月の謎を解く

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The giant impact model

Hartmann-Davis (1975), Cameron-Ward (1976)

- In a later stage of Earth formation, a relatively large body collided obliquely. → large angular momentum → mantle materials are ejected → the Moon is made of rocky materials (no big core).
- **Classic calculation** (1986): **the Moon is mostly from the impactor** 😅
- This is OK if we are concerned only with the **bulk composition**.
- **Recent high-resolution geochemical measurements**

  → (i) **close similarity of isotopes between the Moon and Earth.**

  → **(ii) "wet" Moon**

(i) If the Moon is made mainly of the impactor, why are the isotopic ratios of the Moon so similar to those of Earth ?

→ **isotopic crisis** (how to explain the isotopic similarity together with the large angular momentum?)

(ii) "wet" Moon: Shouldn't water be lost during a giant impact?

Limitations of previous approaches
→ a new approach

- Conventional approaches:
  - **Fate of a giant impact is controlled by the "mechanics" of an impact not by the physical properties of matter** (conventional modeling approach).
  - **Volatile content is determined by the condensation from a gas to solids** (conventional cosmochemistry).

→ Physics and chemistry of matters matter!

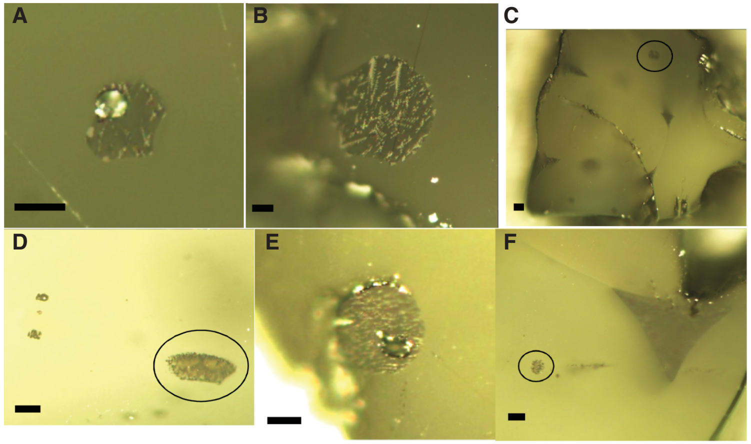
- The role of liquids in volatile retention ("wet Moon")
- The role of liquids in giant impact (→ vaporization → disk formation: isotopic similarity, FeO content)

→ Can solve most of puzzles (?)

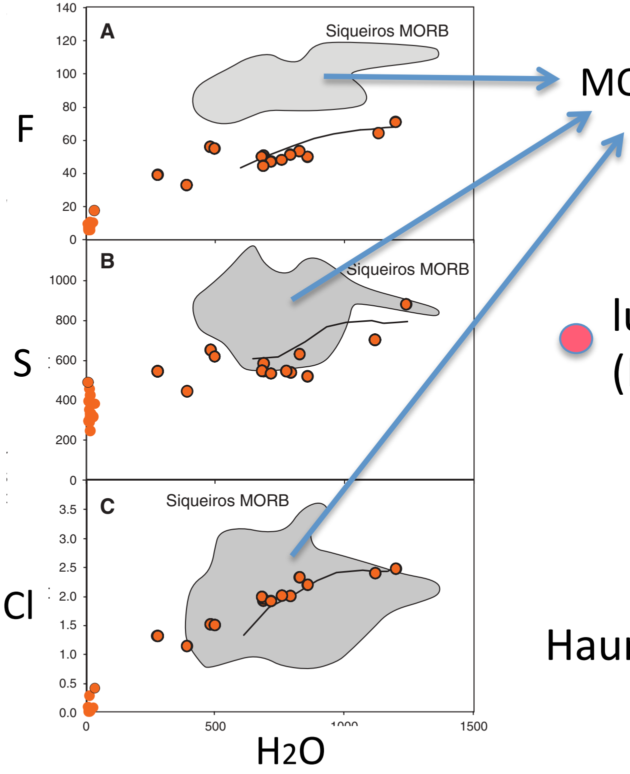
“wet” Moon ?

The “dry Moon” paradigm is challenged by high-resolution chemical analyses.

(sample: 74220 (Apollo 17))



Saal et al. (2008, 2013) (olivine)
Hauri et al. (2011) (olivine)
[Greenwood et al. (2011) (apatite)]



MORB (Earth)

● lunar sample (Moon)

Hauri et al. (2011)

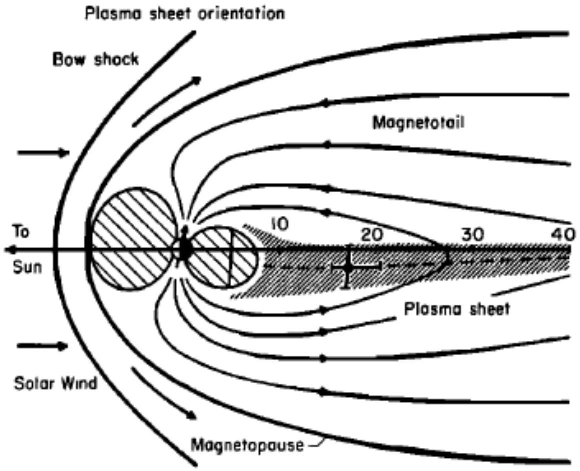
Inclusions in olivine in some lunar rocks show volatile content similar to Earth.

→ Lunar **interior** is as wet as Earth’s upper mantle (depleted but not-so-dry (~100 ppm wt water)).

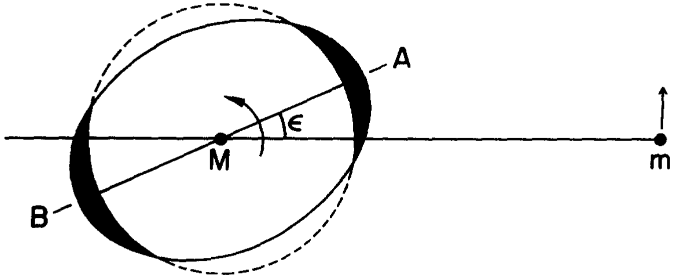
→ **But are these samples representative of the Moon?**

How about geophysical observations?

- **Geophysical observations = global (indirect)**
- Which observations?
  - Seismic wave velocities
  - Electrical conductivity
  - Tidal Q (viscosity)

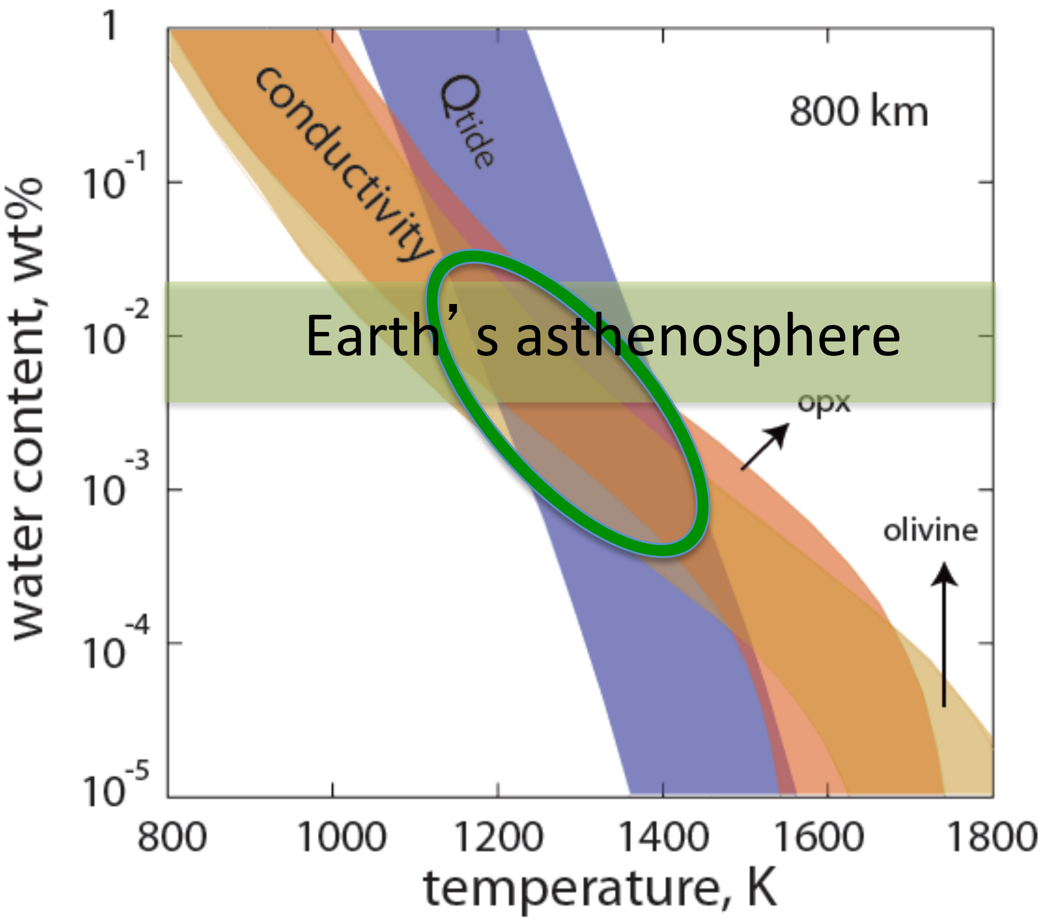


Electro-magnetic induction (electrical conductivity)



Tidal dissipation (viscosity)

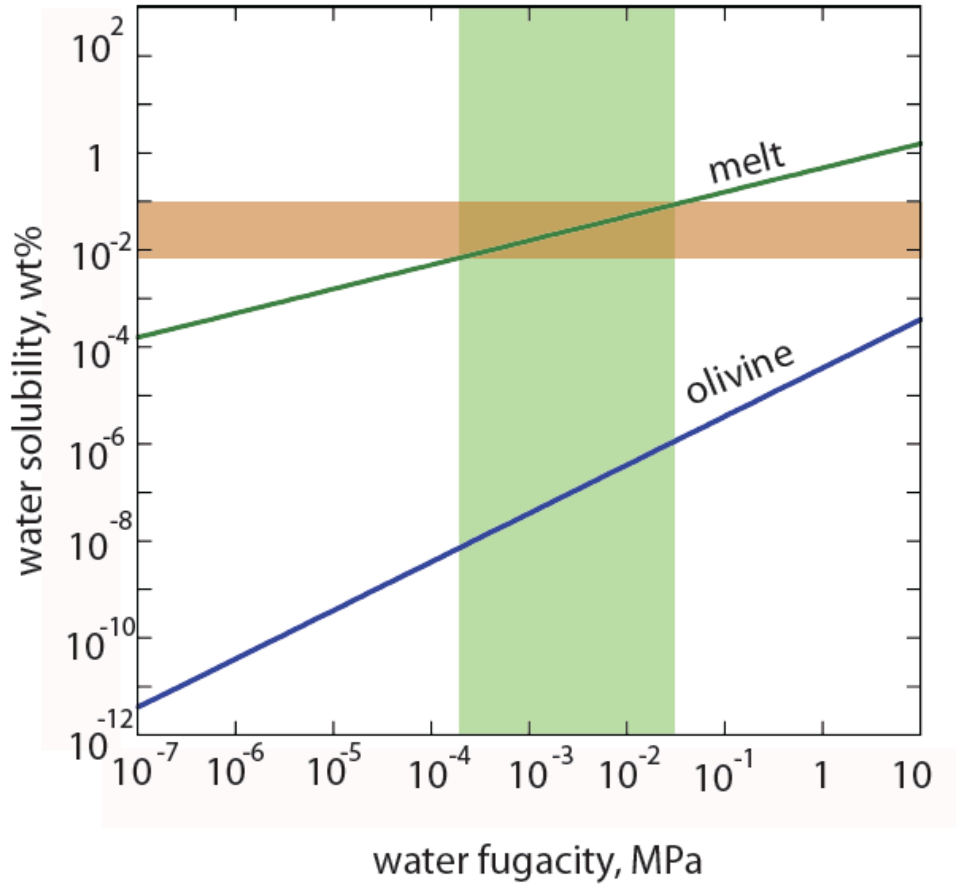
Constraining water content and temperature using both conductivity and tidal Q



Karato (2013)

→Lunar mantle is cooler than Earth's mantle, but its water content is similar to the Earth' s asthenosphere (or slightly less).

Why wasn't water lost during the Moon formation from a high-T gas?



Karato (2013)

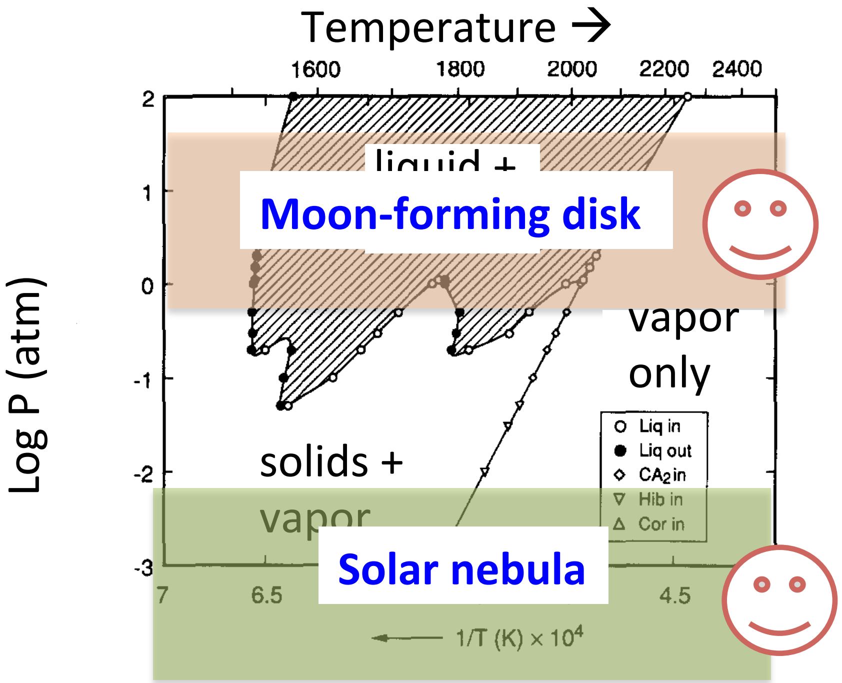
gas→ solid: large water loss

gas→ liquid (melt): not much water loss

What controls the condensation to solid or liquid?

The pressure of a gas determines either liquid or solid condensates (the phase diagram).

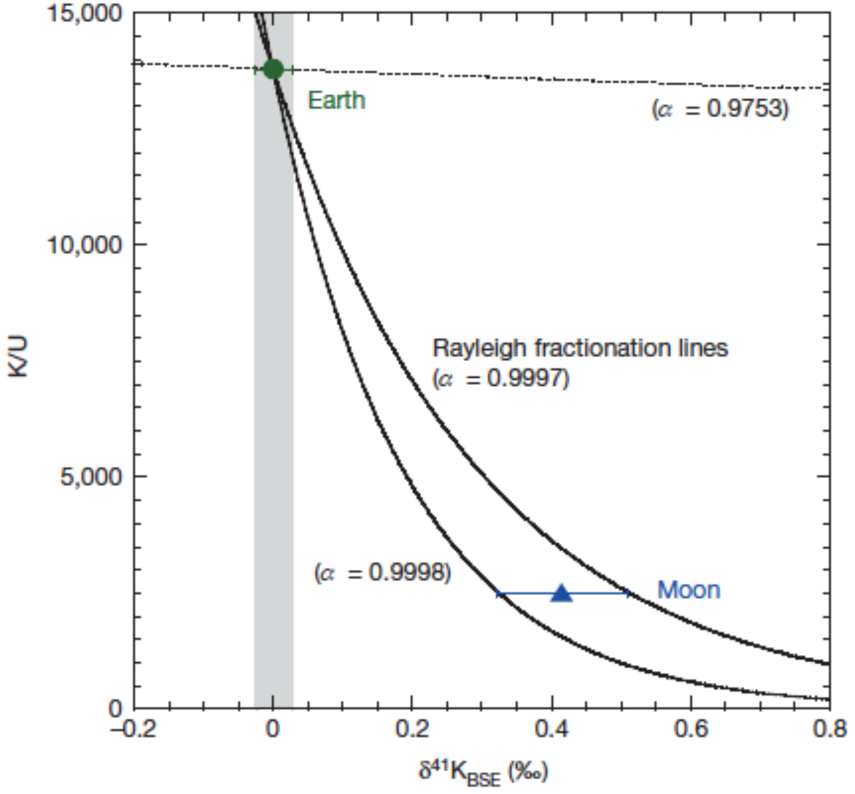
$$P_{disk} \approx \frac{\pi}{2} G\sigma^2 \approx \frac{1}{2\pi} G \frac{M^2}{R^4}$$

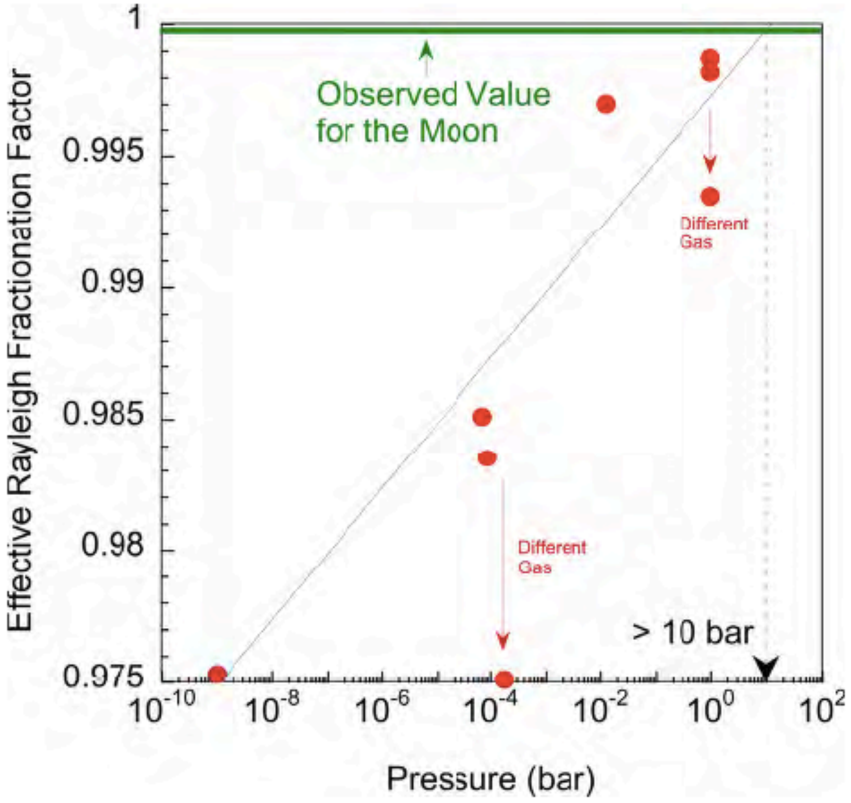


Yoneda-Grossman (1995)

gas → solid: Solar nebula (planet formation) (low P)
gas → liquid: Moon-forming disk (high P)
(because of the small space due to the gravity of Earth)

Support for high-P condensation during the Moon formation

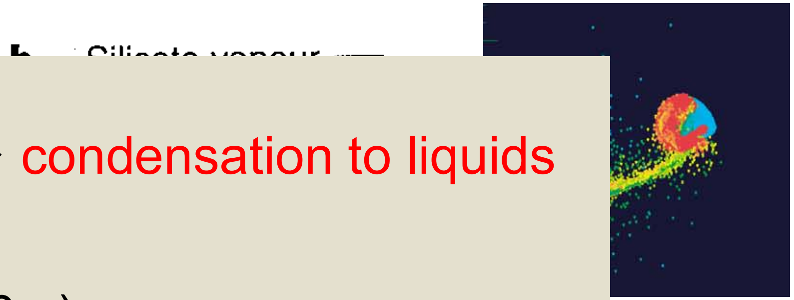




Isotope composition of K of the lunar rocks → condensation in the high-P environment (~1 MPa (~10 bar))

Wang-Jacobsen (2016)

But, liquid should finally solidify. Then all water will be gone 😅. Can the Moon be formed before the complete solidification?



Moon-forming disk

High P (high mass density) → condensation to liquids

and $\tau_{accretion} \leq \tau_{cooling}$

($\tau_{cooling} \approx$ 100 y, $\tau_{acrretion} \approx$ 1-100 y)

→ a large fraction of materials accrete as liquids
→ little depletion in volatiles

Proto-solar nebula

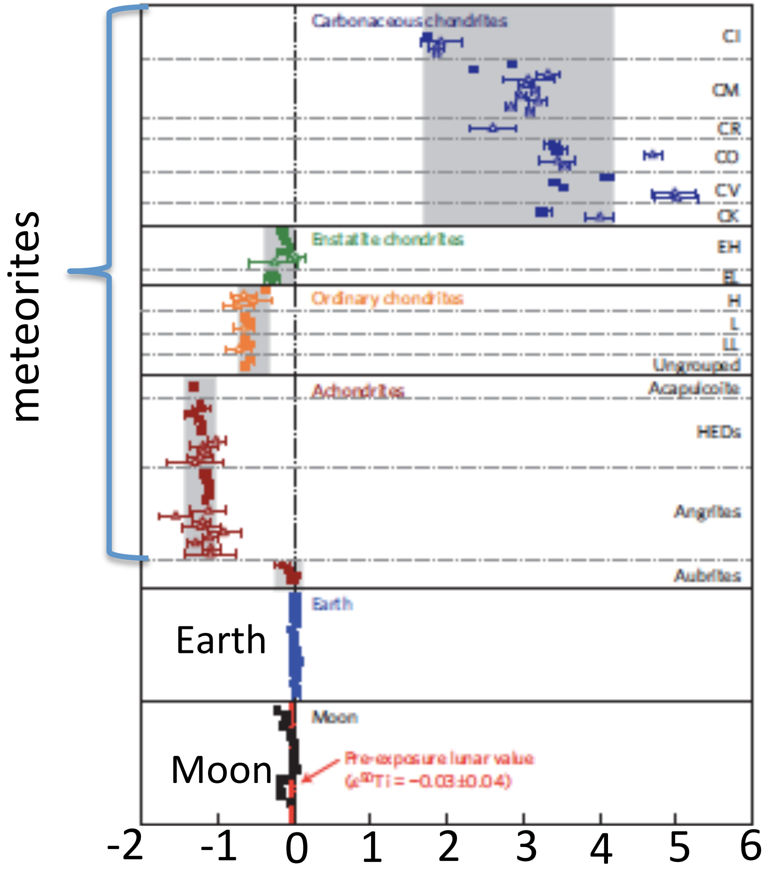
Low P (low mass density) → condensation to solids

[and $\tau_{accretion} \gg \tau_{cooling}$]

→ high degree of depletion in volatiles

The “isotopic crisis”

Very similar isotopic compositions (e.g., Ti, Si, O)



carbonaceous chondrite

enstatite chondrite

ordinary chondrite

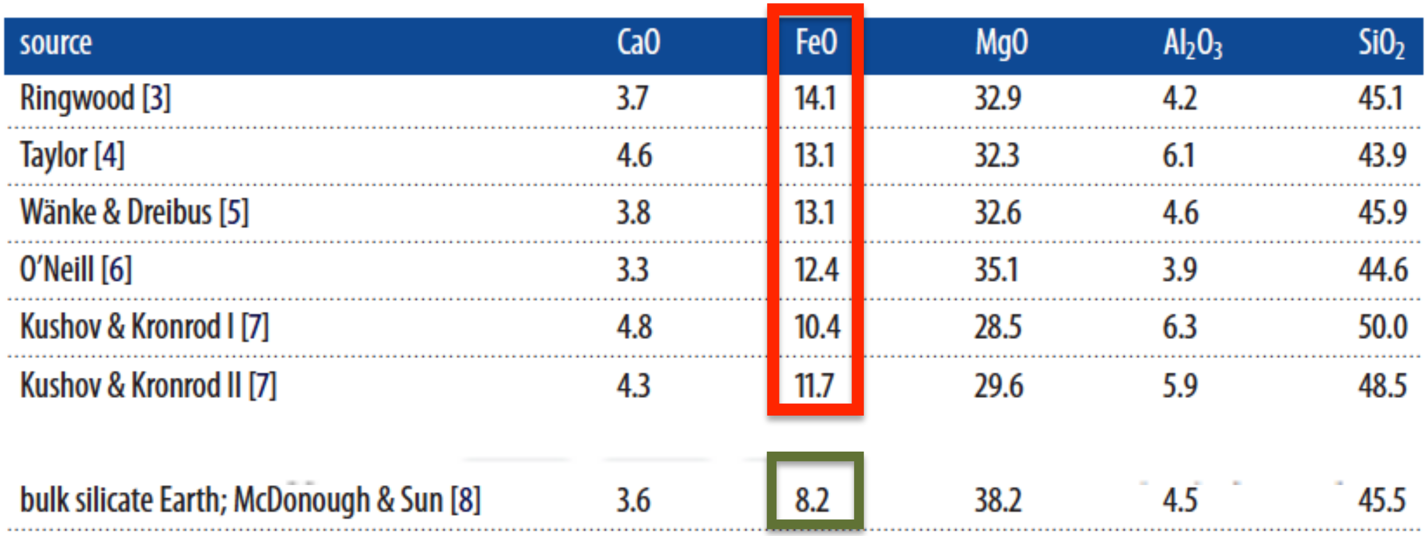
achondrite

(Zhang et al., 2012)

$$\varepsilon^{50}Ti = \left\lfloor \left({}^{50}Ti / {}^{47}Ti\right)_{sample} / \left({}^{50}Ti / {}^{47}Ti\right)_{standard} - 1 \right\rfloor \times 10^{4}$$

Isotopic compositions are different among different meteorites.
But, isotopic composition of the Moon is very similar to that of Earth.

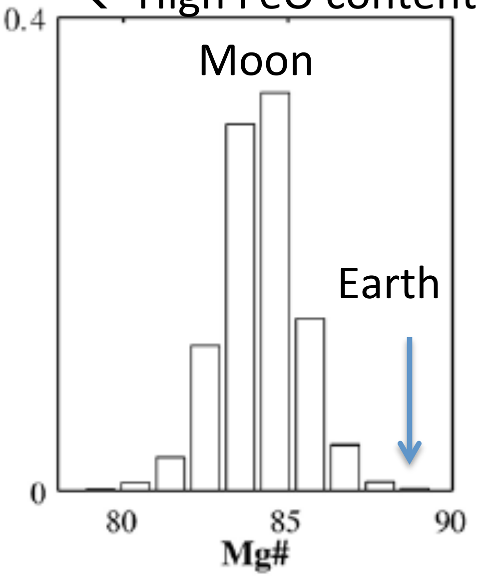
Different FeO content



	source	CaO	FeO	MgO	Al_2O_3	SiO_2
Moon	Ringwood [3]	3.7	14.1	32.9	4.2	45.1
	Taylor [4]	4.6	13.1	32.3	6.1	43.9
	Wänke & Dreibus [5]	3.8	13.1	32.6	4.6	45.9
	O'Neill [6]	3.3	12.4	35.1	3.9	44.6
	Kushov & Kronrod I [7]	4.8	10.4	28.5	6.3	50.0
	Kushov & Kronrod II [7]	4.3	11.7	29.6	5.9	48.5
Earth	bulk silicate Earth; McDonough & Sun [8]	3.6	8.2	38.2	4.5	45.5

Melosh (2014)

← High FeO content



Khan et al. (2006)

Isotope → very similar to Earth

[FeO] → higher than that of (average) Earth

How can we explain both?

[FeO content issue is usually ignored]

What do we need to have after a collision?

Mass balance and the isotopic ratio upon a giant impact

$$\varepsilon_M = \frac{\left(\frac{x}{y}\right)_M}{\left(\frac{x}{y}\right)_E} - 1 = \frac{\xi\left(\frac{\alpha_1}{\alpha_2}-\frac{\beta_1}{\beta_2}\right)(\varepsilon_1-\varepsilon_2)}{\left[\frac{\beta_1}{\beta_2}\left(\frac{\left(\frac{x}{y}\right)_2}{\left(\frac{x}{y}\right)_E}\right)+\xi\right]\left[\frac{\alpha_1}{\alpha_2}\left(\frac{\left(\frac{x}{y}\right)_1}{\left(\frac{x}{y}\right)_E}\right)+\xi\right]} \approx \boxed{(f_E - f_M)}\boxed{(\varepsilon_1-\varepsilon_2)}$$

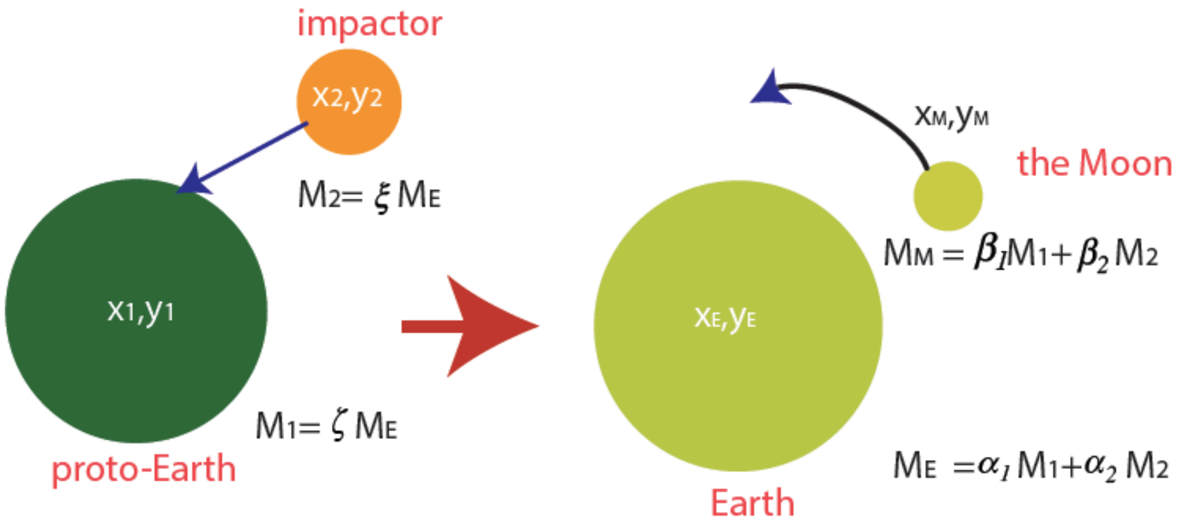
$$\varepsilon_{1,2} = \frac{\left(\frac{x}{y}\right)_{1,2}}{\left(\frac{x}{y}\right)_E} - 1 \quad \left[e.g., \frac{\left(\frac{^{50}Ti}{^{47}Ti}\right)_{1,2}}{\left(\frac{^{50}Ti}{^{47}Ti}\right)_E} - 1\right]$$

isotopic ratio of "1": target

"2": impactor

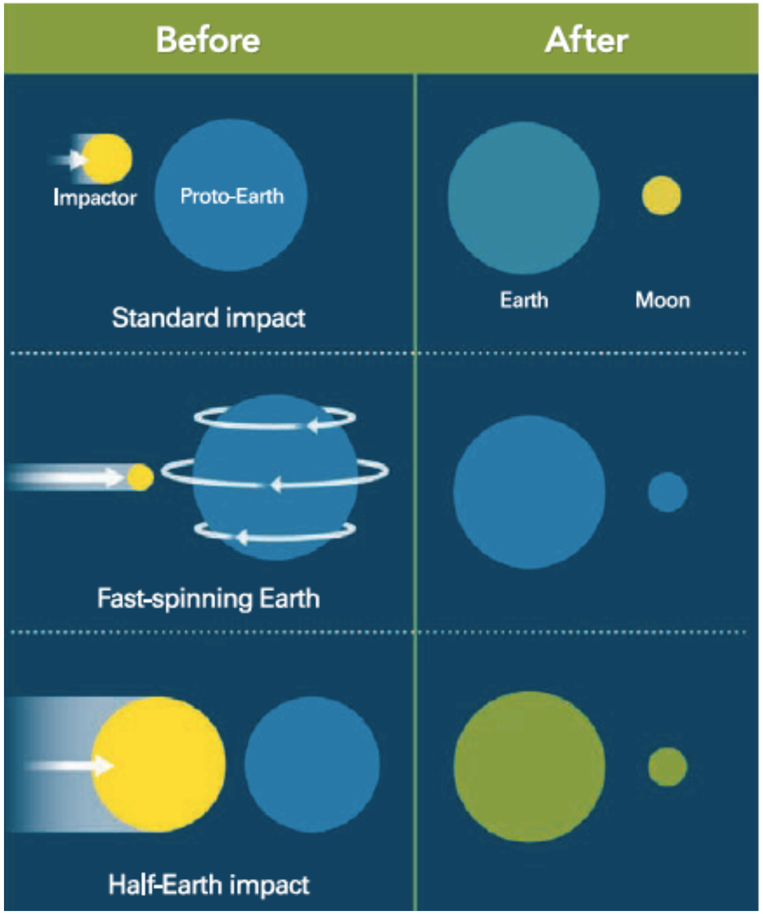
relative to that of Earth

$f_{E,M}$: *target fraction for E(Earth), M(Moon)*



We need to have a small fE-fM to explain small ε_M for large $(\varepsilon_1 - \varepsilon_2)$

How to explain the isotopic similarities?
the isotopic crisis



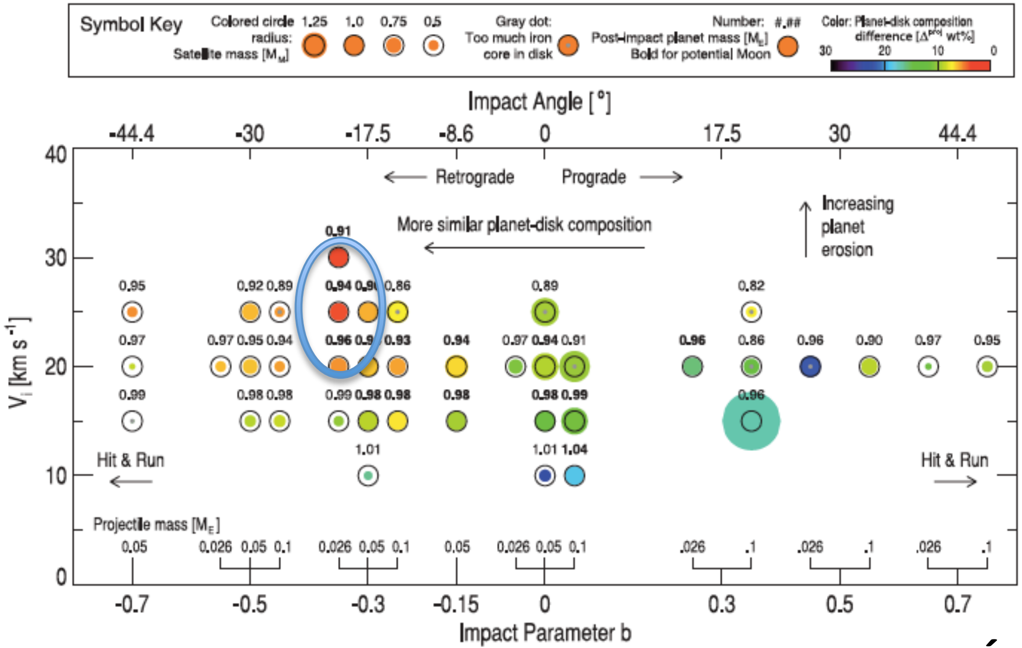
"classic model": oblique collision
Hartmann-Davis (1975), Cameron-Ward (1976)
Benz et al. (1986), Canup (2004)
→ large distortion of an impactor
→ the Moon mainly from the impactor

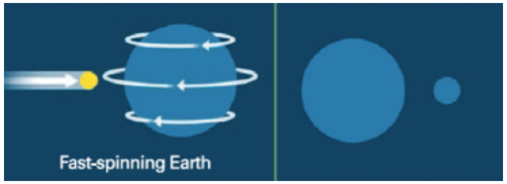
"new models": head-on collision

Ćuk-Stewart (2012)

Canup (2012)

Seek solutions by changing the "**mechanics**" of collision
→ Difficulties in explaining the large **angular momentum** (and FeO content)
→ They (need to) invoke rare events (how probable??)

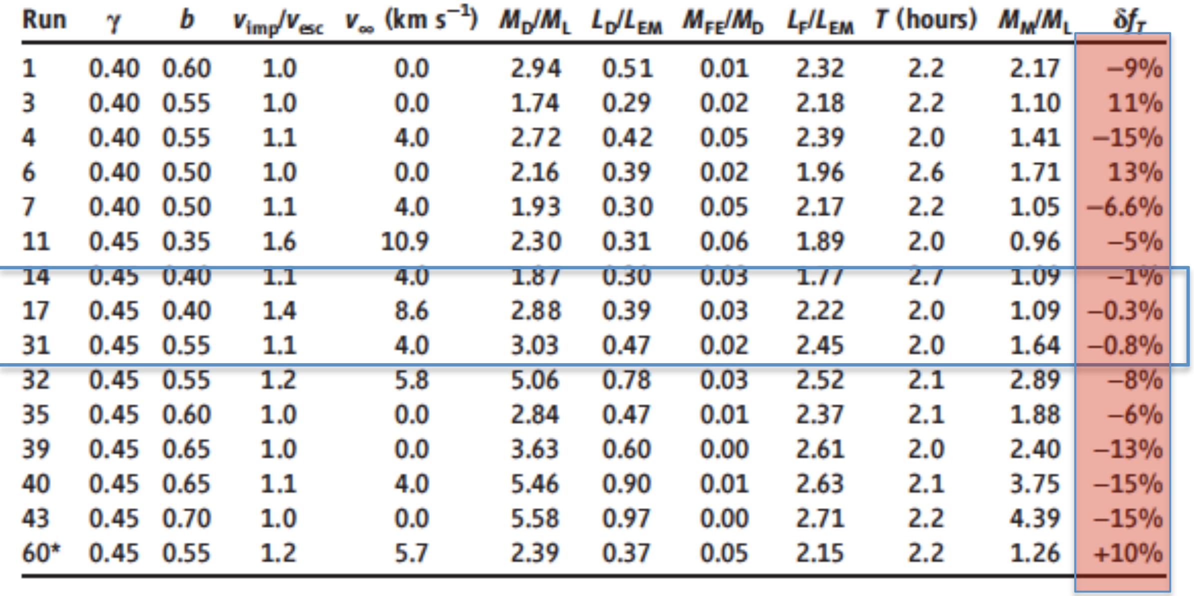




Ćuk-Stewart (2012)

Problems with the Ćuk-Stewart model

1. Only in a small parameter space, can one have the composition similar to Earth (**by chance?**).
2. Predicts FeO content inconsistent with the observation.
3. Angular momentum?



Run	γ	b	v_{imp}/v_{esc}	v_∞ (km s^{-1})	M_D/M_L	L_D/L_{EM}	M_{FE}/M_D	L_F/L_{EM}	T (hours)	M_W/M_L	δf_T
1	0.40	0.60	1.0	0.0	2.94	0.51	0.01	2.32	2.2	2.17	−9%
3	0.40	0.55	1.0	0.0	1.74	0.29	0.02	2.18	2.2	1.10	11%
4	0.40	0.55	1.1	4.0	2.72	0.42	0.05	2.39	2.0	1.41	−15%
6	0.40	0.50	1.0	0.0	2.16	0.39	0.02	1.96	2.6	1.71	13%
7	0.40	0.50	1.1	4.0	1.93	0.30	0.05	2.17	2.2	1.05	−6.6%
11	0.45	0.35	1.6	10.9	2.30	0.31	0.06	1.89	2.0	0.96	−5%
14	0.45	0.40	1.1	4.0	1.87	0.30	0.03	1.77	2.7	1.09	−1%
17	0.45	0.40	1.4	8.6	2.88	0.39	0.03	2.22	2.0	1.09	−0.3%
31	0.45	0.55	1.1	4.0	3.03	0.47	0.02	2.45	2.0	1.64	−0.8%
32	0.45	0.55	1.2	5.8	5.06	0.78	0.03	2.52	2.1	2.89	−8%
35	0.45	0.60	1.0	0.0	2.84	0.47	0.01	2.37	2.1	1.88	−6%
39	0.45	0.65	1.0	0.0	3.63	0.60	0.00	2.61	2.0	2.40	−13%
40	0.45	0.65	1.1	4.0	5.46	0.90	0.01	2.63	2.1	3.75	−15%
43	0.45	0.70	1.0	0.0	5.58	0.97	0.00	2.71	2.2	4.39	−15%
60*	0.45	0.55	1.2	5.7	2.39	0.37	0.05	2.15	2.2	1.26	+10%

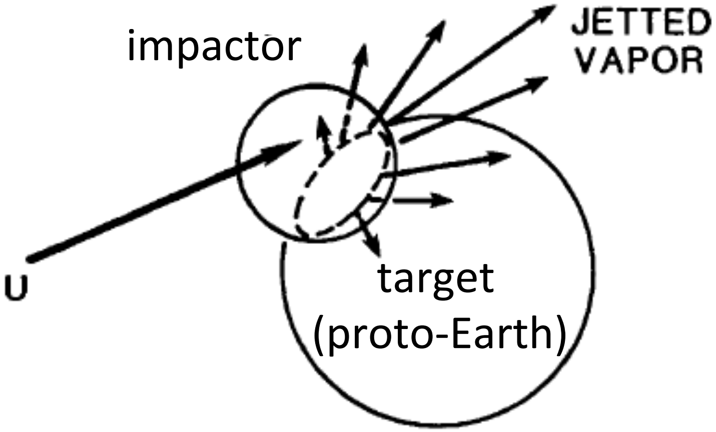
Canup(2012)

Problems with the Canup (2012) model

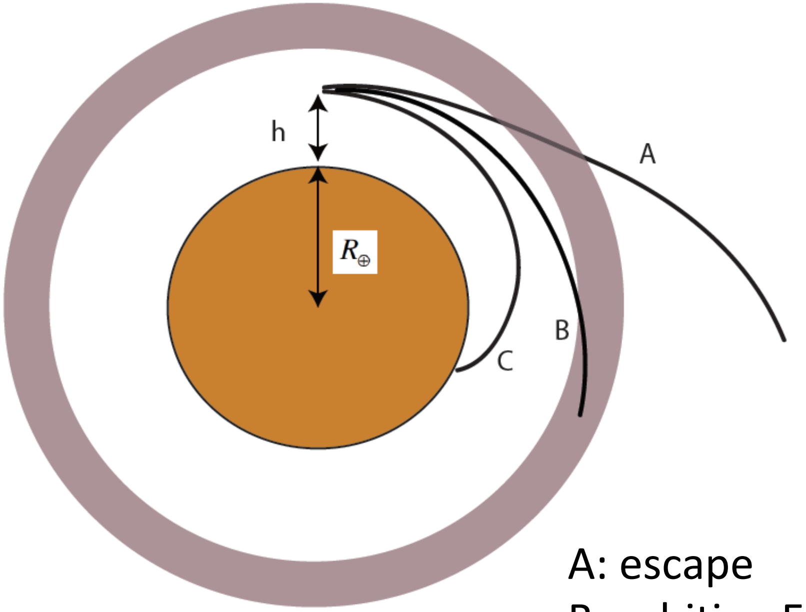
1. Only in a small parameter space one can have composition similar to Earth (**by chance?**).
2. Predicts FeO content inconsistent with the observation.
3. Difficult to explain the large angular momentum

What controls the composition of the Moon formed by a giant impact?

Fate of ejected materials after an impact



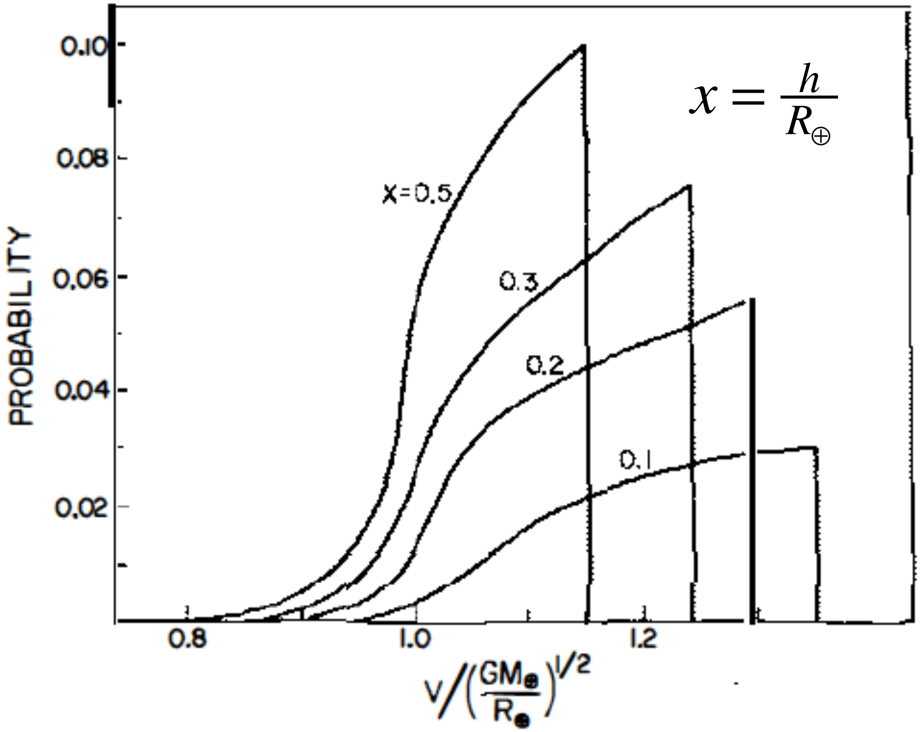
Melosh-Sonett (1986)



A: escape
B: orbiting Earth → Moon
C: re-impact

Collision ejects materials → materials ejected to the relatively high level (and velocity) will become the Moon

To get more proto-Earth materials to the orbit, one needs to have a mechanism to heat the proto-Earth more than the target



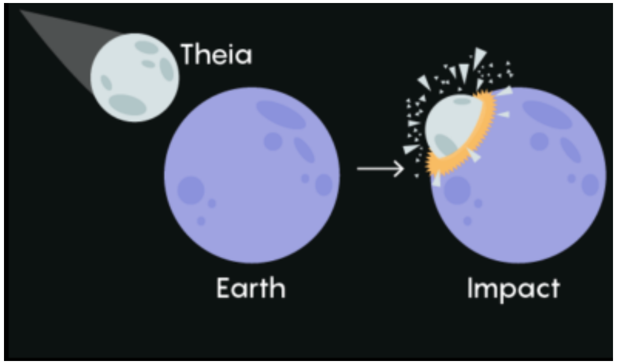
Stevenson (1987)

Shock heating → gas → expand → large $x = \frac{h}{R_\oplus}$ → more chance to get into the proto-Earth surrounding orbit to become the Moon

→ **Is there a physical mechanism to heat the target ()the proto-Earth) more than the impactor?** **→ thermodynamics of matter**

collision → heating: material dependent

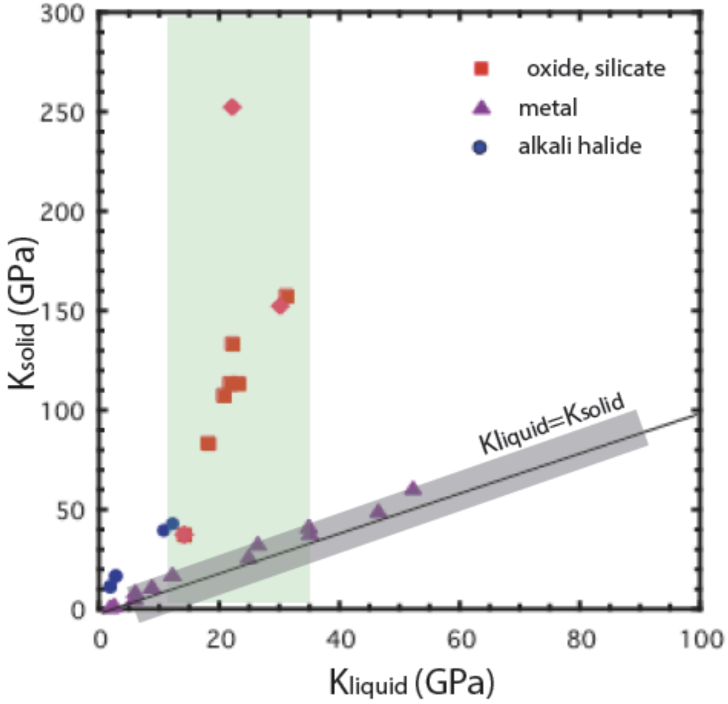
- In all previous studies, the same materials properties (**equation of state**) was used for the proto-Earth and the impactor (Theia).
- Proto-Earth is likely covered by a **magma ocean** (liquids), but an impactor is likely solids.
- Solids and liquids have very different equation of state (next slide; Jing-Karato, 2011). → different degree of heating



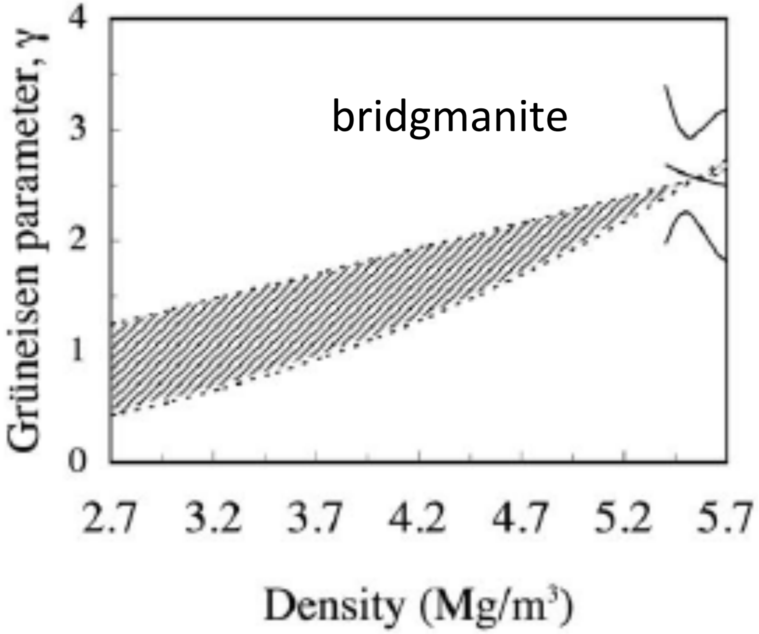
Unique compressional properties of liquids

→ When a liquid (magma ocean) collides a solid body

→ the liquid will be heated more than solid.



(modified from Jing-Karato (2011))

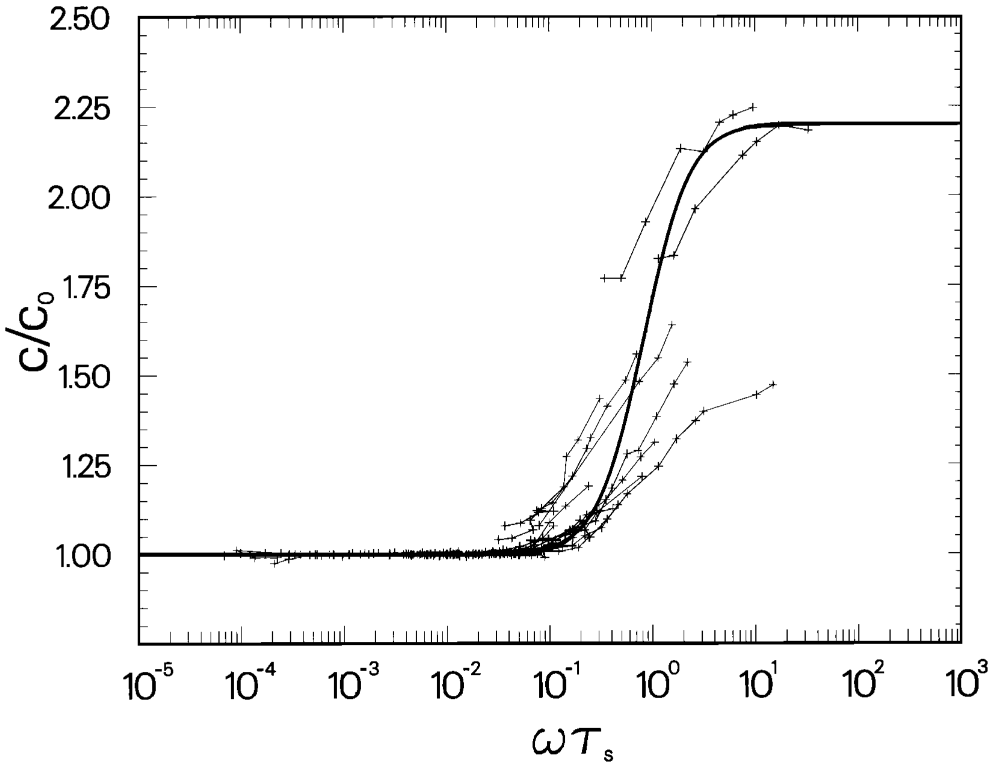


(Mosenfelder et al. (2007))

Bulk moduli of complex liquids (silicate, oxide melts) have little correlation with those of corresponding solids → little role of chemical bonding (internal energy)

Grüneisen parameter **decreases** with compression in **solids**, but it **increases** with compression in **liquids** → intense heating upon compression $\gamma = \left(\frac{\partial \log T}{\partial \log \rho}\right)_{ad}$

Unique compressional properties of **liquids** cont.



Rivers and Carmichael (1987)

frequency-dependent sound velocity

→ compression involves some viscous (time-dependent) processes

Compression of solids changes mostly their internal energy. Resistance for compression of complex liquids has no correlation with that of corresponding solids.

→ Compression of liquids changes entropy

$$P = -\left(\frac{\partial F}{\partial V}\right)_T = -\left(\frac{\partial U}{\partial V}\right)_T + T\left(\frac{\partial S}{\partial V}\right)_T$$

solids

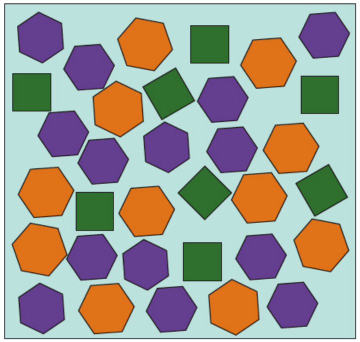
liquids

→ upon compression, much of the free energy change is change in entropy → high degree of heating

→ liquids expand and go to the orbit (to form the Moon)

solid

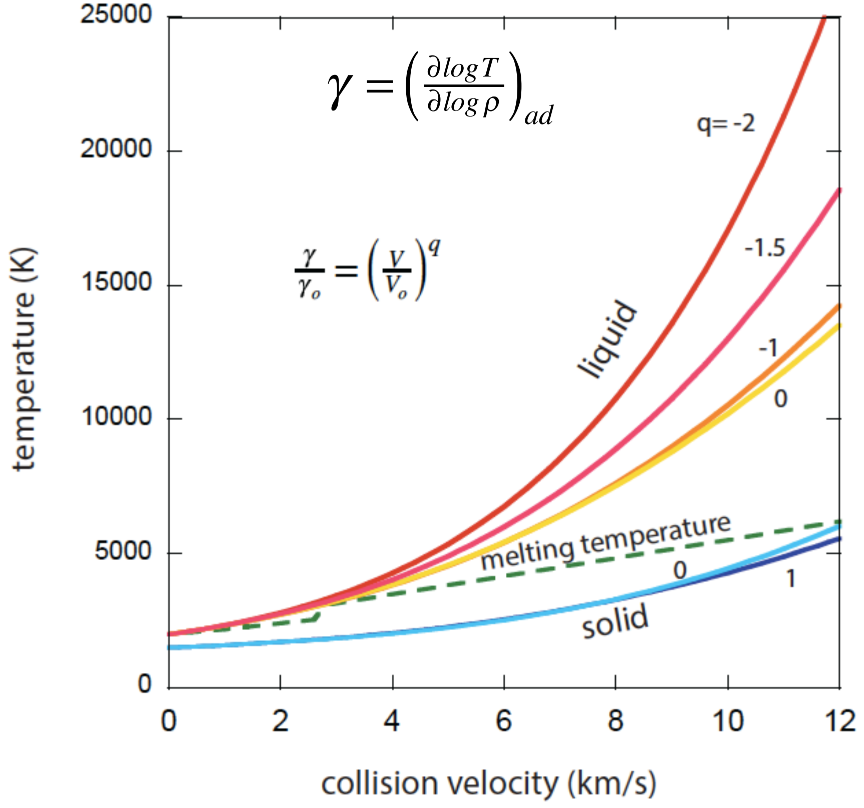
liquid



A simple analytical model
(a “flat Earth” model)

Karato (2014)

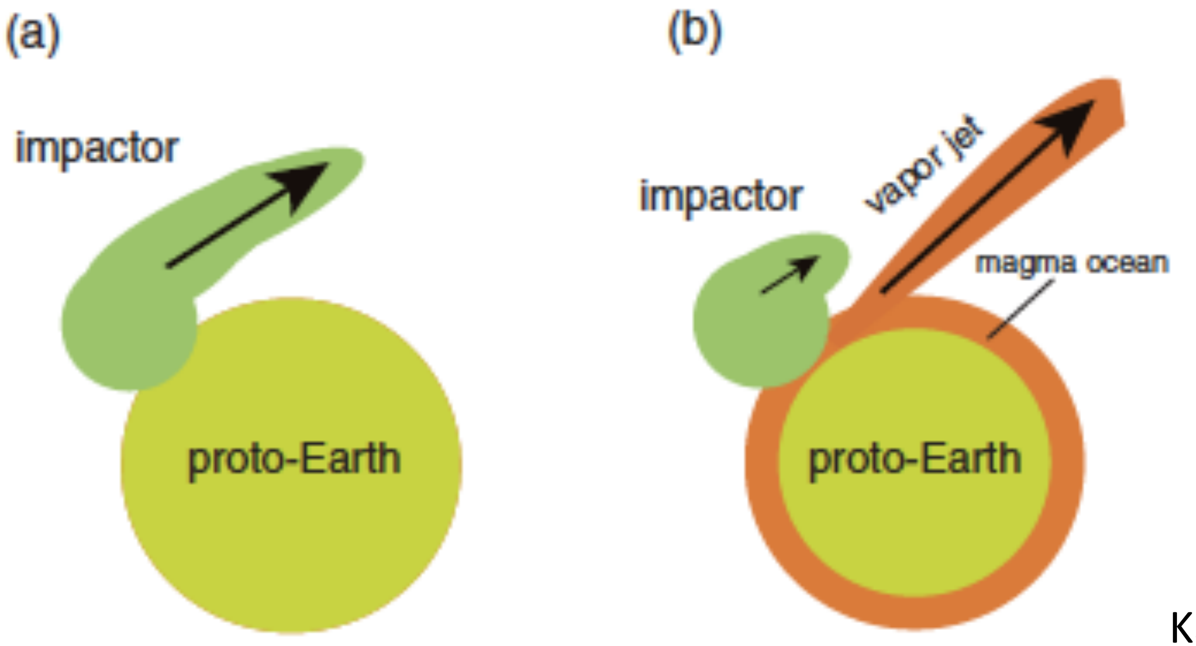
$$dT = \left\{-\frac{T\gamma}{V} + \frac{1}{2C_v}\left[\left(P - P_o\right) + \left(V_o - V\right)\frac{dP}{dV}\right]\right\}dV$$



heating of liquids >> heating of solids

→ more materials go to the orbit from the magma ocean

→ the Moon is mainly from the magma ocean of the proto-Earth?



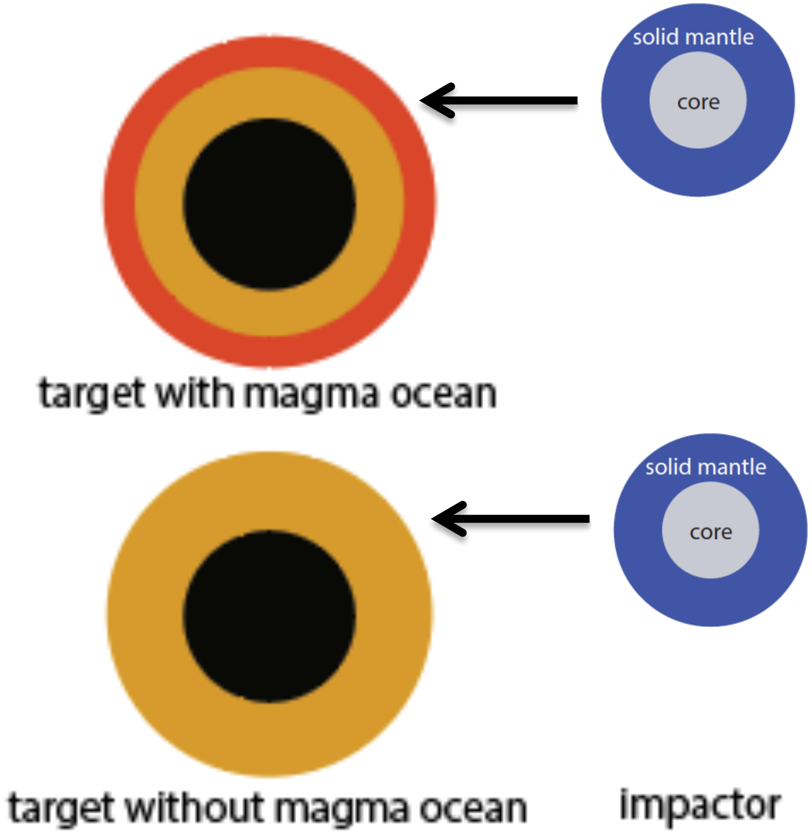
Karato (2014)

(a) conventional model: impactor is distorted by an oblique collision → most materials for the Moon are from the impactor
(b) with a **magma ocean** on the target (proto-Earth) → most of ejected materials are from the magma ocean → explain the isotopic similarity, FeO-enrichment (together with large angular momentum)?

Beyond the flat Earth

3-D numerical modeling using a modified SPH*

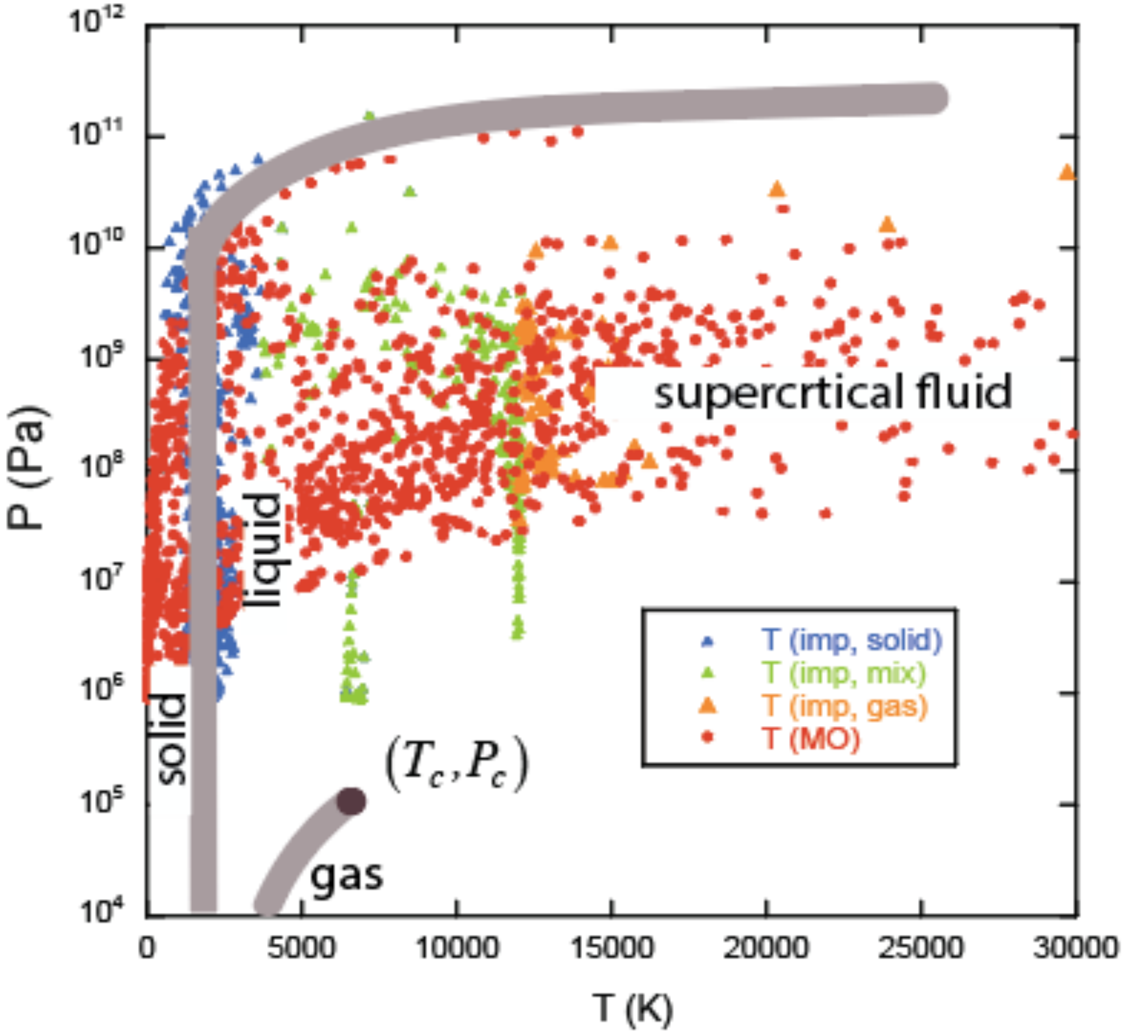
(K (京)-computer, RIKEN, Kobe, Japan)



Hosono et al. (2019)

*: A standard SPH (Smoothed Particle Hydrodynamics) code cannot treat a density discontinuity properly. → "DISPH" (density-independent SPH)

P-T conditions after a giant impact



Consequence of a collision depends strongly on **EoS (equation of state)**

(Hosono et al. 2019)

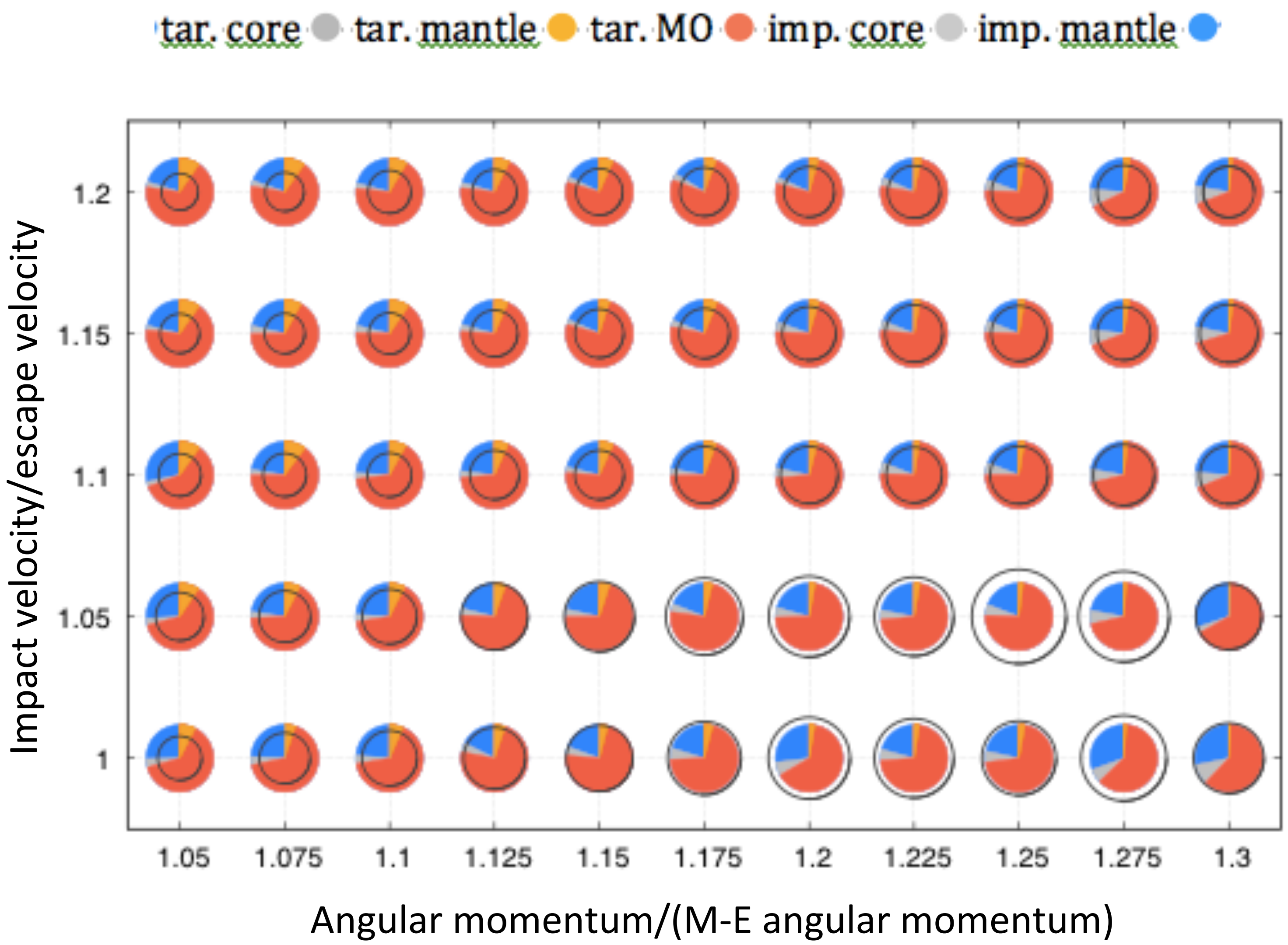
DISPH

Standard SPH

Solid + gas EoS
(Tillotson EoS)

Mie-Grüneisen EoS
for magma ocean

Hard-Sphere EoS
for magma ocean

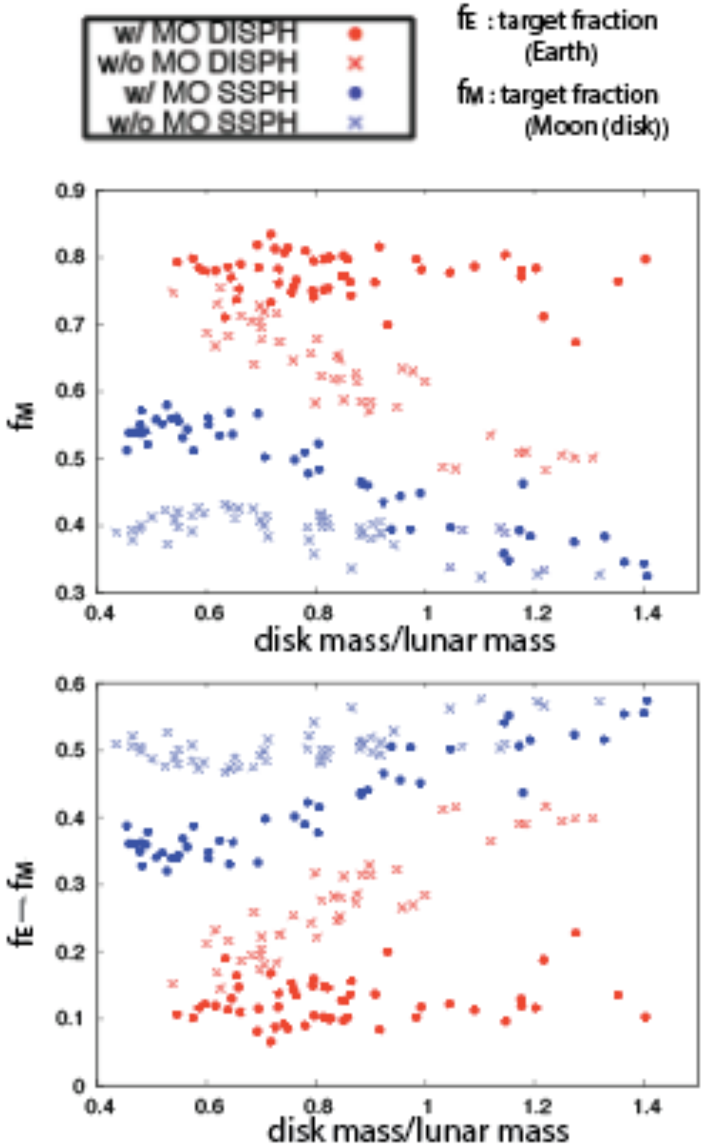


(Hosono et al. 2019)

Target (proto-Earth) materials form a large fraction of the Moon.

Composition of the Moon (and Earth)

after a giant impact that produces required angular momentum



A small $f_E - f_M$ is needed to explain the similar isotopic composition between the Moon and Earth.

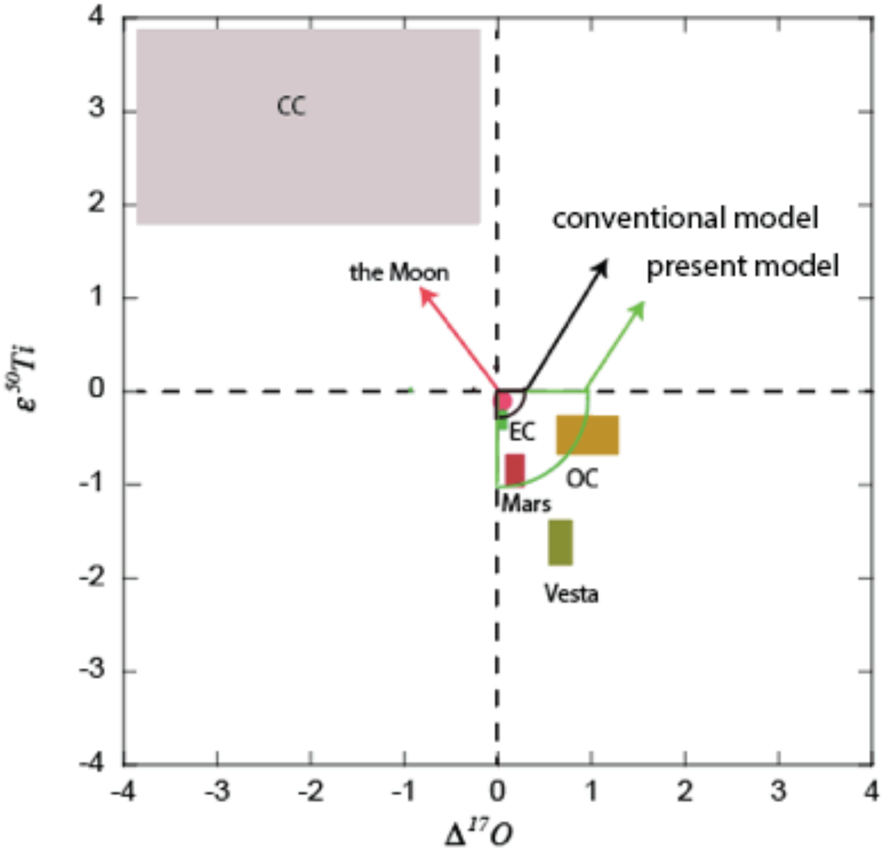
In a conventional model with an oblique collision (that explains large angular momentum), $f_E - f_M$ is too large
→ Needs to invoke “unusual” collision conditions.

With a magma ocean + improved SPH (DISPH), $f_E - f_M$ can be reduced substantially. → the composition and the angular momentum of the Moon can be explained more naturally.

(Hosono et al. 2019)

What type of impactors?

A comparison to isotopic observations



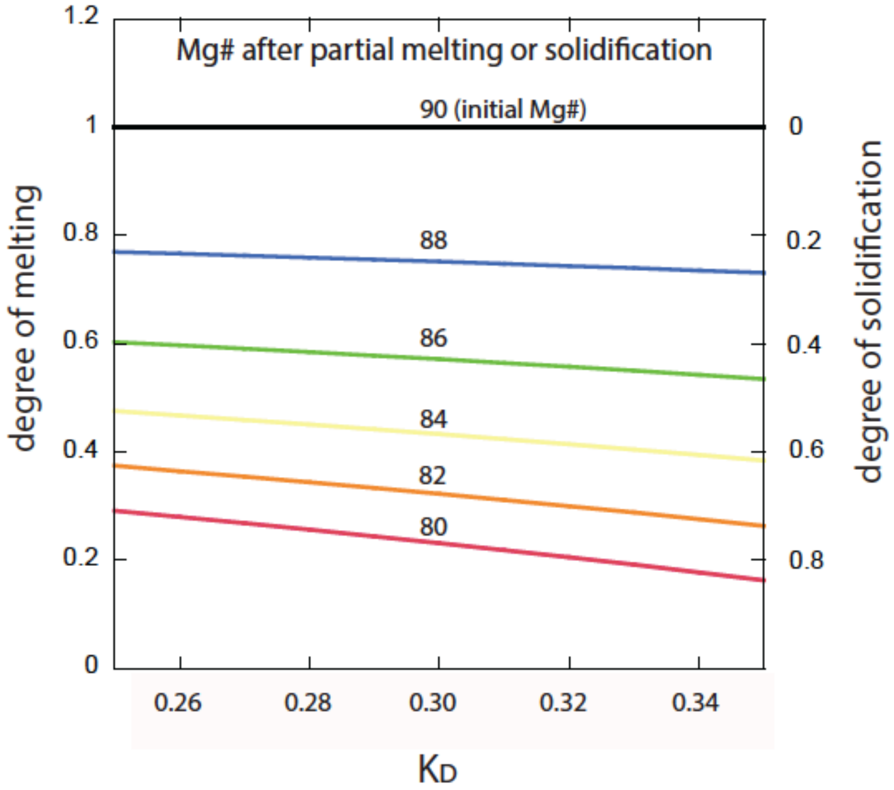
CC: carbonaceous chondrite
EC: enstatite chondrite
OC: ordinary chondrite

(Hosono et al. 2019)

- For a conventional model, impactor has to be very similar to Earth.
- For the present magma ocean model, a broader range of materials are acceptable as an impactor.

→ The Moon as observed can be explained more naturally.

The magma ocean model also explains high FeO of the Moon.



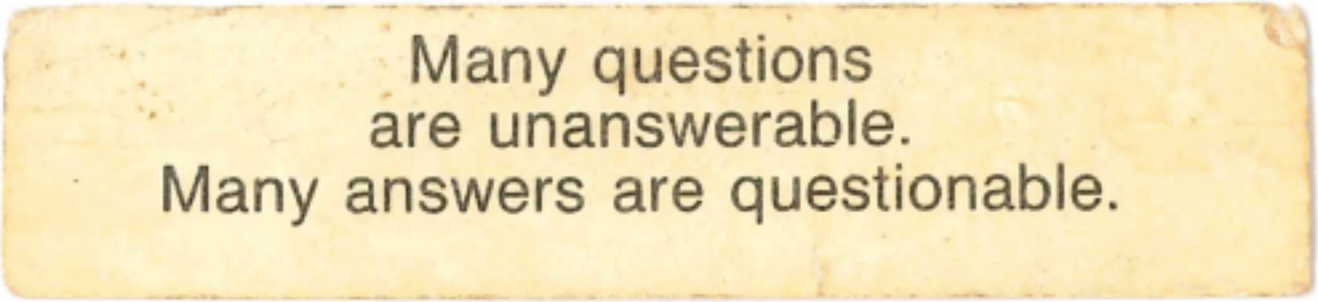
FeO goes more to the melt (but little change in isotopic composition)
→ magma ocean will have higher FeO than the bulk of Earth
→ If the ejected materials are mostly from the magma ocean, this explains the high FeO of the Moon

Conclusions

- Many "puzzles" of the lunar composition can be understood as a **natural consequence of the Moon formation** (giant impact model) if the importance of **liquids** is included in the model.
- Isotopic compositions and FeO content: A giant impact → **magma ocean materials on the proto-Earth materials are heated more than the impactor → majority of the disk (to become the Moon) was the magma ocean materials → isotopic similarity, FeO content difference.**
- "wet Moon": When the hot disk gas cooled**, condensation occurred to liquids** (not to solids) due to high P of the disk (~1 MPa**) → only small loss of volatiles.**

What's next?

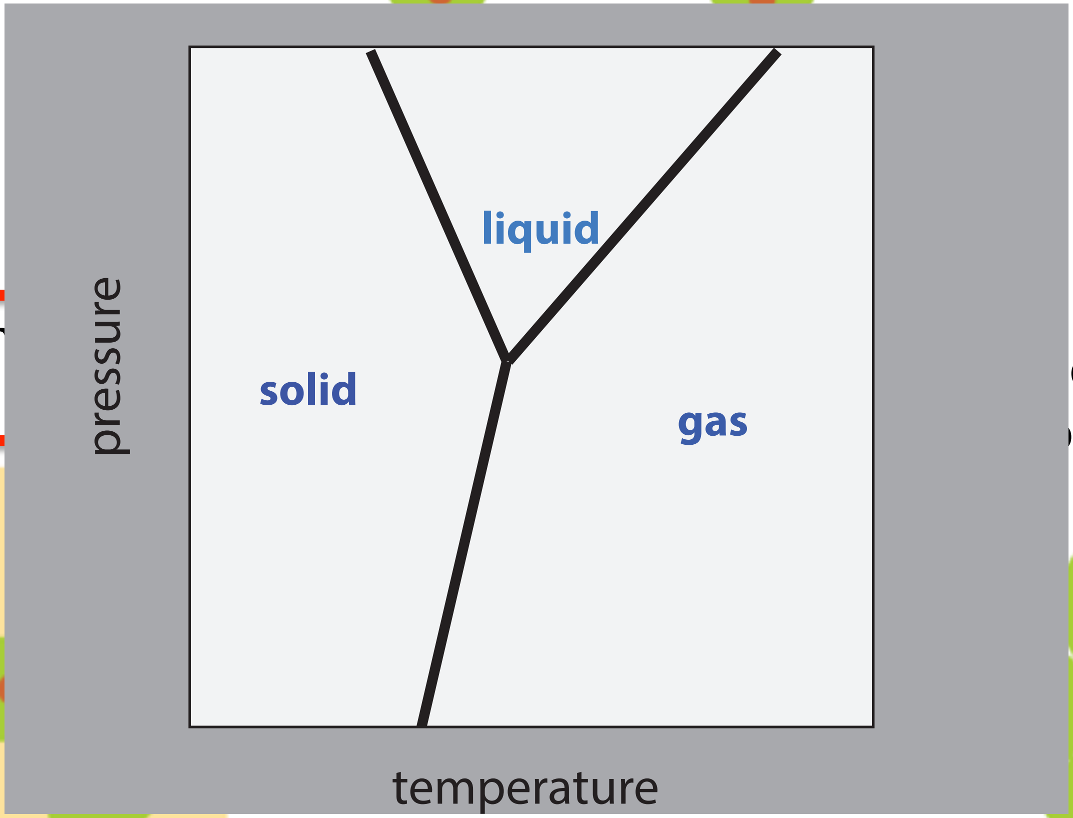
- **How often is the Moon-type satellite formed?**
  - **Is the Earth-Moon system so "rare" that Earth-like planet will be very unique ("Rare Earth" hypothesis)?**
  - **Sometimes, mantle of the target planet is stripped off (Mercury).**
    - **When is a satellite formed? When does mantle materials escape?**
- Our model explain isotopic similarities (Ti, Si, O ---). But some isotopes show differences (e.g., Zn, K).
  - What do they tell us about the Moon formation?
- **Explain the major element chemistry (FeO, Al_2O_3, CaO)**
- **More complete equation of state (limit of the hard-sphere model):** the hard-sphere equation of state will not work at very high degree of compression.



Many questions
are unanswerable.
Many answers are questionable.

Earth (terrestrial planets) lost most of volatiles during formation, but the Moon did not lose much water: **why?**

Moon form
(cosmoche



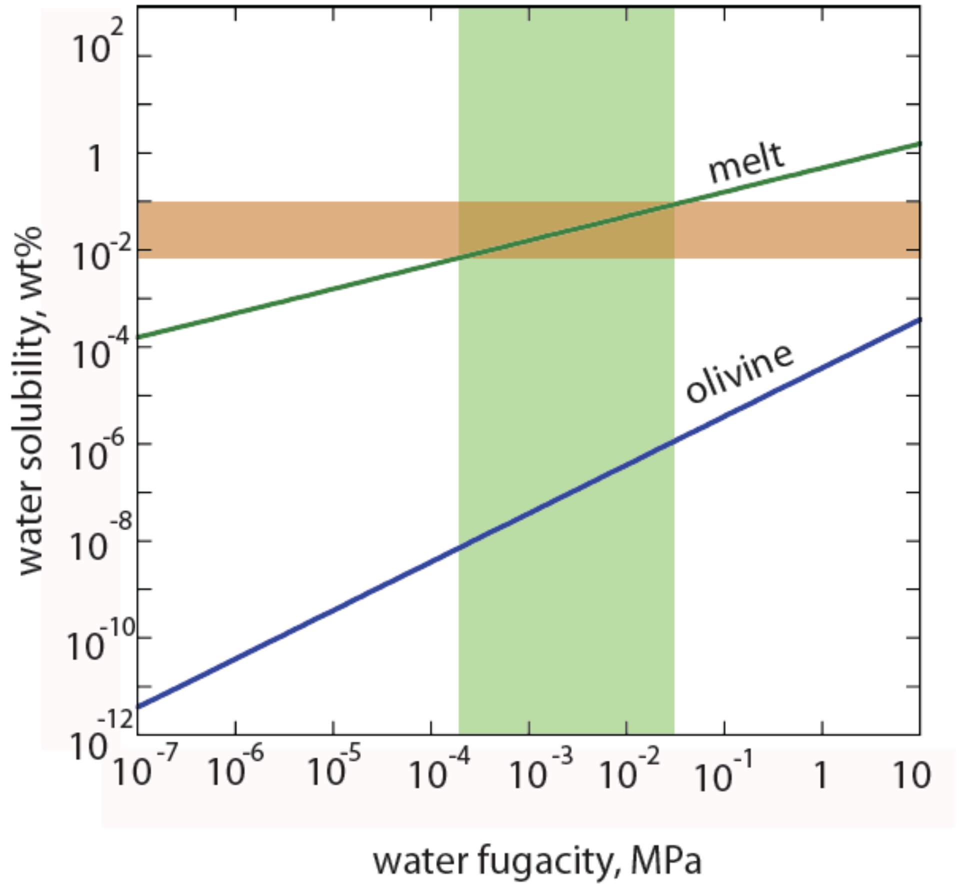
of)
chemistry

Liquid
(loves volatiles)

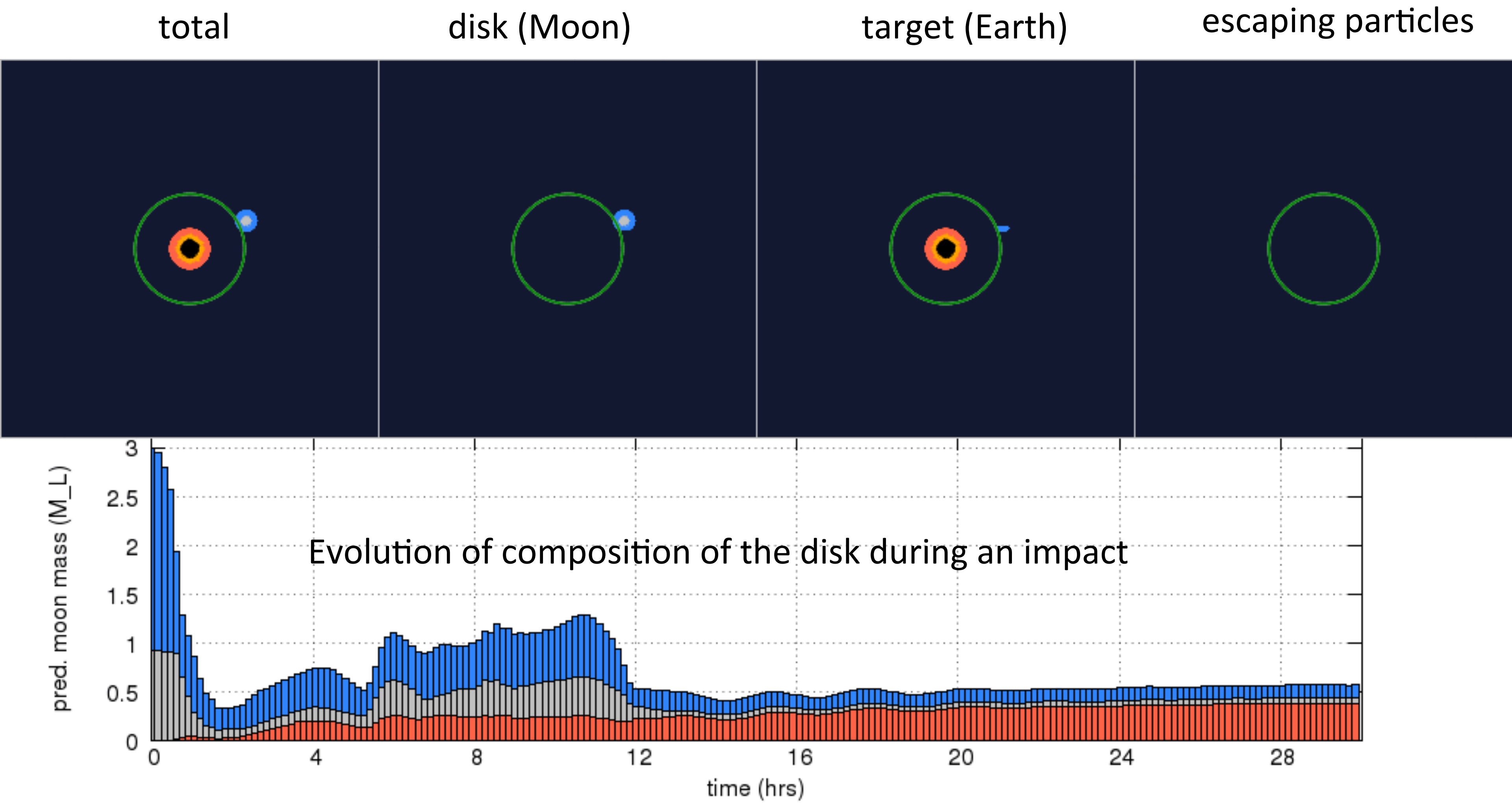
geology (petrology)
[partial melting]

Solid
(doen’t like volatiles)

Not much water loss due to the condensation to **liquid** (major water loss due to the condensation to solid)



Karato (2013)



Evolution of composition of the disk during an impact

- 🟦 impactor (rock)
- ⬜ impactor (Fe core)
- 🟥 target magma ocean

(Hosono et al. 2019)

A giant impact model

- **Explains the large angular momentum + chemistry of the Moon (lack of a Fe-rich core)**
- **Giant impact model is questioned by the results of modern geochemical measurements.**
  - **Unexpected observations**:

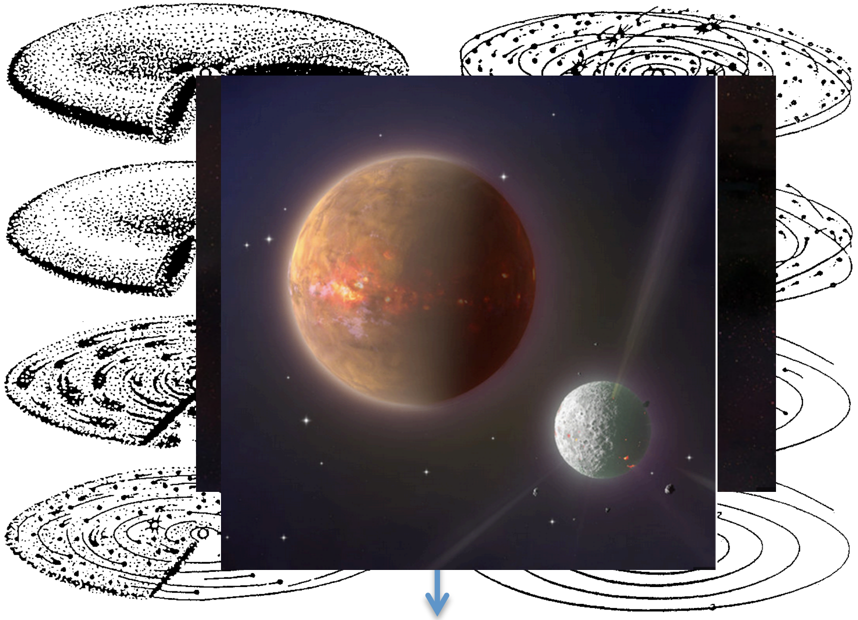
    (1) "wet" Moon?

    (2) very close agreement in (most) isotopic ratios

    (higher FeO content)

Background

Planetary formation and formation of the Moon



Hayashi (林忠四郎) and Safronov

Star formation: gravitational collapse of a molecular cloud

→ heating

→ formation of a hot nebular gas disk

→ cooling (by radiation)

→ **condensation** → dust formation

→ gravitational instability

→ "planetesimals"

→ planetesimal size increases

→ big ones get hot (a magma ocean)

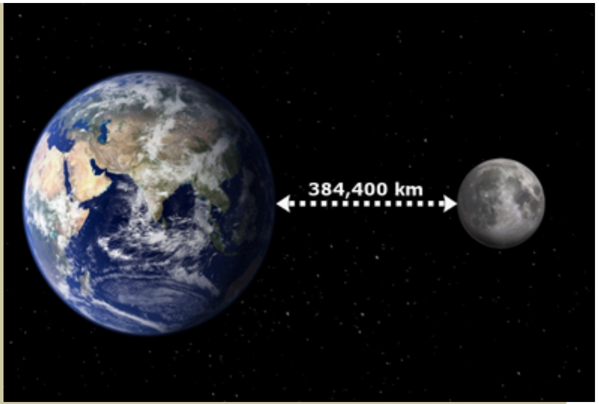
→ small ones remain cold ($M_c \approx 2M_{Mars}$)

→ **collision of big bodies (giant impacts)**

→ hot gas → cooling → **condensation**

→ the Moon

Key features of the Moon

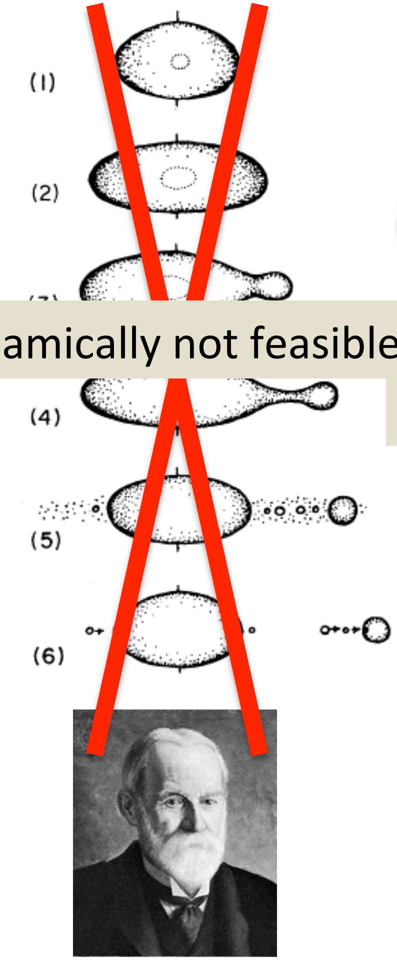


- The Moon is a relatively large planetary body (~1/4 of the Earth size, ~1/100 of the Earth mass) yet composition is nearly homogeneous (very small core), and made mostly of rocks.
- **Most of other planetary bodies of this size are differentiated: mantle-core structure (e.g., parent bodies of iron meteorites).**
- Formed in the later stage of planetary formation (~50-60 Ma).
- The Earth-Moon system has large angular momentum.

→ How can we explain **rocky composition** of the Moon (and **the large angular momentum**)? [need to understand how composition is controlled during the complex processes of Moon formation]

Models of the Moon formation

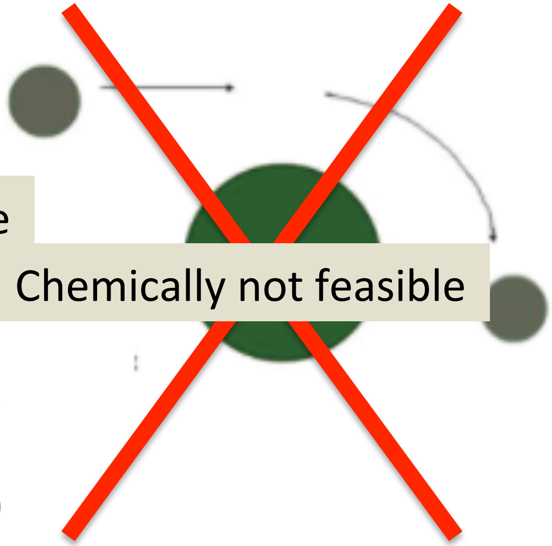
Fission model



Dynamically not feasible

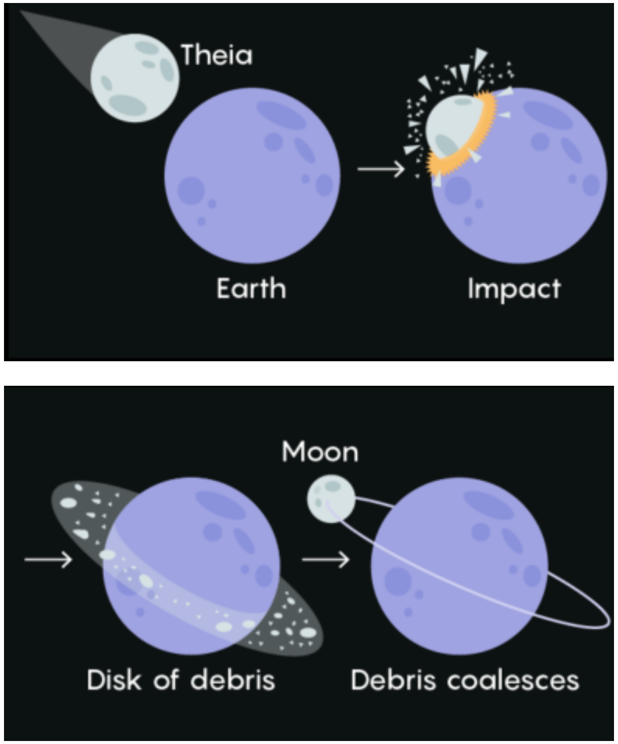
George Darwin (1845-1912)

Capture model



Chemically not feasible

A giant impact model



Hartmann-Davis (1975)
Cameron-Ward (1976)