回転系での縞状構造の形成と消失について

竹広真一

京都大学数理解析研究所

2024 年 3 月 17 日

「GFD セミナー 2024 特別編」@休暇村支笏湖

Outline

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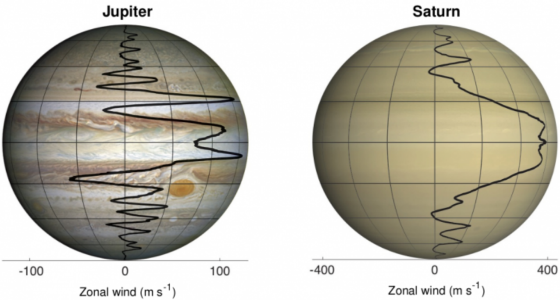
Surface flows of gas giant planets

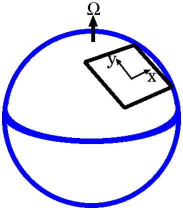


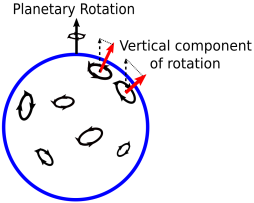
Kaspi et al. (2020)

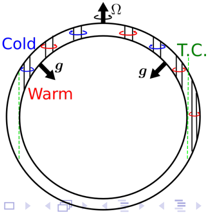
- Surface flows of Jupiter and Saturn
  - Broad prograde jets around the equator (equatorial superrotation)
  - Nnarrow alternating jets in mid-/high-latitudes (band structure)
- Origin of the surface jets
  - Convective motions in the "deep" region
  - Fluid motions in the "shallow" weather layer
- Formation of band structure
  - Explanation by forced turbulence models of several rotating systems

Zonal flow formation in rotating systems : outline

- Forced turbulence models of several rotating systems
  - Initially, band structure emerges
  - But jets merge after long time
  - 2D spectrum view : dumbbell feature
- Important concepts
  - Potential vorticity (PV), β effect (spatial gradient of PV)
  - Anisotropic inverse cascade
- Examples of rotating systems
  - 2D β plane
  - 2D rotating sphere
  - 2D columnar vortices inside TC excited by thermal convection in a rapidly rotating (thin) spherical shell







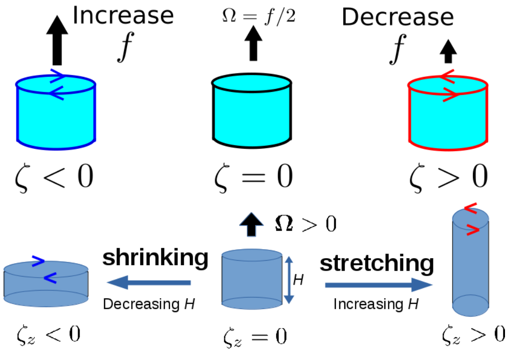
Potential Vorticity (PV) conservation

- Conservation of PV $\sim$ local angular momentum conservation

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla\right) q = 0, \quad q = \frac{f+\zeta}{H}.$$

(f: vorticity of planetary rotation, ζ: vorticity of flows, H: height of fluid layer)

- β effect : Variation of ambient PV $f/H \Rightarrow$ vorticity of flow ζ
- Result of conservation of PV : Rossby waves



Outline

β plane model: Governing equations

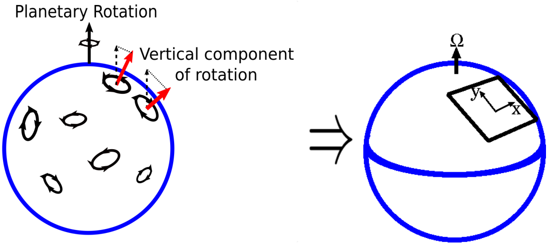
- 2D barotopic vorticity equation

$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi, \nabla^2\psi) + \beta\frac{\partial \psi}{\partial x} = F - D, \quad u = -\frac{\partial \psi}{\partial y}, \ v = \frac{\partial \psi}{\partial x}.$$

  - ψ : stream function, F : small scale forcing, D : dissipation
  - Without forcing and dissipation $\Rightarrow$ Hasegawa-Mima eq.
- Conservation of PV

$$\frac{\partial q}{\partial t} + J(\psi, q) = F - D, \quad q = \nabla^2\psi + \beta y$$

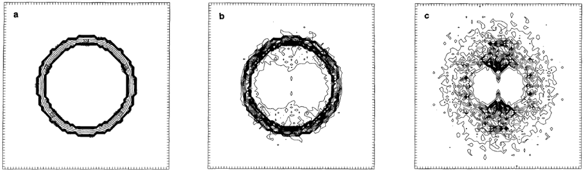
- Only vertical component of vorticity affects the dynamics
  - $f = 2\Omega \sin\varphi$ varies with latitude φ, $\beta = df/dy = 2\Omega\cos\varphi/a$



β plane model : pioneering studies

- Rhines (1975)
  - observation of band structure formation
  - order estimation of band width (Rhines scale)
- Vallis and Maltrud (1993)
  - 2D kinetic energy spectrum
  - Explanation of dumbbell shape spectrum

Stream function
Rhines (1975)



Time development of 2D kinetic spectrum
(Vallis and Maltrud 1993)

Rhines scale

- Variation of f (and/or H) : β effect $\Rightarrow$ emergence of zonal band structure
  - Conservation of PV : difficult to move in latitudinal direction
    $\Rightarrow$ Dominance of zonal flow, band structure
  - Rhines (1975) :
    advection of PV of flow (Jacobian) $\sim$ advection of environment PV (β term)

$$|J(\psi, \nabla^2\psi)| \sim \left|\beta \frac{\partial \psi}{\partial x}\right| \Rightarrow \frac{U^2}{L^2} \sim \beta U \quad U : \text{Flow amplitude}$$

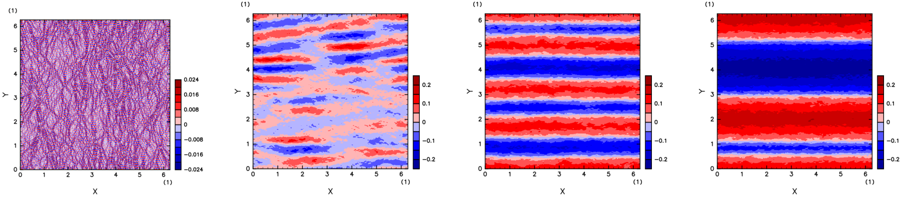
$$\text{Rhines scale :} \quad L_\beta = \sqrt{\frac{U}{\beta}}$$

  - Rhines scale is frequently used for estimation of band width
  - It can be derived by the homogeneity of PV.

$$\zeta \sim \delta f = \beta L \Rightarrow \frac{U}{L} \sim \beta L$$

β plane model : Numerical example

Snapshots of x-velocity

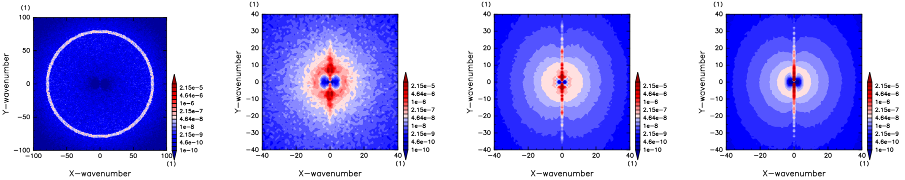


$t = 10^1$ $t = 10^3$ $t = 10^5$ $t = 5 \times 10^6$

- Traditional knowledge and understanding
  - Small scale forcing, generation of small vortices
    $\Rightarrow$ larger vortices by N.L.T : inverse cascade
    $\Rightarrow$ inverse cascade stops at Rhines scale (dominance of β term)
    $\Rightarrow$ band structure is formed
  - Rhines scale $\sim$ expected band width
- Longer time integration : jets merge, broader band structure appears
- Rhines scale is not good indicator for band width

β plane model : example of 2D-spectrum

2D kinetic energy spectrum



$t = 10^1$ $\quad$ $t = 10^3$ $\quad$ $t = 10^5$ $\quad$ $t = 5 \times 10^6$

red : k_E, blue : k_R, green : k_H

- Inhibit of fluid motion in “dumbbell shape region”
- Anisotropic inverse cascade occurs around k_y axis:
  $\Rightarrow$ transition to larger scale structures in y direction

β plane model : 2D kinetic energy spectrum

- Derivation of dumbbell like structure (Vallis and Maltrud, 1993)

  Turbulent frequency $\omega_t \sim$ frequency of Rossby waves, $\omega_R = -\dfrac{\beta k_x}{k_x^2 + k_y^2}$

- T.F.= energy dissipation rate ε

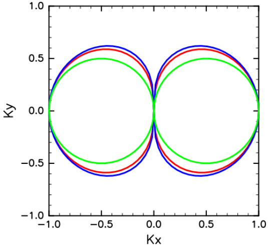
$$k_E = \left(\frac{\beta^3}{\varepsilon}\right)^{1/5}, \quad \begin{cases} k_{E,x} & = k_E \cos^{8/5}\theta \\ k_{E,y} & = k_E \sin\theta \cos^{3/5}\theta \end{cases}$$

- T.F. = velocity scale U

$$k_R = \sqrt{\frac{\beta}{U}} = \frac{1}{L_R}, \quad \begin{cases} k_{R,x} & = k_R \cos^{3/2}\theta \\ k_{R,y} & = k_R \sin\theta \cos^{1/2}\theta \end{cases}$$

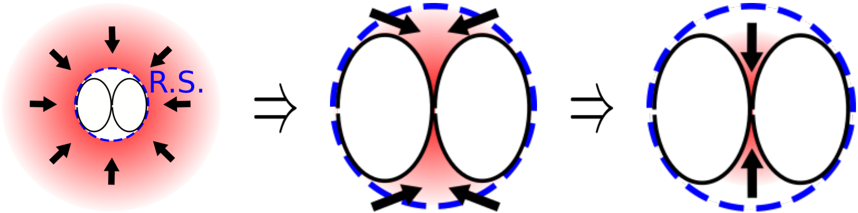
- T.F = vorticity scale Z

$$k_H = \frac{\beta}{Z} = \frac{1}{L_Z}, \quad \begin{cases} k_{H,x} & = k_H \cos^2\theta \\ k_{H,y} & = k_H \sin\theta \cos\theta \end{cases}$$



$(k_{E,x}, k_{E,y})$
$(k_{R,x}, k_{R,y})$
$(k_{H,x}, k_{H,y})$

β plane model : summary



- Homogeneous inverse cascade of small scale eddies
- Inhibit of fluid motion in "dumbbell shape region":
  $\Rightarrow$ homogeneous inverse cascade stops at Rhines scale
- Anisotropic inverse cascade towards k_y axis:
  $\Rightarrow$ band structure with multiple zonal jets appears
- Anisotropic inverse cascade along k_y axis:
  $\Rightarrow$ merger of thin multiple jets
  $\Rightarrow$ eventually, a few jets with maximum width appear

β plane model : Effects of friction

- General dissipation term : $D = -\nu_n(-\nabla^2)^n\zeta$
  - $n = 1$: normal viscosity, $n > 1$: hyper-viscosity
  - $n = 0$: Linear (Rayleigh,Ekman) friction, $n < 0$: hypo-viscosity
- Balance between energy input ε and dissipation:

$$E_k \sim \frac{\varepsilon}{2\nu_n k^{2n}}.$$

  - $n \geq 1$: E_k increases as k decreases $\Rightarrow$ inverse cascade may not stop
  - $n \leq 0$: E_k is constant or decreases as k decreases $\Rightarrow$ inverse cascade may stop
- Spatial scale by dissipation:

$$L_\nu = \left(\frac{\varepsilon}{\nu_n\beta^2}\right)^{1/2(2-n)}, \quad k_\nu = \left(\frac{\nu_n\beta^2}{\varepsilon}\right)^{1/2(2-n)},$$

For Ekman friction ($n = 0$) (Danilov and Guraeie, 2002, Vallis 2006),

$$L_E = \left(\frac{\varepsilon}{\nu_0\beta^2}\right)^{1/4}, \quad k_E = \left(\frac{\nu_0\beta^2}{\varepsilon}\right)^{1/4}.$$

Outline

Rotating sphere model : Governing equations

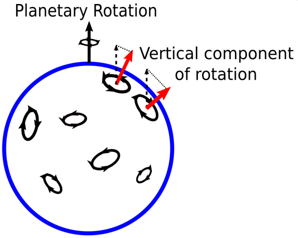
- 2D barotropic vorticity equation

$$\frac{\partial \zeta}{\partial t} + \frac{1}{a^2} J(\psi, \zeta) + \frac{2\Omega}{a} \frac{\partial \psi}{\partial \lambda} = F - D, \qquad u_\lambda = -\frac{\partial \psi}{\partial \varphi}, u_\varphi = \frac{1}{\cos \varphi} \frac{\partial \psi}{\partial \lambda}.$$

  - λ : longitude, φ : latitude, $\mu = \sin \varphi$.
  - $\psi(\lambda, \mu, t)$: stream function, $\zeta(\lambda, \mu, t) = \nabla^2 \psi$: vorticity
  - $J(\psi, \zeta) = (\partial_\lambda \psi)(\partial_\mu \zeta) - (\partial_\mu \psi)(\partial_\lambda \zeta)$
  - F : small scale forcing, D : dissipation

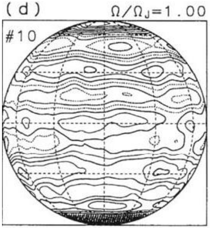
- Conservation of PV

$$\frac{\partial q}{\partial t} + J(\psi, q) = F - D,$$
$$q = \nabla^2 \psi + 2\Omega \sin \varphi$$

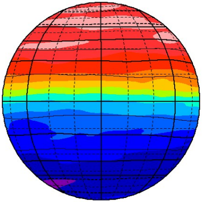


Rotating sphere model : pioneering studies

- Williams (1978)
  - 8-fold and equatorial symmetry
  - Random forcing is not isotropic
- Nozawa and Yoden (1997)
  - Whole sphere, isotropic random forcing
  - Various scale of random forcing and rotation
  - Band widths consistent with Rhines scale
- Obuse et al. (2019)
  - Long time integration of Nozawa and Yoden (1997)
  - Merging of zonal jets ⇒ Broader band structure appears



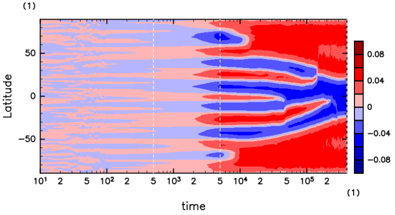
NY1997 : stream function



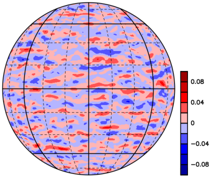
Obuse et al. (2000)
final mean zonal flows

Rotating sphere model : Numerical example

Time evolution of mean flow

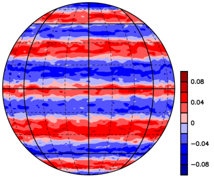


Snapshots of azimuthal velocity



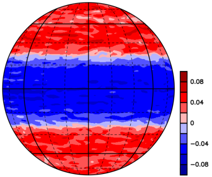
$t = 5 \times 10^2$

$\Rightarrow$



$t = 5 \times 10^3$

$\Rightarrow$



$t = 4 \times 10^5$

- Jets merge, broader band structure appears
- Seems to be a similar story with β plane

- Derivation of dumbbell like structure (Huang et al. 2001)
  Turbulent frequency $\omega_t \sim$ frequency of Rossby waves,
  $\omega_R = -\dfrac{2\Omega m}{n(n+1)}$
- T.F.= energy dissipation rate ε

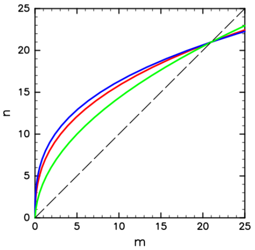
$$m_E(n) = \frac{\varepsilon^{1/3}}{2\Omega} n^{5/3}(n+1).$$

- T.F. = velocity scale U

$$m_R(n) = \frac{U}{2\Omega a} n^2(n+1)$$

- T.F = vorticity scale Z

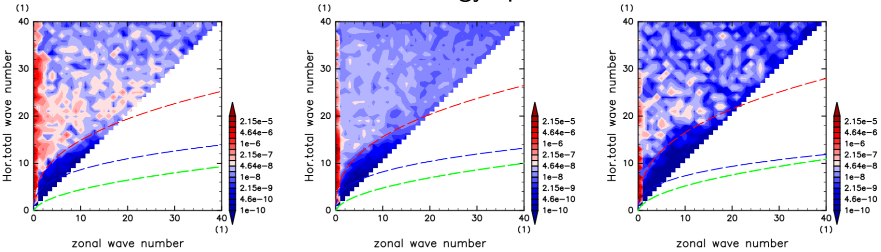
$$m_H(n) = \frac{Z}{2\Omega} n(n+1).$$



m_E, m_R, m_H

Rotating sphere model : example of 2D-spectrum

2D kinetic energy spectrum



$t = 10^3$ $\quad$ $t = 10^5$ $\quad$ $t = 5 \times 10^6$

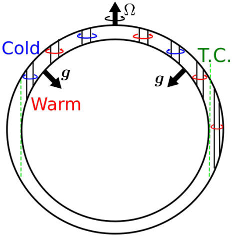
red :m_E, blue :m_R, green:m_H

- Inhibit of fluid motion in “dumbbell shape region”
- Anisotropic inverse cascade occurs around n axis:
  $\Rightarrow$ transition to larger scale structures in latitudinal direction

Outline

Spherical shell model : outline

- Rapidly rotating thin spherical shell model
  = 2D turbulence forced by thermal convection in a spherical shell
- Outside TC (around equator, not focused on)
  - Single equatorial prograde jet emerges outside TC
- Inside TC (mid-/high latitudes)
  - Small scale convective motion is excited due to fast rotation
  - Convective motion induces 2D axial vortices
  - Multiple zonal alternating jets appears initially
  - After long time integration, jets merge, broader band structure appears

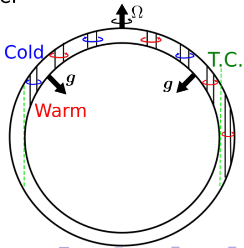


- Boussinesq fluid in a rotating spherical shell.
  - Scaling: the shell thickness, rotation period, temperature difference.

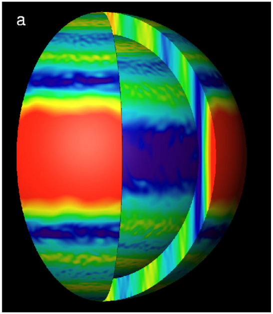
$$
\begin{aligned}
&\nabla\cdot\boldsymbol{u} = 0,\\
&\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + 2\boldsymbol{k}\times\boldsymbol{u} + \nabla p - \mathrm{E}\boldsymbol{d}(\boldsymbol{u}) = \frac{\mathrm{Ra}\mathrm{E}^2}{\mathrm{Pr}}\frac{\boldsymbol{r}}{r_o}T,\\
&\frac{\partial T}{\partial t} + (\boldsymbol{u}\cdot\nabla)T = \frac{\mathrm{E}}{\mathrm{Pr}}d(T).
\end{aligned}
$$

  - $\boldsymbol{d}(\boldsymbol{u})$, $d(T)$: Viscous/diffusion terms including hyper viscosity depending on the total horizontal wavenumber

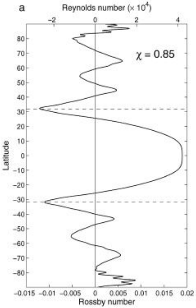
- Parameters:
  - Prandtl number: $\mathrm{Pr} = \nu/\kappa$
  - Rayleigh number: $\mathrm{Ra} = \dfrac{\alpha g_o \Delta T D^3}{\kappa\nu}$
  - Ekman number: $\mathrm{E} = \dfrac{\nu}{\Omega D^2}$
  - radius ratio: $\chi = r_i/r_o$



- Heimpel and Aurnou (2007)
  - Equatorial prograde jet, band structure in middle and high latitudes



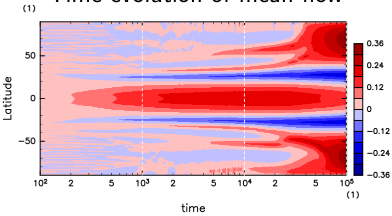
longitudinal velocity



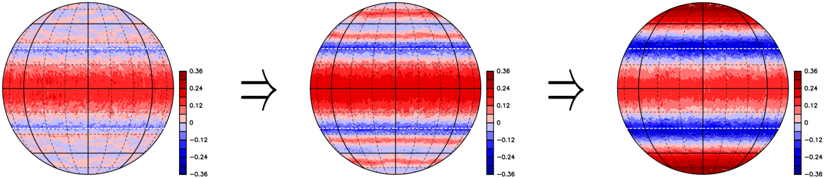
Mean zonal flows

  - 1/8 sector domain : not whole shell! (resolution $128 \times 512 \times 64$)
  - Short integration time : about 1600 rotation (0.02 viscous diffusion time)

Time evolution of mean flow



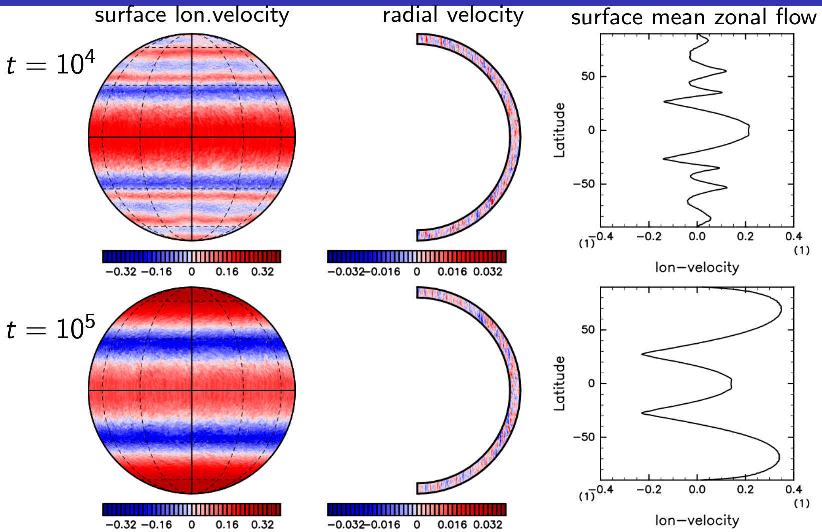
Snapshots of azimuthal velocity



$t = 10^3$ $\Rightarrow$ $t = 10^4$ $\Rightarrow$ $t = 10^5$

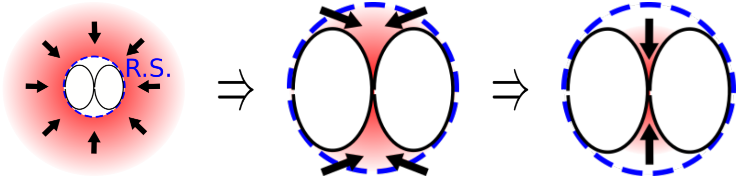
- Time development of azimuthal velocity for $\chi = 0.9,\ \mathrm{Pr} = 0.1,\ \mathrm{E} = 3 \times 10^{-6},\ \mathrm{Ra}^* = 0.05.$
- Broader band structure appears in mid-/high-atitudes

Spherical shell model : Numerical example



- Convective structure for $\chi = 0.9$, $\mathrm{Pr} = 0.1$, $\mathrm{E} = 3 \times 10^{-6}$, $\mathrm{Ra}^* = 0.05$.
- Broader band structure appears in mid-/high-latitudes

Outline



- Forced turbulence models of several rotating systems (β-plane, rotating sphere, rapidly rotating spherical shell)
  - Initially, band structure emerges
  - But jets merge after a long time
- Rhines scale : not valid for band width estimation
  - Scale at homogeneous inverse cascade stops
- 2D spectral view : dumbbell structure
  - Inhibit of fluid motion in "dumbbell shape region"
  - Anisotropic inverse cascade progress slowly
  - Single band with largest width will eventually appear
- Hypo-viscosity may prevent jets from merging

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