Deepconv, A Numerical Model for Moist Convection in Planetary Atmosphere

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And deepconv development group of GFD-Dennou-Club
History of Deepconv

- anelastic, mass stream function
  - Nakajima 1984-86 On the Earth’s convective clouds
- anelastic, pressure eq
  - Nakajima 1989-94 On the Earth’s convective clouds
  - Nakajima 1995-2000 application to Jupiter’s H2O cloud
  - Nakajima 1998 ‘generic’ convections with phase change in GFD
  - Odaka 1998-2002 application to dry convection on Mars

- Thermochemical package (“Oboro”)
  - Sugiyama 2006, for Jupiter’s H2O, NH3, NH4SH clouds
- Quasi-compressible eq. (“Arare”)
  - Sugiyama 2009, on Jupiter’s convective clouds
  - Odaka 2005- on Mars’s convection
  - Yamashita -2015 CO2 condensation cloud on ancient Mars
Basic equations (1)

Equations of Motion

\[
\begin{align*}
\frac{\partial u}{\partial t} &= -\left( (\bar{u} + u) \frac{\partial u}{\partial x} + (\bar{v} + v) \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) - w \frac{\partial \bar{u}}{\partial z} \\
&\quad - c_{pd} \bar{v} \frac{\partial \pi}{\partial x} + \text{Turb}.u + \text{Turb}.\bar{u} \\
\frac{\partial v}{\partial t} &= -\left( (\bar{u} + u) \frac{\partial v}{\partial x} + (\bar{v} + v) \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) - w \frac{\partial \bar{v}}{\partial z} \\
&\quad - c_{pd} \bar{v} \frac{\partial \pi}{\partial y} + \text{Turb}.v + \text{Turb}.\bar{v} \\
\frac{\partial w}{\partial t} &= -\left( (\bar{u} + u) \frac{\partial w}{\partial x} + (\bar{v} + v) \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) - c_{pd} \bar{v} \frac{\partial \pi}{\partial z} + \text{Turb}.w \\
&\quad + \left( \frac{\dot{\theta}}{\theta} + \frac{\sum q_{v}/M_{v}}{1/M_d + \sum \bar{q}_{v}/M_{v}} - \frac{\sum q_{v} + \sum q_{c} + \sum q_{r}}{1 + \sum \bar{q}_{v}} \right) g
\end{align*}
\]

Pressure Equation

\[
\begin{align*}
\frac{\partial \pi}{\partial t} &= -\left\{ \frac{C_{s}^2}{c_{pd} \bar{v}} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{C_{s}^2}{c_{pd} \bar{v}} \frac{\partial \bar{v} \omega}{\partial z} \right\} \\
&\quad - \left( (\bar{u} + u) \frac{\partial \pi}{\partial x} + (\bar{v} + v) \frac{\partial \pi}{\partial y} + w \frac{\partial \pi}{\partial z} \right) + \frac{R_{ed} \pi}{c_{ed}} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \\
&\quad + \frac{C_{s}^2}{c_{pd} \bar{v}} \left\{ \frac{\dot{\theta}}{\bar{v}} - \left( \frac{\sum \bar{q}_{v} + \sum \bar{q}_{c} + \sum \bar{q}_{r}}{1 + \sum \bar{q}_{v}} \right) \right. \left. - \frac{\sum \bar{q}_{v}/M_{v}}{1/M_d + \sum \bar{q}_{v}/M_{v}} \right\} \quad (1.4)
\end{align*}
\]
Basic equations (2)

Thermodynamic Equation

\[
\frac{\partial \theta}{\partial t} = - \left( (\bar{u} + u) \frac{\partial \theta}{\partial x} + (\bar{v} + v) \frac{\partial \theta}{\partial v} + w \frac{\partial \theta}{\partial z} \right) - w \frac{\partial \bar{\theta}}{\partial x} + \frac{1}{\pi} (Q_{\text{end}} + Q_{\text{rad}} + Q_{\text{dis}}) + \text{Turb}.\bar{\theta} + \text{Turb}.\theta
\]

(1.5)

Equation of state

\[
\rho = \frac{p}{R_d T} \left( \frac{1/M_d}{1/M_d + \sum q_v/M_v} \right) \left( 1 + \sum q_v + \sum q_c + \sum q_r \right)
\]

\[
= \frac{p}{R_d T_v} = \frac{p_0 \pi c_v}{R_d \theta_v}
\]

(1.11)

Definition of Virtual Temperature

\[
\theta_v = \frac{\theta}{\left( \frac{1/M_d}{1/M_d + \sum q_v/M_v} \right) \left( 1 + \sum q_v + \sum q_c + \sum q_r \right)}
\]

(1.12)

Sound velocity

\[
C_s^2 = \frac{c_{pd}}{c_{vd}} R_d \pi \theta_v
\]

(1.13)
Basic equations (3)

Thermodynamic eq

\[
\frac{\partial \theta}{\partial t} = - \left( (\bar{u} + U) \frac{\partial \theta}{\partial x} + (\bar{v} + V) \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) - w \frac{\partial \theta}{\partial x} + \frac{L}{c_p \theta} (CN_{vc} - EV_{cv} - EV_{rv}) \\
+ \frac{1}{\pi} (Q_{rad} + Q_{dis}) + Turb.\theta + Turb.\theta
\]

Conservation eq. for vapor, cloud, and rain

\[
\frac{\partial q_v}{\partial t} = - \left( (\bar{u} + U) \frac{\partial q_v}{\partial x} + (\bar{v} + V) \frac{\partial q_v}{\partial y} + w \frac{\partial q_v}{\partial z} \right) - w \frac{\partial q_v}{\partial x} - (CN_{vc} - EV_{cv} - EV_{rv}) \\
+ Turb.q_v + Turb.q_v,
\]

\[
\frac{\partial q_c}{\partial t} = - \left( (\bar{u} + U) \frac{\partial q_c}{\partial x} + (\bar{v} + V) \frac{\partial q_c}{\partial y} + w \frac{\partial q_c}{\partial z} \right) + (CN_{vc} - EV_{cv} - CN_{cr} - CL_{cr}) \\
+ Turb.q_c,
\]

\[
\frac{\partial q_r}{\partial t} = - \left( (\bar{u} + U) \frac{\partial q_r}{\partial x} + (\bar{v} + V) \frac{\partial q_r}{\partial y} + w \frac{\partial q_r}{\partial z} \right) + (CN_{cr} + CL_{cr} - EV_{rv}) + PR_r
\]

(a) H₂O condensation

<table>
<thead>
<tr>
<th>Vapor</th>
<th>H₂O(g)</th>
<th>EV_rv</th>
<th>EV_cv</th>
<th>CN_vc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>H₂O(s)</td>
<td>CN_cr</td>
<td>CL_cr</td>
<td>H₂O(s)</td>
</tr>
</tbody>
</table>

(b) NH₃ condensation

<table>
<thead>
<tr>
<th>Vapor</th>
<th>NH₃(g)</th>
<th>EV_rv</th>
<th>EV_cv</th>
<th>CN_vc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>NH₃(s)</td>
<td>CN_cr</td>
<td>CL_cr</td>
<td>NH₃(s)</td>
</tr>
</tbody>
</table>

(c) NH₄SH reaction

<table>
<thead>
<tr>
<th>Vapor</th>
<th>H₂S(g), NH₃(g)</th>
<th>EV_rv</th>
<th>EV_cv</th>
<th>CN_vc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain</td>
<td>NH₄SH(s)</td>
<td>CN_cr</td>
<td>CL_cr</td>
<td>NH₄SH(s)</td>
</tr>
</tbody>
</table>
Grid configuration

- Horizontal Arakawa-C grid
- Vertical Lorenz grid
- Centered difference
  - 2nd / 4th order
Coding style: use F90 array functions

\begin{equation}
\pi_{i(u),j,k} \equiv \frac{\pi_{i+1,j,k} + \pi_{i,j,k}}{2}
\end{equation}

\begin{equation}
u_{i,j,k} \equiv \frac{u_{i(u),j,k} + u_{i-1(u),j,k}}{2}
\end{equation}

\[\zeta \Delta t = \nu \left( \partial_{z_2} \zeta / \partial x_2 + \partial_{x_2} \zeta / \partial z_2 \right)\]
Back to ”moderate” way

- With extensive usage of F90 array functions, the model runs like a turtle.
  - Back to more F77 like style
  - Improve the speed by 100 times

- With extensive dependence of Oboro thermodynamic package, the model loses flexibility.
  - “molecular weight of non-condensible component” can be specified again.
  - Helpful for “mechanistic experiments.”
Example 1: Jupiter clouds of three components: H2O, NH3, NH4SH
Set-up of the experiments

- **Boundary conditions**
  - Horizontal boundary is cyclic. Stress free condition and \( w = 0 \) are given at the lower and upper boundaries.
  - Temperature and mixing ratios of vapor at the lowest level are fixed.

- **Initial condition**
  - Random potential temperature perturbation (\( \Delta \theta_{\text{max}} = 0.1 \, \text{K} \)) is given to seed convective motion.
Jupiter’s “Convective” clouds

Convective clouds

(Vasavada and Showman, 2005)
Results: Animation
Temporal variation

- The convective activity of the whole layer is not steady but quasi-periodic with a period of about 40 days.

- Overall temperature of the cloud layer synchronizes with the intermittent convective activity.
  - We will refer the time when the active cloud convection occurs as `active period' (A) and the other as `quiet period' (Q).
Period of the intermittency

- The period of the quasi-periodic cycle is about 2 times larger than that of the previous study.

<table>
<thead>
<tr>
<th>Conversion rate from cloud to rain (100 sec)</th>
<th>period in the numerical experiment (day)</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.4</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>60.2</td>
<td>1.80</td>
</tr>
<tr>
<td>100</td>
<td>62.5</td>
<td>1.85</td>
</tr>
</tbody>
</table>
Example 2: Mars condensation of major component (CO2)
Ancient warm and moist Mars?
Condensation convection of CO$_2$ and cloud type (Colaprete et al. 2003)

- Whether supersaturation is allowed or not is key point.
  - No: Stratiform cloud  Yes: Convective cloud
CO$_2$ ice cloud microphysics

- Condensation/Evaporation rate (Tobie et al., 2003)

\[ M_{\text{cond}} = 4\pi r \rho N_{\downarrow} \times k R \theta \Pi_{\uparrow 2} \Pi_{\uparrow 2} / L_{\uparrow 2} \ (S-1) \]

- For $1 < S < S_{\text{cr}}$, condensation does not occur if cloud density is less than a threshold value.
- We assume the value of threshold as $1.0e-6 \ \text{kg/m}^3$

- Gravitational settling rate

\[ M_{\text{fall}} = \partial / \partial z \left( \rho \downarrow s \ V_{\text{term}} \right) \]

\[ V_{\text{term}} = (1 + 4/3 \ \text{Kn}) \times r_{\uparrow 2} \ \rho \downarrow l / \eta \]

\[ S = \frac{p}{p_*} : \text{Saturation ratio} \]
\[ p : \text{Pressure} \quad p_* : \text{Saturation vapor pressure} \]
\[ \rho : \text{Gas density} \quad \eta : \text{viscosity coefficient} \]
\[ N*: \text{Number density of condensation nuclei} \]
\[ r : \text{Radius of cloud particle} \quad \rho_{\downarrow} : \text{CO2 ice density} \]
\[ \text{Kn} : \text{Knudsen number for cloud particle} \]
\[ k : \text{Thermal diffusion coefficient} \quad R : \text{Gas constant} \]
Setup of experiments

- Domain size:
  - 100km in horizontal direction (grid spacing: 500m)
  - 80km in vertical direction (grid spacing: 400m)
- Initial temperature and cooling profile
  - Based on Kasting (1991)

- Critical saturation ratio (Scr): 1.0, 1.35 (Glandorf et al., 2002)
- Number density of condensation nuclei
  - 5.0x10^8 /kg
  (Tobie et al., 2003: Forget et al., 2013)

Initial temperature and cooling profile

- Iso-thermal
- Newtonian cooling
- Uniform cooling (-0.1 K/day)
- Dry adiabat

Ts = 273K
Ps = 2.0x10^5 Pa
Circulation features and cloud distribution: $\text{Scr}=1.0$ (100 days)

Cloud is generated by gravity wave
Circulation features and cloud distribution: $\text{Scr}=1.35$ (143 days)

- Vertical velocity
- Potential temperature ($\theta - \Theta(z)$)
- Cloud density
- Saturation ratio

Buoyancy flow occurs above condensation region.

The dry convection is activated due to evaporation cooling of dropped cloud particle.