

Deepconv, A Numerical Model for Moist Convection in Planetary Atmosphere

K. Sugiyama, M. Odaka, T. Yamashita,
K. Nakajima, Y.-Y. Hayashi,
And deepconv development group of
GFD-Dennou-Club



History of Deepconv

- anelastic, mass stream function
 - Nakajima 1984-86 On the Earth's convective clouds
- anelastic, pressure eq
 - Nakajima 1989-94 On the Earth's convective clouds
 - Nakajima 1995-2000 application to Jupiter's H₂O cloud
 - Nakajima 1998 'generic' convections with phase change in GFD
 - Odaka 1998-2002 application to dry convection on Mars
- Thermochemical package ("Oboro")
 - Sugiyama 2006, for Jupiter's H₂O, NH₃, NH₄SH clouds
- Quasi-compressible eq. ("Arare")
 - Sugiyama 2009, on Jupiter's convective clouds
 - Odaka 2005- on Mars's convection
- Yamashita -2015 CO₂ condensation cloud on ancient Mars



Basic equations (1)

Equations of Motion

$$\begin{aligned}\frac{\partial u}{\partial t} = & - \left((\bar{u} + u) \frac{\partial u}{\partial x} + (\bar{v} + v) \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) - w \frac{\partial \bar{u}}{\partial z} \\ & - c_{pd} \bar{\theta}_v \frac{\partial \pi}{\partial x} + Turb.u + Turb.\bar{u}\end{aligned}\quad (1.1)$$

$$\begin{aligned}\frac{\partial v}{\partial t} = & - \left((\bar{u} + u) \frac{\partial v}{\partial x} + (\bar{v} + v) \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) - w \frac{\partial \bar{v}}{\partial z} \\ & - c_{pd} \bar{\theta}_v \frac{\partial \pi}{\partial y} + Turb.v + Turb.\bar{v}\end{aligned}\quad (1.2)$$

$$\begin{aligned}\frac{\partial w}{\partial t} = & - \left((\bar{u} + u) \frac{\partial w}{\partial x} + (\bar{v} + v) \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) - c_{pd} \bar{\theta}_v \frac{\partial \pi}{\partial z} + Turb.w \\ & + \left(\frac{\theta}{\bar{\theta}} + \frac{\sum q_v/M_v}{1/M_d + \sum \bar{q}_v/M_v} - \frac{\sum q_v + \sum q_c + \sum q_r}{1 + \sum \bar{q}_v} \right) g\end{aligned}\quad (1.3)$$

Pressure Equation

$$\begin{aligned}\frac{\partial \pi}{\partial t} = & - \left\{ \frac{\overline{C_s^2}}{c_{pd} \bar{\theta}_v} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{\overline{C_s^2}}{c_{pd} \bar{\rho} \bar{\theta}_v^2} \frac{\partial \bar{\rho} \bar{\theta}_v w}{\partial z} \right\} \\ & - \left((\bar{u} + u) \frac{\partial \pi}{\partial x} + (\bar{v} + v) \frac{\partial \pi}{\partial y} + w \frac{\partial \pi}{\partial z} \right) + \frac{R_d \pi}{c_{vd}} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \\ & + \frac{\overline{C_s^2}}{c_{pd} \bar{\theta}_v} \left\{ \frac{\dot{\theta}}{\bar{\theta}} - \left(\frac{\sum \dot{q}_v + \sum \dot{q}_c + \sum \dot{q}_r}{1 + \sum \bar{q}_v} - \frac{\sum \dot{q}_v/M_v}{1/M_d + \sum \bar{q}_v/M_v} \right) \right\}\end{aligned}\quad (1.4)$$



Basic equations (2)

Thermodynamic Equation

$$\begin{aligned}\frac{\partial \theta}{\partial t} = & - \left((\bar{u} + u) \frac{\partial \theta}{\partial x} + (\bar{v} + v) \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) - w \frac{\partial \bar{\theta}}{\partial x} + \frac{1}{\bar{\pi}} (Q_{cnd} + Q_{rad} + Q_{dis}) \\ & + Turb.\bar{\theta} + Turb.\theta\end{aligned}\quad (1.5)$$

Equation of state

$$\begin{aligned}\rho &= \frac{p}{R_d T} \left(\frac{1/M_d}{1/M_d + \sum q_v/M_v} \right) \left(1 + \sum q_v + \sum q_c + \sum q_r \right) \\ &= \frac{p}{R_d T_v} = \frac{p_0 \pi^{c_{vd}/R_d}}{R_d \theta_v}\end{aligned}\quad (1.11)$$

Definition of Virtual Temperature

$$\theta_v = \frac{\theta}{\left(\frac{1/M_d}{1/M_d + \sum q_v/M_v} \right) (1 + \sum q_v + \sum q_c + \sum q_r)} \quad (1.12)$$

Sound velocity

$$C_s^2 = \frac{c_{pd}}{c_{vd}} R_d \pi \theta_v \quad (1.13)$$



Basic equations (3)

Thermodynamic eq

$$\begin{aligned} \frac{\partial \theta}{\partial t} = & - \left((\bar{u} + u) \frac{\partial \theta}{\partial x} + (\bar{v} + v) \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) - w \frac{\partial \bar{\theta}}{\partial x} + \frac{L}{c_{pd}\bar{\pi}} (CN_{vc} - EV_{cv} - EV_{rv}) \\ & + \frac{1}{\bar{\pi}} (Q_{rad} + Q_{dis}) + Turb.\bar{\theta} + Turb.\theta \end{aligned} \quad (1.14)$$

Conservation eq. for vapor, cloud, and rain

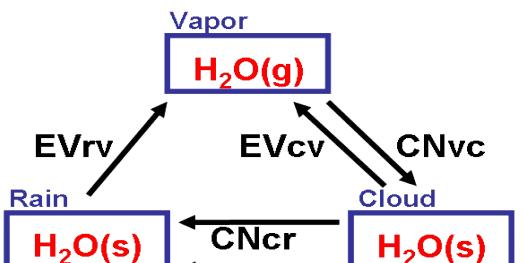
$$\begin{aligned} \frac{\partial q_v}{\partial t} = & - \left((\bar{u} + u) \frac{\partial q_v}{\partial x} + (\bar{v} + v) \frac{\partial q_v}{\partial y} + w \frac{\partial q_v}{\partial z} \right) - w \frac{\partial \bar{q}_v}{\partial x} - (CN_{vc} - EV_{cv} - EV_{rv}) \\ & + Turb.q_v + Turb.\bar{q}_v, \end{aligned} \quad (1.15)$$

$$\begin{aligned} \frac{\partial q_c}{\partial t} = & - \left((\bar{u} + u) \frac{\partial q_c}{\partial x} + (\bar{v} + v) \frac{\partial q_c}{\partial y} + w \frac{\partial q_c}{\partial z} \right) + (CN_{vc} - EV_{cv} - CN_{cr} - CL_{cr}) \\ & + Turb.q_c, \end{aligned} \quad (1.16)$$

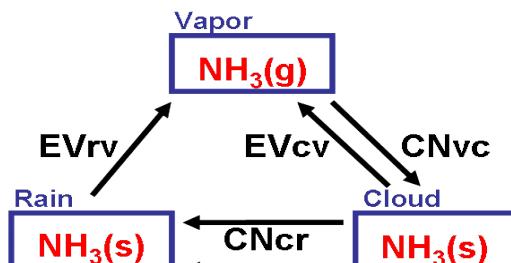
$$\frac{\partial q_r}{\partial t} = - \left((\bar{u} + u) \frac{\partial q_c}{\partial x} + (\bar{v} + v) \frac{\partial q_c}{\partial y} + w \frac{\partial q_c}{\partial z} \right) + (CN_{cr} + CL_{cr} - EV_{rv}) + PR_r$$

Thermodynamic quantities are provided by Oboro (thermochemical package) automatically.

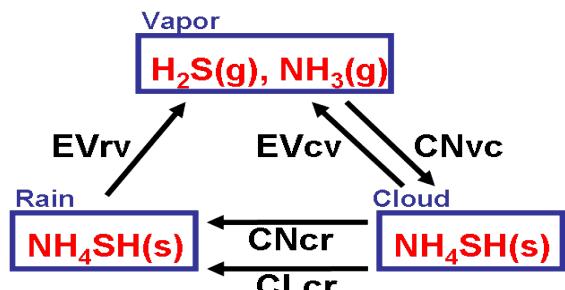
(a) H_2O condensation



(b) NH_3 condensation

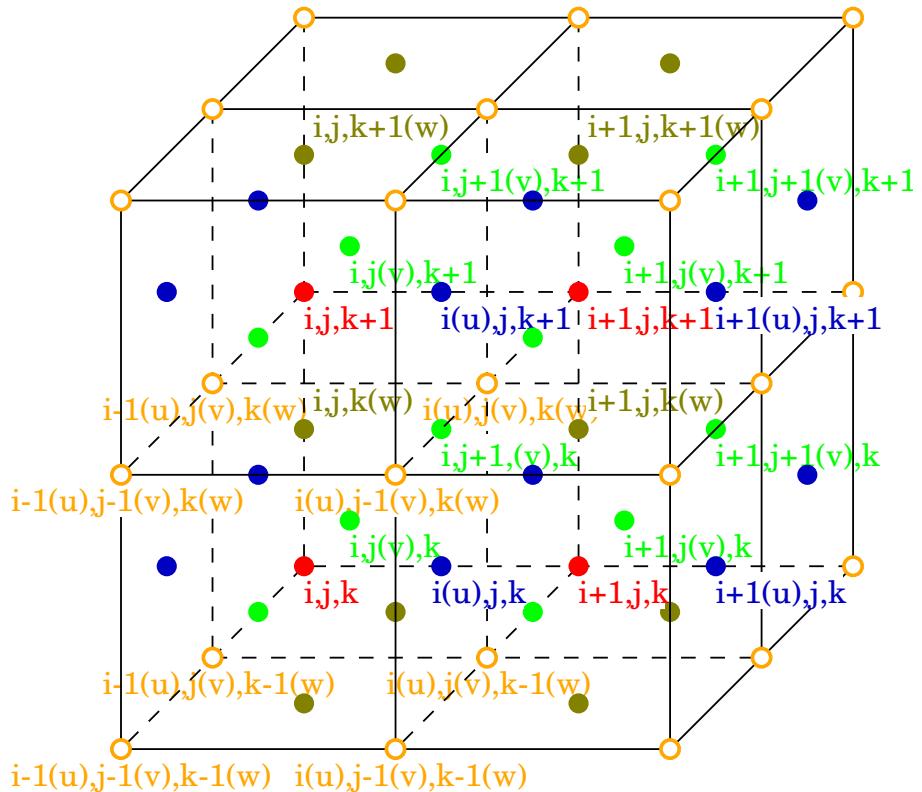


(c) NH_4SH reaction



Grid configuration

- Horizontal Arakawa-C grid
- Vertical Lorenz grid
- Centered difference
 - 2nd / 4th order



Coding style : use F90 array functions

```
program diffuse_2d
use gridset, only :: DimXMin...
use differentiate_center4,only:xz_dx_pz...
...
do it=1,nt
    write(*,*) '*it = ',it
    xz_ZetaA = xz_ZetaN
    & + dt * nu * (
    &     xz_dx_pz(pz_dx_xz(xz_ZetaN)) &
    &     + xz_dz_xr(xr_dz_xz(xz_ZetaN))&
    & )
    call BoundaryXcyc_xz( xz_ZetaA )
    call BoundaryZCyc_xz( xz_ZetaA )
    xz_ZetaN = xz_ZetaA
end do
```

$$\pi_{i(u),j,k} \equiv \frac{\pi_{i+1,j,k} + \pi_{i,j,k}}{2}$$

$$u_{i,j,k} \equiv \frac{u_{i(u),j,k} + u_{i-1(u),j,k}}{2}$$

$$\zeta_t = \nu (\partial^2 \zeta / \partial x^2 + \partial^2 \zeta / \partial z^2)$$

Back to "moderate" way



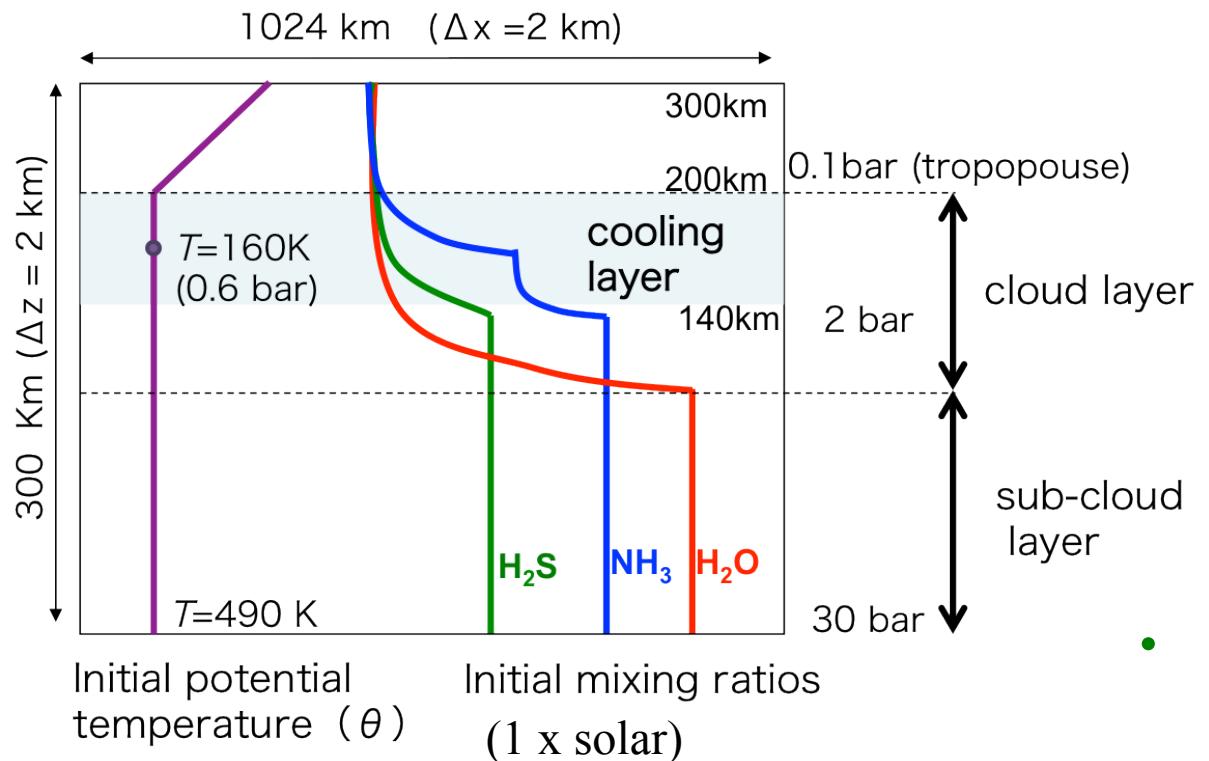
- With extensive usage of F90 array functions, the model runs like a turtle.
 - Back to more F77 like style
 - Improve the speed by 100 times
- With extensive dependence of Oboro thermodynamic package, the model loses flexibility.
 - “molecular weight of non-condensible component” can be specified again.
 - Helpful for “mechanistic experiments.”



Example 1 : Jupiter clouds of three components: H_2O , NH_3 , NH_4SH

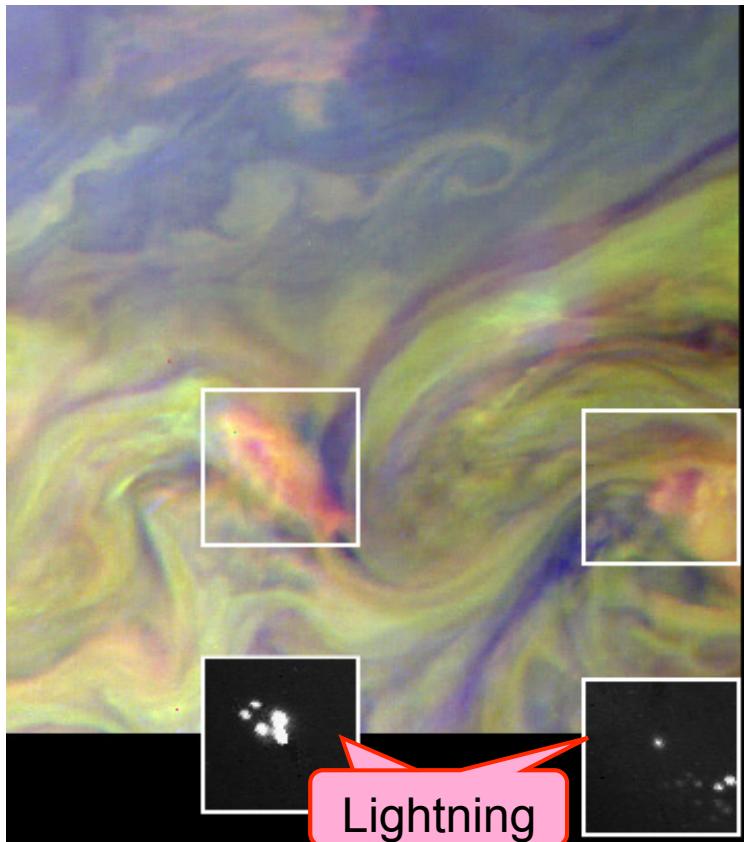


Set-up of the experiments



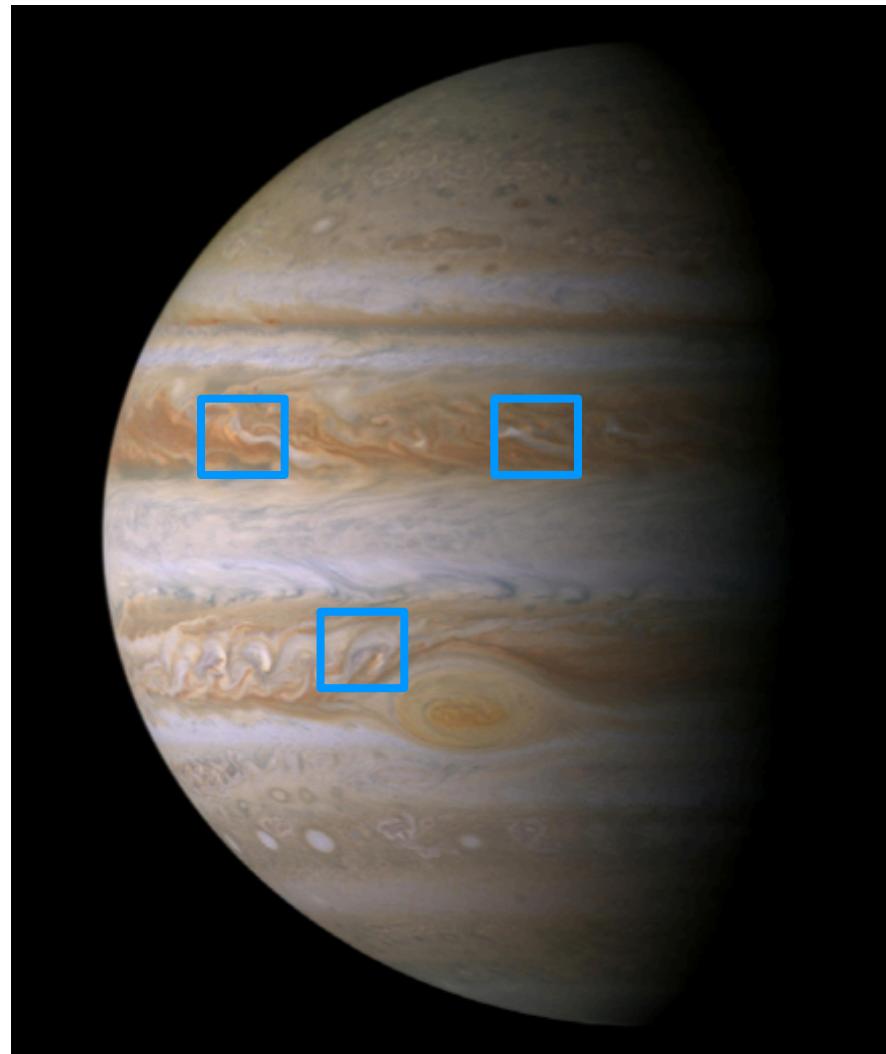
- **Boundary conditions**
 - Horizontal boundary is cyclic. Stress free condition and $w = 0$ are given at the lower and upper boundaries.
- Temperature and mixing ratios of vapor at the lowest level are fixed.
- **Initial condition**
 - Random potential temperature perturbation ($\Delta\theta_{\max} = 0.1 \text{ K}$) is given to seed convective motion.

Jupiter's “Convective” clouds

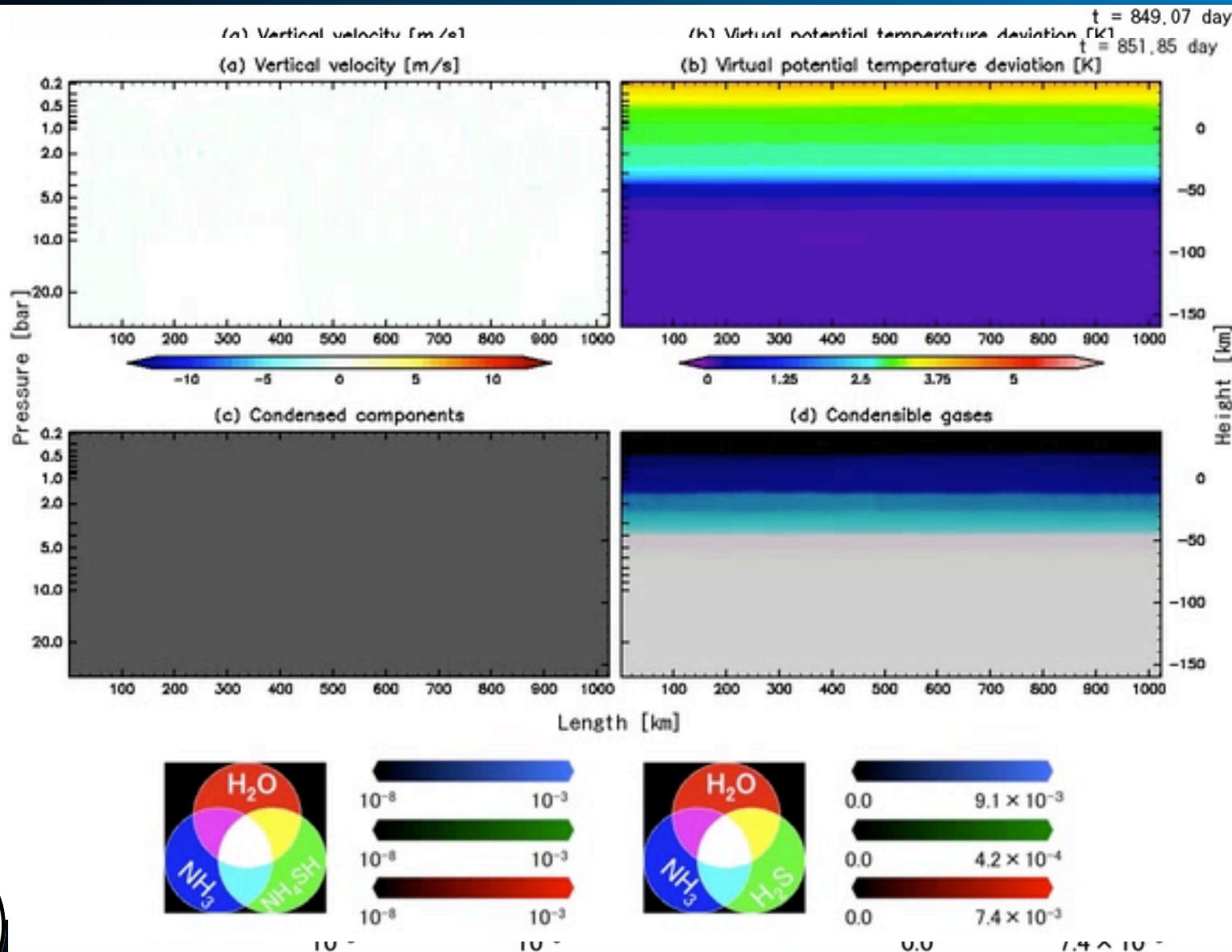


Convective clouds

(Vasavada and Showman, 2005)

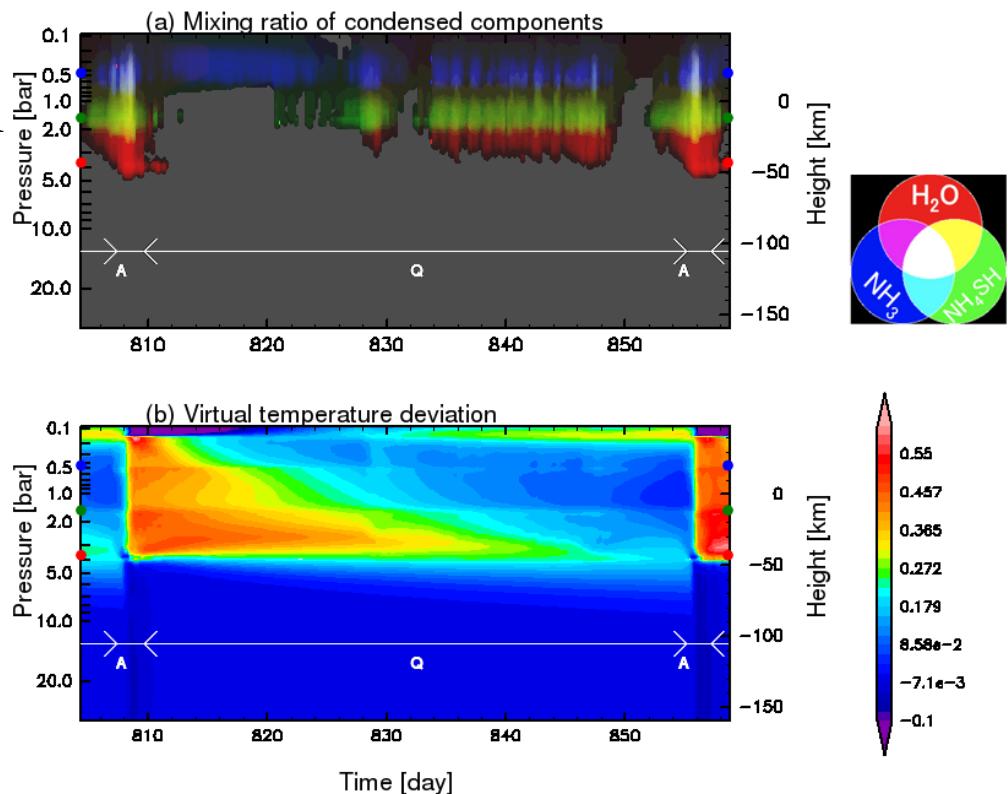


Results: Animation



Temporal variation

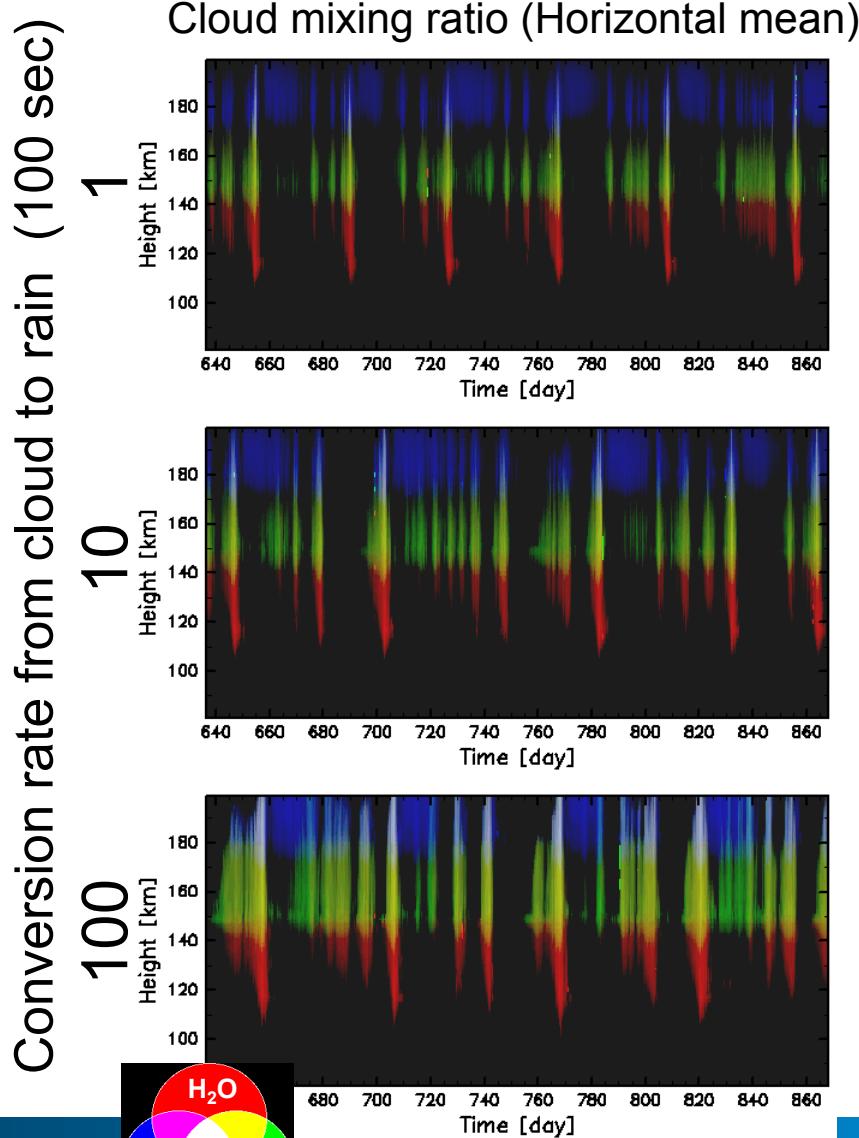
- The convective activity of the whole layer is not steady but quasi-periodic with a period of about 40 days.
- Overall temperature of the cloud layer synchronizes with the intermittent convective activity.
 - We will refer the time when the active cloud convection occurs as 'active period' (A) and the other as 'quiet period' (Q).



Period of the intermittency

- The period of the quasi-periodic cycle is about 2 times larger than that of the previous study.

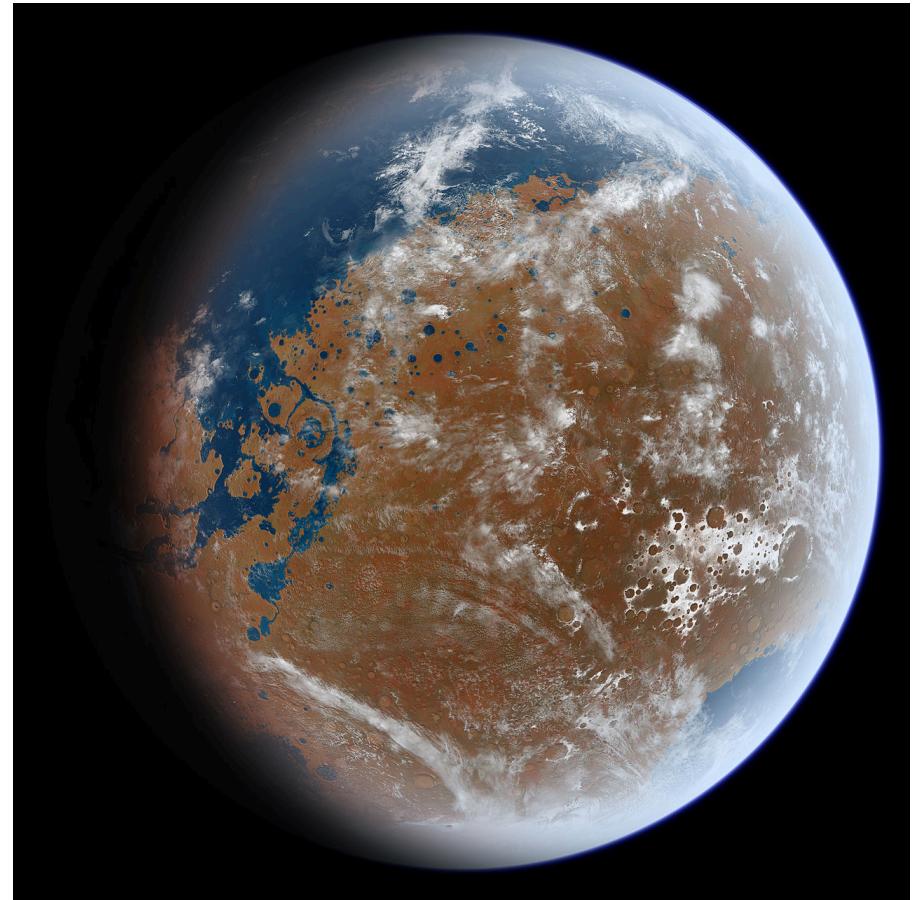
Conversion rate from cloud to rain (100 sec)	period in the numerical experiment (day)	ratio
1	36.4	1
10	60.2	1.80
100	62.5	1.85



Example 2 : Mars condensation of major component (CO₂)

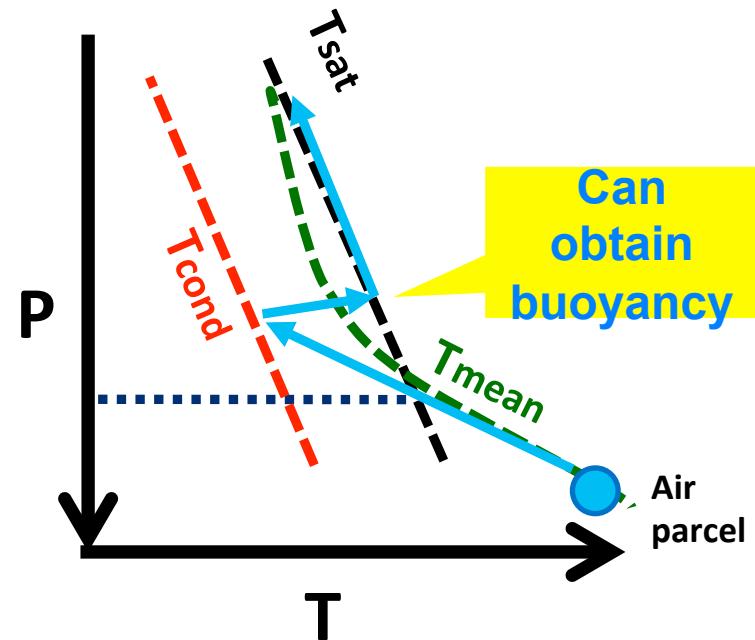
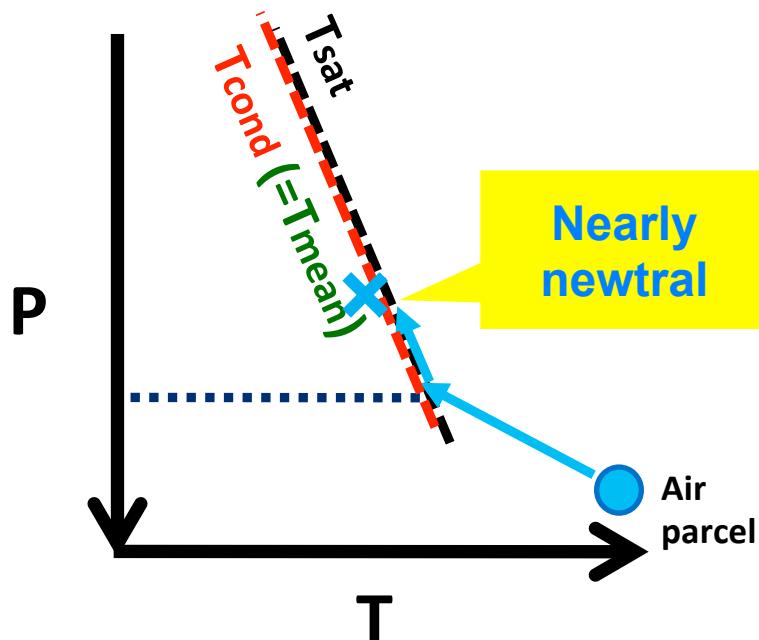


- Ancient warm and moist Mars?



Condensation convection of CO₂ and cloud type (Colaprete et al. 2003)

- Whether supersaturation is allowed or not is key point.
 - No: Stratiform cloud Yes: Convective cloud



CO₂ ice cloud microphysics

- Condensation/Evaporation rate (Tobie et al., 2003)

$$M \downarrow cond = 4\pi r \rho N \downarrow * k R \theta \tau^2 \Pi \tau^2 / L \tau^2 (S - 1)$$

- For $1 < S < S_{cr}$, condensation does not occur if cloud density is less than a threshold value
- We assume the value of threshold as $1.0 \text{e-}6 \text{ kg/m}^3$

- Gravitational settling rate

$$M \downarrow fall = \partial / \partial z (\rho \downarrow s V \downarrow term)$$

$$V \downarrow term = (1 + 4/3 K \downarrow n) 2r \tau^2 g \rho \downarrow I / 9 \eta$$

$S = \frac{p}{p_*}$: Saturation ratio

p : Pressure p_* : Saturation vapor pressure

ρ : Gas density η : viscosity coefficient

N^* : Number density of condensation nuclei

r : Radius of cloud particle ρ_I : CO₂ ice density

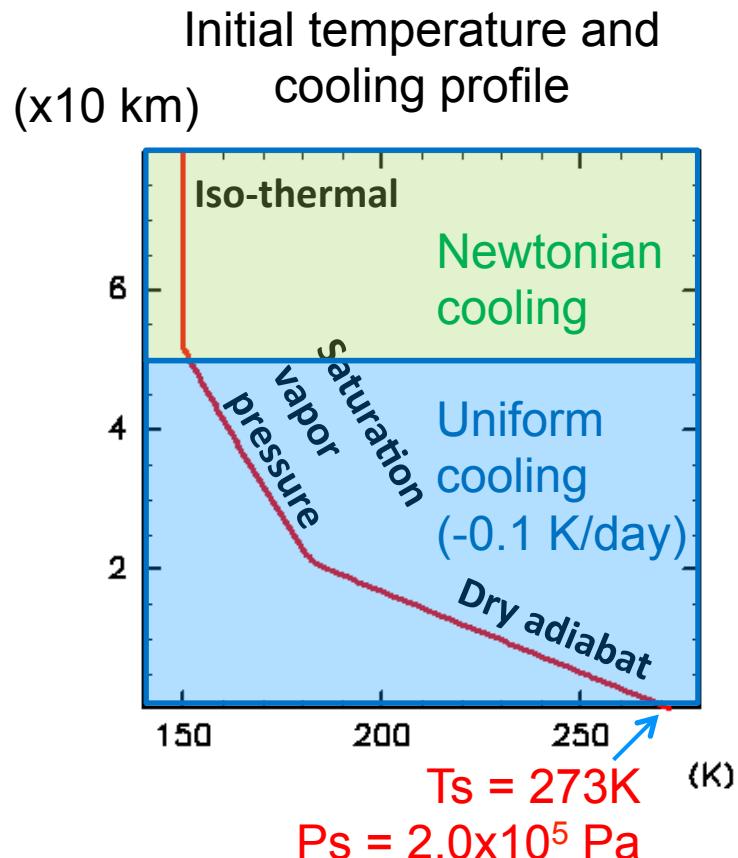
Kn : Knudsen number for cloud particle

k : Thermal diffusion coefficient R : Gas constant

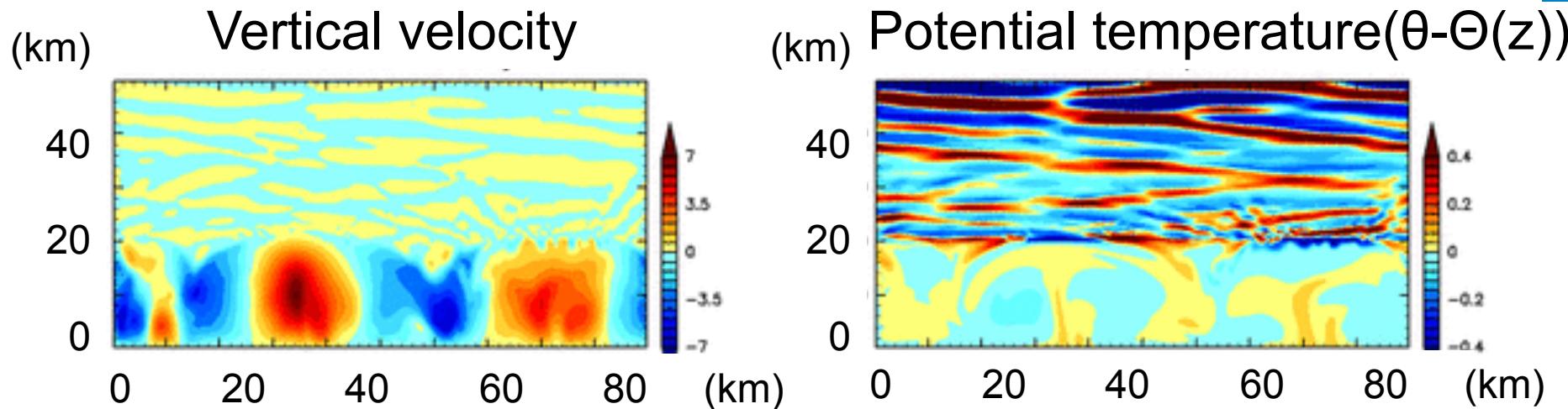


Setup of experiments

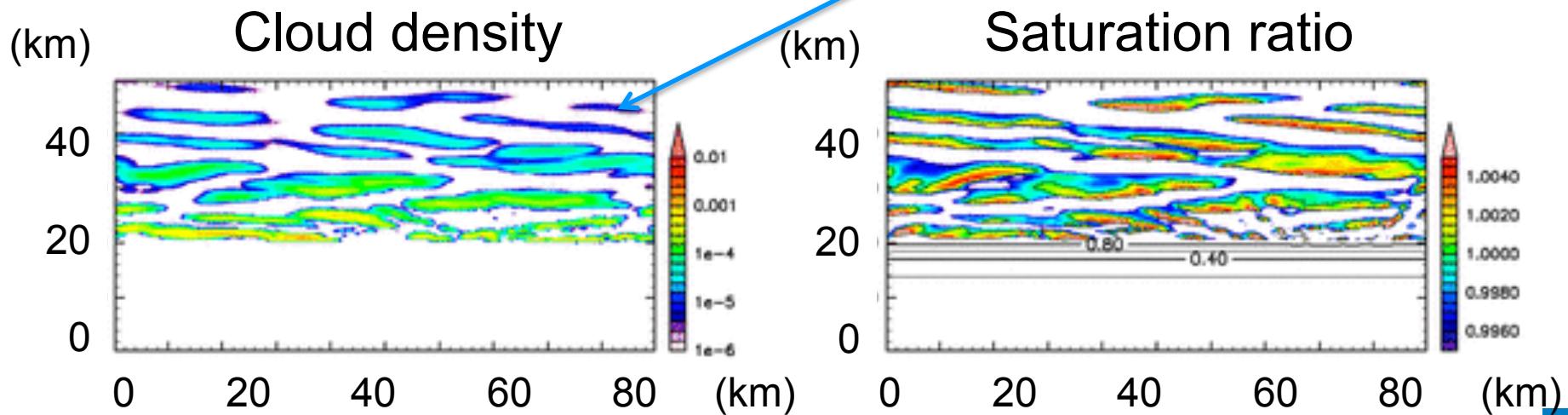
- Domain size:
 - 100km in horizontal direction (grid spacing: 500m)
 - 80km in vertical direction (grid spacing: 400m)
- Initial temperature and cooling profile
 - Based on Kasting (1991)
- Critical saturation ratio (Scr): 1.0, 1.35 (Glandorf et al., 2002)
- Number density of condensation nuclei $5.0 \times 10^8 / \text{kg}$ (Tobie et al., 2003; Forget et al., 2013)



Circulation features and cloud distribution: Scr=1.0 (100 days)

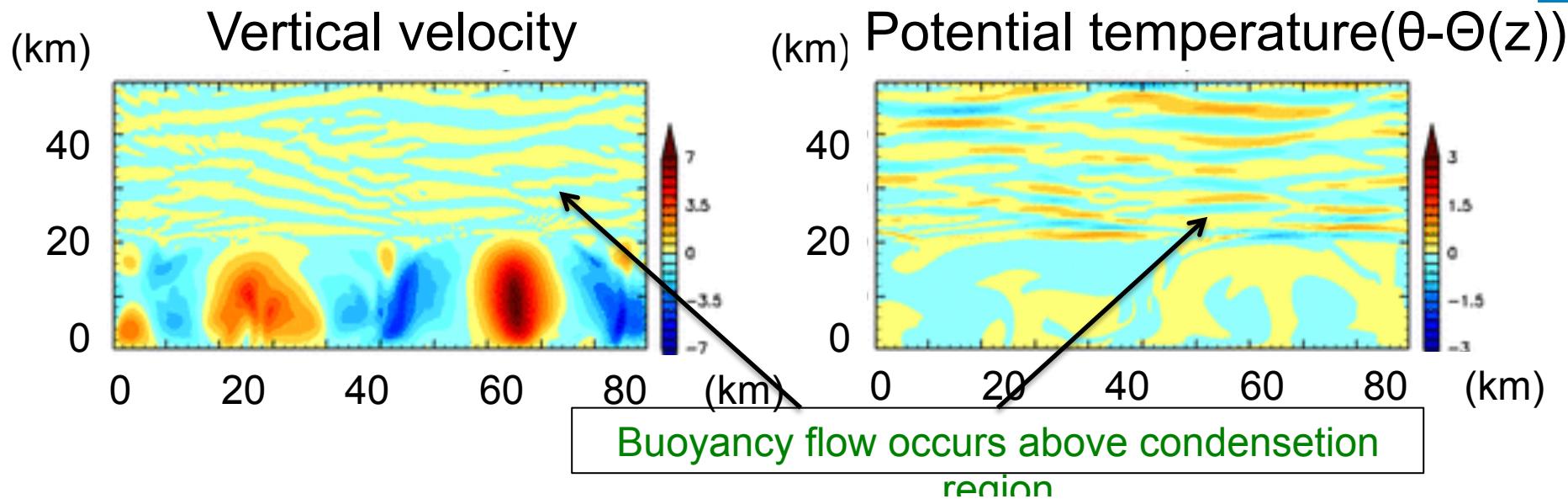


Cloud is generated by gravity wave



Circulation features and cloud distribution: Scr=1.35

(143 days)



The dry convection is activated due to evaporation cooling of dropped cloud particle

Cloud density

