The Southern ocean dynamics and its implications for global climate

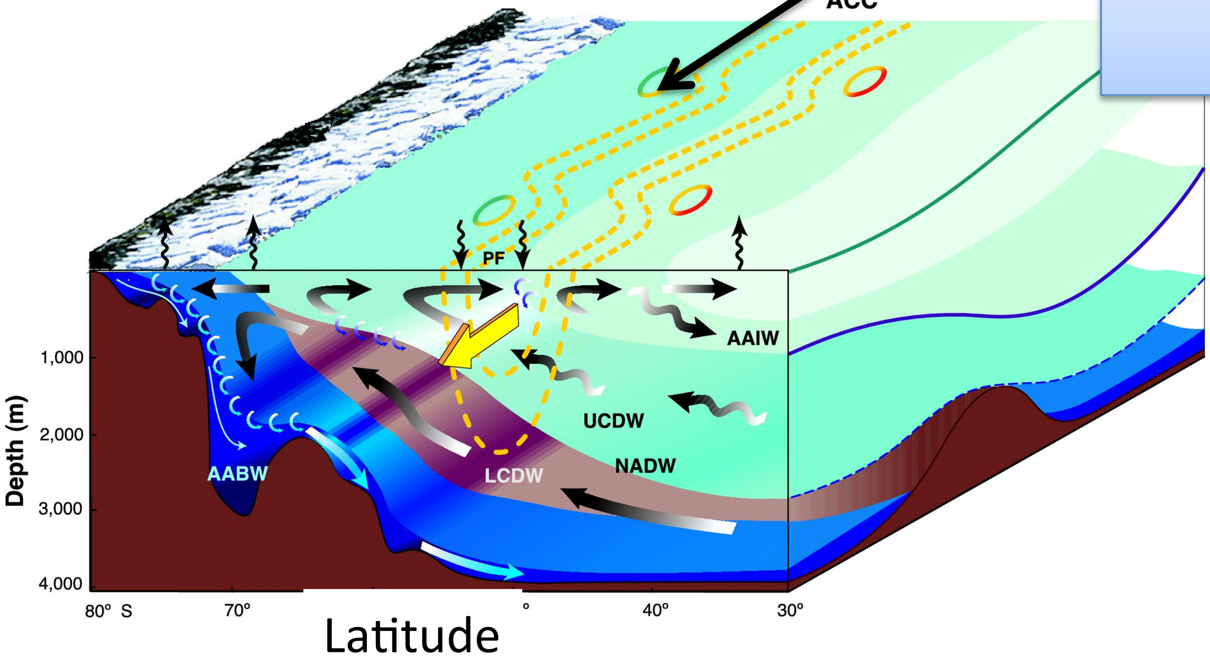
Lecture 2

David Ferreira
University of Reading

Outline

1. Southern Ocean role in global climate: past present and future
2. Impact of the Ozone hole on ocean and sea ice
3. Eddy-compensation problem and eddy diffusivities
4. Estimating eddy diffusivity with an adjoint method

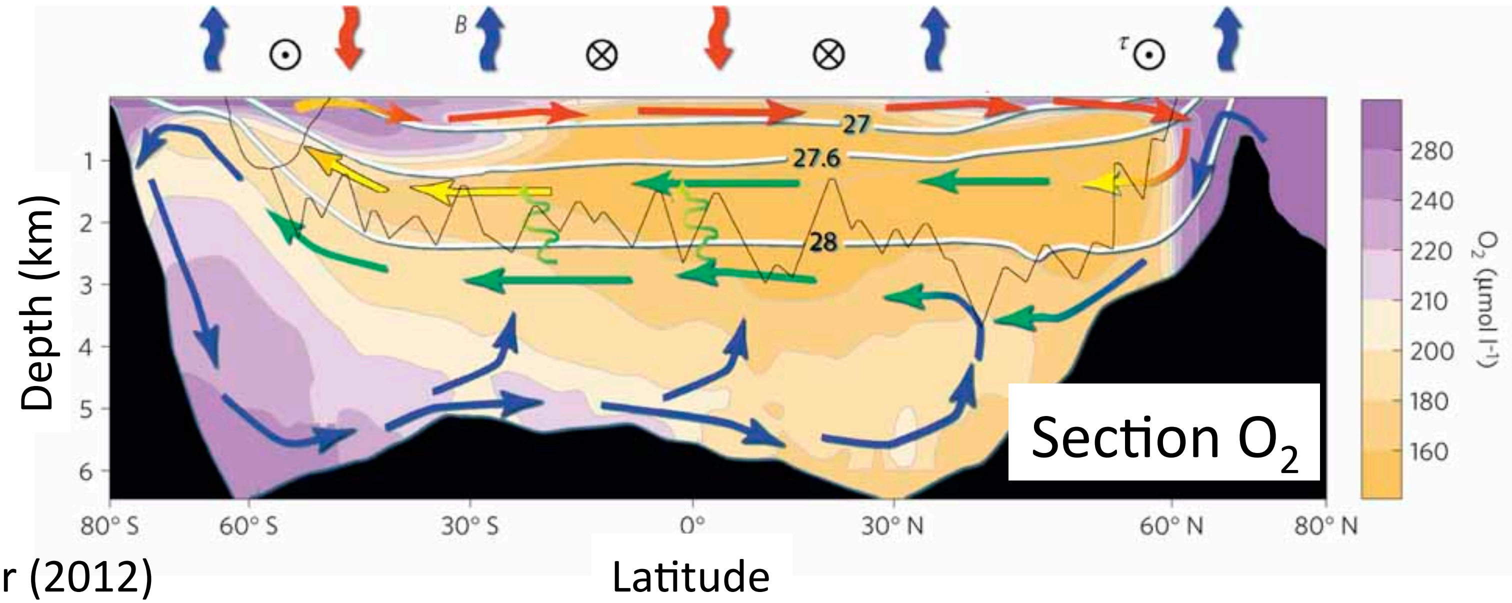
The mean state of the Southern Ocean



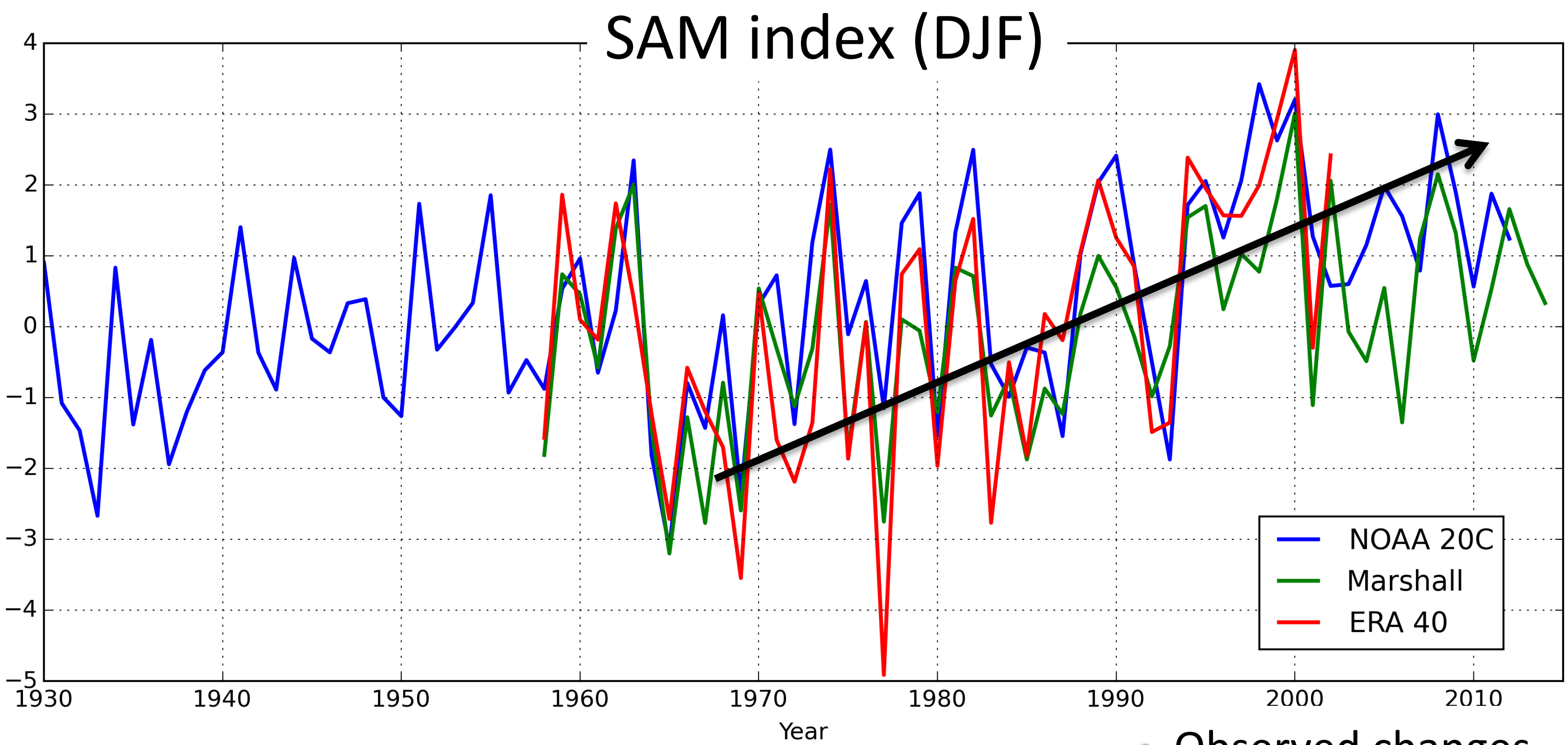
→ SO circulation is largely wind driven

Global MOC pulled and pushed

see Samuels and Toggweiler, 95

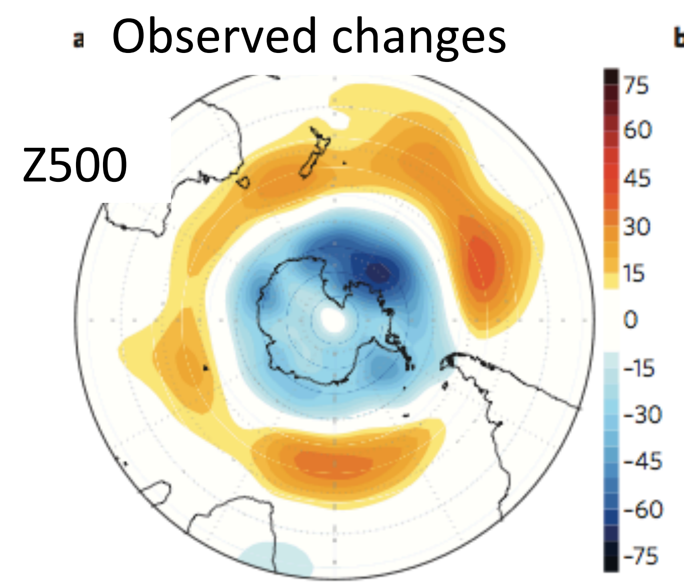


Marshall and Speer (2012)



- Thomas et al. (2015):
  SAM trend : not outside natural variability of CMIP5 models
  Jet magnitude and location: outside natural variability of CMIP5 models

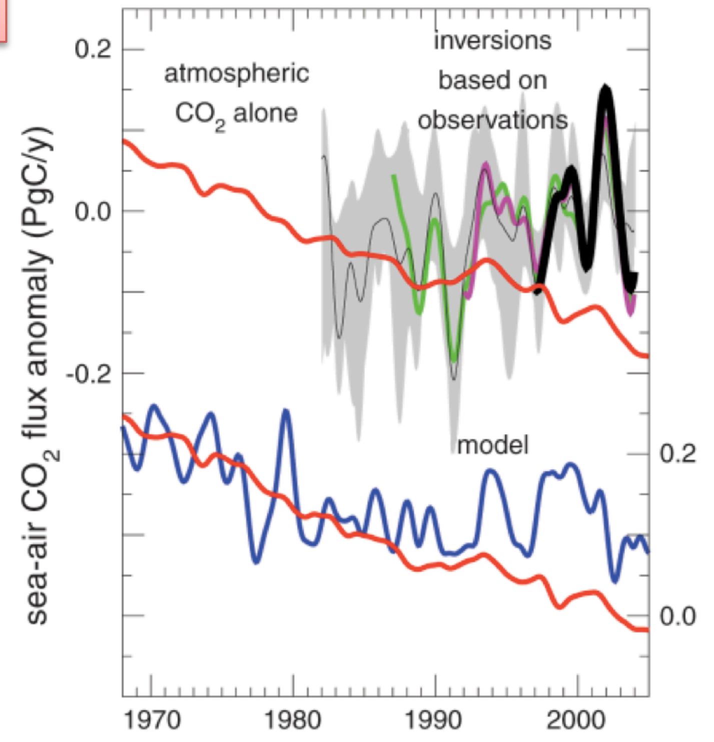
- Trends predicted to continue in 21st century

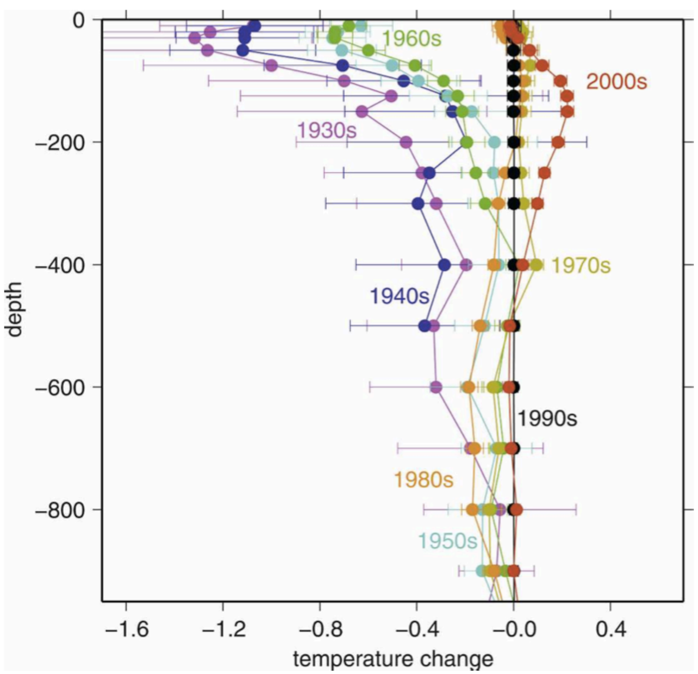


Heat and Carbon uptake

- 75% of ocean heat uptake in SO (Frolicher et al. 2015)
- Half of anthropogenic CO_2 emissions absorbed by oceans
- 40% in the SO (in observations and CMIP5 models, Sabine et al., 04)
- Southern Ocean sink could be saturating (Le Quere et al., 07, but Landschutzer et al. 15)
- Possibly because of strengthening winds

SO CO_2 flux



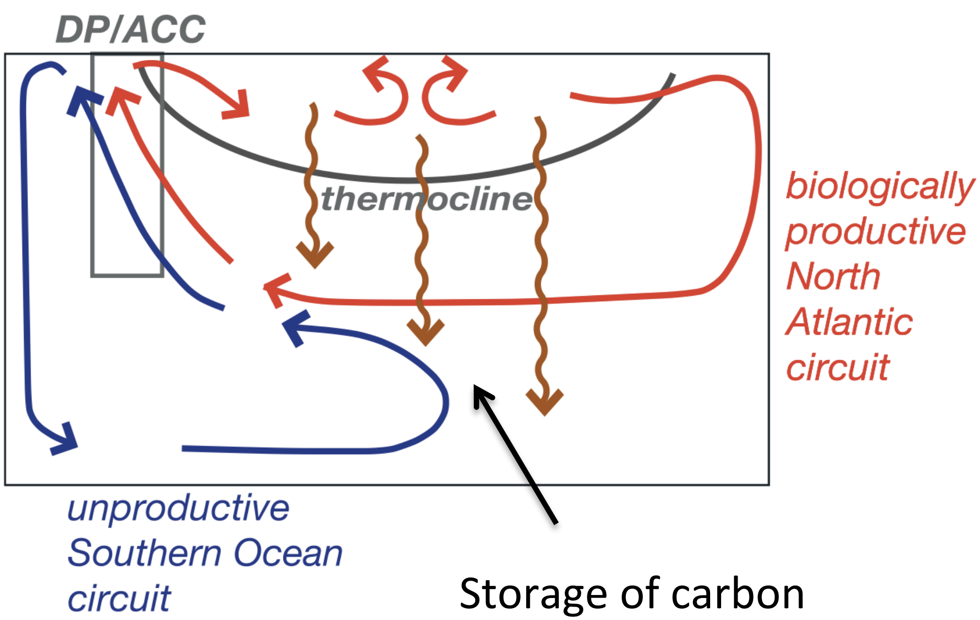


- SO warming: explained by shift of ACC fronts and of winds?
- SO warming less than other oceans: signature of the MOC?

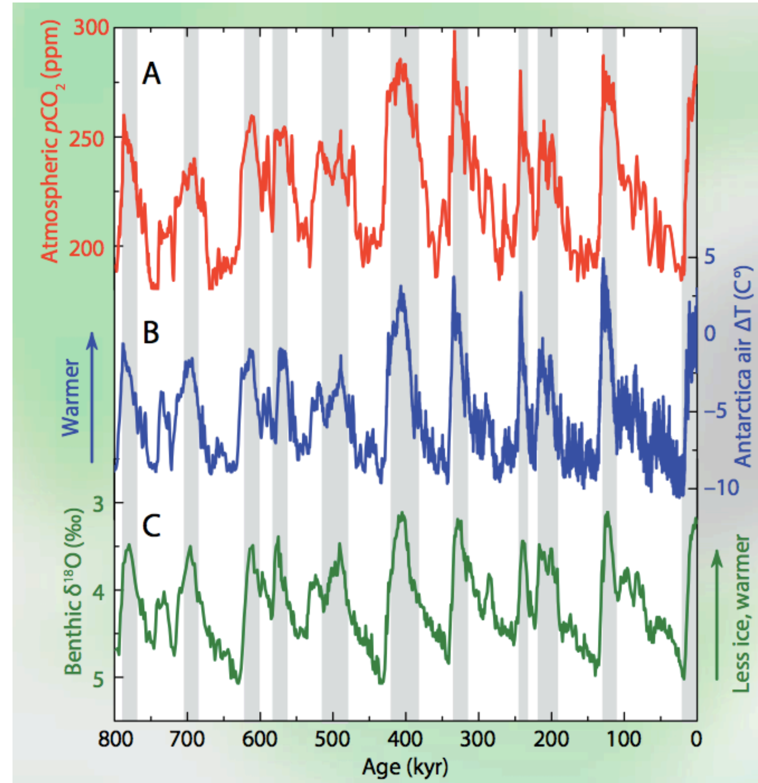
Gille (2008)

Paleoclimate perspective

SH jet stream might have been more equatorward and/or stronger at Last Glacial Maximum



Storage of carbon in deep ocean

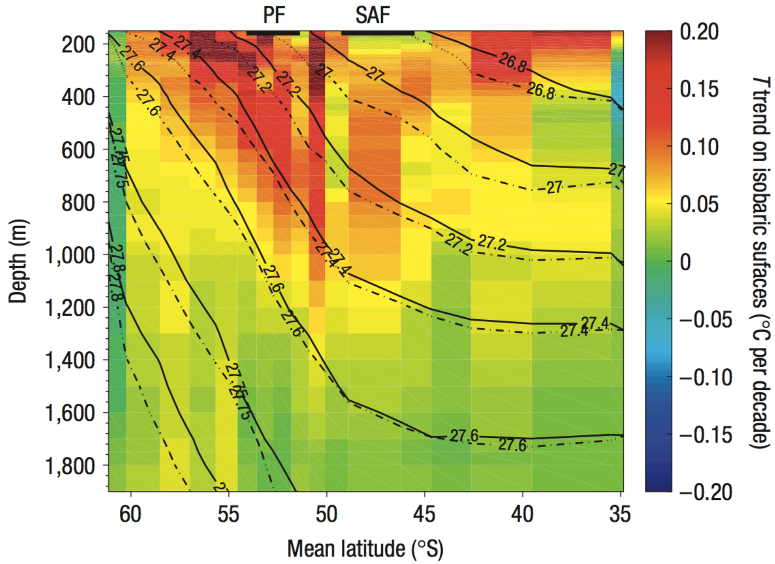


At LGM, weaker bottom MOC → accumulation of DIC

Toggweiler et al. 2006; Toggweiler 2009; Toggweiler and Russel 2008

No change in slope detected in Obs:

40 years changes

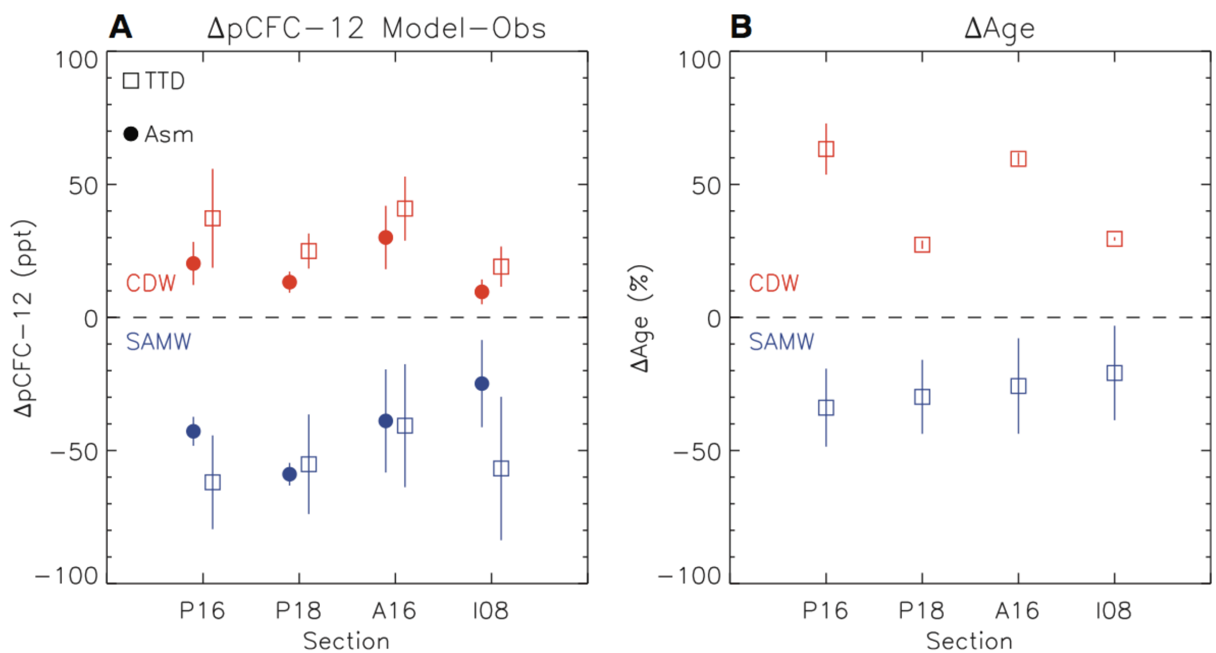


Boning et al. 08

Present day observations
(reminder winds have been changing over the 30 years)

CFC and age did change

Waugh et al. 2013



Outline

1. Southern Ocean role in global climate: past present and future

→ How does the Southern Ocean adjust to changing winds?

2. Impact of the Ozone hole on ocean and sea ice

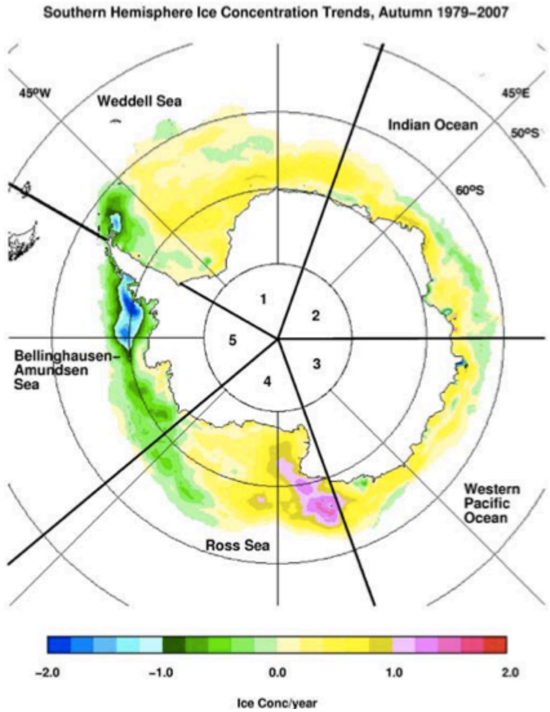
3. Eddy-compensation problem and eddy diffusivities

4. Estimating eddy diffusivity with an adjoint method

Collaborators:
John Marshall (MIT), Cecilia Bitz (UW), Susan Solomon (MIT), Alan Plumb (MIT), Ute Hausmann (MIT), Yavor Kostov (Oxford), Kyle Armour (UW), Marika Holland (NCAR)

1

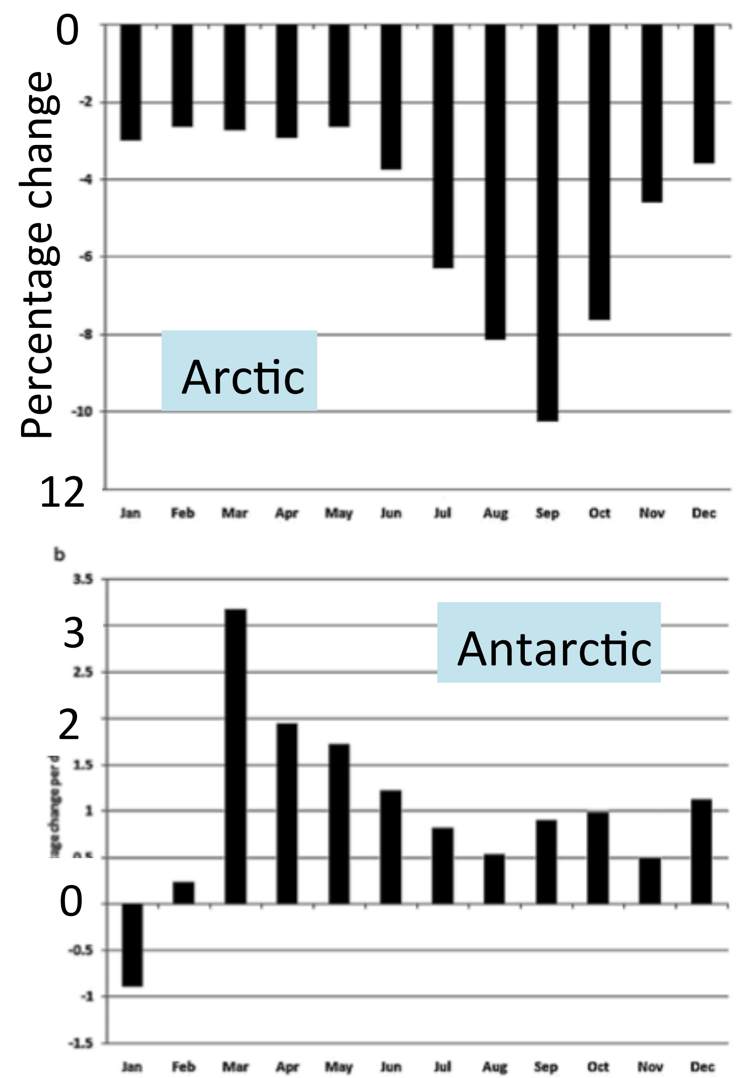
Spatial Pattern of Autumn sea ice concentration changes over 1979-2007



Southern Ocean Sea ice cover increased over last 3 decades

- Sea Ice extent: +0.5 Million km^2/30y (Parkinson and Cavalieri, 12)
- Not reproduced by models (Turner et al. 12)
- Various possible contributing factors: Goosse et al. (2009), Holland and Kwok (2012), Bintanja et al. (2013) …

Turner et al. (2009)

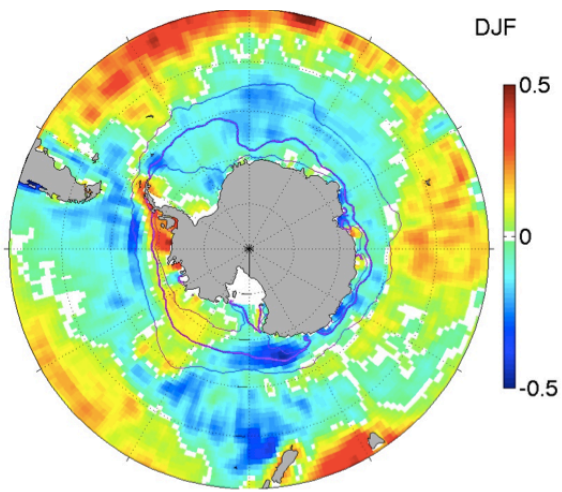


Monthly trends in Sea ice extent for 1979-2007

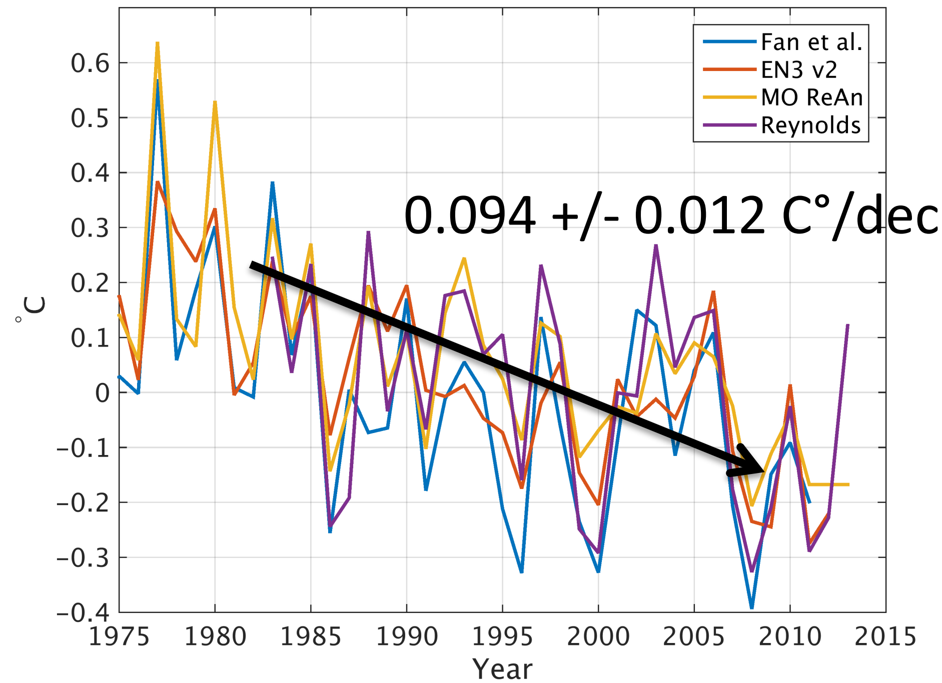
1

Southern Ocean SST cooled over last 3 decades

Reynolds
1982-2013
K/decade



SST (70-50°S) DJF

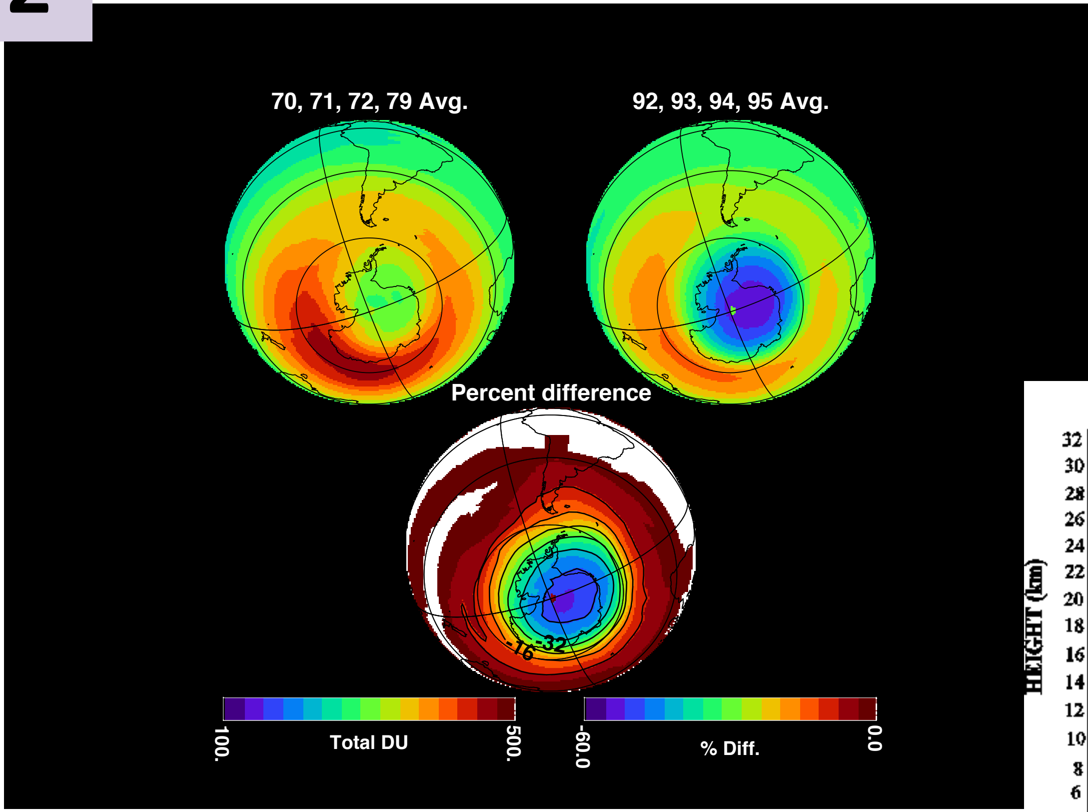


<u>Southern Ocean cooling and sea ice expansion:</u>

- Not reproduced by models (Turner et al. 12)
- Various possible contributing factors: Goosse et al. (2009), Holland and Kwok (2012), Bintanja et al. (2013) …

2

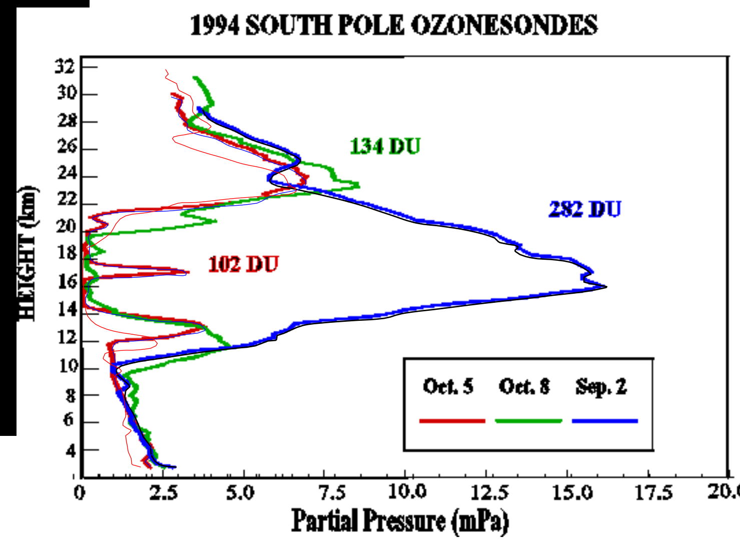
Observed Ozone change



Link with sea ice trends ?

Is there a role for the ocean in the climate response to ozone depletion?

Antarctic Ozone Measurements vs Altitude



Ozone partial pressure (mPA) profiles at the South Pole, measured by balloon–borne ozonesondes. The profile on 2 September 1994, before depletion began, is compared with profiles for 5 October and 8 October 1994, during the minimum ozone period.

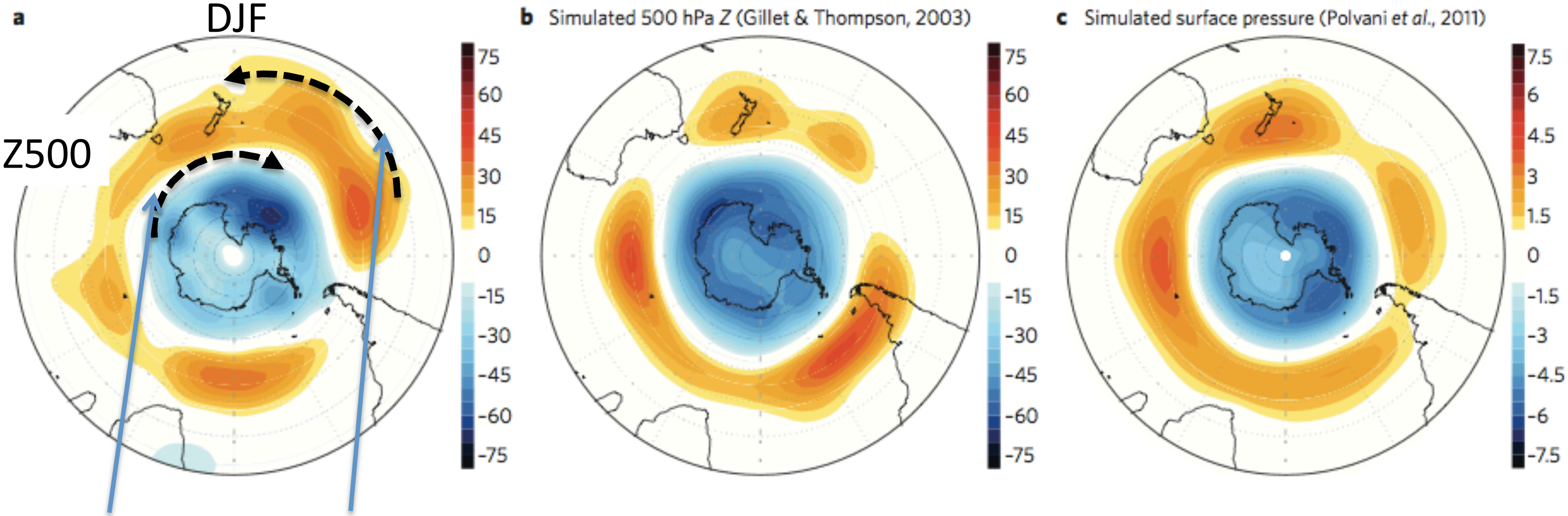
(from NOAA/CMDL home page http://www.cmdl.noaa.gov)

3 Atmospheric Trends

Observed changes
now minus pre-OH
DJF

Model response to Ozone depletion, DJF

Z500



Stronger Westerlies

Weaker Westerlies

Thompson et al. 11

Observed decadal trend similar to a positive **S**outhern **A**nnular **M**ode, **SAM**

Mechanism from ozone depletion to a SAM-like response not fully understood

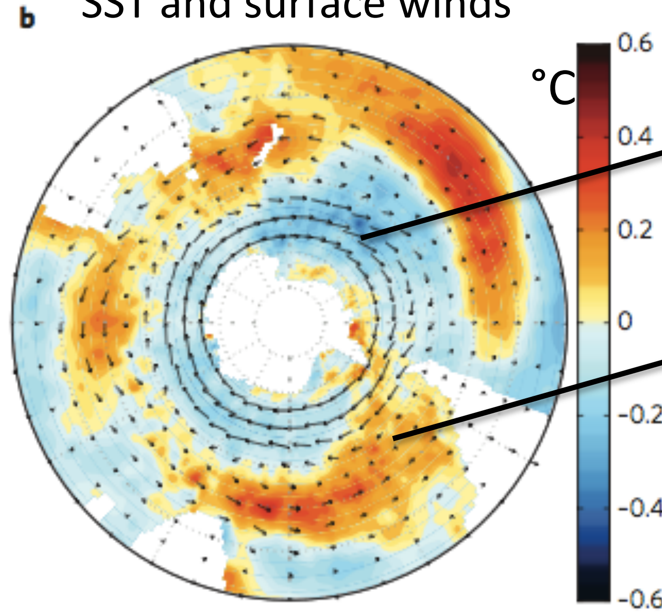
4

SST signature of a +SAM

Air-sea fluxes and mixed layer dynamics

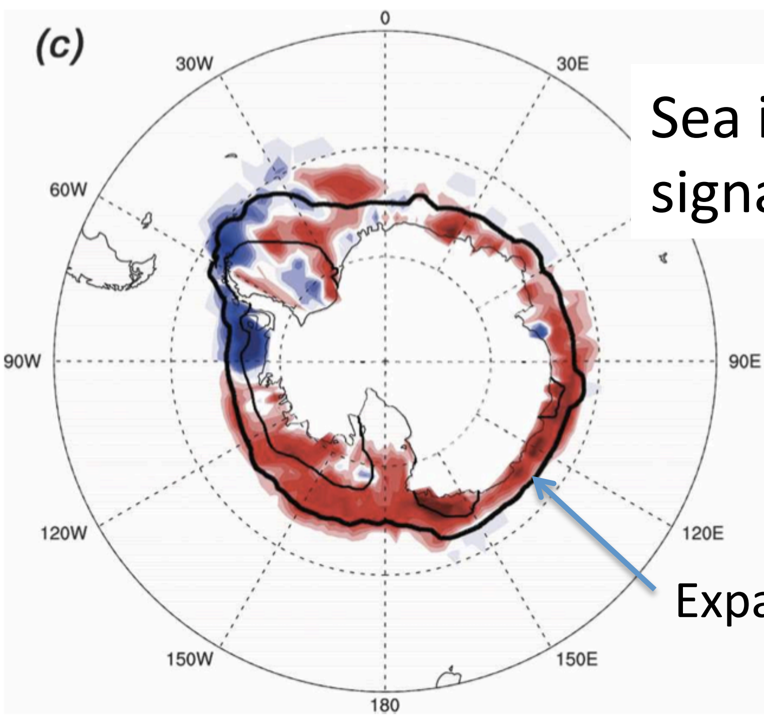
SST and surface winds

Ciasto and Thompson 2008



Cooling around Antarctica.

Warming in mid-latitudes



Sea ice concentration signature of a +SAM

Expansion

Sequence:
Ozone depletion:
- ->decadal positive SAM trend
- -> cooling around Antarctica
- -> sea ice expansion

(Goose et al. 2009)

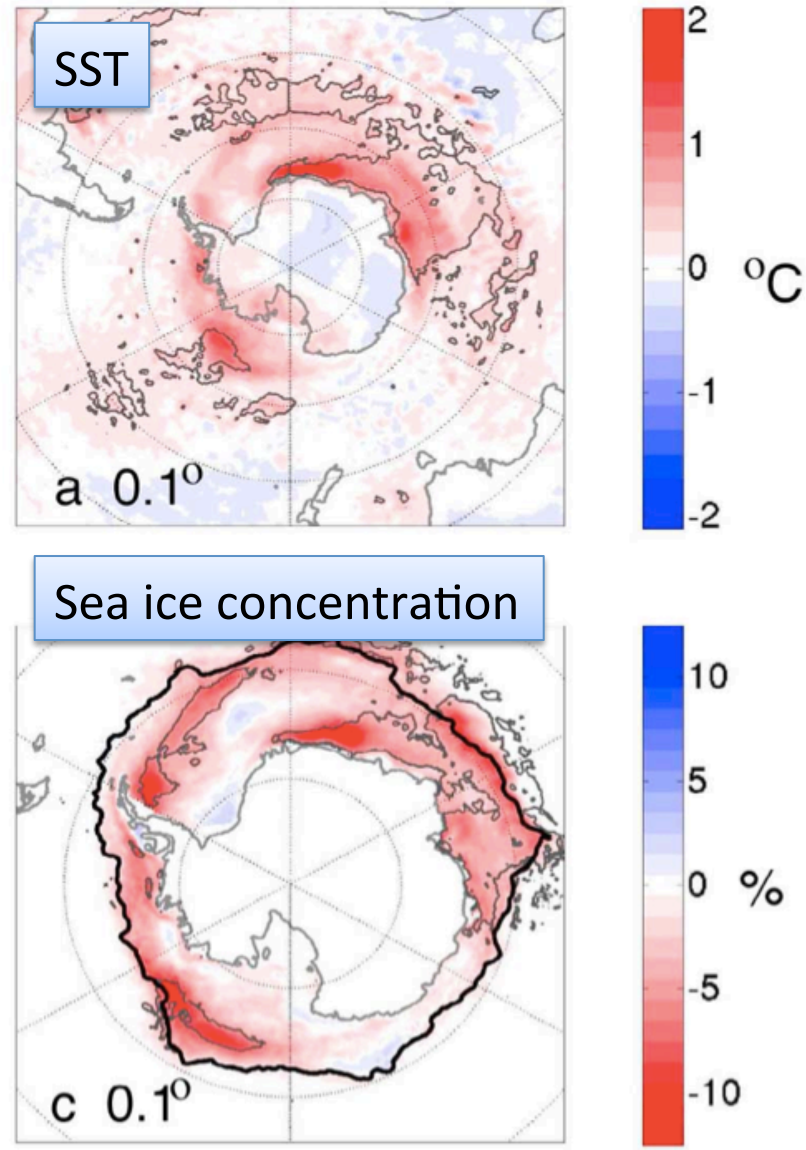
4 On the GCM side

The response to Ozone depletion in coupled climate models: **a sea-ice loss**

Two examples with IPCC-class coupled GCMs:
- Sigmond and Fyfe, 2010
- Bitz and Polvani, 2012 (@ 2 resolutions)

-> SAM-like atmospheric response
-> warming of Southern Ocean,
-> a sea-ice loss

Bitz and Polvani 2012



Conundrum:

a) Ozone depletion contributes to the observed sea ice expansion

Ozone depletion:
-> positive SAM trend
-> cooling around Antarctica
-> sea ice expansion

Each step backed up by observations and models

b) Ozone depletion does not contribute to the observed sea ice expansion: from coupled GCMs

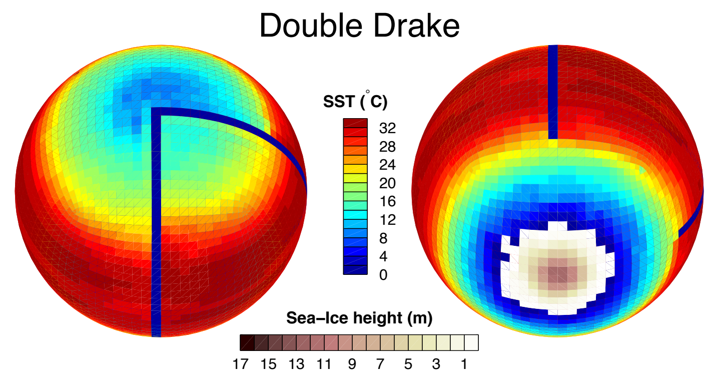
Questions:

- Can we reconcile the coupled model response to Ozone depletion with the observed SST/SAM relationship ?
- Does Ozone depletion contribute to the observed sea ice expansion?
- Is there a role for the Southern Ocean dynamics?

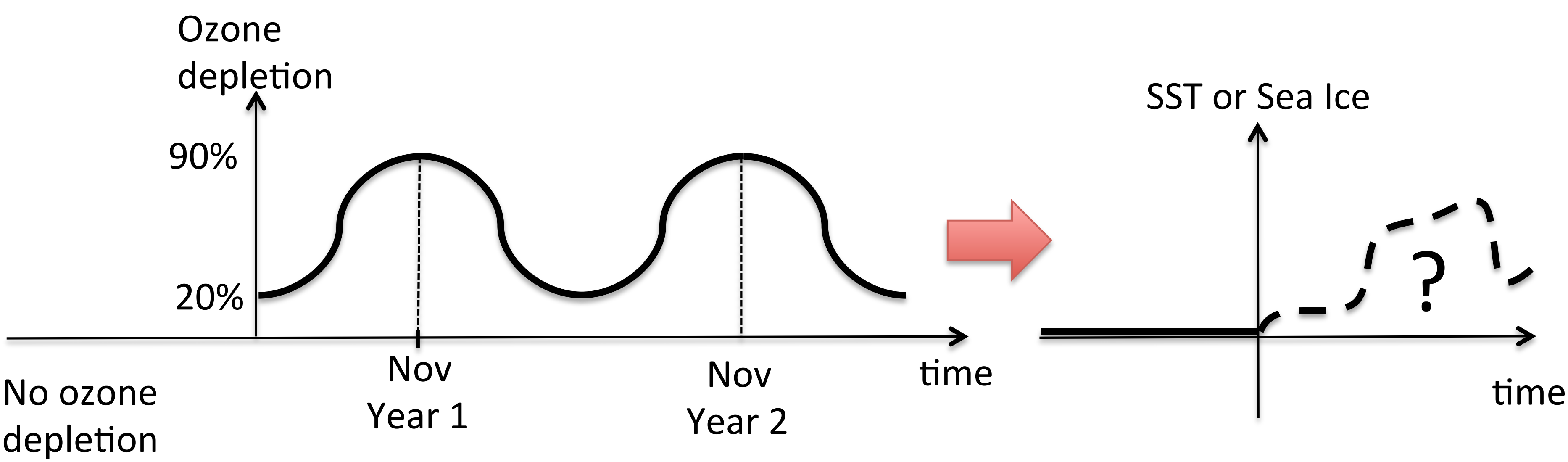
Transient adjustment to a step change in stratospheric ozone

→ two coupled GCMs : idealized MITgcm, CCSM3.5

→ Ozone forcing : seasonal, mid-90s depletion



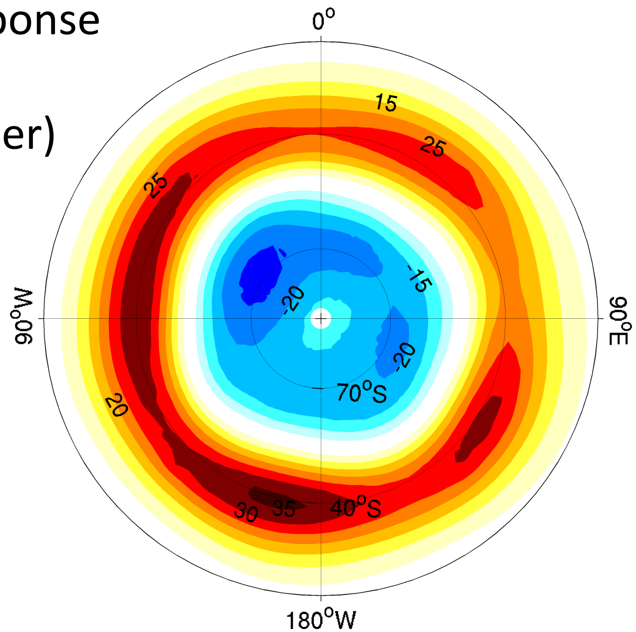
A repeated mid 90s Ozone hole:



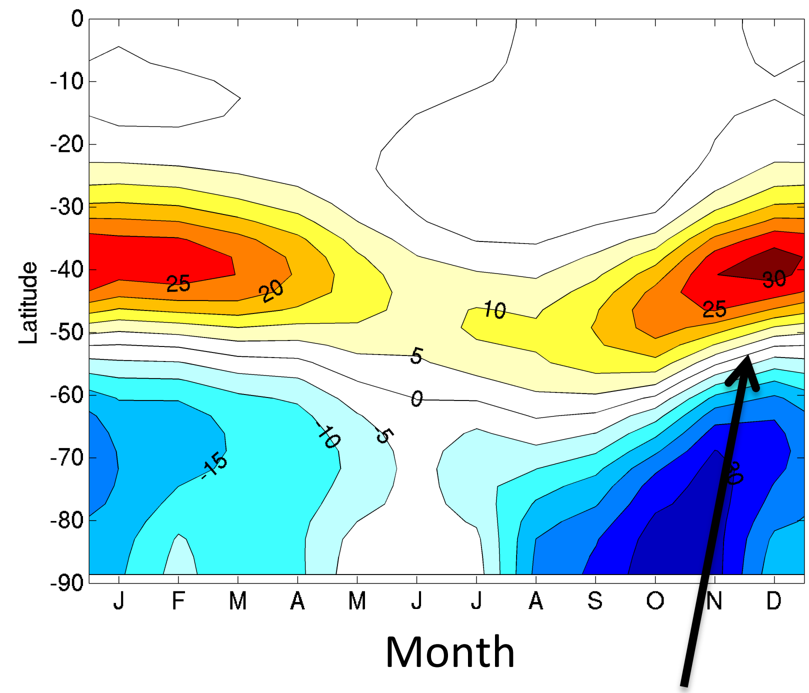
A seasonal SAM-like response to Ozone depletion

Similar to that seen in other (more sophisticated) atmospheric models

Z500 response
DJF
(in meter)



Z500 climatological response



Similar to SAM pattern:
- 1^{st} EOF of Z500 mb
- explains 34 % of variance

Summer response

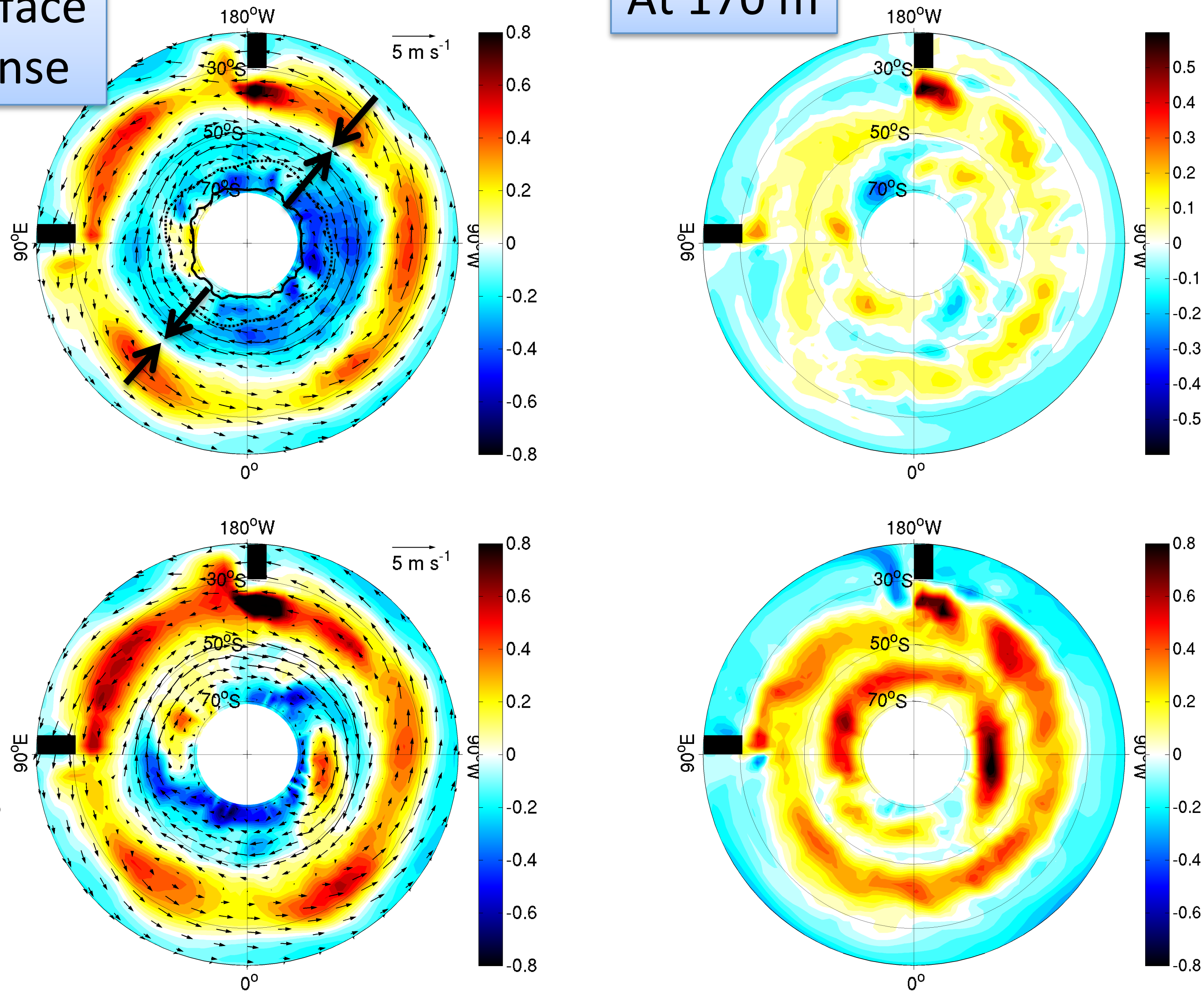
SST and Surface Wind response

ved correlations

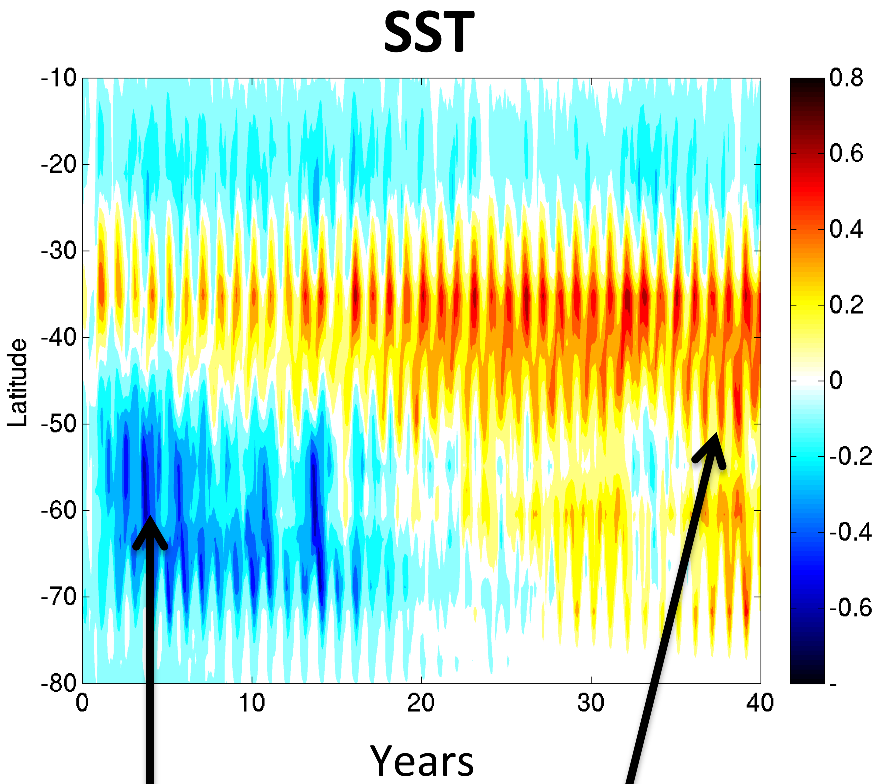
At 170 m

After
1-5 years

After
15-20 years



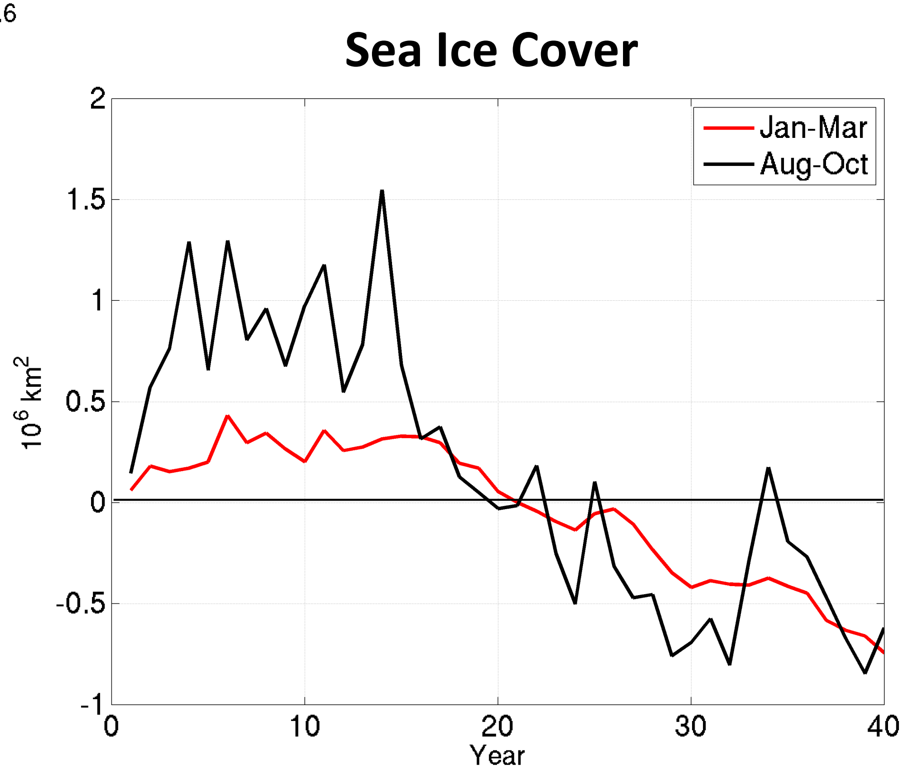
The Transient response



SST response evolves from a dipole response to a wide spread warming

Fast response: SST dipole

Slow response: widespread warming



Sea ice expansion for the first 20 years

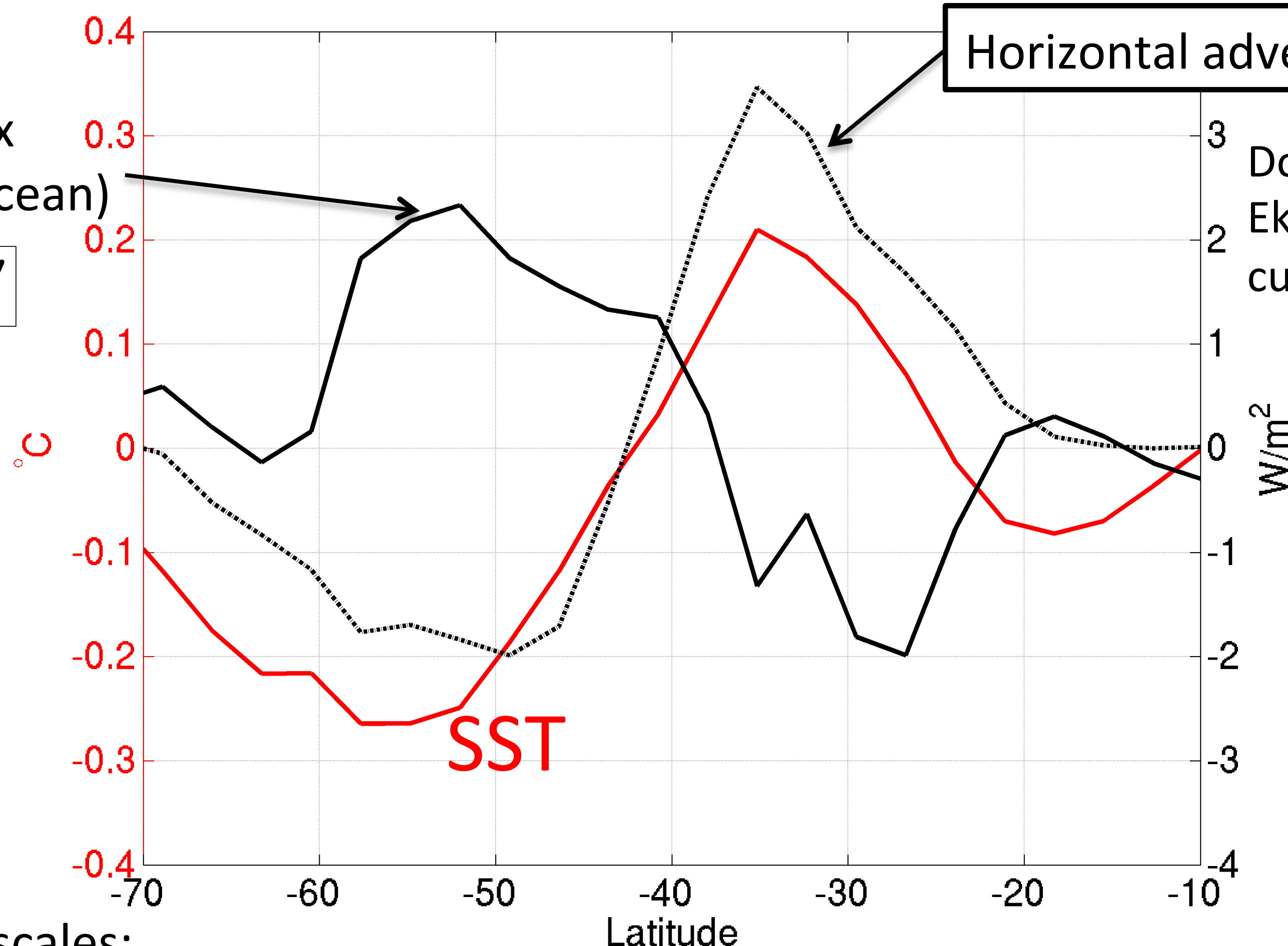
The fast response

$$-\rho_o c_P h_S v'_{res} \frac{\partial \overline{T}}{\partial y}$$

Horizontal advection

Air-sea Flux
(positive into ocean)

$$F' = -\lambda T'$$



Dominated by Ekman currents

Vertical advection is negligible

<u>On short timescales:</u>
Quasi-equilibrium between Ekman advection and air-sea flux damping

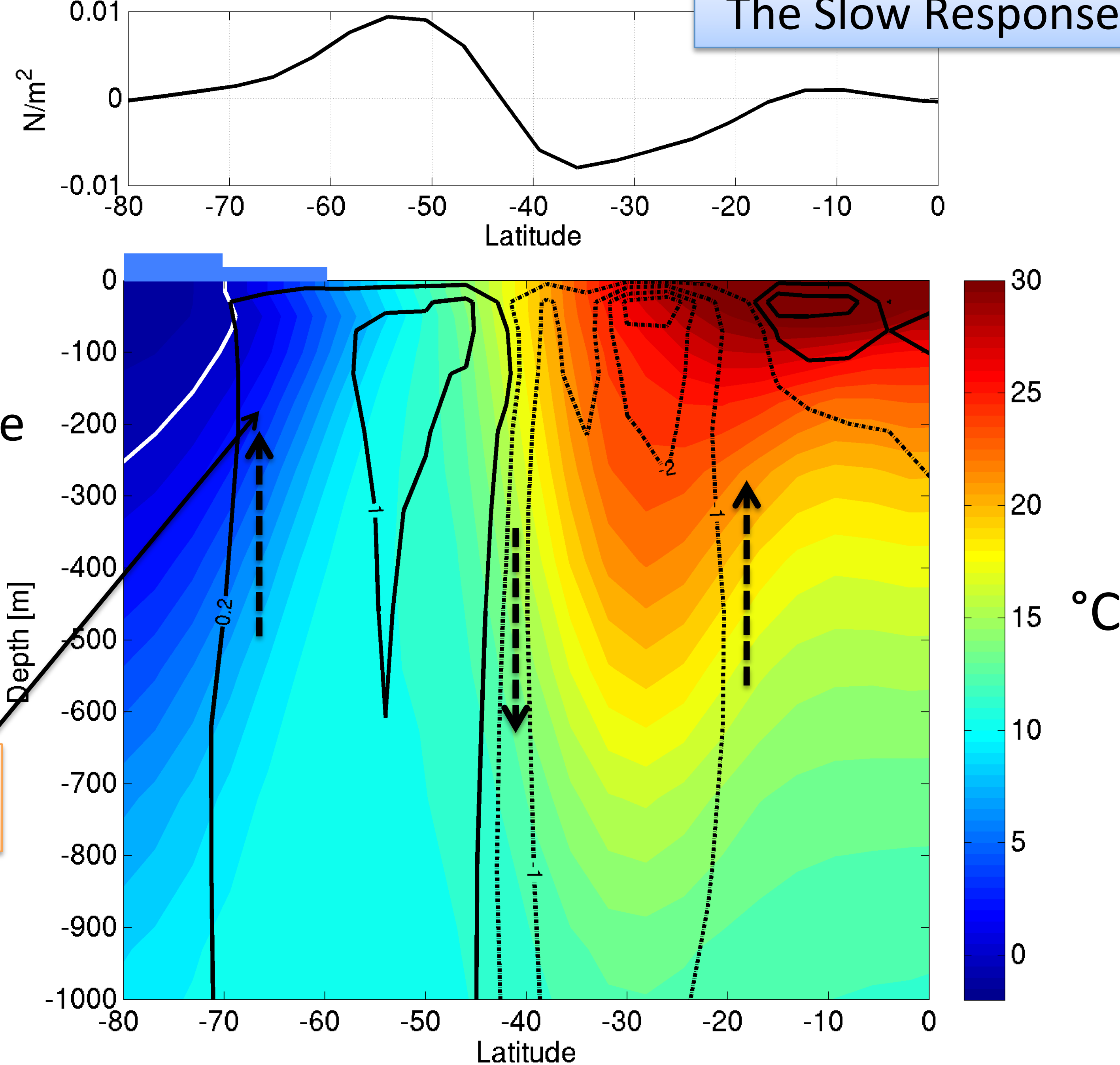
The Slow Response

τ_x: response

Control mean state

Ψ_{res}: response

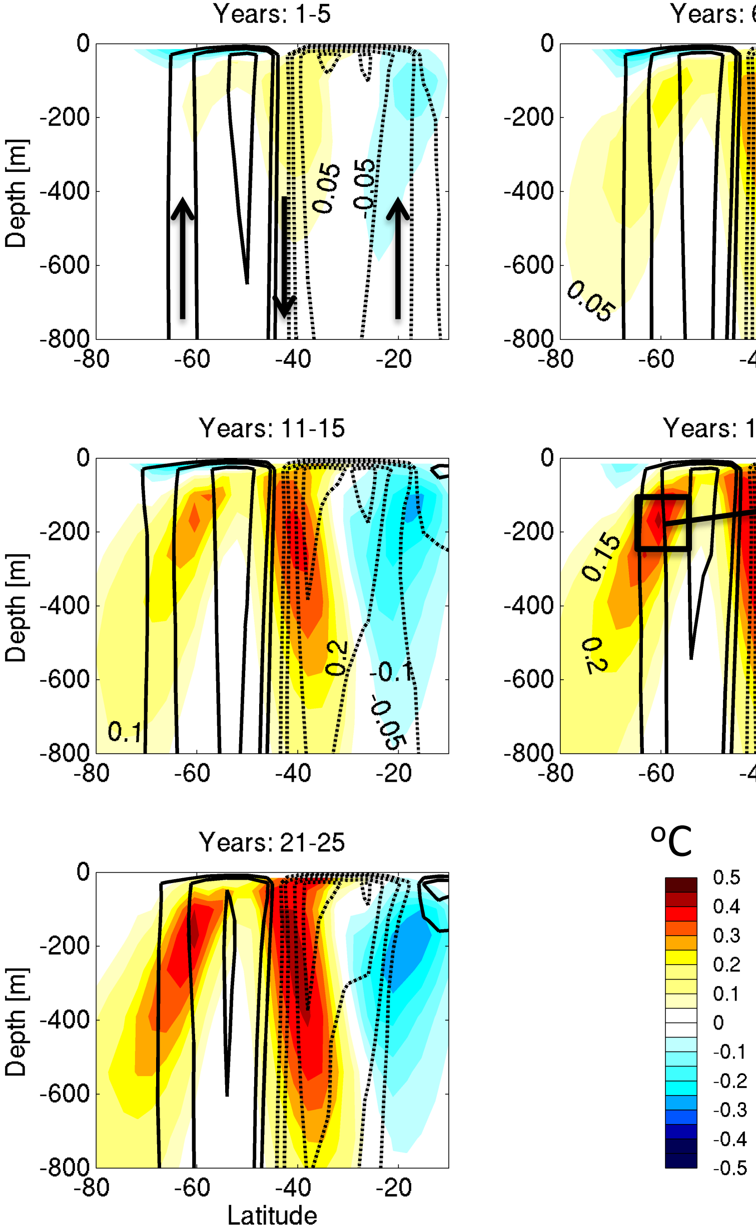
θ_o : control



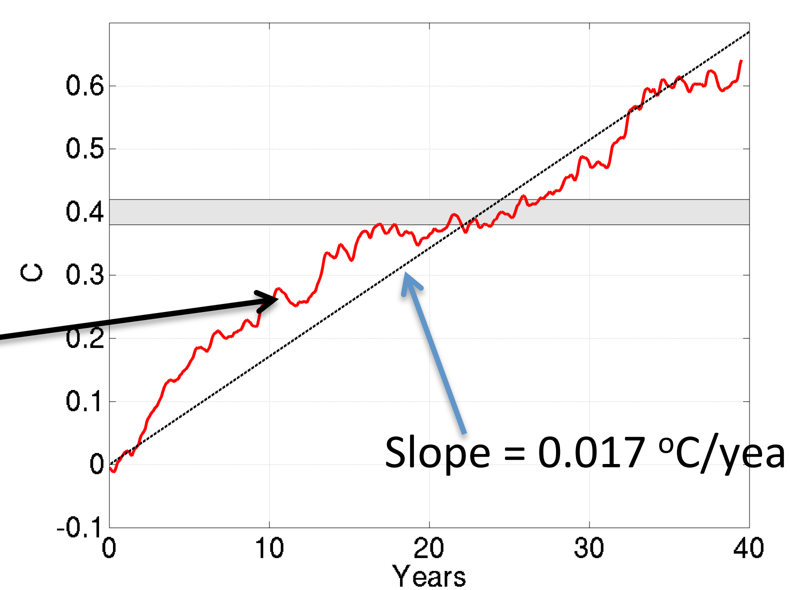
Temperature inversion

The Slow Response

MOC and Temp. response



Temperature timeseries @ 60°S, 170m



$$\frac{\partial T'_{sub}}{\partial t} \approx -w'_{res} \frac{\partial \overline{T}_{sub}}{\partial z}$$

dT/dz =-0.019 °C/m

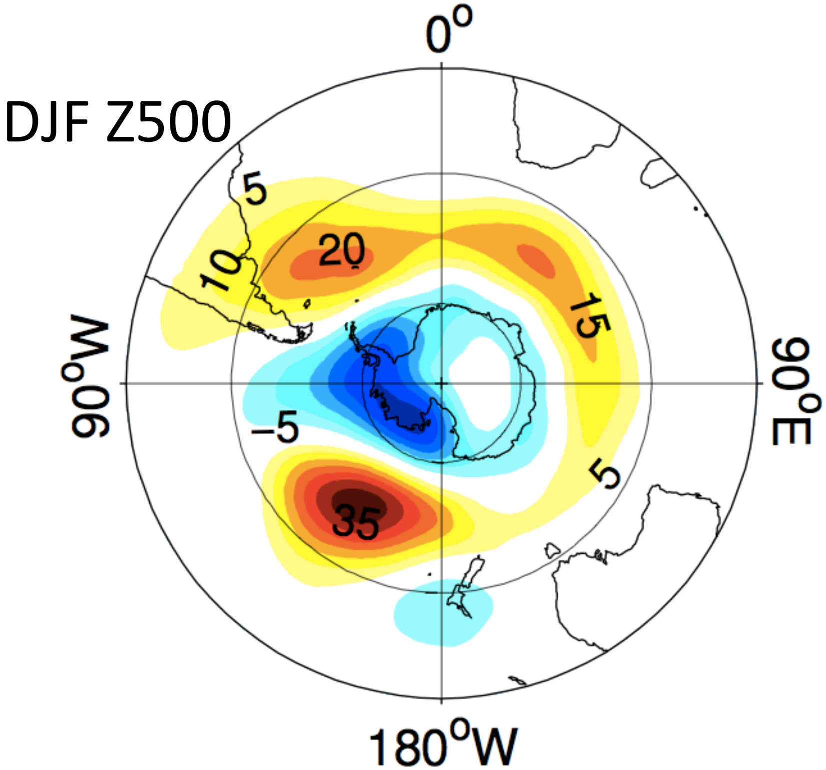
W'_{res} ~ 1 m/year

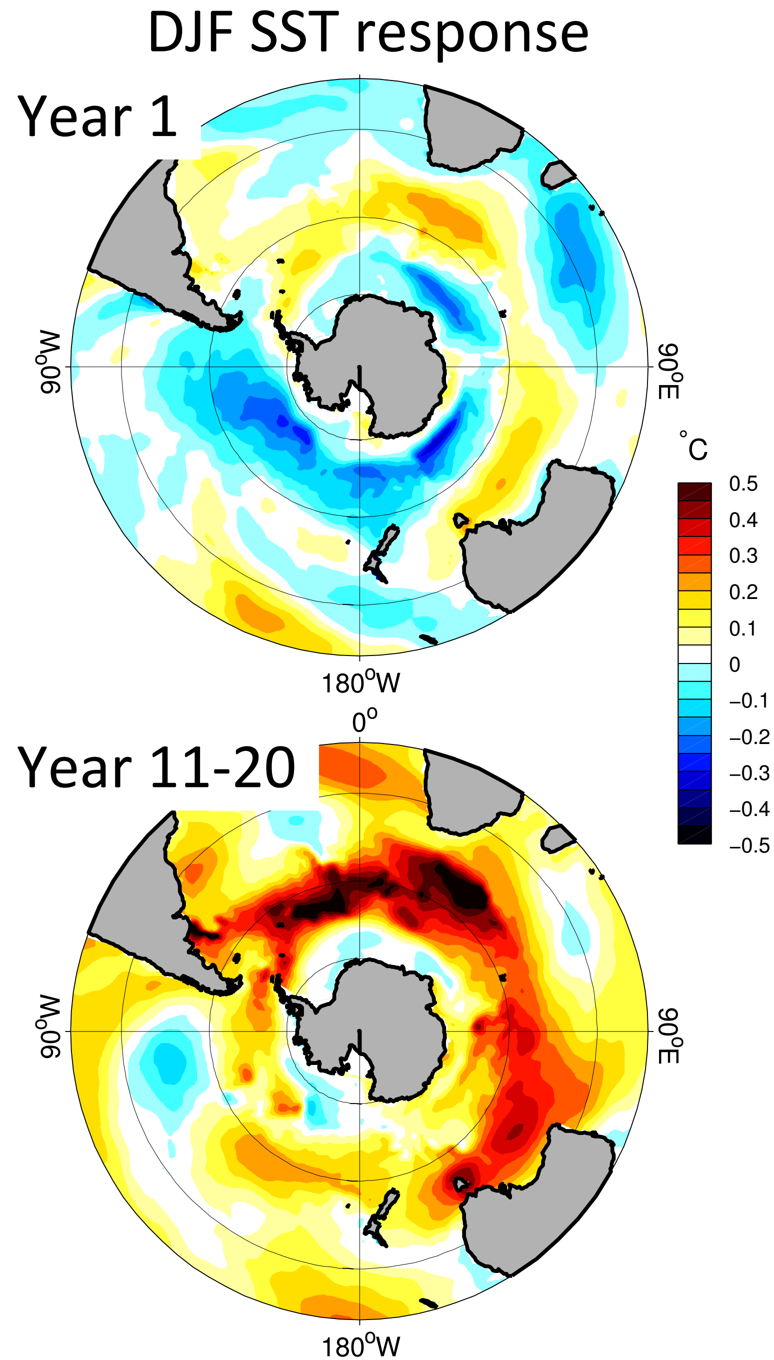
Importantly: $W'_{res} = W'_{eul} + W'_{eddy}$

W'_{eul} ~ 2 m/year

W'_{eddy} ~-1 m/year

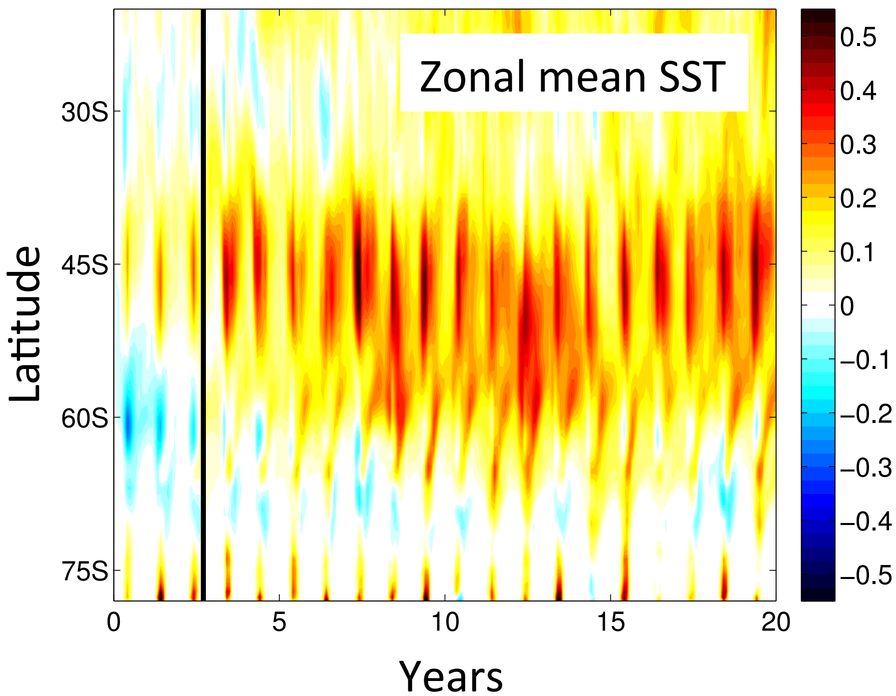
An IPCC-class model: CCSM3.5





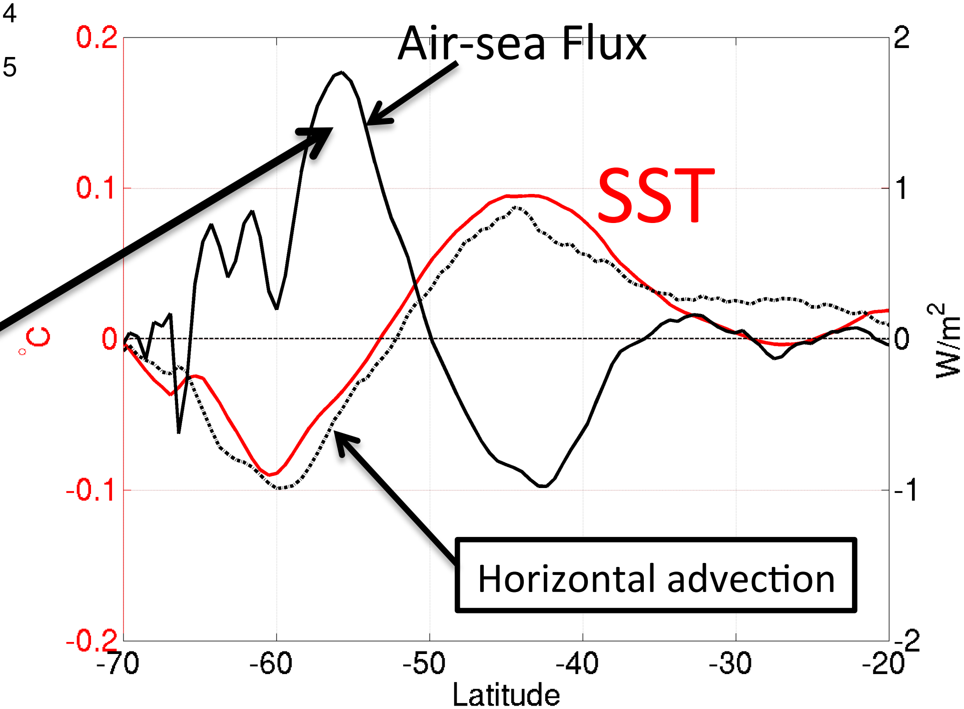
- Follows same sequence of events of in the MITgcm

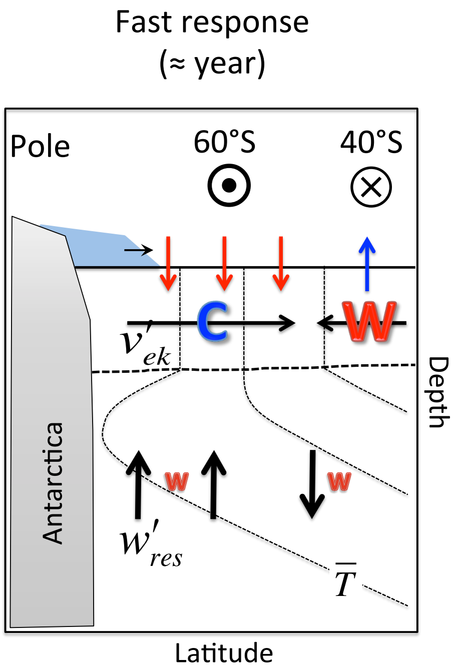
CCSM3.5 surface heat balance



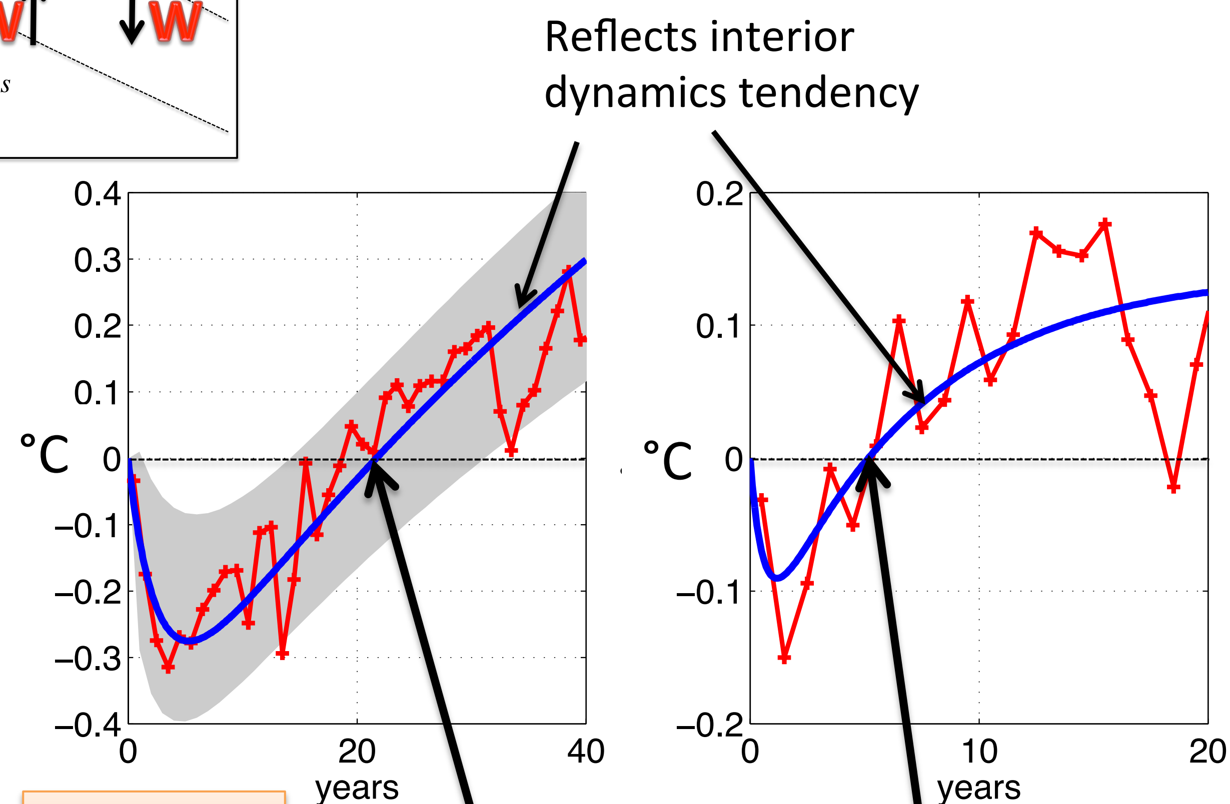
- Heat flux anomaly larger than in the MITgcm around 60-50°S
- Negative SSTA around 60-50°S reverses very rapidly
- But negative SSTA lingers for decades around 70°S

SW cloud feedback associated with the SAM response





Sea Surface Temperature Response (50-70°S)



Reversal timescale:

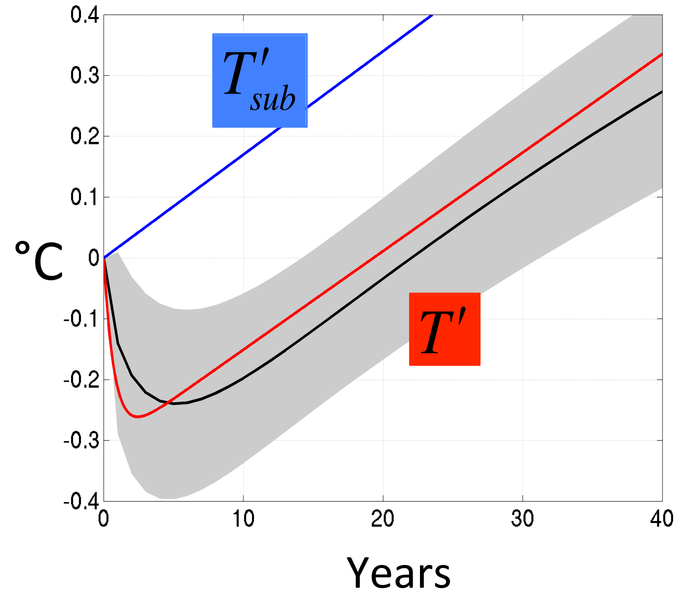
$t_R = 20 \pm 8$ years

$t_R = 3 - 5$ years

A toy model

for 70-50°S

$$\begin{cases} \partial_t T' = -v'_{res}\partial_y \overline{T} + F'_a - \lambda T' + \Lambda_e T'_{sub} & \text{~0-50 m} \\ \partial_t T'_{sub} = -w'_{res}\partial_z \overline{T_{sub}} - \lambda_{sub} T'_{sub} & \text{~100 m} \end{cases}$$



Best fit parameters

	Air-sea damping		Atm. forcing		λ_{sub}^{-1}	Λ_e^{-1}	$-w'_{res}\partial_z\overline{T}_{sub}$
	λ^{-1}	λ_F	$\tilde{F}$	$\tilde{F}_F$			
	year	W m^{-2} K^{-1}	°C year^{-1}	W m^{-2}	year	year	°C year^{-1}
MITgcm	2.6	1.5	-0.18	-0.7	78	1.5	0.014
CCSM3.5	0.59	6.7	-0.27	-1.1	6.8	0.36	0.027

Cloud effect ?
Zonal asymmetry ?

SAM response
Eddy compensation

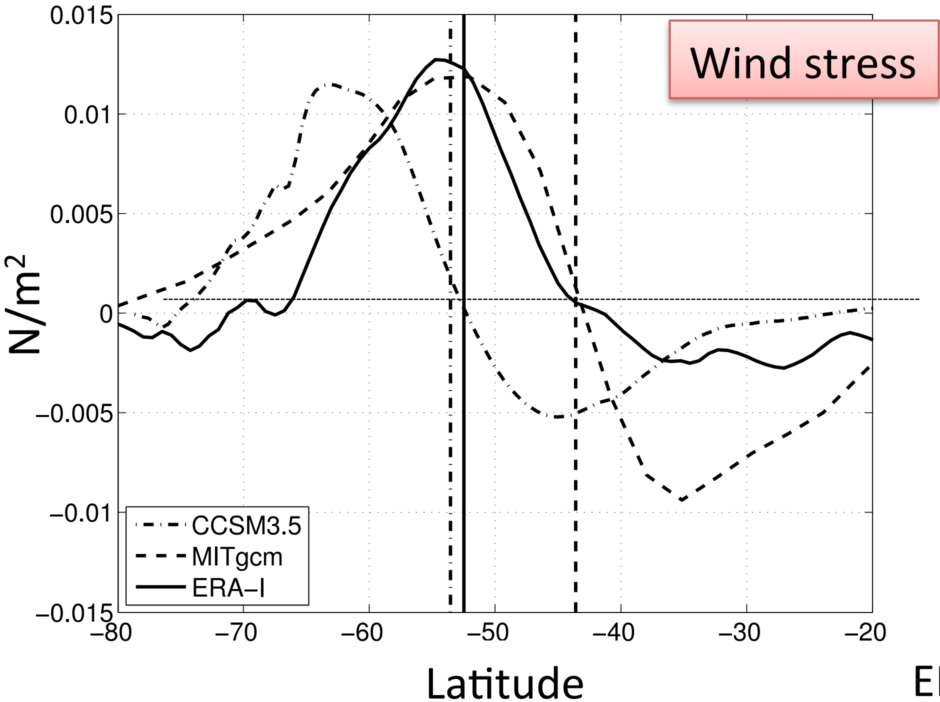
Sources of discrepancy

1. **Heat flux feedback :**
   - Atlantic: typically 15-20 W/m^2/K locally, 5-10 W/m^2/K in zonal mean (Frankignoul et al. 98, 04)
   - Few estimates for SO, south of 40°S, but about 4 W/m^2 (Hausmann et al. 2015)
   - Cloud effect associated to a SAM: 3-4 W/m^2 at TOA (Grise et al., 2013)

2. **Shape of the wind stress response:**
   -

$$w'_{Eul} \propto \partial_y \left(\frac{\tau'_x}{\rho_O f} \right)$$

3. **Eddy parameterization:**
   - Both models have coarse resolution and rely on Gent and McWilliams scheme



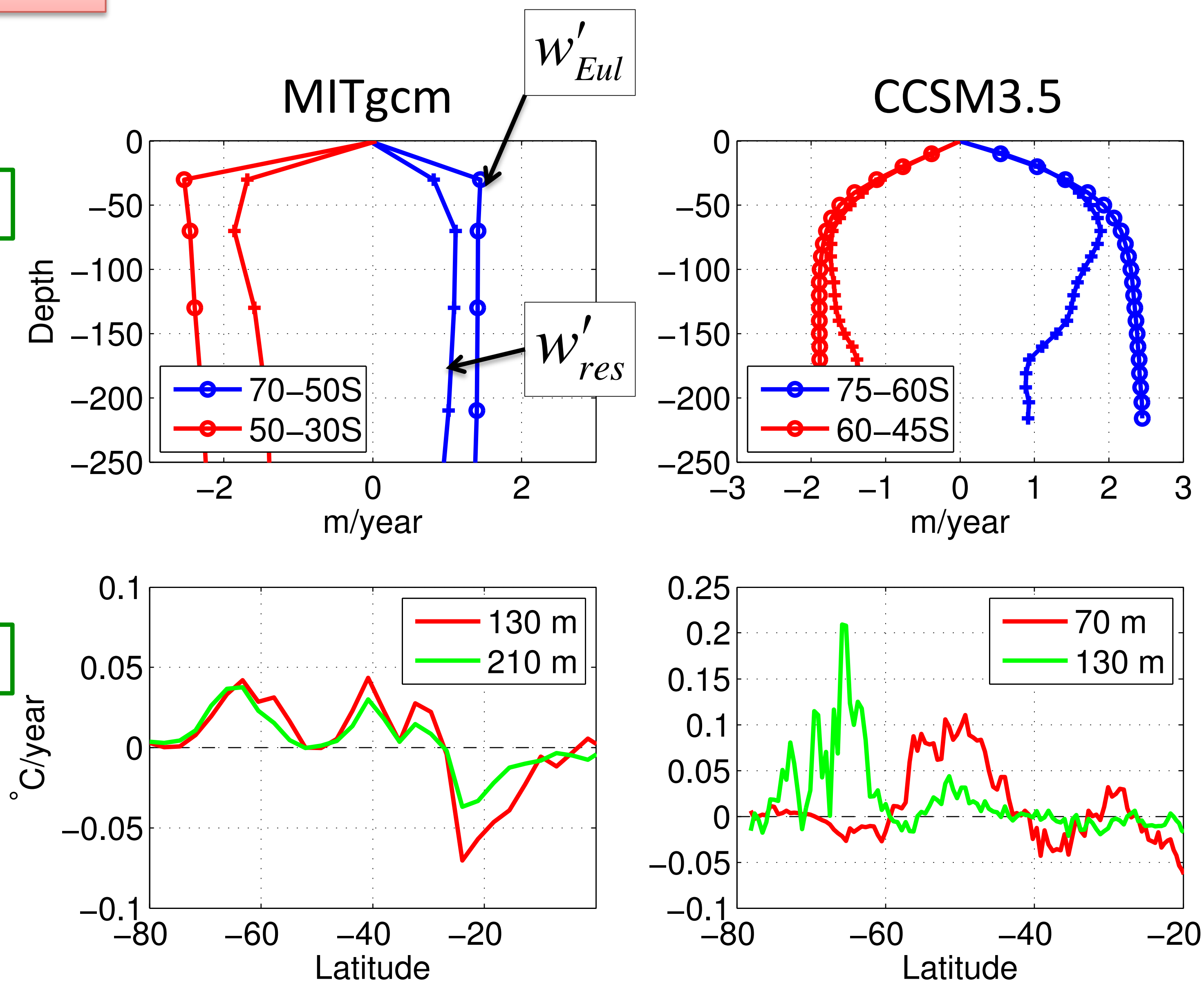
ERA-Interim: (1995-2004) minus (1980-1989)

Vertical advection

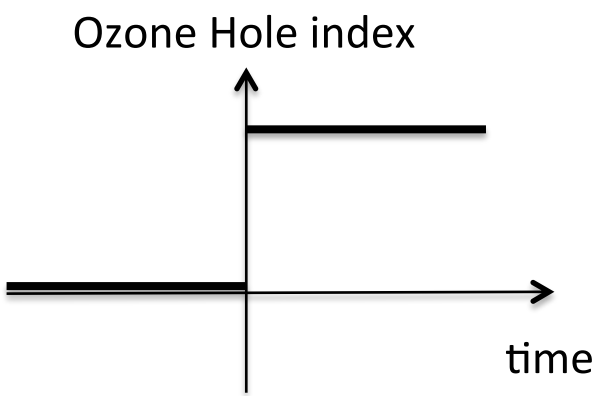
Vertical velocity

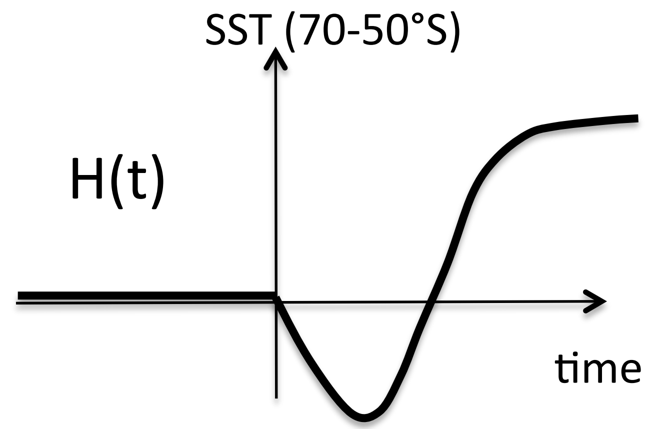
Vertical advection

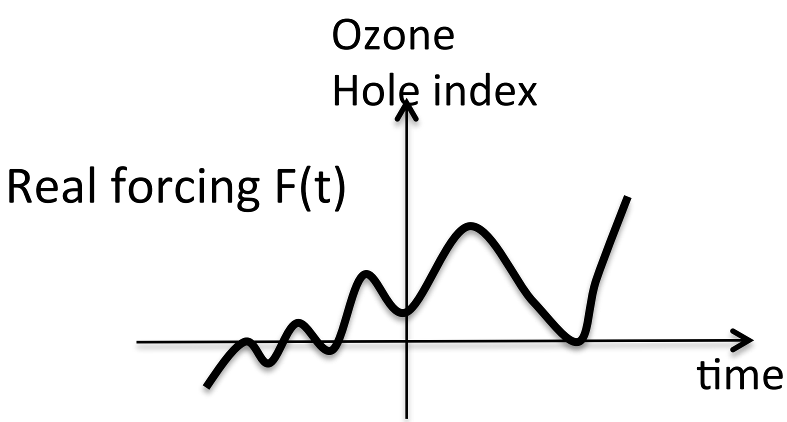
$$-w'_{res}\frac{\partial \overline{T}_{sub}}{\partial z}$$



What does the step-function response tell us about the response to ozone hole and recovery?







Response to "Real" forcing:

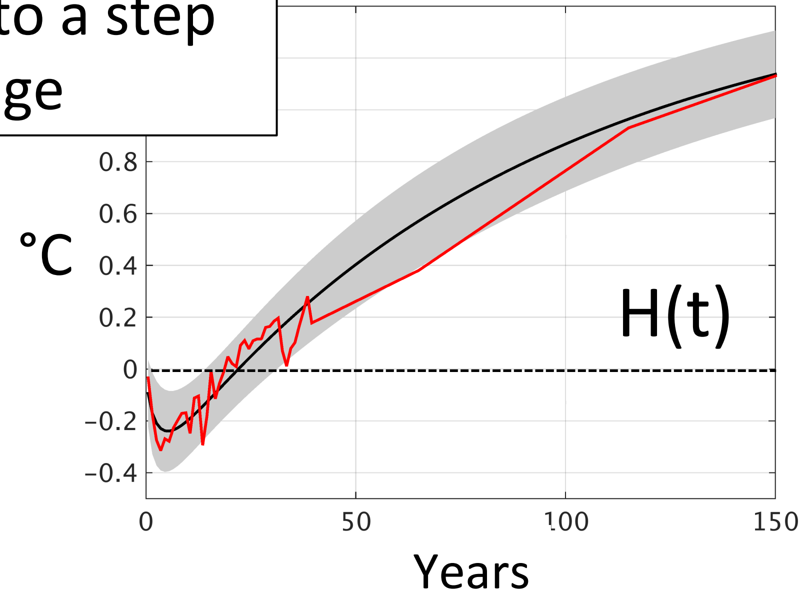
$$SST(t) = \int_{o}^{t} H(t-t') \frac{\partial F}{\partial t}(t') dt'$$

for a ***linear*** system

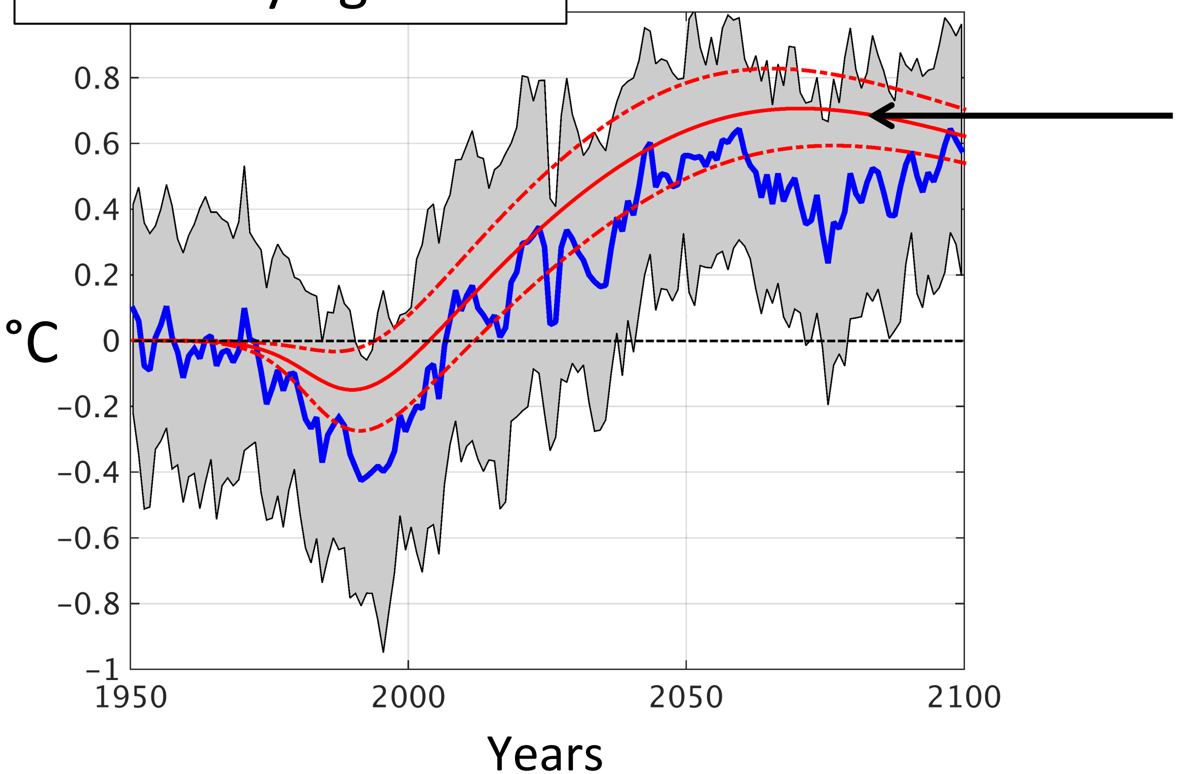
→ Test this in a model by comparing linear prediction and "true" response to a time-varying forcing

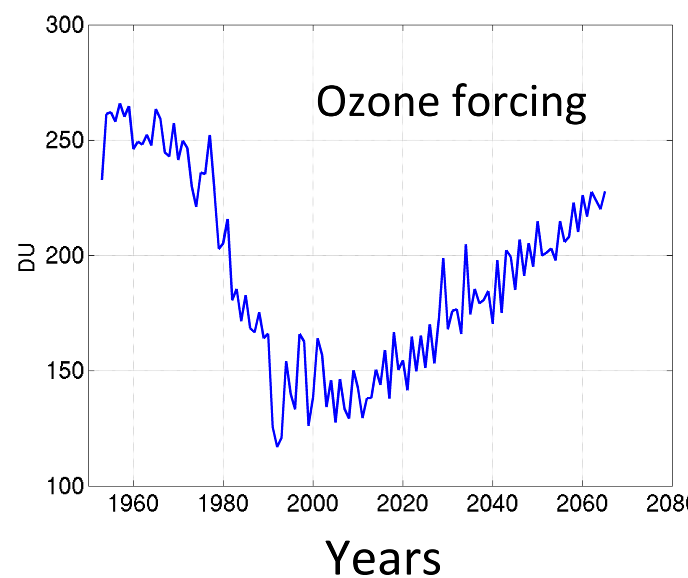
SST (70-50°S)

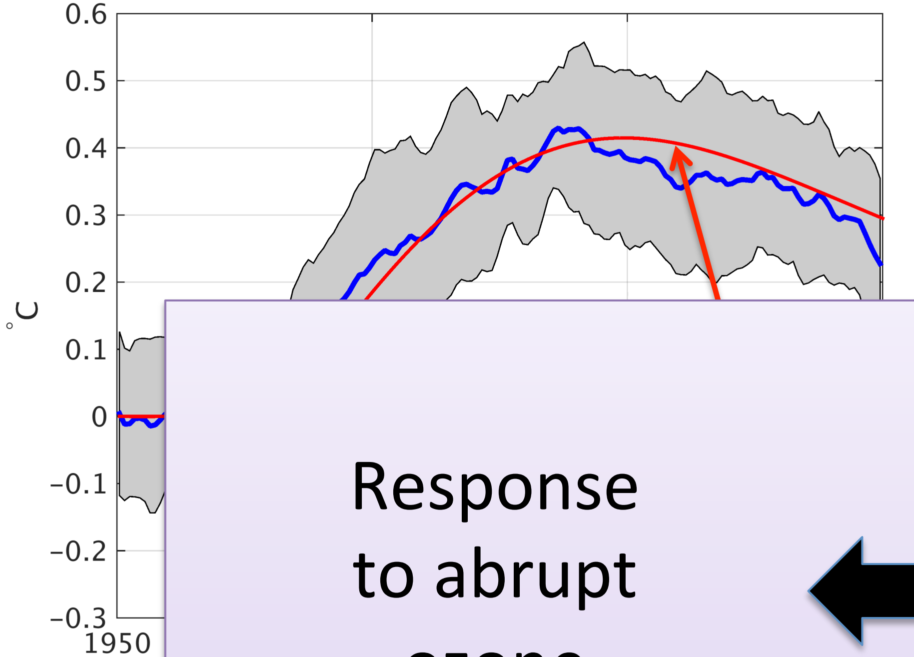
Response to a step change



Response to the time-varying ozone





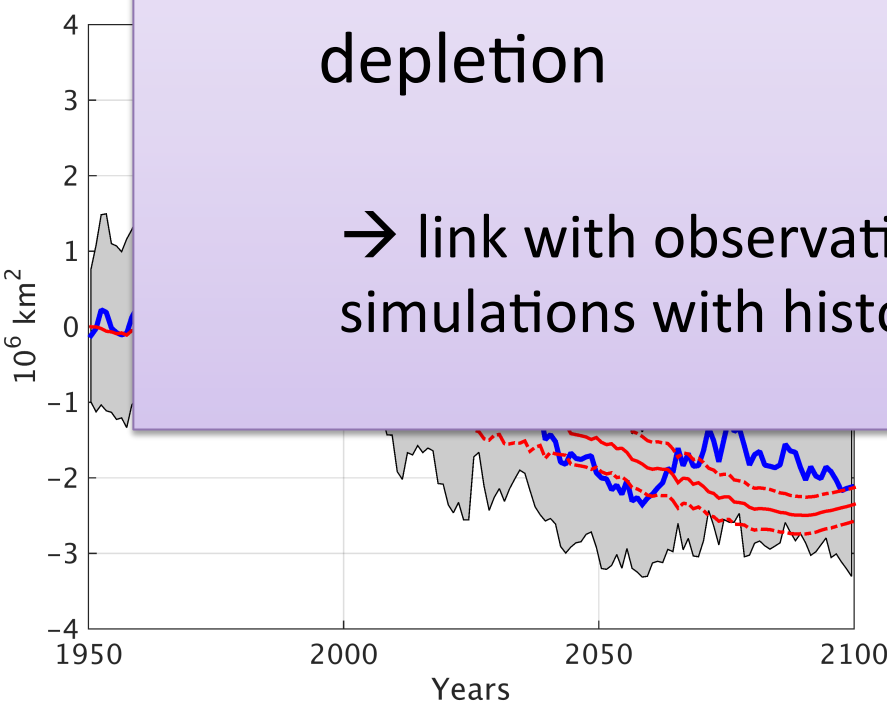


Subsurface temperature response to the time-

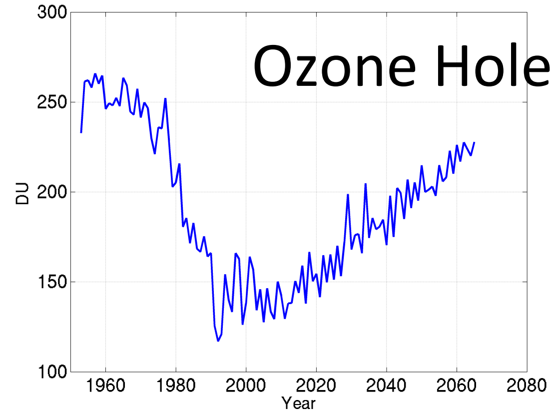
Response to abrupt ozone depletion

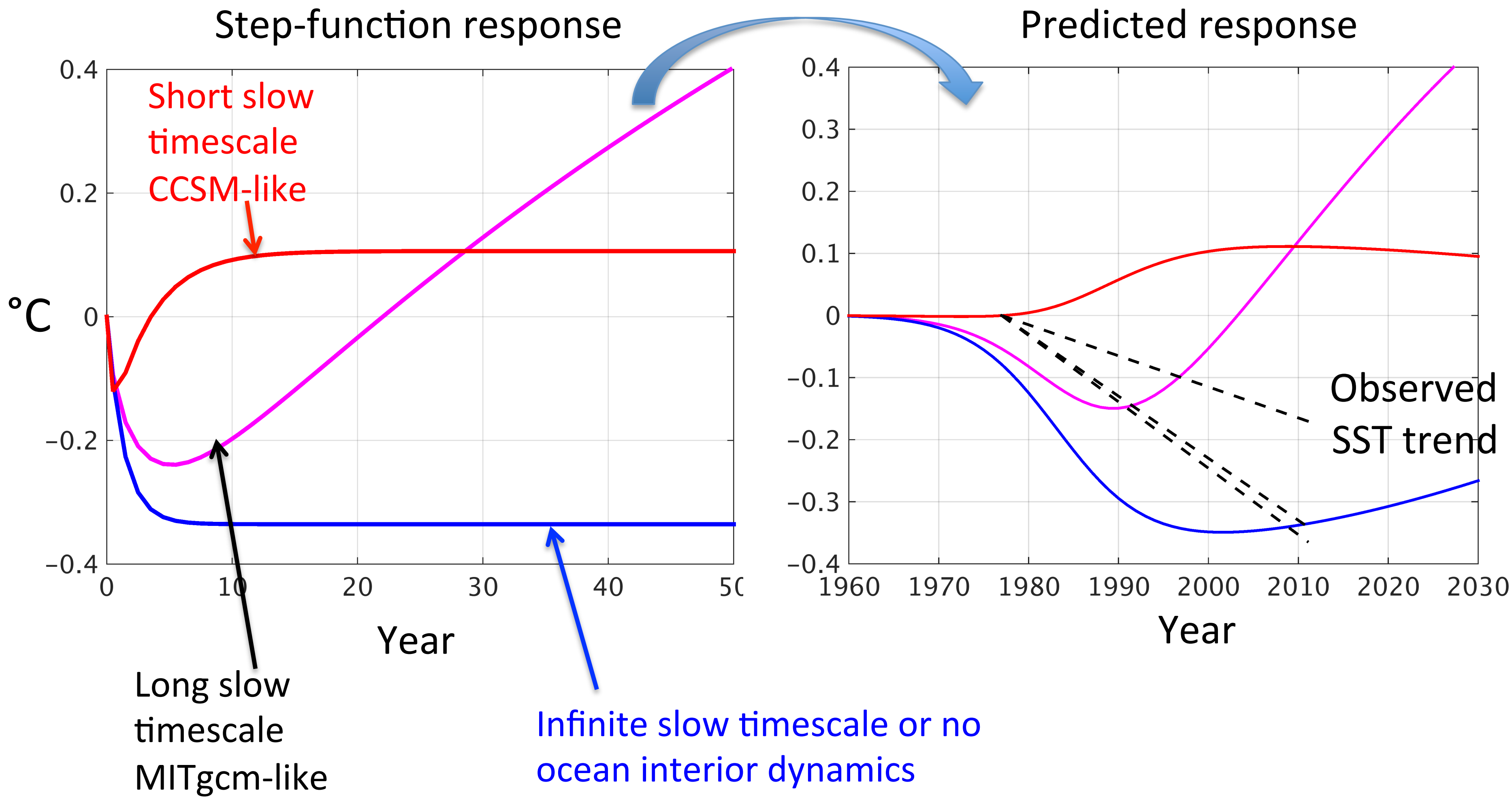
Response to to time-varying ozone

→ link with observations and model simulations with historical/future scenarios

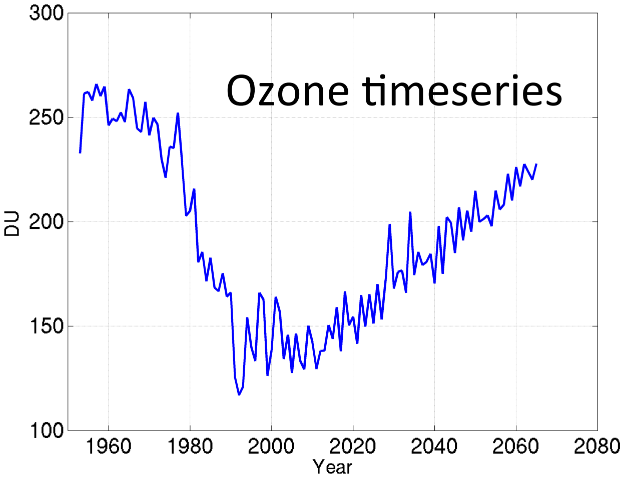


SST (70-50°S)

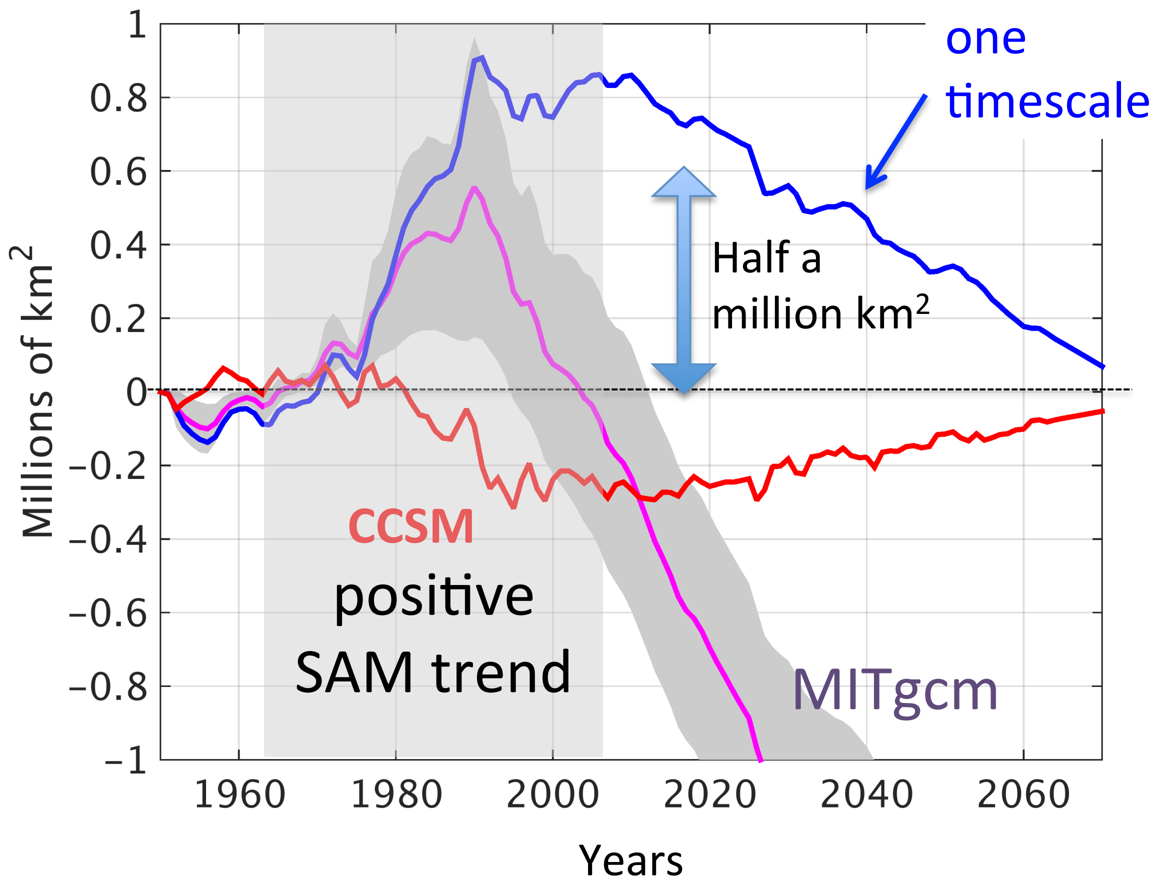

Ozone Hole and recovery
300
250
200
150
100
DU
1960
1980
2000
2020
2040
2060
2080
Year


Step-function response
Predicted response
Short slow
timescale
CCSM-like
°C
0.4
0.2
0
−0.2
−0.4
0
10
20
30
40
50
Year
Long slow
timescale
MITgcm-like
Infinite slow timescale or no
ocean interior dynamics
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
1960
1970
1980
1990
2000
2010
2020
2030
Year
Observed
SST trend

Implications for Sea Ice cover



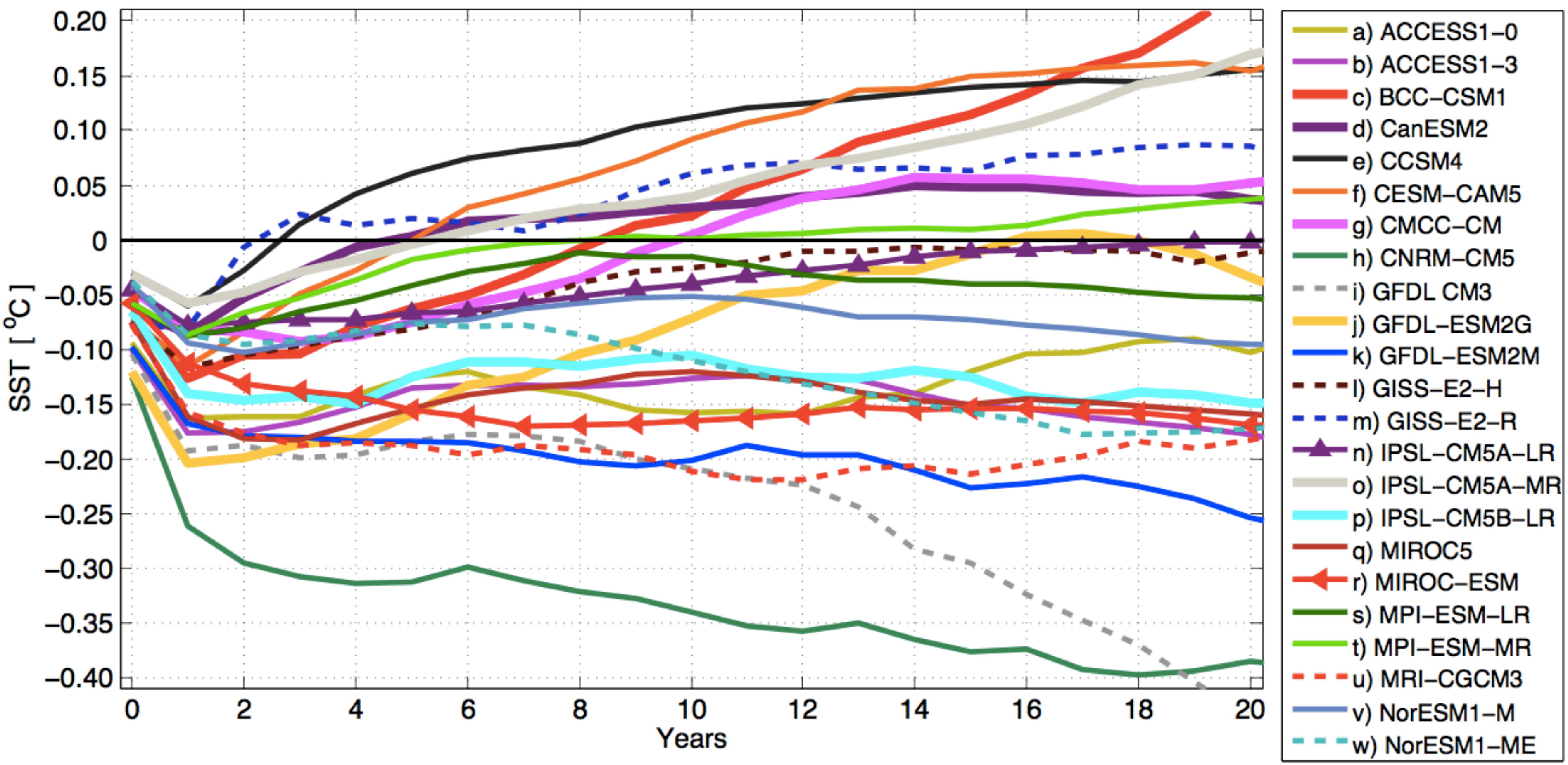
Prediction for Sea Ice Area response to Ozone change (Annual mean)



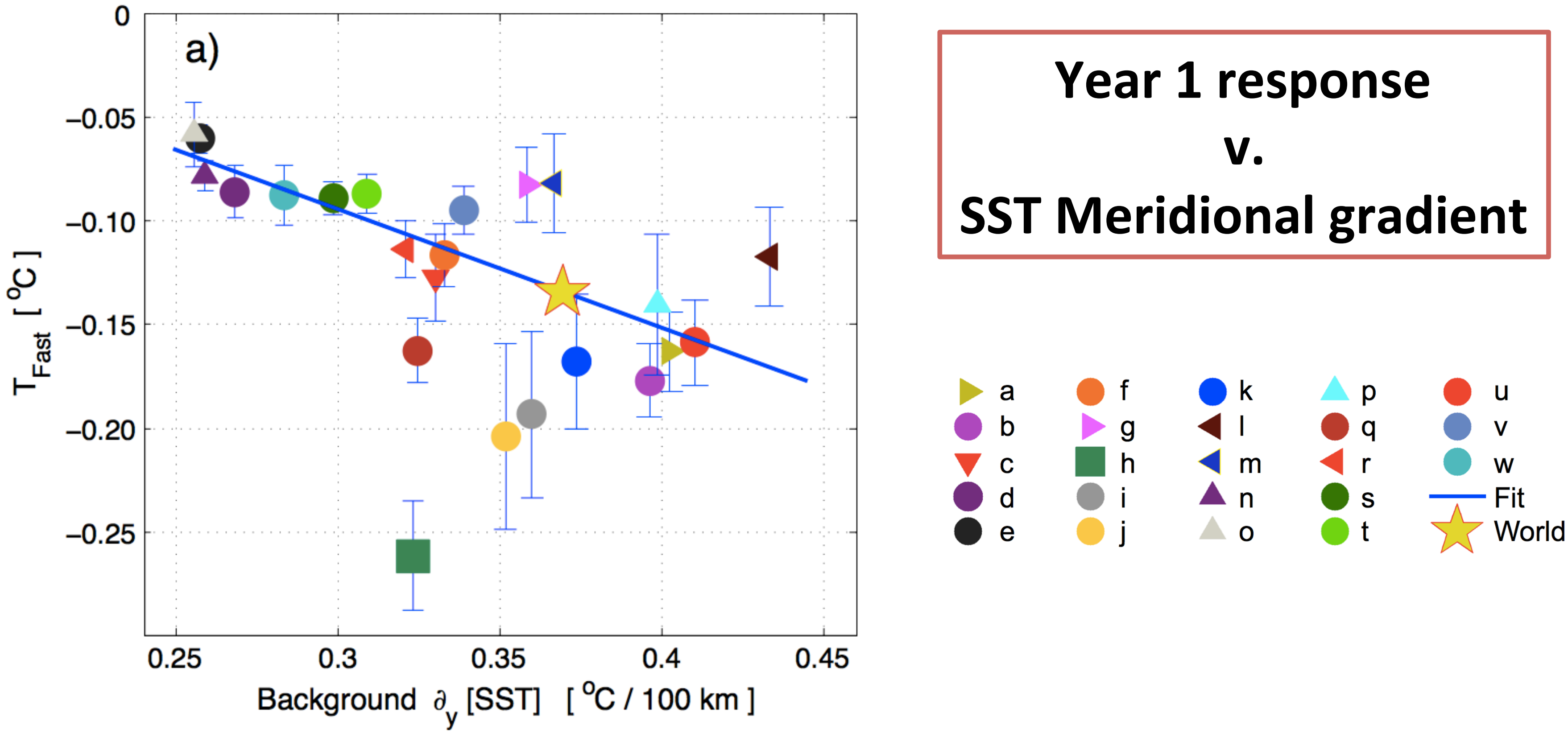
Ferreira and Marshall, in prep

CMIP5: model response to a 1σ SAM

SST (70-50°S)



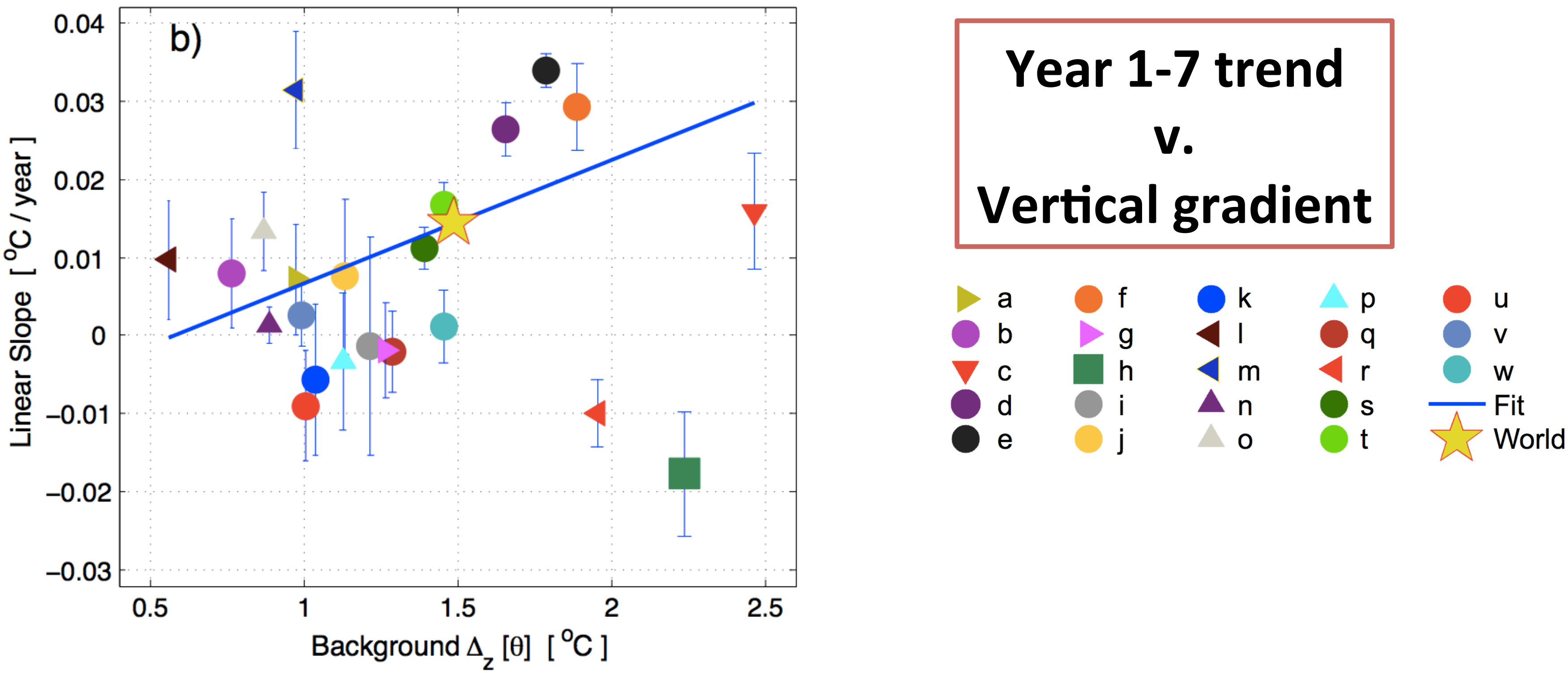
Kostov et al. (2016, submitted)



Year 1 response
v.
SST Meridional gradient

Short timescale response

$$\frac{dSST'}{dt} \approx \frac{[\tau_x']}{\rho_0 f Z_{Ek}} \partial_y \overline{[SST]} + F,$$

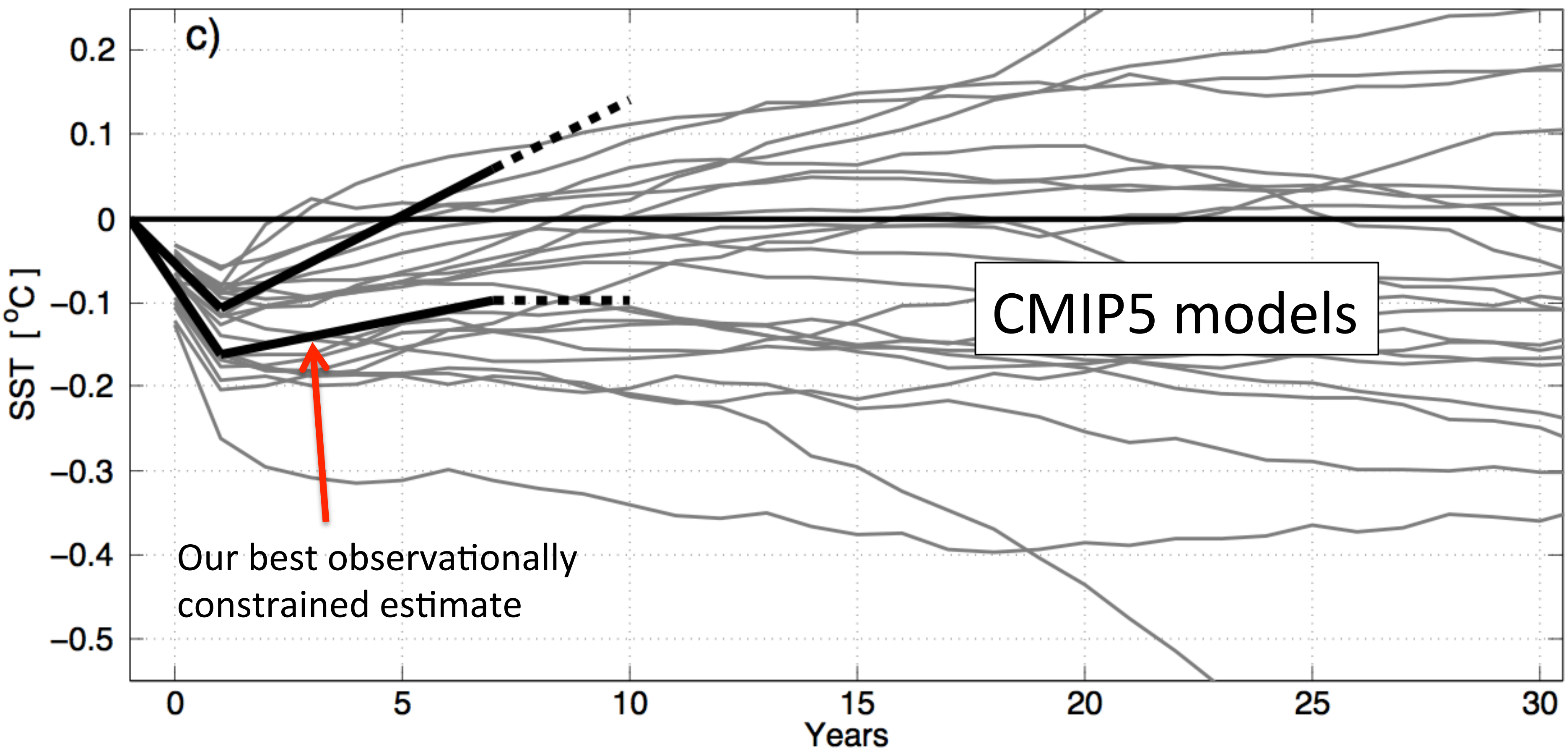


Year 1-7 trend
v.
Vertical gradient

Medium-range timescale response (before equilibrium)

$$\left.\frac{dSST'}{dt}\right|_{t=t_{lin}} \approx -t_{lin}\gamma\frac{dT'_{sub}}{dt} \approx -t_{lin}\gamma\frac{\delta}{\rho_0}\left(\frac{\partial}{\partial y}\left[\frac{\tau'_x}{f}\right]\right)\frac{\overline{\Delta_z[\theta]}}{Z_{sub}}$$

Observationally-constrained two-timescale response to a 1σ +SAM (annual mean)



Conclusion I

- Ocean/Sea Ice response to abrupt ozone depletion has 2 phases: cooling and then warming
- robust, but, large uncertainties in transition timescale: 3 to 20 years
- No inconsistency between expectations from observed correlation and coupled GCMs' response to ozone depletion
- The two timescales emerge through ocean dynamics

Ferreira et al. 2015, JClim

Conclusion II

- Does ozone depletion contribute to the sea ice expansion?

  we don't know: depends on the 2-timescale:

  → "fast" model (CCSM-like) : no

  → "slow" model (MITgcm-like) : yes

  → Real world? see Kostov et al. (2015) in prep.

- Ocean dynamics →multiple timescales (month-decade):

  goes beyond the SAM/ozone context

- Implications for interpreting observations and models:

  <u>*Response to a SAM trend is not a trend*</u>

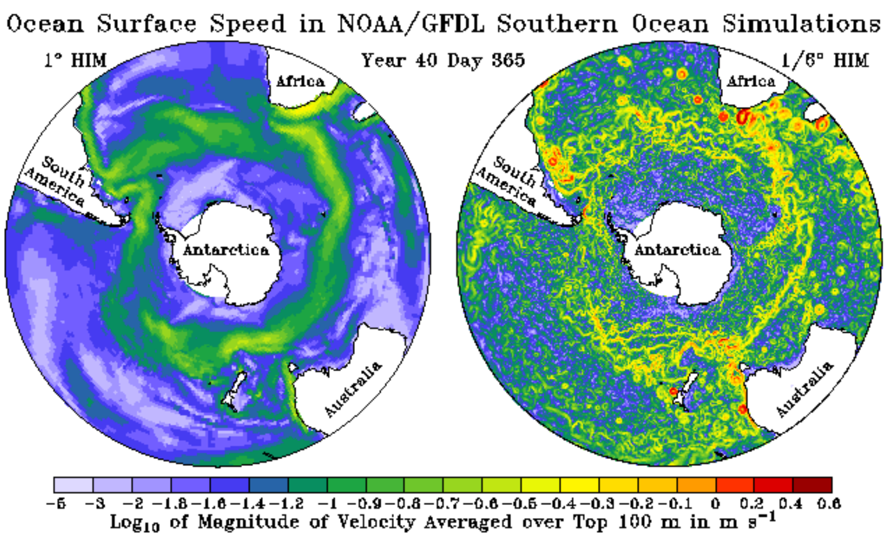
  <u>*Long time memory and lag in the response*</u>

  no unique relationship SAM-SST(/sea ice) relationship on all timescales

Ferreira and Marshall, in prep; Kostov et al., 2016, Clim. Dyn.

Outline

1. Southern Ocean role in global climate: past present and future
2. Impact of the Ozone hole on ocean and sea ice
3. Eddy-compensation problem and eddy diffusivities
4. Estimating eddy diffusivity with an adjoint method



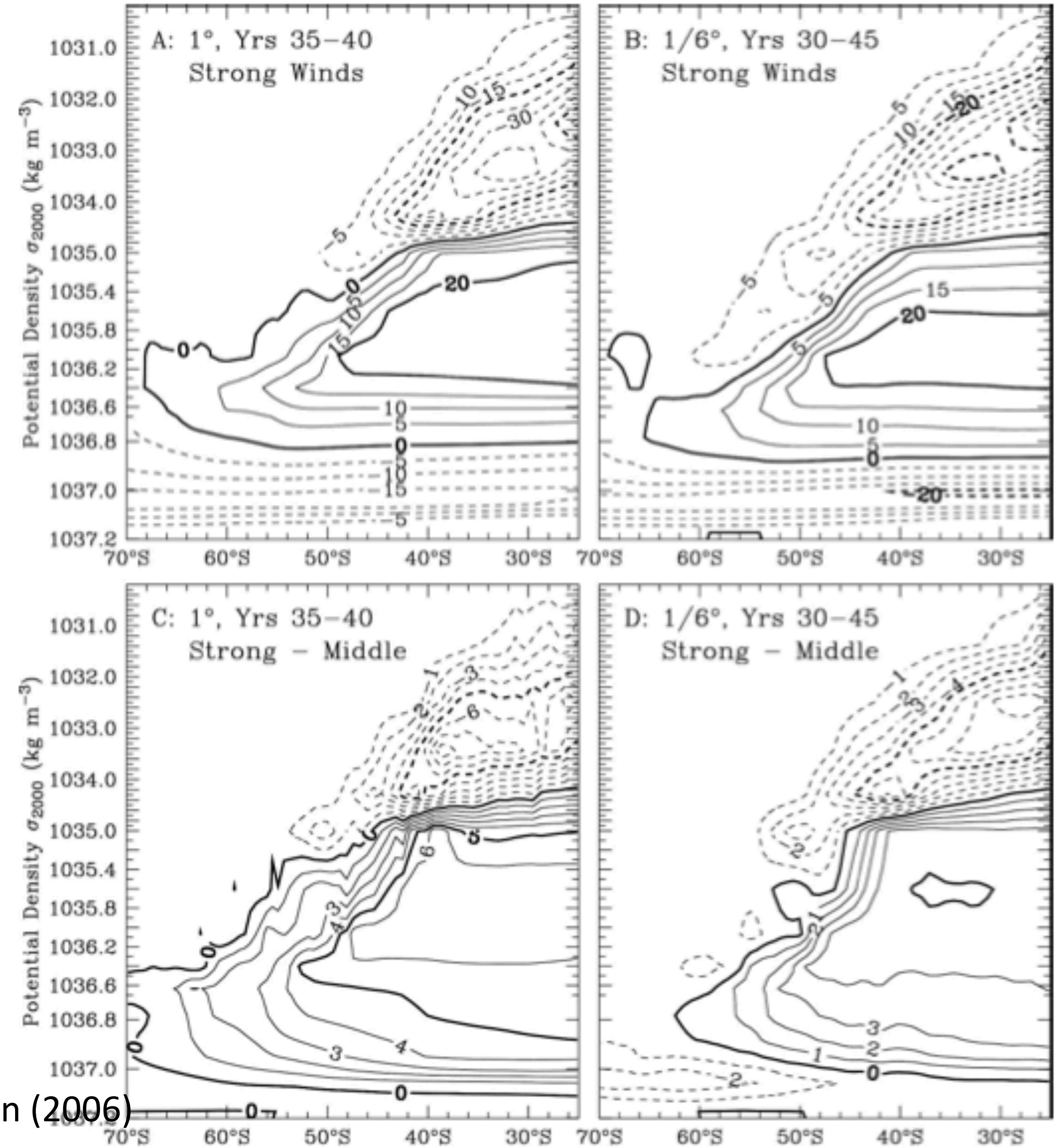
Hallberg and Gnanadesikan (2006)

Overturning in density coordinate

Reference state

Eddy compensation

Response to stronger winds weaker at high-resolution



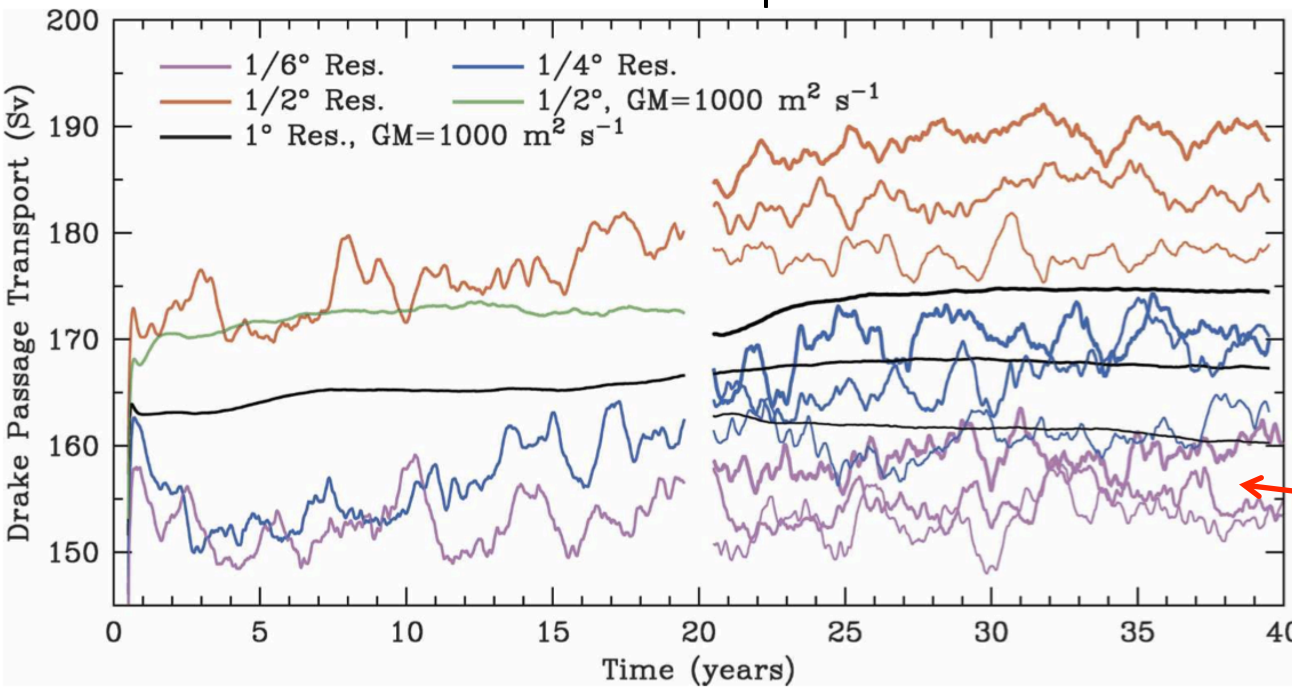
Hallberg and Gnanadesinkan (2006)

How sensitive the SO circulation is to wind changes?

A lot if thinking (e.g. Toggweiler and others) based on Ekman dynamics
But not so much if eddies are included in the picture.

Hallberg and Gnanadesinkan (2006)

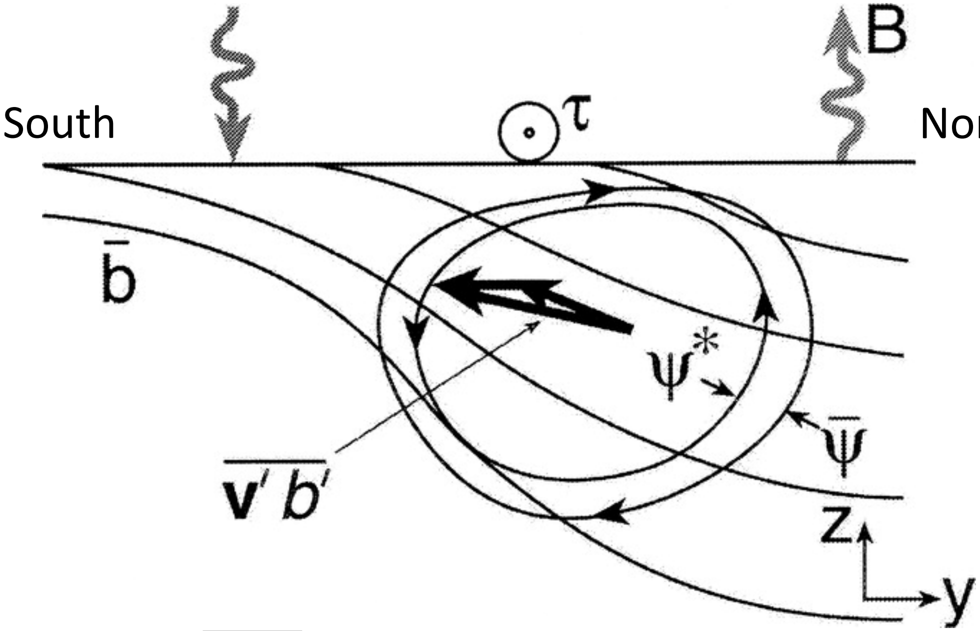
Zonal ACC transport



Eddy-saturation

Change winds

Southern Ocean Overturning

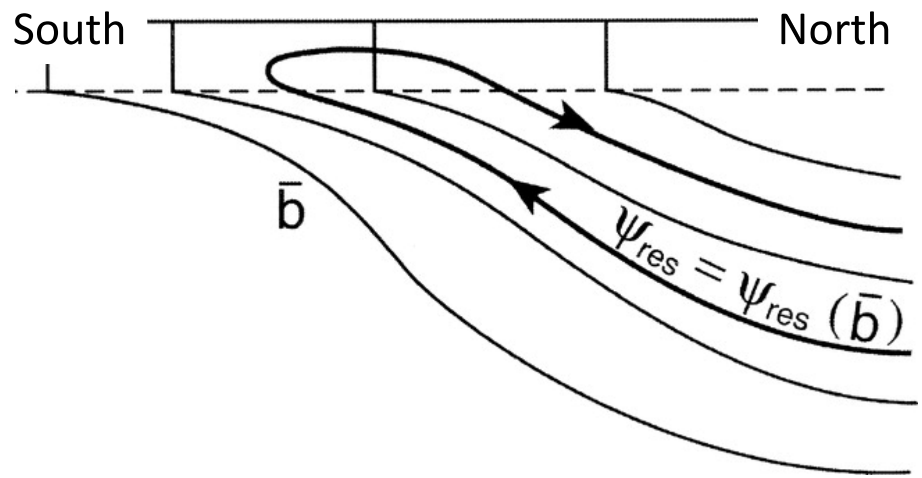


Eddy buoyancy flux are adiabatic

→ Advective effect

→ Eddy-induced circulation

$$\Psi^* = \frac{\overline{v'b'}}{N^2} = K_b \times S_y$$



<u>In short :</u>

- Surface winds put potential energy into the Southern Ocean
- Eddies remove potential energy

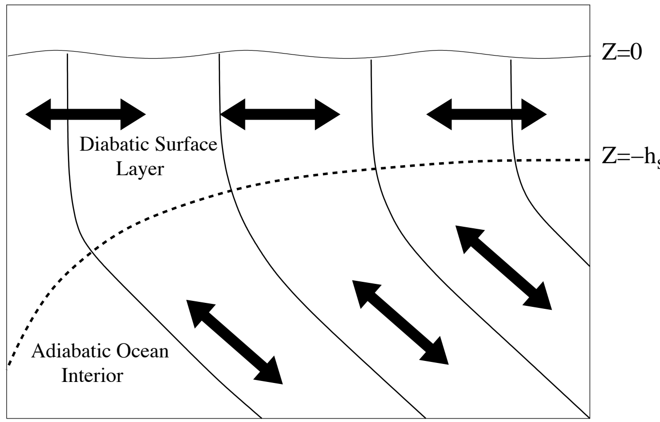
→ net MOC = small residual betweenwind-driven and eddy-induced MOCs

Andrews and McIntyre (1978), Treguier et al. (1997), Marshall and Radko (2003)

Large Scale Ocean Dynamics

Momentum :

$$\frac{D}{Dt}\overline{\mathrm{u}} + \ldots = \frac{\partial \tau_x^w}{\partial z} - \partial_y\left(\overline{\mathrm{u}'\mathrm{v}'}\right)$$



Buoyancy :

$$\frac{D}{Dt}\overline{\mathrm{b}} = \frac{\partial}{\partial z}\left(k_b\frac{\partial\overline{\mathrm{b}}}{\partial z}\right) + Q_{dia} - \nabla.\left(\overline{\mathrm{v}'\mathrm{b}'}\right)$$

along isopycnal flux in ocean interior

- Eddies appear as a forcing on the rhs of both the momentum and the buoyancy equation

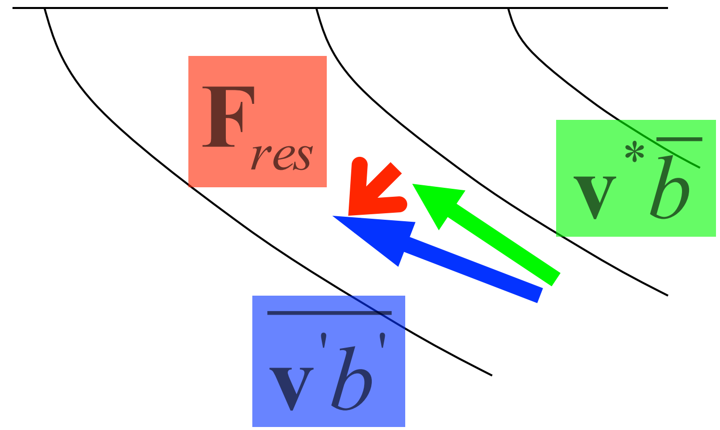
- Eddy buoyancy fluxes are nearly adiabatic

Large Scale Ocean Dynamics

Andrews and McIntyre (1978)

The mean buoyancy equation:

$$\frac{\partial \bar{b}}{\partial t} + \bar{\mathbf{v}}.\nabla \bar{b} = -\nabla.\overline{\mathbf{v}'b'}$$



$$\nabla.\overline{\mathbf{v}'b'} = \mathbf{v}^*.\nabla \bar{b} + \nabla.\mathbf{F}_{res}$$

where $\mathbf{v}^* = -\nabla \times \Psi^*$ and $\Psi^* = \left(\frac{\overline{v'b'}}{\bar{b}_z}, -\frac{\overline{u'b'}}{\bar{b}_z}, 0 \right)$

Residual mean buoyancy equation

$$\frac{\partial \bar{b}}{\partial t} + \mathbf{v}_{res}.\nabla \bar{b} = -\nabla.\mathbf{F}_{res} \quad \text{where} \quad \mathbf{v}_{res} = \mathbf{v}^* + \bar{\mathbf{v}}$$

Framework for eddy effects on large-scale ocean dynamics

$$\partial_t \overline{C} + (\overline{\mathbf{v}} + \mathbf{v}^*).\nabla\overline{C} = \nabla.(K\nabla\overline{C})$$

C is a tracer

Eddy effect in two places

Advective effect

Mixing effect: along isopycnal

$$\mathbf{v}^* = -\vec{k} \times \nabla\Psi^*$$

K : symmetric tensor

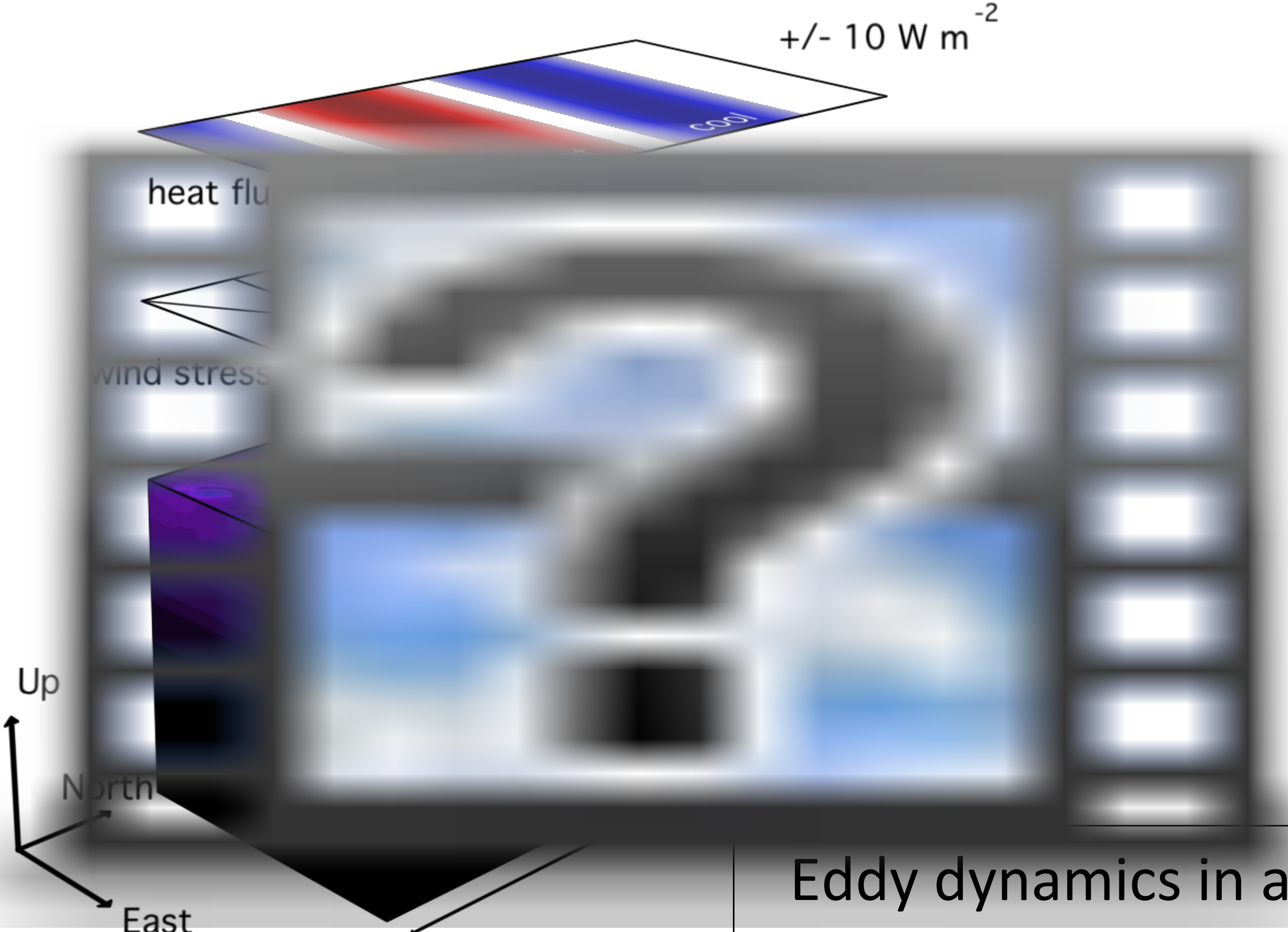
Gent and McWilliams scheme:

$$\Psi^* = \frac{\overline{v'b'}}{N^2} = \frac{-K_{GM}\overline{b}_y}{N^2} = K_{GM} \times S_y$$

Redi isopycnal mixing:

Mixing in the ocean is mainly along isopycnal

$$\nabla.(K\nabla\overline{C}) = \nabla.(K_{iso}\nabla_\rho\overline{C})$$



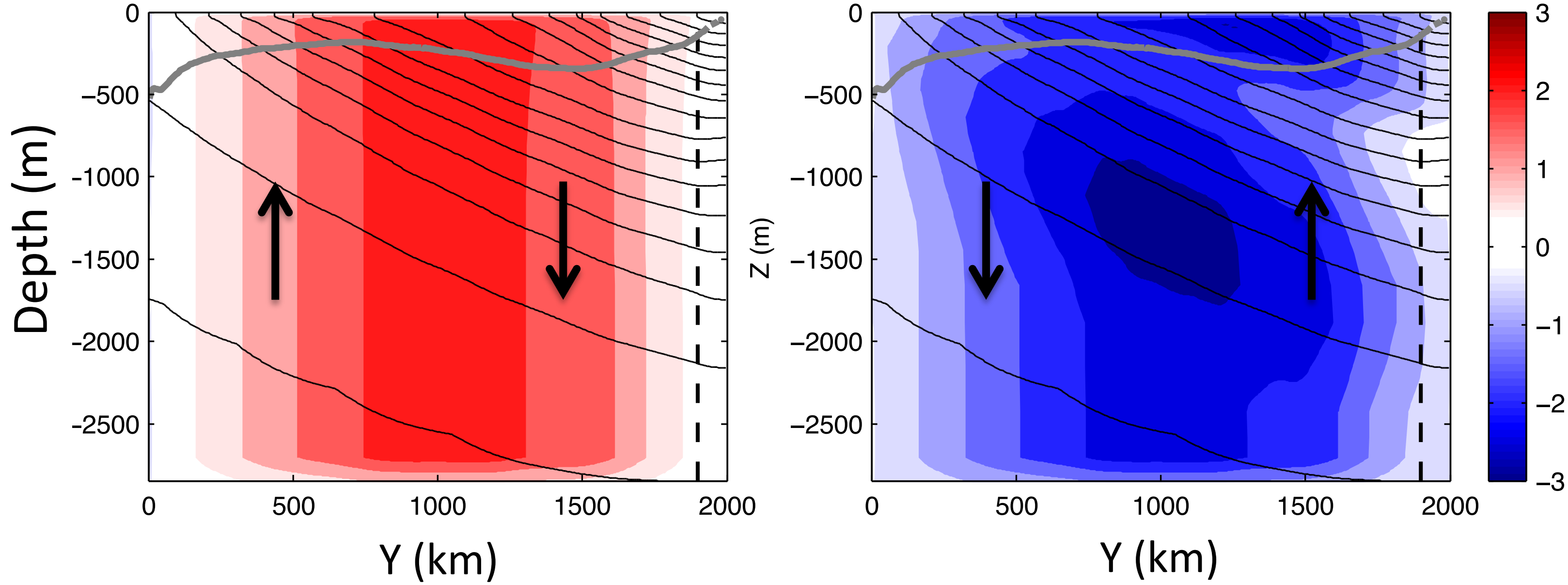
Eddy dynamics in an idealized channel

Idealized channel

$$\overline{\Psi} = \frac{\overline{\tau}_x}{\rho_o f}$$

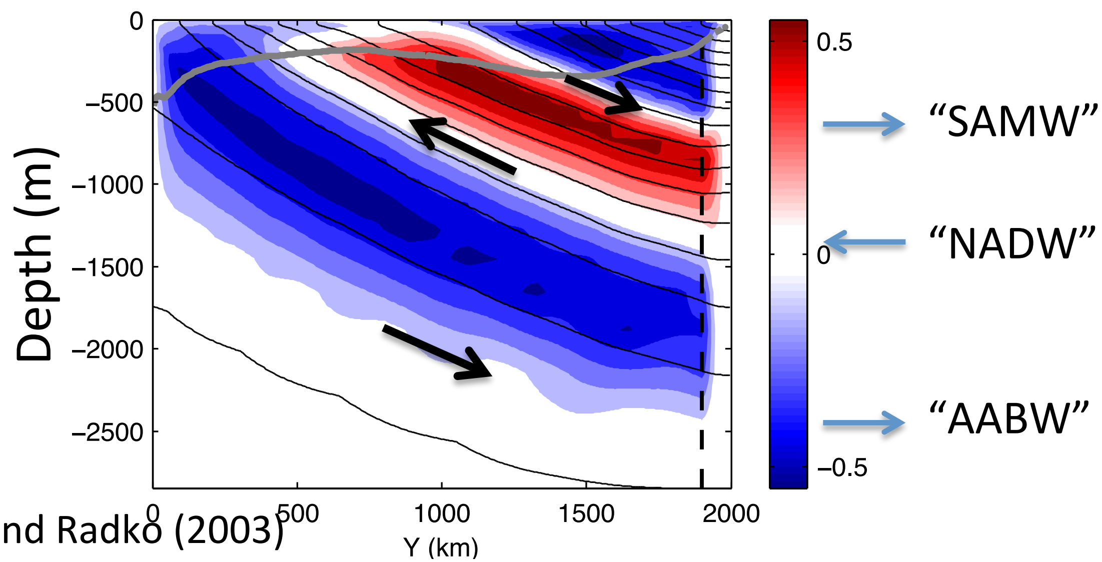
Eddy-induced circulation

$$\Psi^* = \frac{\overline{v'b'}}{N^2}$$



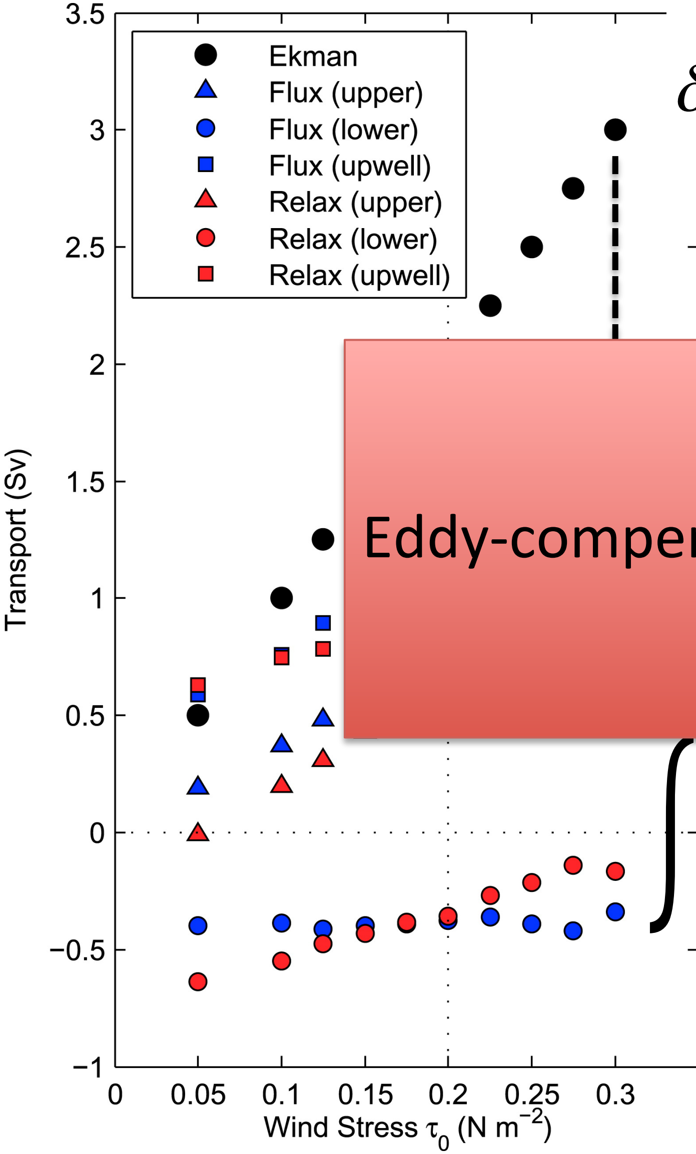
Residual-mean circulation

$$\Psi_{res} = \overline{\Psi} + \Psi^*$$



Andrews and McIntyre (1978), Marshall and Radko (2003)

Equilibrium response to wind changes



$$\delta\overline{\Psi} = \frac{\delta\overline{\tau_x}}{\rho_O f}$$

- Ψ_{res} does not change much with wind

reases

ne

2006)

$\delta\Psi_{res}$

Eddy-compensation:

$$\Psi_{res} = \overline{\Psi} + \Psi^*$$

$$\delta\Psi_{res} = \delta\overline{\Psi} + \delta\Psi^* \approx 0$$

- Important paleoclimate implications (Toggweiler et al. 2006)

Abernathey et al. 2012

Equilibrium response to wind changes

$$\delta\Psi^* = \delta(K_b \times S_y)$$

$$\Psi^* = \frac{\overline{v'b'}}{N^2} = K_b \times S_y$$

$$S_y \times \delta K_b$$

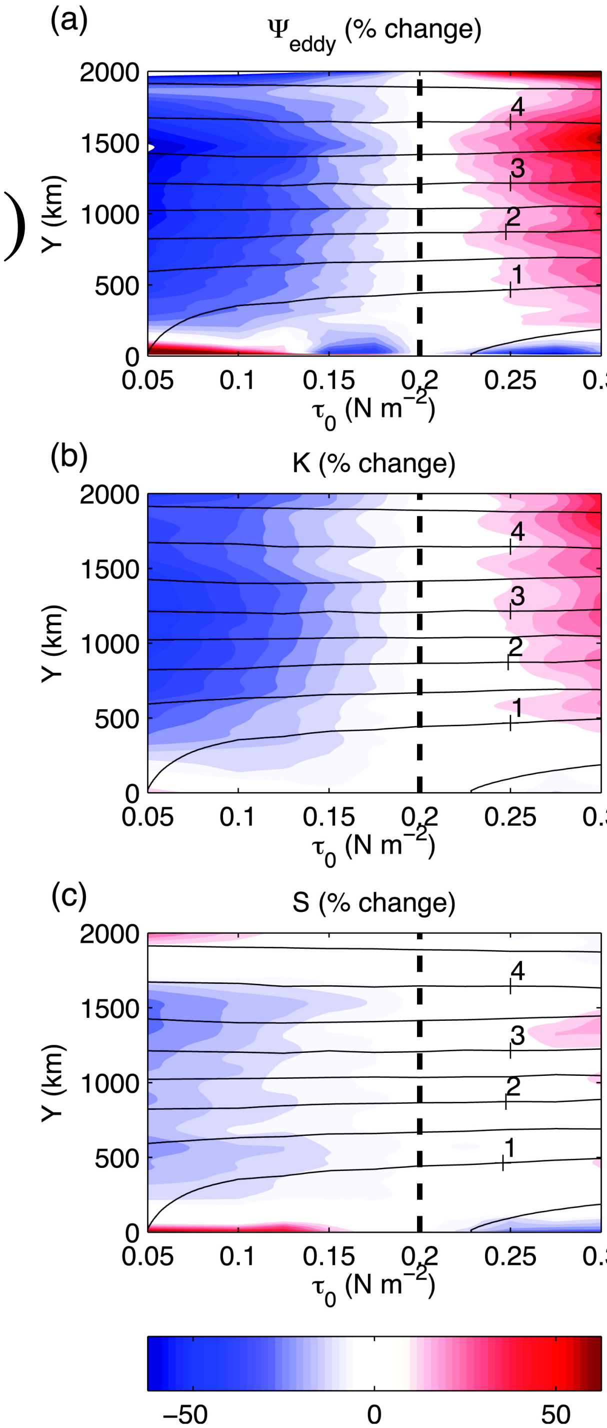
$$K_b \times \delta S_y$$

Residual-mean MOC nearly constant

because the diffusivity ***increases***:

$$K_b \propto \sqrt{\tau} \propto \sqrt{EKE}$$

and nearly constant slopes



Abernathey et al. 2012

$K_{iso}(T)$

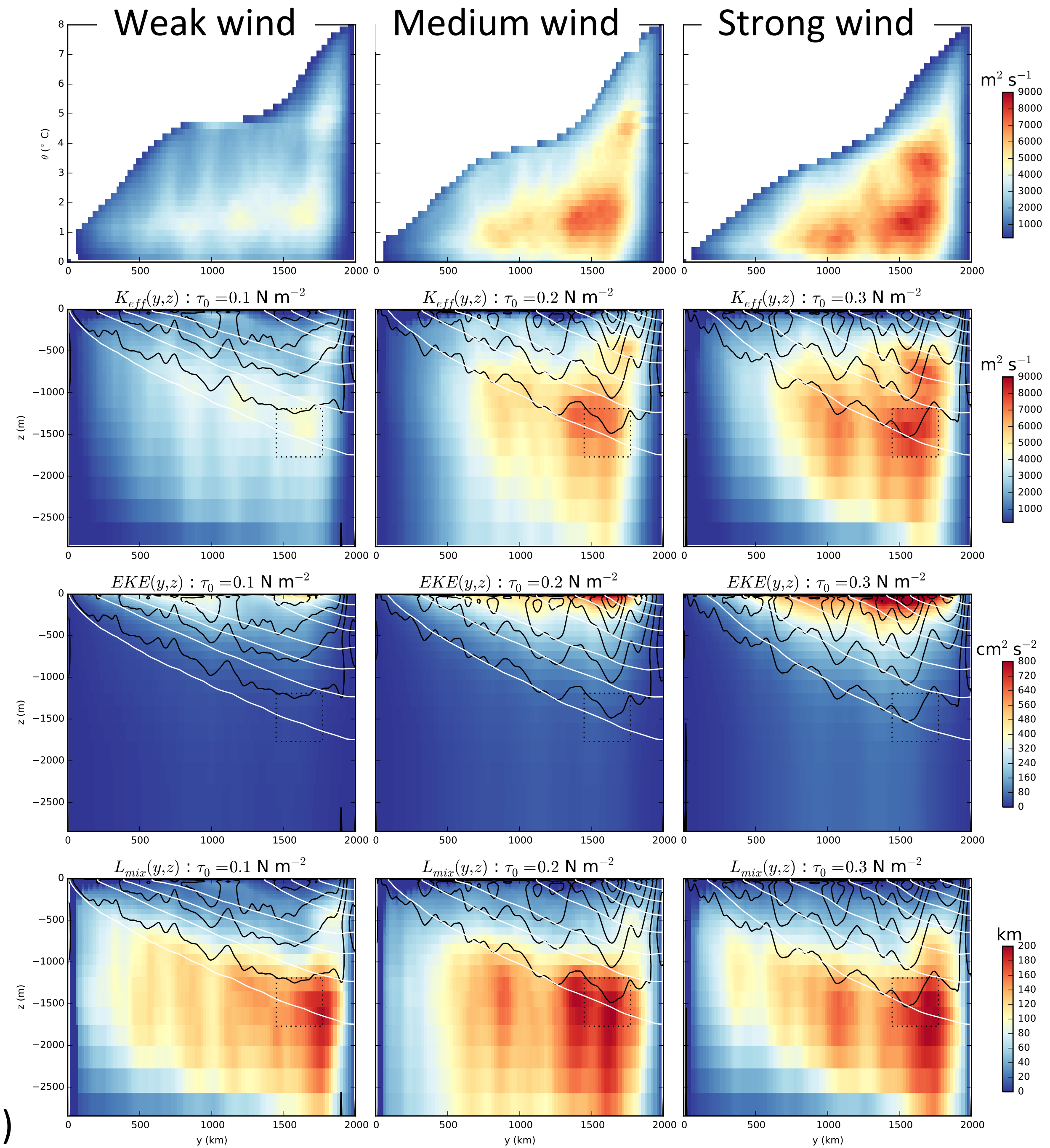
What about isopycnal mixing?

$K_{iso}(z)$

$$K_{eff} \propto U_{rms} \times L_{mix}$$

EKE

L_{mix}

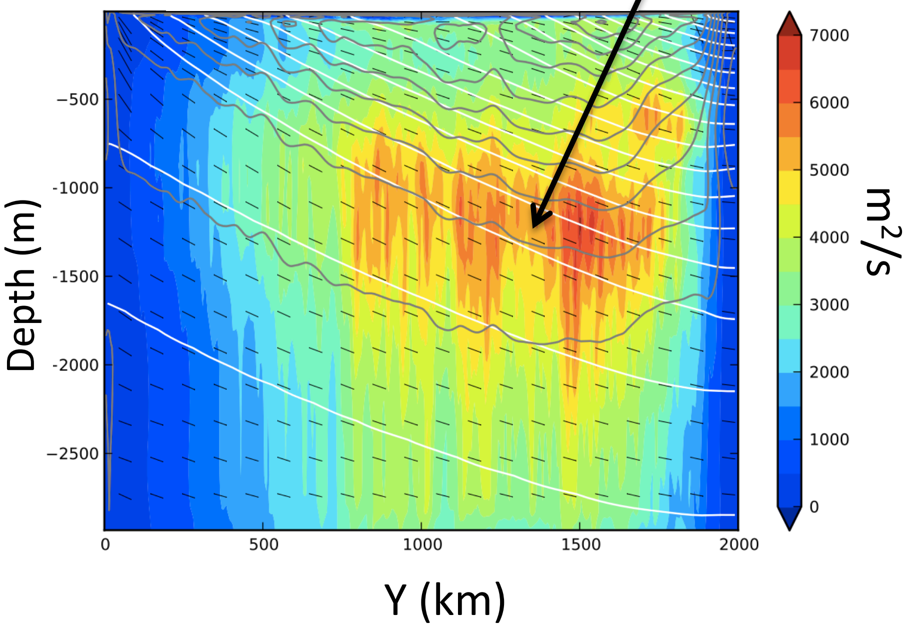


Abernathey and Ferreira (2015)

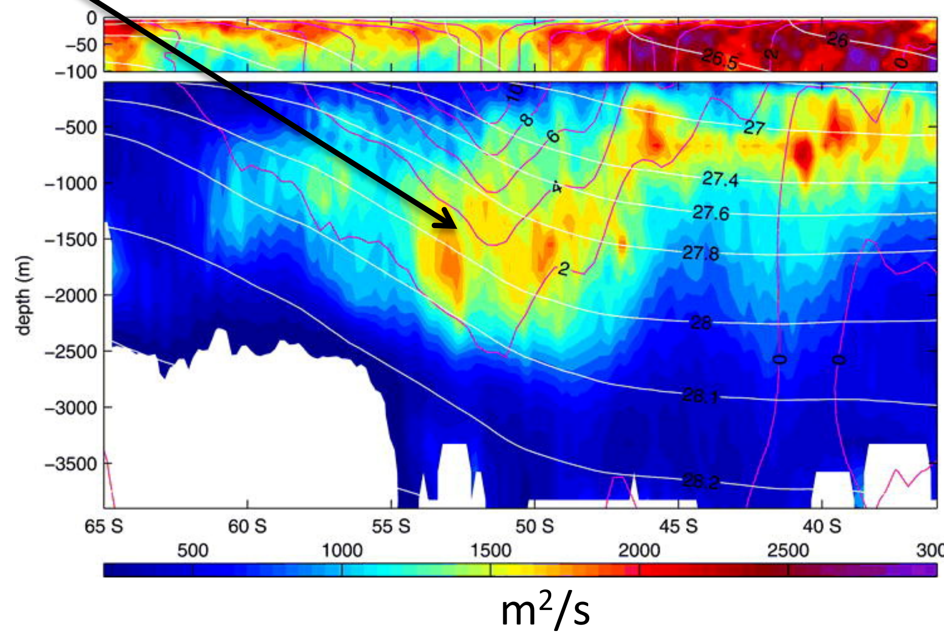
Spatial variations of effective eddy diffusivity

Maximum at steering level @ U(z)≈C

Idealized Channel



Ocean Reanalysis SOSE (Mazloff et al. 08)

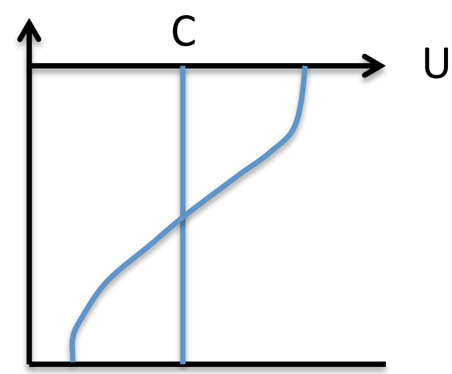


$$K_{eff} = \Gamma \times U_{rms} \times L_{mix}$$

$$L_{mix} = \frac{L_{eddy}}{1 + d_1 \times [c - U(z)]^2 / EKE}$$

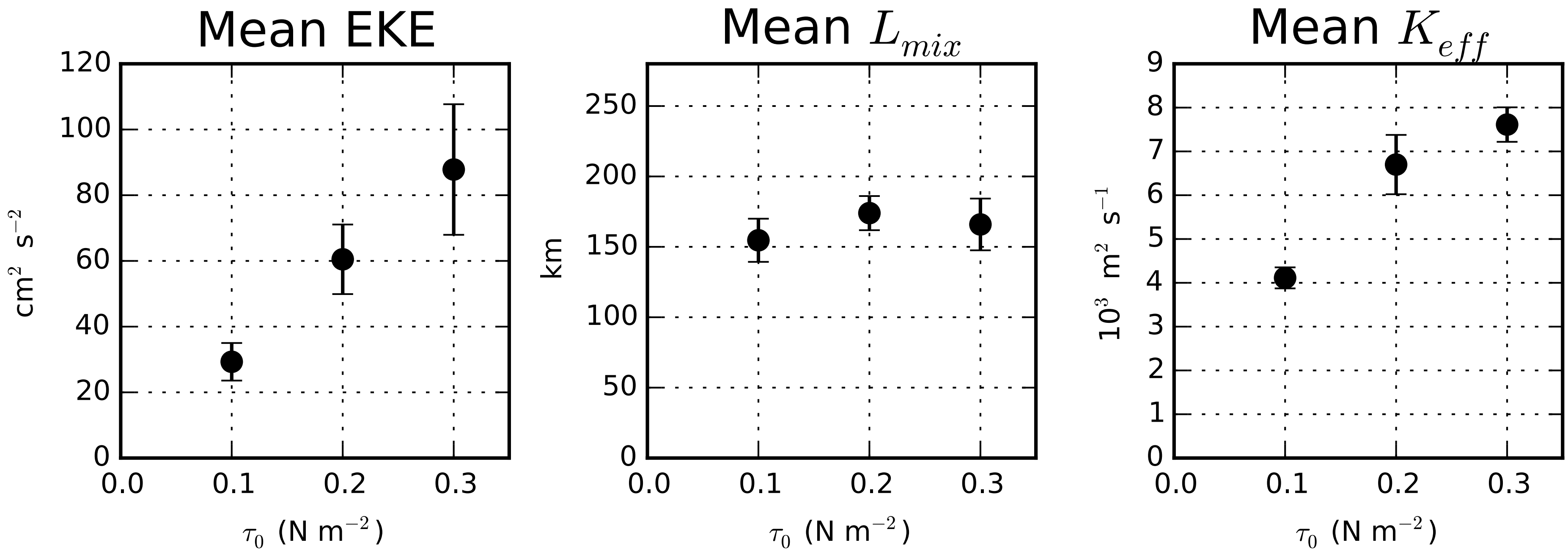
$$U_{rms} = \sqrt{2\mathrm{EKE}}$$

c = phase of eddies



Green (1970), Abernathey et al. (2010, 2013), Ferrari and Nikurashin (2011)

Isopycnal mixing varies with wind stress too



$$K_{Iso} \propto \sqrt{EKE} \times L_{mix}$$

$$L_{mix} = \frac{L_{eddy}}{1 + d_1\left[c - U(z)\right]^2}$$

Green (1970)

→ Does matter for uptake of tracer along isopycnal

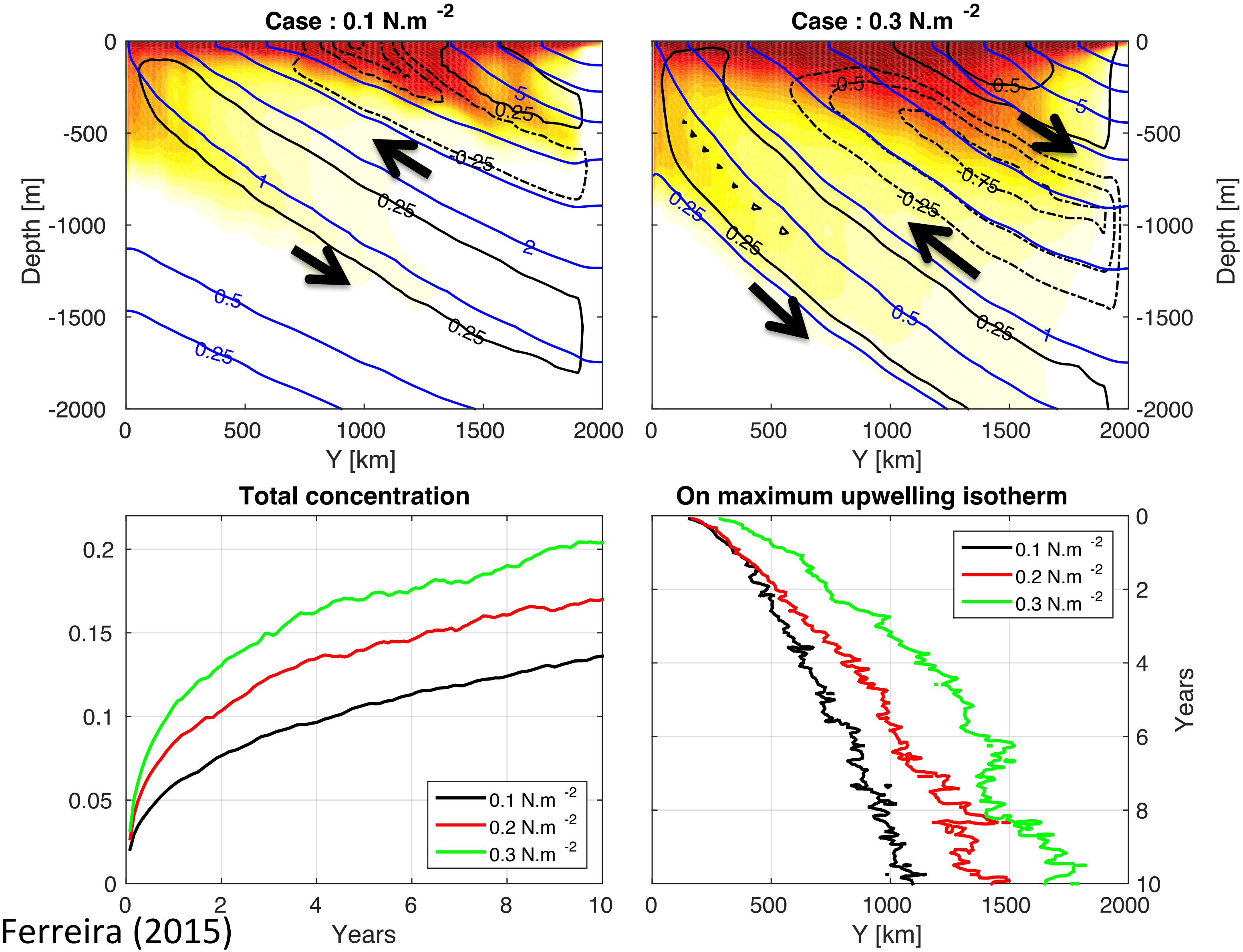
→ Could be strong enough to overcome advective effects

Abernathey and Ferreira (2016)

Effect on idealized ventilation tracer

C=0 at t=0
C→1 at surface

After 5 years



Abernathey and Ferreira (2015)

What do current ocean models do ?

- Advective effect: Gent and McWilliams 90, buoyancy diffusivity K_{GM}:

$$\Psi^* = K_{GM} \times S$$

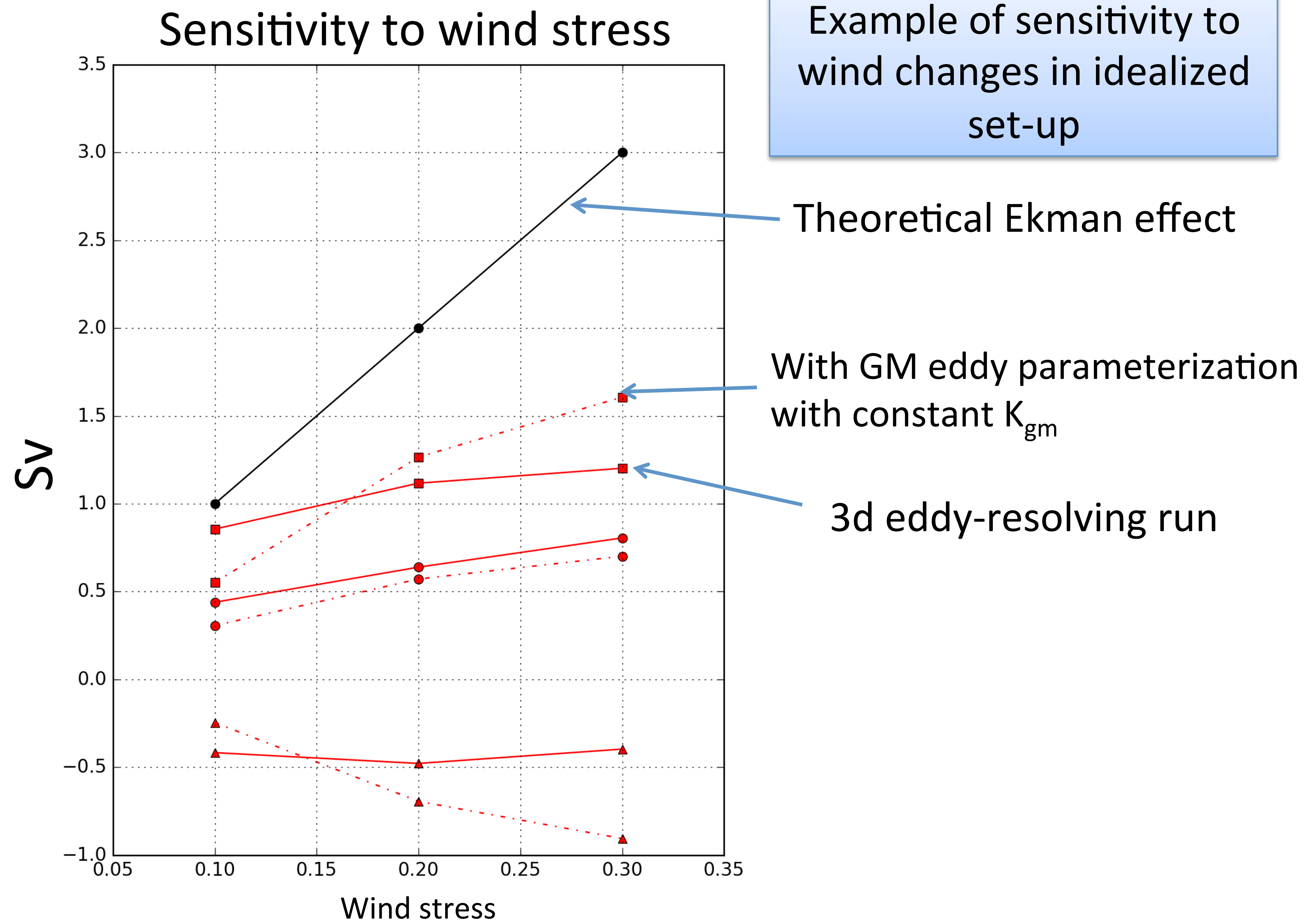
- Often K_{GM} is constant and uniform (~500 to 2000 m^2/s)

- Some exceptions:
  - Visbeck et al. (1997): K_{GM} scales as Eady Growth rate
  - CCSM4: K_{GM} scales as N^2 (Ferreira et al., 2005)

- Isopycnal mixing for tracer other than buoyancy, $K_{iso} = K_{GM}$ but not genrally true

→ OGCMs cannot reproduce the ***equilibrium*** "eddy-compensation" response to wind changes

→ Likely they are also unfit for the ***transient*** (interannual to decadal) response



Example of sensitivity to wind changes in idealized set-up

Summary

- SO MOC is *weakly* sensitive to wind changes because of the counter-acting advective effect of eddies
  - →Associated eddy diffusivity scales with EKE or wind
- Isopycnal mixing is *very* sensitive to wind changes
- The state of the SO is likely strongly sensitive to winds changes, but in the way it was initially envisioned
- Feedbacks on carbon and heat uptake is unclear
- Eddy parameterizations do not have the proper physics

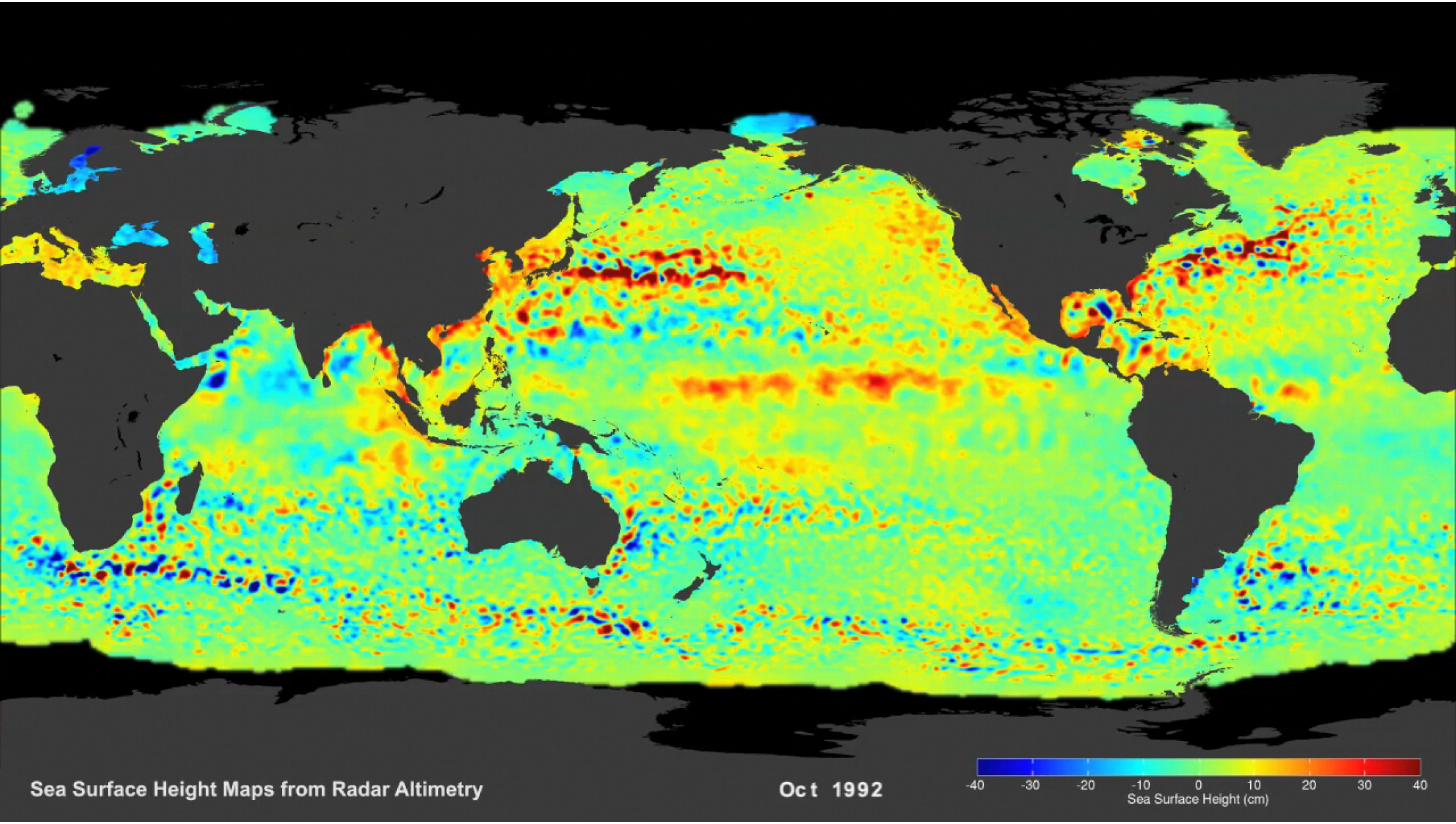
Outline

1. Southern Ocean role in global climate: past present and future
2. Impact of the Ozone hole on ocean and sea ice
3. Eddy-compensation problem and eddy diffusivities
4. Estimating eddy diffusivity with an adjoint method

→ Eddy are very difficult to observe in the ocean (requires high rate of sampling in both time and space

→Satellite only provides information for the surface ocean

→ Eddies are everywhere


Sea Surface Height Maps from Radar Altimetry
Oct 1992
-40
-30
-20
-10
0
10
20
30
40
Sea Surface Height (cm)

Residual Mean Formulation

Andrews and McIntyre (1976)

Buoyancy :

$$\frac{\partial \overline{b}}{\partial t} + (\underbrace{\overline{\mathrm{v}} + \vec{\mathrm{v}}^*}_{\vec{\mathrm{v}}_{res}}).\nabla \overline{b} = Q_{dia}$$

What are the optimal Eddy Stresses ?

Momentum :

$$\frac{\partial \vec{\mathrm{v}}_{res}}{\partial t} + \vec{\mathrm{v}}_{res}.\nabla \vec{\mathrm{v}}_{res} + \ldots = \frac{\partial \vec{\tau}_{wind}}{\partial z} + \frac{\partial \vec{\tau}^{e}}{\partial z}$$

In the ocean interior:

$$\tau^e = (\tau^e_x, \tau^e_y) = \rho_o f \left(\frac{\overline{v'b'}}{\overline{b}_z}, -\frac{\overline{u'b'}}{\overline{b}_z} \right)$$

and

$$\tau^e_{x,y}\Big|_{bottom}^{top} = 0$$

Equations have a familiar form but :

- interpretations of terms is different
- eddies appear as a stress in the momentum equation

→ Adiabatic eddy forcing

Opttimization Procedure to estimate eddy effects: MITgcm 4x4 degree

Cost Function :

$$J = \frac{1}{N}\sum \frac{(\bar{T}_{Mod} - T_{Lev})^2}{\sigma_T^{\ 2}} + \frac{\lambda}{N}\sum_{|\tau^e|>0.4} (|\tau^e| - 0.4)^2$$

Control parameters :

$$\left.\begin{matrix} \tau^e_x(x,y,z) \\ \tau^e_y(x,y,z) \end{matrix}\right\} \tau^e_{x,y}\Big|_{bottom}^{top} = 0$$

First Guess : a GM-like eddy stress

$$\tau^e_x = +f\rho_o \kappa S_y \qquad \kappa = 1000\ \mathrm{m^2 s^{-1}}$$

$$\tau^e_y = -f\rho_o \kappa S_x$$

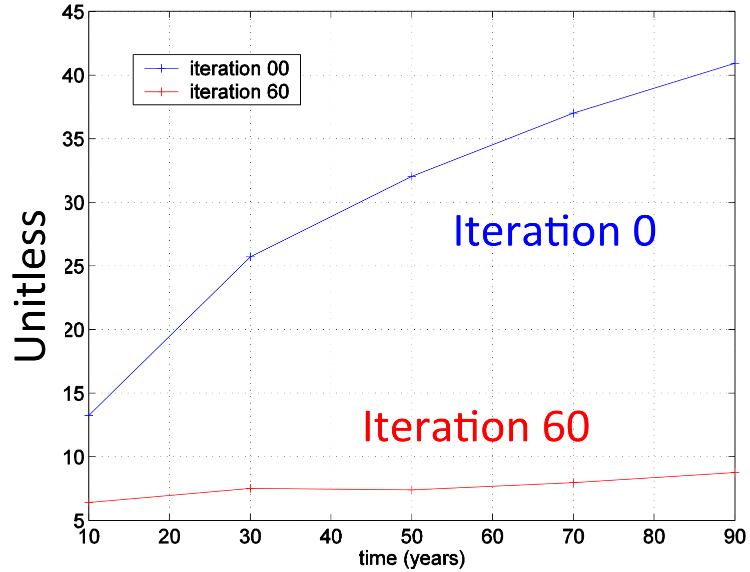
Optimization procedure:

$$T_{Lev} \xrightarrow{\text{100 years}} T_{Mod} \rightarrow J \xrightarrow{\text{Adjoint}} \frac{\partial J}{\partial \tau^e_{x,y}}$$

$$\frac{\partial J}{\partial \tau^e_{x,y}} \rightarrow \delta\tau^e_{x,y} \rightarrow T_{Lev}$$

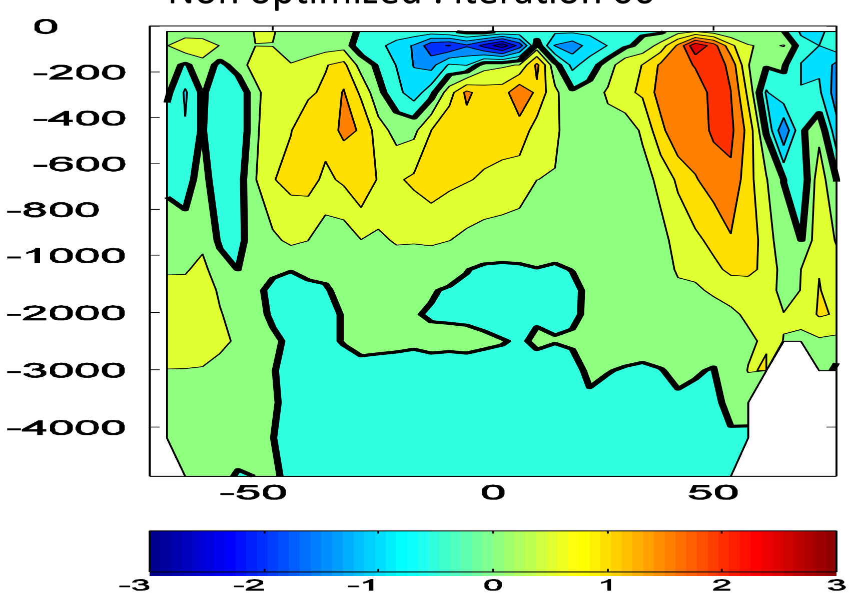
After 60 iterations, the solution converges and the cost function is decreased by 80 %

Cost Function : departure from observations

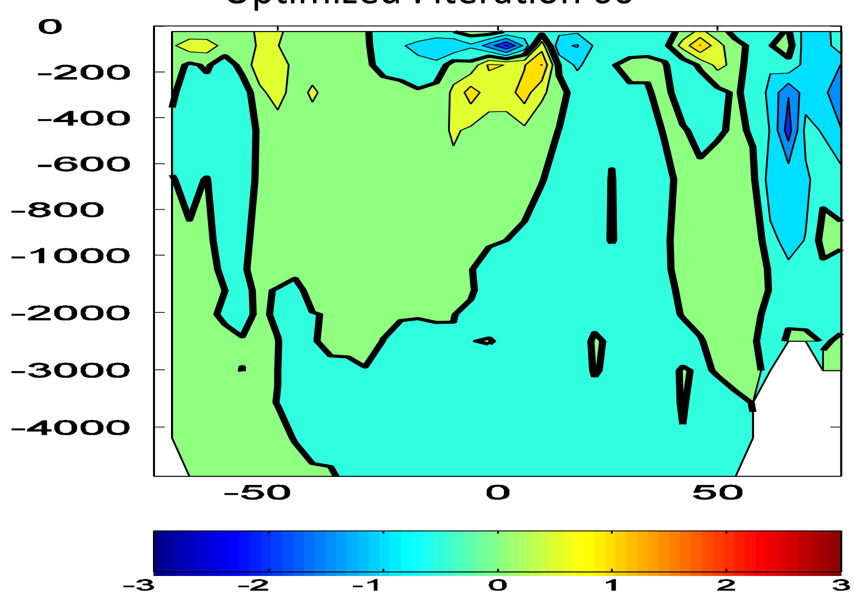


Zonal Mean Temperature Error :

Non optimized : iteration 00



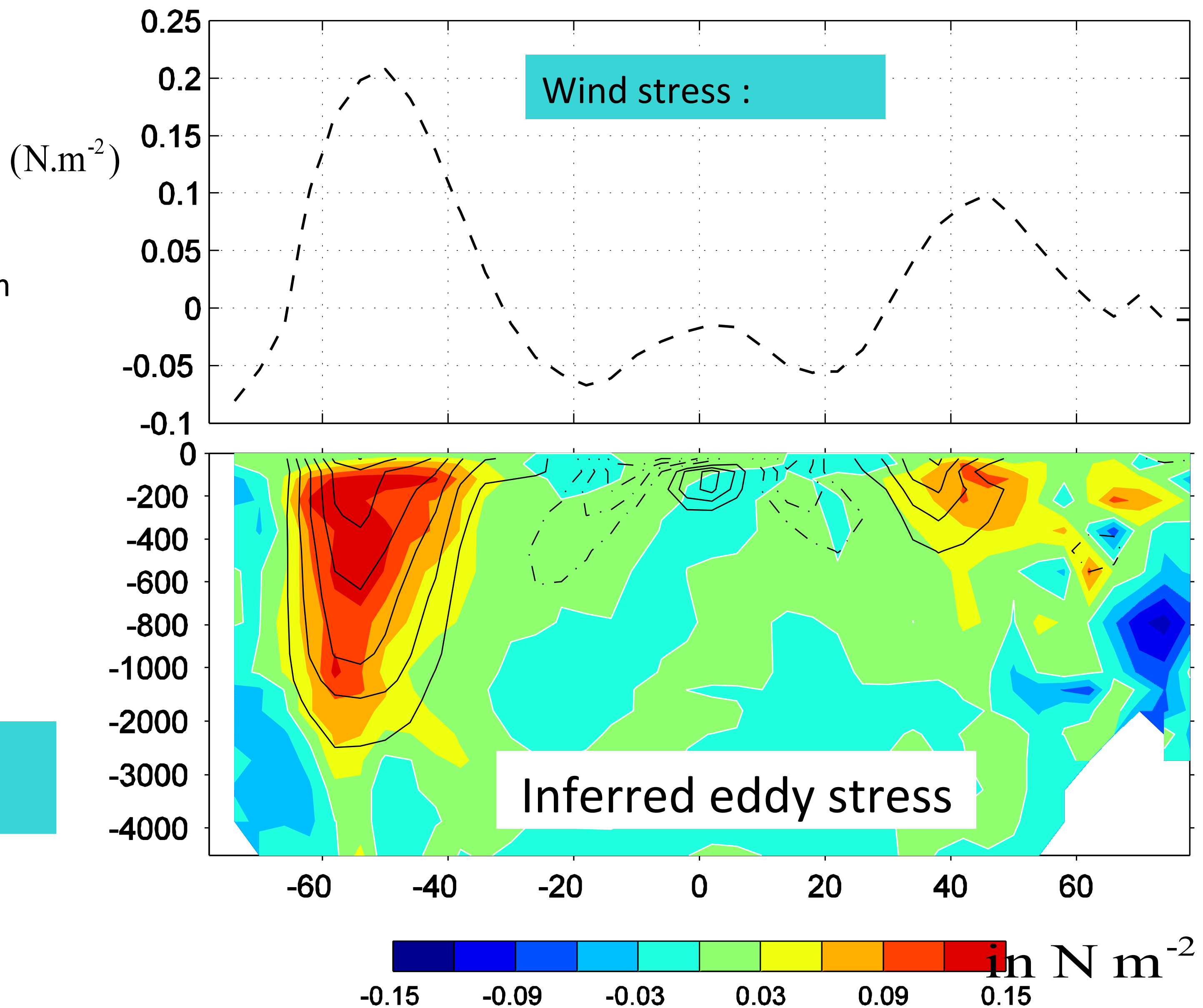
Optimized : iteration 60



MIT GCM at 4x4 degree

Ferreira et al. 2005, JPO

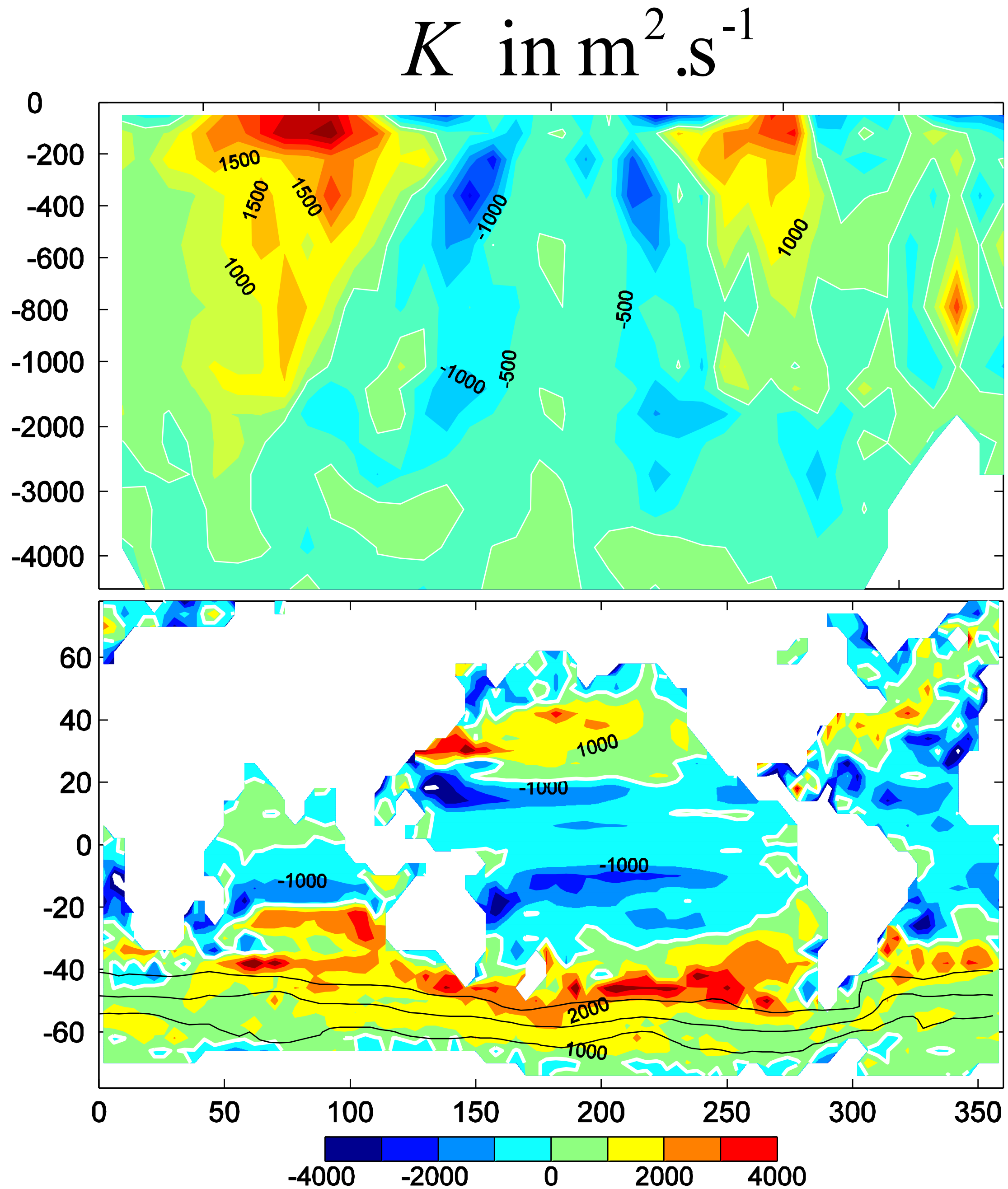
The input of momentum by the wind at the surface is carried downward by eddy stress



Eddy Stress and velocity :

Inferred eddy diffusivity

$$(\tau_x^e, \tau_y^e) = \rho_o f K \left(S_y, -S_x \right)$$



The eddy diffusivity is:

- surface intensified,
- large on the equator flank of the ACC.

Eddy closure

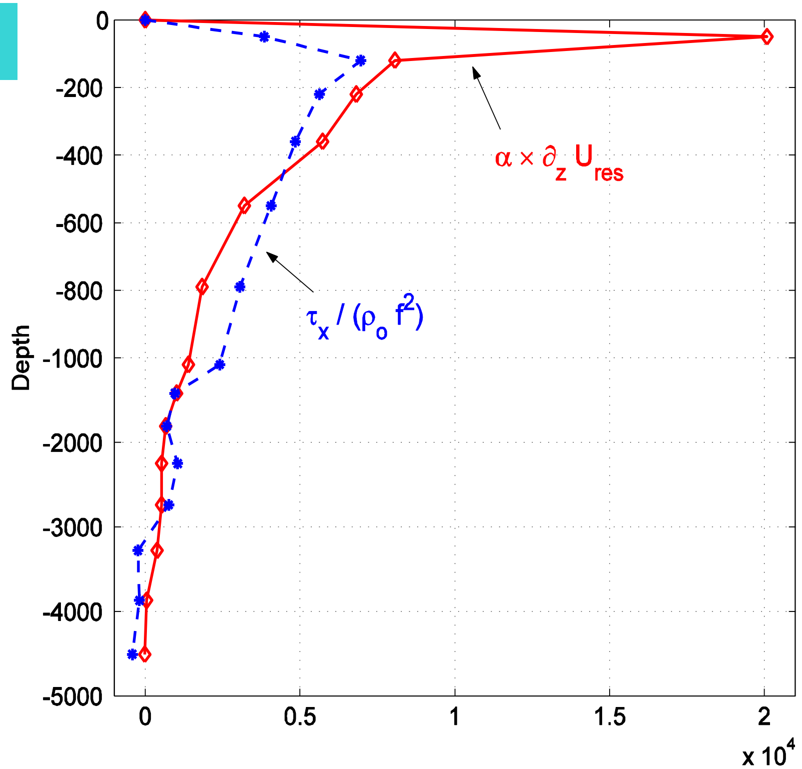
Eddy Stress as a vertical mixing of momentum

$$\tau^e \propto \frac{\partial u}{\partial z}$$

Down-gradient eddy buoyancy flux
+ thermal wind relationship:

$$\tau_x^{\ e} = \rho_o f \frac{\overline{v'b'}}{\overline{b}_z} = \rho_o f^2 K \frac{\overline{b}_y}{\overline{b}_z} = \rho_o f^2 \frac{K}{N^2} u_z$$

(Rhines and Young, Greatbach, GM90)



$$\Rightarrow \quad \frac{1}{\rho_o}\frac{\partial \tau^e}{\partial z} = \frac{\partial}{\partial z}\left(\upsilon_e \frac{\partial \mathbf{v}_{res}}{\partial z}\right) \quad \text{with} \quad \upsilon_e = \alpha f^2 \qquad (\upsilon_e \approx 1-3\,\mathrm{m}^2.\mathrm{s})$$

Implicit N^2 -dependent diffusivity $\quad K \propto N^2$

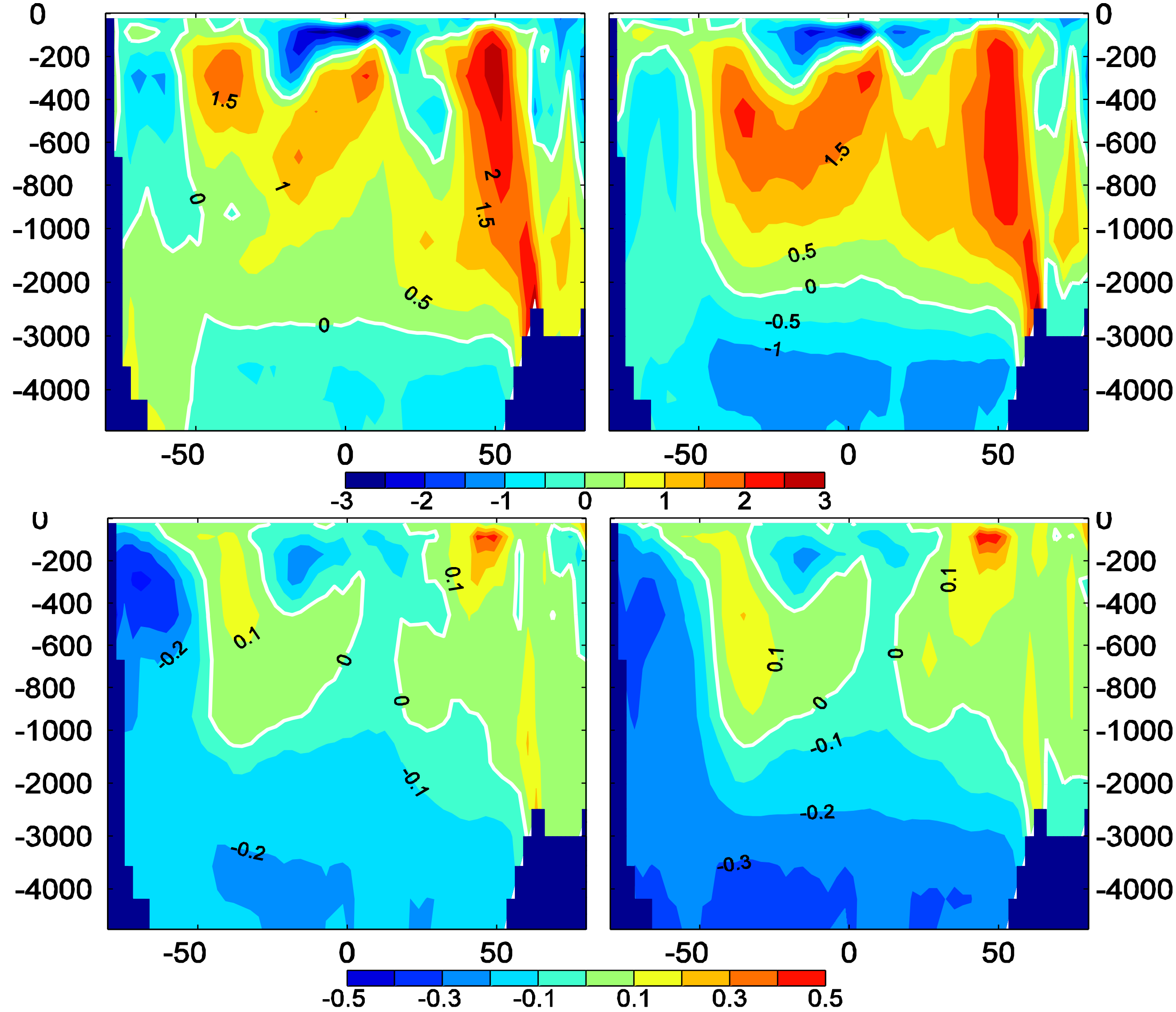
Test with the MIT GCM at 2.8 degree resolution

Residual model

Eulerian model (with GM and K=1000 m^2/s)

Zonal mean temperature error (C)

Zonal mean salinity error (psu)



Summary

- Residual mean ocean in which the resolved circulation is the sum of the Eulerian and eddy-induced circulations: Eddy terms appear as an eddy stress in the residual momentum equation and a residual flux in the buoyancy equation.
- We can estimate subgrid scale processes using adjoint technics,
- Eddy diffusivity is surface intensified,
- A simple eddy closure: the vertical divergence of the eddy stress is parameterized as a vertical mixing of momentum.
- N^2–dependent eddy diffusivity in the Gent and McWilliams scheme

$$K = K_{ref} \frac{N^2}{N^2_{ref}}$$