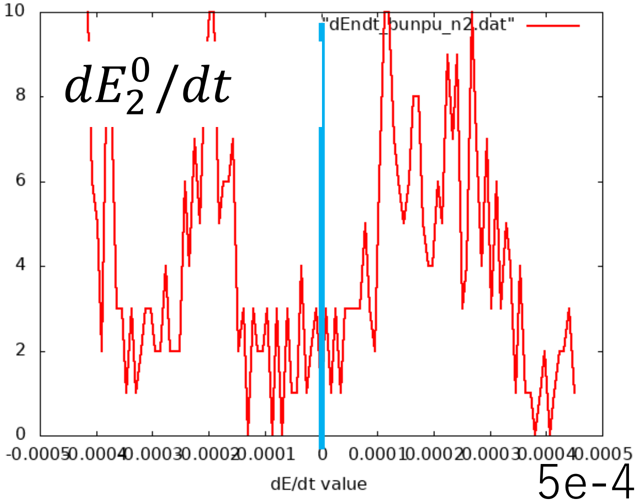
n=41~50

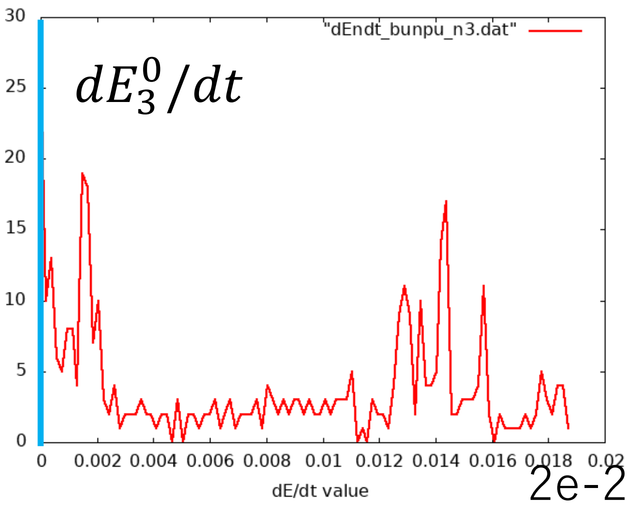


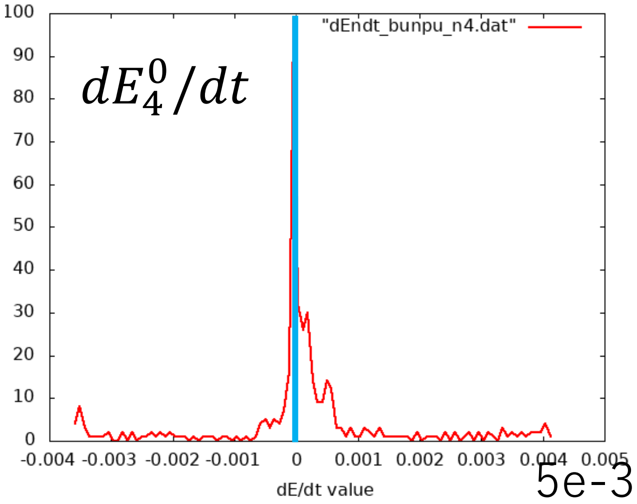
Triads including waves with n:even take nonzero H_{kns}^{j0-j} values

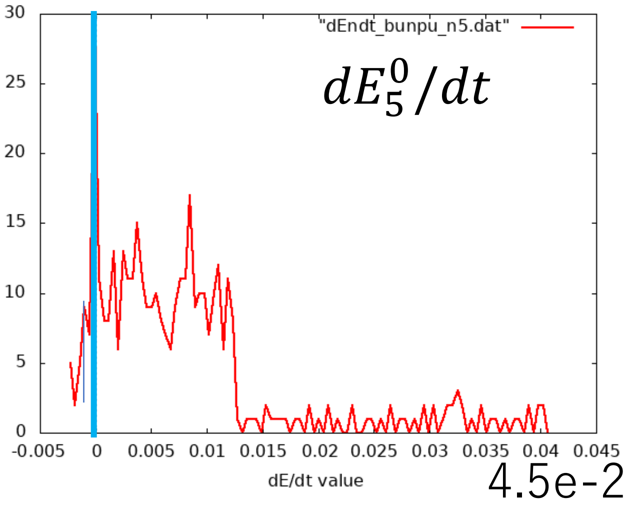
Time variation of dE_ℓ^0/dt

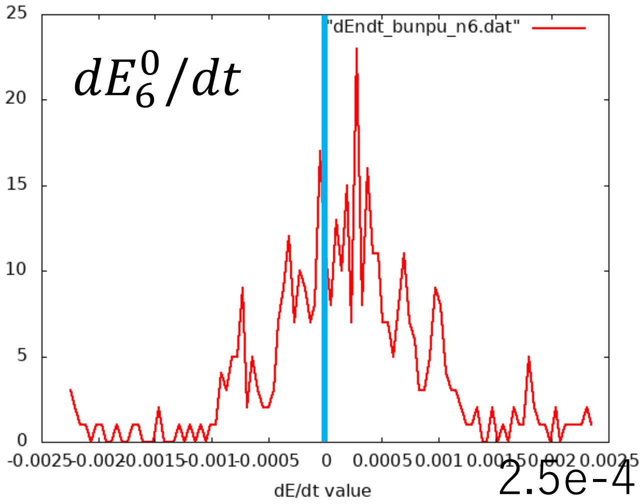
Number distribution of dE_ℓ^0/dt at $t \in [0,0.8]$ (400 data, 100 bins)

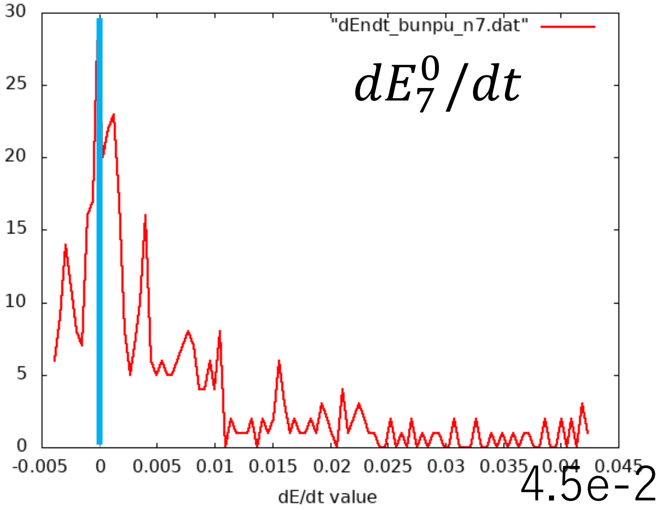












ℓ =odd:

Nonsymmetric distribution

→ positive energy accumulation

ℓ =even:

Symmetric distribution

→ cancel out in time

Three wave interaction and resonance

Nonlinear interaction of three Rossby waves $Y_k^q \times Y_m^r \to Y_n^p$ is resonant interaction when They satisfy

$$p = q + r,$$
$$\frac{p}{n(n+1)} = \frac{q}{k(k+1)} + \frac{r}{m(m+1)},$$
$$|k - m| \leq n \leq k + m,$$
$$n + k + m = \text{odd},$$
$$n, k, m > 0,\ k \neq m,\ n \neq k,\ n \neq m,$$
$$p \neq 0,\ q \neq 0,\ r \neq 0.$$

Rossby waves

$$\begin{cases} Y_n^m(\lambda, \mu) \exp(i\omega t), \\ \omega = \dfrac{-2m\Omega}{n(n+1)} \end{cases}$$

In fact,

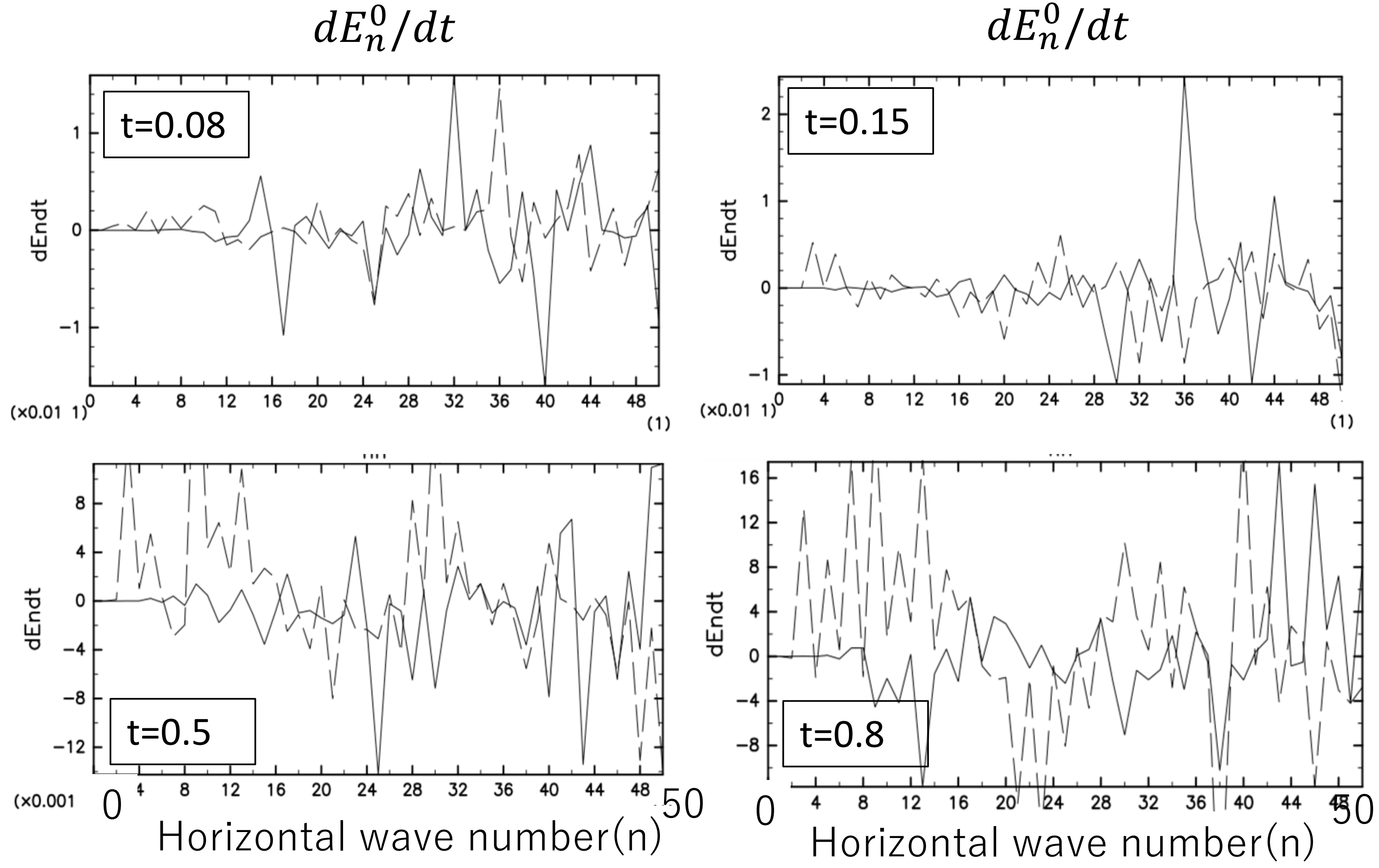
$Y^0_{\ell=odd}$ can be a member of resonant triads (**resonant waves**)
$Y^0_{\ell=even}$ are not resonant waves

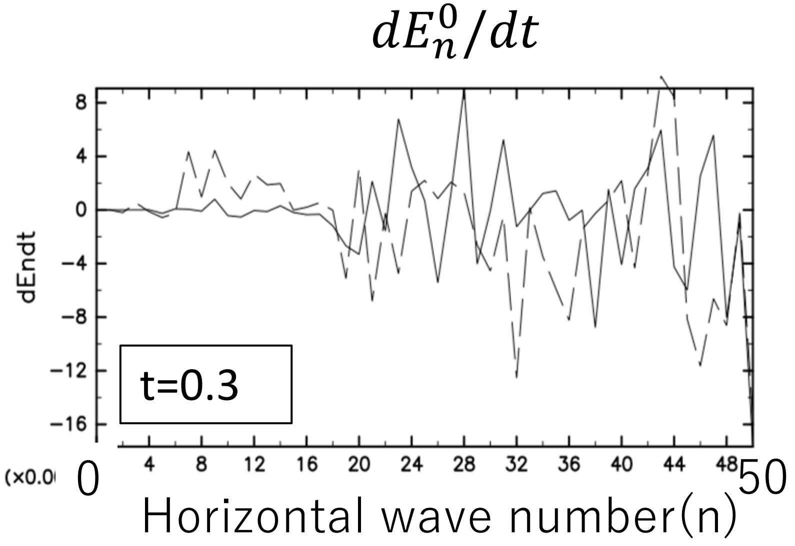
No energy is transferred to zonal waves by resonant interactions
(Reznik et.al. 1993, Obuse et al. 2019)

To zonal modes by near resonant interaction?

$\left(\frac{dE_n^0}{dt}\right)$ by near-resonant and non-resonant interactions $\left(|\omega_C - (\omega_A + \omega_B)| \lesseqgtr 1.1Ro\right)$

※Solid line : non-resonant, Dashed line: near-resonant





Near-resonant interaction is predominant in the low wavenumber region.

Nonlinear interaction on a rotating sphere

- Energy accumulates on odd total wavenumber modes, which constitutes an invariant set of the nonlinear interaction.

- Further, the energy is concentrated in zonal flow modes through near-resonant interaction. The zonal flow modes are energetically isolated from non-zonal flow modes.

- The effective energy transfer is nonlocal in the wavenumber space.

- Reference

Yusuke Hagimori, Kiori Obuse and Michio Yamada:
Effect of non-local near-resonant interactions of Rossby waves on formation of large-scale zonal flows in unforced two-dimensional turbulence on rotating sphere. (2024, to appear in Physics of Fluids)

Fin