Atmospheric and interior dynamics on Giant Planets

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Some Jupiter characteristics:

- Rotation: 9.92 hours
- Radius: 70000 km
- Max wind: 140 m/s
- Surface gravity: 23 m/s²
- Obliquity: 3°
- No solid surface

An ideal Geophysical Fluid Dynamical (GFD) system:

1. Small Rossby number
2. No continental boundaries, topography, air-sea interaction, seasons, biology, etc.
3. Deformation radius is small relative to the size of the planet
1. What drives the multiple zonal jets and equatorial superrotation?

Strong eastward winds around the equator with alternating zonal jets toward the poles.

2. Why is the meridional thermal emission flat?

- Emitted radiation is stronger than absorbed indicating a substantial internal heat source.
- Flat emission spectra indicates there may be a mechanism that transfers heat poleward.
Two general mechanistic approaches: Shallow vs. Deep models

**Shallow Models**
- **Superrotation:** Schneider & Liu (2009), Scott & Polvani (2009), Lian & Showman (2009)

**Deep Models**
Shallow vs. deep approaches

- Shallow models

- Deep models
Rossby waves

\[ q = \bar{q} + q' \]

planetary vorticity  relative vorticity

Fig. 12.4 The momentum transport in physical space, caused by the propagation of Rossby waves away from a source in mid-latitudes. The ensuing bow-shaped eddies are responsible for a convergence of momentum, as indicated in the idealization pictured.

From Vallis (2006)
**Shallow models**

**Figure 1:** Hovmöller diagrams of zonal and vertical averaged eddy momentum flux convergence for different rotation rate $-80$ to $80$.

- Zonal wind $[\text{ms}^{-1}]$
- Latitude

**Figure 2:** Zonal wind and eddy momentum flux convergence as a function of latitude.

**Figure 3:** Eddy momentum flux convergence on the poleward and equatorward flanks for all the jets for the run with rotation rate $8$ Earth's rotation rate. Each dot represents a jet in certain time. The black line represents a line where the eddy momentum flux convergence on the poleward flank (EMCP) is equal to the eddy momentum flux convergence on the equatorward flank (EMCE). The percentage of the jets which expirenced a poleward bias (dots which lies below the black line) is presented at the title. The red line represents the mean ratio between the eddy momentum flux convergence on the equatorward and poleward flanks (EMCE/EMCP). The slope of the red line is also presented at the title.

**Figure 4:** Shallow models - Eddy momentum flux convergence.
Shallow models

Recently 3D GCMs have been able show superrotation, multiple jets and consistent jets for all four giant planets

Lian & Showman 2010

Liu & Schneider 2010
Deep Models

• Heuristic models of columnar convection based on lab convection on experiments. Busse, 1970; Busse 1976; Ingersoll & Pollard, 1982.

• Boussinesq convection models
  Sun et. Al., 1993; Zhang & Schubert, 1997; Christensen 2002; Busse, 2002; Heimpel et al., 2005; Aurnou et al., 2007; Heimpel & Gomez-Perez, 2011;

• Compressible convection models (no MHD)
  Evonuk et al. 2007; Glatzmair et al. 2009; Kaspi et al. 2009; Jones et. al., 2009; Gastine & Wicht 2012

Christensen, 2002  Heimpel & Gomez-Perez, 2011  Kaspi et al., 2009  Heimpel et al., 2005
A Deep Anelastic GCM

• Stripped down version of the MITgcm – uses dynamical core

• Deep dynamics – full sphere instead of spherical shell

• Non-Hydrostatic

• Non-Boussinesq (Anelastic  \( \nabla \cdot (\bar{\rho} u) = 0 \))

• Gravity field and thermodynamic properties vary with depth

• Forced by internal heating and surface cooling

• Uses a hydrogen equation of state (Saumon, 1995)

\[ \rho' = \rho'(s', p') \]

• Set by only 3 control parameters:

\[ \text{Pr} = \frac{\nu}{\kappa} \quad \text{Ek} = \frac{\nu}{\Omega H^2} \quad \text{Ra}_F = \frac{H^3}{\nu \kappa^2} \int \frac{\alpha_{TG} gF}{\rho \rho'} \, dr \]
Model equations:

Momentum conservation

\[
\frac{Du}{Dt} + \frac{uw}{r} - \frac{uv}{r} \tan \theta - 2\Omega \sin \theta v + 2\Omega \cos \theta w = -\frac{1}{\bar{\rho} r \cos \theta} \frac{\partial p'}{\partial \phi} + \nu \nabla^2 u
\]

\[
\frac{Dv}{Dt} + \frac{wv}{r} + \frac{u^2}{r} \tan \theta + 2\Omega \sin \theta u = -\frac{1}{\bar{\rho} r} \frac{\partial p'}{\partial \theta} + \nu \nabla^2 v
\]

\[
\frac{Dw}{Dt} - \frac{u^2}{r} - \frac{v^2}{r} - 2\Omega \cos \theta u = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial r} - \frac{\rho'}{\bar{\rho}} g + \nu \nabla^2 w
\]

Mass conservation

\[
w \frac{\partial \bar{\rho}}{\partial r} + \bar{\rho} \nabla \cdot \bar{u} = 0
\]

Thermodyn’ equation

\[
\frac{Ds'}{Dt} + \kappa \nabla^2 s' = \frac{Q}{T}
\]

Equation of State

\[
\rho'(p, s) = \left( \frac{\partial \rho}{\partial s} \right)_p s' + \left( \frac{\partial \rho}{\partial p} \right)_s p'
\]

Solve coupled nonlinear system of 6 equations for the unknowns \( u, v, w, \rho', s', p' \)
Basic balances

Geostrophic:

\[-2\Omega \sin \theta v + 2\Omega \cos \theta w = -\frac{1}{\bar{\rho} r \cos \theta} \frac{\partial p'}{\partial \phi}\]

\[2\Omega \sin \theta u = -\frac{1}{\bar{\rho} r} \frac{\partial p'}{\partial \theta}\]

Hydrostatic:

\[-2\Omega \cos \theta u = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial r} - \frac{\rho'}{\bar{\rho}} g\]

\[-\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial r} \left(-\frac{\rho'}{\bar{\rho}} g\right)\]

Zonally averaged fields of the vertical momentum equation on a meridional slice (multiplied by \(\bar{\rho}\))

Thermal Wind:

\[
\frac{\partial u}{\partial z} = \frac{g}{2\Omega \bar{\rho}} \frac{1}{r} \frac{\partial \rho'}{\partial \theta} - \frac{1}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial r} u \sin \theta
\]
The effect of rotation on internal convection

*For demonstration heat flux is applied at the bottom boundary only (axisymmetric simulations)

** Heat flux is held constant and therefore slow rotation runs are more supercritical
The effect of rotation: 2D Boussinesq example

Fast Rotation \( \frac{2\pi}{\Omega} = 10hr \)

Slow Rotation \( \frac{2\pi}{\Omega} = 100hr \)
Angular momentum: \[ M = \Omega r^2 \cos^2 \theta + ur \cos \theta \approx \Omega r^2 \cos^2 \theta \]

Angular momentum conservation:

\[ \frac{\partial M}{\partial t} + \frac{1}{\rho} \nabla \cdot (\rho \vec{u}M) = \nabla^2 M \]

\[ \vec{u} \cdot \nabla M \approx \theta \nabla^2 M \]

For fast rotation there is no flow across angular momentum surfaces except near the boundaries.

Streamfunction: \[ \nabla \times \Psi = \vec{\rho} \vec{u} \]
The effect of rotation: 3D Boussinesq example

\[ \frac{2\pi}{\Omega} = 10 \text{hr} \]

Eastward flow

Westward flow

Modified Rayleigh number

\[ Ra_F^* = Ra_F E k^3 P r^{-2} \]

\[ Ra_F^* = \frac{1}{\Omega^3 H^2 M} \int \frac{\alpha T g F}{C_p} r^2 dr \]
**Boussinesq vs. Anelastic**

For a Boussinesq fluid zonal velocity forms along Taylor columns; for the anelastic case the flow has a baroclinic structure which is aligned with the axis of rotation.

\[
\text{Zonally averaged zonal velocity}
\]

\[
\text{Anelastic} \\
\text{Boussinesq}
\]

\[
2\Omega \frac{\partial}{\partial z} (\bar{\rho} u) = \frac{\partial}{\partial \theta} \frac{\bar{\rho}'}{\bar{s}'} = \alpha_s \bar{\rho}g \frac{1}{r} \frac{\partial s'}{\partial \theta} + \beta \bar{\rho}g \frac{1}{r} \frac{\partial \bar{p}'}{\partial \theta}
\]
Variation of the zonal wind with depth

Since the flow is to leading order geostrophic thermal wind balance holds:

In terms of entropy perturbations:

\[
\frac{\partial u}{\partial z} = \alpha_s \frac{g}{2\Omega} \frac{1}{r} \frac{\partial s'}{\partial \theta}
\]

Based on SCVH equation of state

For ideal gas:

\[
\alpha_s = \frac{1}{c_p}
\]
The zonally averaged zonal velocity for different geometries (from a spherical shell to a full sphere)

3D anelastic cases

Aspect ratio: \[ D = \frac{r_o - r_i}{r_o} \]
Results: overview

Meridional section
- Zonal velocity
- Entropy
- 2D streamfunction

Equatorial section
- Zonal velocity
- Entropy
- 2D streamfunction

1 Bar Surface:
- Zonal velocity
- Entropy
- Vertical vorticity
Results: overview

Entropy anomaly on the equatorial plane

- Zonal velocity
- Entropy
- 2D streamfunction

Equatorial Plane

- Zonal velocity
- Entropy
- 2D streamfunction

Surface

- Zonal velocity
- Entropy
- Vertical vorticity
Results: overview

- Zonal velocity
- Entropy
- 2D streamfunction

Equatorial Plane

- Zonal velocity
- Entropy
- 2D streamfunction

Surface

- Zonal velocity
- Entropy
- Vertical vorticity
2D streamfunction on the equatorial plane

- Zonal velocity
- Entropy
- 2D streamfunction

Eq. streamfunction Hovmoller Diag.

- Zonal velocity
- Entropy
- 2D streamfunction

$c_\phi = 250 \text{ m/s}$

Equatorial Plane

- Zonal velocity
- Entropy

Surface

- Zonal velocity
- Entropy
- Vertical vorticity
streamfunction on surfaces of constant latitudinal angle
Angular momentum fluxes

Angular momentum:

\[ M = \Omega r^2 \cos^2 \theta + u r \cos \theta \]

Angular momentum conservation:

\[ \frac{\partial M}{\partial t} + \frac{1}{\rho} \nabla \cdot (\rho \bar{u} \bar{M}) + \frac{1}{\rho} \nabla \cdot (\rho u' \bar{M}') = \nu \nabla^2 \bar{M} \]

Heat fluxes

\[ u = \bar{u} + u' \]

Mean zonal velocity

Eddy zonal velocity
Columnar barotropic model (Ingersoll & Pollard, 1982)

mom:
\[
\frac{\partial u}{\partial t} - \frac{uw}{r} - 2\Omega w = -\frac{1}{r} \frac{\partial \Phi'}{\partial \phi} \\
\frac{\partial v}{\partial t} = -\frac{\partial \Phi'}{\partial z} \\
\frac{\partial w}{\partial t} + \frac{u^2}{r} + 2\Omega u = -\frac{\partial \Phi'}{\partial r}
\]

mass:
\[
\frac{\bar{\rho}}{r} \left( \frac{\partial u}{\partial \phi} + \frac{\partial (rw)}{\partial \phi} \right) + w \frac{\partial \bar{\rho}}{\partial r} + \frac{\partial (\bar{\rho}v)}{\partial z} = 0
\]

Deriving a vorticity equation by assuming a perturbation streamfunction of the form:
\[
\varphi = \hat{\varphi}(r) e^{ik(\phi-ct)}
\]

Gives an equivalent barotropic equation
\[
\left(\bar{u} - c\right) \left( \frac{d^2 \hat{\varphi}}{dr^2} - k^2 \hat{\varphi} \right) + \left( -\frac{d^2 \bar{u}}{dr^2} \right) \hat{\varphi} = 0
\]
Columnar barotropic model  (Ingersoll & Pollard, 1982)

mom: \[
\frac{\partial u}{\partial t} - \frac{uw}{r} - 2\Omega w = -\frac{1}{r} \frac{\partial \Phi'}{\partial \phi} \\
\frac{\partial v}{\partial t} = \frac{\partial \Phi'}{\partial z} \\
\frac{\partial w}{\partial t} + \frac{u^2}{r} + 2\Omega u = -\frac{\partial \Phi'}{\partial r}
\]

mass: \[
\frac{\bar{\rho}}{r} \left( \frac{\partial u}{\partial \phi} + \frac{\partial (rw)}{\partial \phi} \right) + w \frac{\partial \bar{\rho}}{\partial r} + \frac{\partial (\bar{\rho}v)}{\partial z} = 0
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\[
\left( \bar{u} - c \right) \left( \frac{d^2 \hat{\varphi}}{dr^2} - k^2 \hat{\varphi} \right) + \left( 2\Omega \frac{dM}{dr} - \frac{d^2 \bar{u}}{dr^2} \right) \hat{\varphi} = 0
\]

\[
M = \int_{-h}^{h} \bar{\rho} dz
\]

\[
B(r)
\]
Columnar barotropic model (Ingersoll & Pollard, 1982)

mom: \( \frac{\partial u}{\partial t} - \frac{uw}{r} - 2\Omega w = -\frac{1}{r} \frac{\partial \Phi'}{\partial \phi} \)
\( \frac{\partial v}{\partial t} = -\frac{\partial \Phi'}{\partial z} \)
\( \frac{\partial w}{\partial t} + \frac{u^2}{r} + 2\Omega u = -\frac{\partial \Phi'}{\partial r} \)

mass: \( \frac{\bar{\rho}}{r} \left( \frac{\partial u}{\partial \phi} + \frac{\partial (rw)}{\partial \phi} \right) + w \frac{\partial \bar{\rho}}{\partial r} + \frac{\partial (\bar{\rho}v)}{\partial z} = 0 \)

Deriving a vorticity equation by assuming a perturbation streamfunction of the form:
\[ \varphi = \hat{\varphi}(r)e^{ik(\varphi - \omega t)} \]

Gives an equivalent barotropic equation
\[ \left( \bar{u} - c \right) \left( \frac{d^2 \hat{\varphi}}{dr^2} - k^2 \hat{\varphi} \right) + \left( \frac{2\Omega}{M} \frac{dM}{dr} - \frac{d^2 \bar{u}}{dr^2} \right) \hat{\varphi} = 0 \]
\[ M = \int_{-h}^{h} \bar{\rho}dz \]
\[ B(r) \]
Thin spherical shell

- Planetary vorticity grows with latitude
- Beta is positive
- Rossby wave propagates **westward**

Sphere interior

- Planetary vorticity decreases towards the axis of rotation
- $B(r)$ is negative
- Rossby wave propagates **eastward**

Analytic solutions to linear convection in a rotating sphere. (Zhang and Schubert, 1997)

Rossby wave mechanism on the equatorial plane (Busse, 1986, 2002).
Thin spherical shell

- Planetary vorticity grows with latitude
- Beta is positive
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Sphere interior

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Analytic solutions to linear convection in a rotating sphere. (Zhang and Schubert, 1997)

Rossby wave mechanism on the equatorial plane (Busse, 1986, 2002).
The zonal velocity:

Splitting the total velocity into a zonal mean and a zonally asymmetric part:

\[
u(r,0,\phi) = u'(r,\phi) + \bar{u}(r)\]

Weakly nonlinear solution (small Rayleigh and Prandtl numbers)
The zonal velocity:

Splitting the total velocity into a zonal mean and a zonally asymmetric part:

Weakly nonlinear solution (small Rayleigh and Prandtl numbers)

\[ \bar{u}(r, \theta) + u'(r, \theta, \phi) \]

Evolves from random initial condition

\[ \frac{\partial \bar{u}}{\partial t} \approx -\frac{1}{\rho} \nabla \cdot (\bar{\rho} u' \bar{u}) + \nu \nabla^2 \bar{u} \]
Zonal Momentum balance after the instability:

\[
\frac{\partial \bar{u}}{\partial t} + \frac{1}{\rho} \nabla \cdot (\rho u' \bar{u}) - 2\Omega \sin \theta \bar{v} + 2\Omega \cos \theta \bar{w} = \nu \nabla^2 \bar{u}
\]
Turbulent Run:

Weakly Nonlinear Run:

Equatorial Streamfunction:

Surface Zonal velocity:
Annulus Model

Solve for:

\[
\frac{Dq}{Dt} = 0
\]

\[
\nabla \cdot (uH) = 0 \quad \rightarrow \quad uH = \nabla \times \psi
\]

\[
q = \frac{\zeta + f}{H}
\]

\[
\zeta = \nabla \cdot \frac{1}{H} \nabla \psi
\]

We use a channel model and map it to an annulus using the following mapping:

\[
r = r_0 e^M
\]

\[
\theta = -\frac{x}{M}
\]

\[
Z = z
\]

Examples of runs with opposite linear profiles of \( H(r) \)
Annulus Model

Solve for:
\[
\frac{Dq}{Dt} = 0 \\
\nabla \cdot (uH) = 0 \quad \Rightarrow \quad uH = \nabla \times \psi
\]

pv:
\[
q = \frac{\zeta + f}{H}
\]

vorticity
\[
\zeta = \nabla \cdot \frac{1}{H} \nabla \psi
\]

We use a channel model and map it to an annulus using the following mapping:

\[
r = r_0 e^M \\
\theta = -\frac{x}{M} \\
Z = z
\]

Examples of runs with opposite linear profiles of \(H(r)\)
Annulus Model

Solve for:
\[
\frac{Dq}{Dt} = 0 \quad , \quad \nabla \cdot (uH) = 0 \quad \Rightarrow \quad uH = \nabla \times \psi
\]

pv:
\[
q = \frac{\zeta + f}{H}
\]

vorticity
\[
\zeta = \nabla \cdot \frac{1}{H} \nabla \psi
\]

We use a channel model and map it to an annulus using the following mapping:

Examples of runs with opposite linear profiles of \( H(r) \)
We can solve for the eigenmodes in the presence of friction

\[ \frac{\partial \zeta}{\partial t} + uH \cdot \nabla q = v \nabla^2 \zeta \]

\[ \zeta = \nabla \cdot \frac{1}{H} \nabla \psi \equiv M \psi \]

\[ q = \frac{\zeta + f}{H} \]

Assume solution of the form:

\[ \zeta = e^{ik(x - ct)}z_0(y) \]

Solve for the eigenvectors \( z_0 \):

\[ \left[ M^{-1} \frac{\partial q}{\partial y} - i \nu \left( -k^2 + \frac{\partial}{\partial y} \left( -k^2 + \frac{1}{H} \frac{\partial}{\partial y} \right) \right) \right] z_0 = -cz_0 \]

**Ocean Analogy:**
Refraction of surface gravity waves as they approach a sloping beach

![Ocean Analogy](image)
Conclusion

1. We present a new GCM for giant planets which is deep, anelastic and has realistic thermodynamics.

2. Not adopting the traditional “shallow” approximation makes the basic dynamical balances different (e.g. thermal wind) and has interesting consequences on the dynamics (e.g. opposite planetary vorticity gradient).

3. Compressible effects cause shear in the zonal winds resulting in weak interior flow, while an equivalent Boussinesq system has constant velocity along the direction of the axis of rotation.

4. Shallower systems result in narrower equatorial superrotation although this relation is not linear.

5. In the linear limit the system develops convection columns with counterrotating circulation, and as the system becomes more turbulent these columns become all rotating in the direction of the planetary rotation.