Mean zonal flows induced by Boussinesq thermal convection in rotating spherical shells as an application to the gas giant planets

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Outline

Background

Formulation

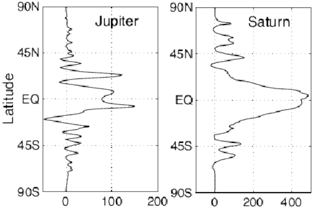
Our calculations

Summary

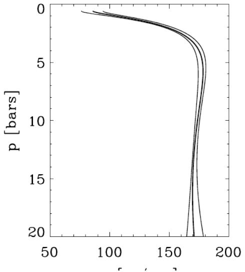
Outline

Surface flows of gas giant planets

- Surface flows of Jupiter and Saturn are characterized by the broad prograde jets around the equator and the narrow alternating jets in mid- and high-latitudes.
- It is not yet clear whether those surface jets are produced by convective motions in the "deep" region, or are the result of fluid motions in the "shallow" weather layer.



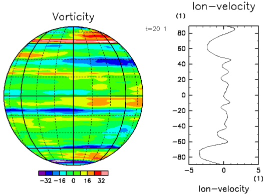
(Sukoriansky et al., 2002)

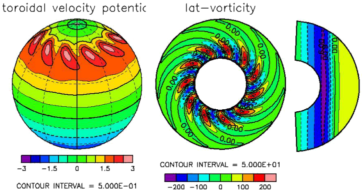


(Vasavada and Showman, 2005)

“Deep” models and “Shallow” models

- “Shallow” models
  - 2D turbulence on a rotating sphere
  - Primitive model
    - Result: Narrow alternating jets in mid- and high-latitudes.
    - Problem: the equatorial jets are not necessarily prograde
  - Recent improvements
    - Strong Newtonian cooling (shallow water)
    - MHD drag (primitive model)
- “Deep” models
  - Convection in rotating spherical shells
    - Result: Produce equatorial prograde flows easily
    - Problem: difficult to generate alternating jets in mid- and high-latitudes
  - Recent improvements : thin spherical shell





Outline

Model setup

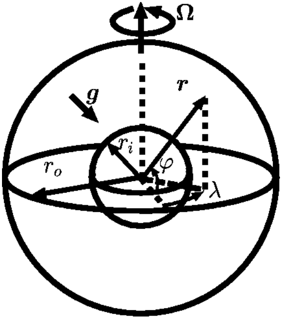
- Boussinesq fluid in a rotating spherical shell.
  - scaling: the shell thickness, viscous diffusion time, temperature difference.

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\mathrm{E}\left\{\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nabla^2\boldsymbol{u}\right\} + 2\boldsymbol{k}\times\boldsymbol{u} + \nabla p = \frac{\mathrm{Ra}\mathrm{E}^2}{\mathrm{Pr}}\frac{\boldsymbol{r}}{r_o}T,$$

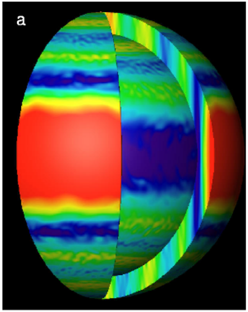
$$\frac{\partial T}{\partial t} + (\boldsymbol{u}\cdot\nabla)T = \frac{1}{\mathrm{Pr}}\nabla^2 T.$$

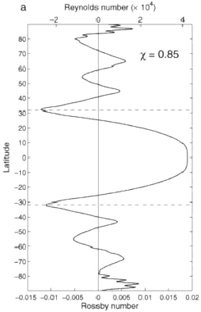
- Parameters:
  - Plandtl number: $\mathrm{Pr} = \dfrac{\nu}{\kappa}$
  - Rayleigh number: $\mathrm{Ra} = \dfrac{\alpha g_o \Delta T D^3}{\kappa\nu}$
  - Ekman number: $\mathrm{E} = \dfrac{\nu}{\Omega D^2}$
  - radius ratio: $\eta = \dfrac{r_i}{r_o}$



“Thin” spherical shell model

- Heimpel and Aurnou(2007) (hereafter, HA2007)
  - “Thin” spherical shell model with large Rayleigh number, small Ekman number.
  - Prograde jets and alternating jets in mid- and high-latitudes can produce simultaneously
  - However, eight-fold symmetry in the longitudinal direction is assumed.





Outline

Purpose

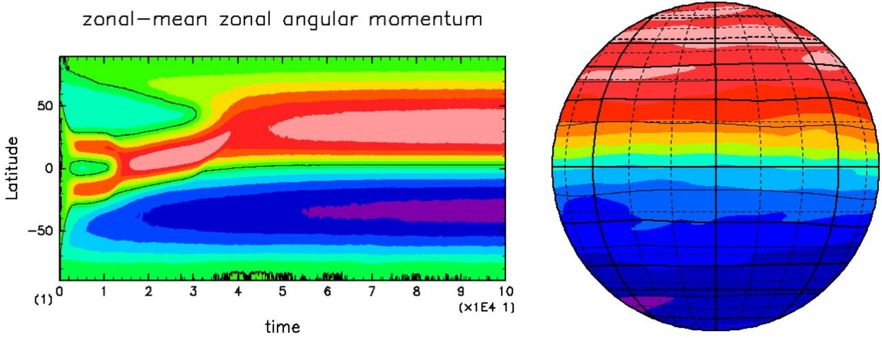
- HA2007 explains that the banded structure in high latitudes is caused by forced 2D turbulence on the β plane (Rhiens effect)
- Forced 2D barotropic turbulence on a rotating sphere :
  banded structure disappears after long time integration
  (Obuse et al. 2010).
- Question : does the banded structure of HA2007 disappear after long time integration or not?

In the present study:

- Let us perform long time integration of thermal convection in a rotating thin spherical shell.
- Calculate in the whole domain.

Forced 2D barotropic turbulence on a rotating sphere

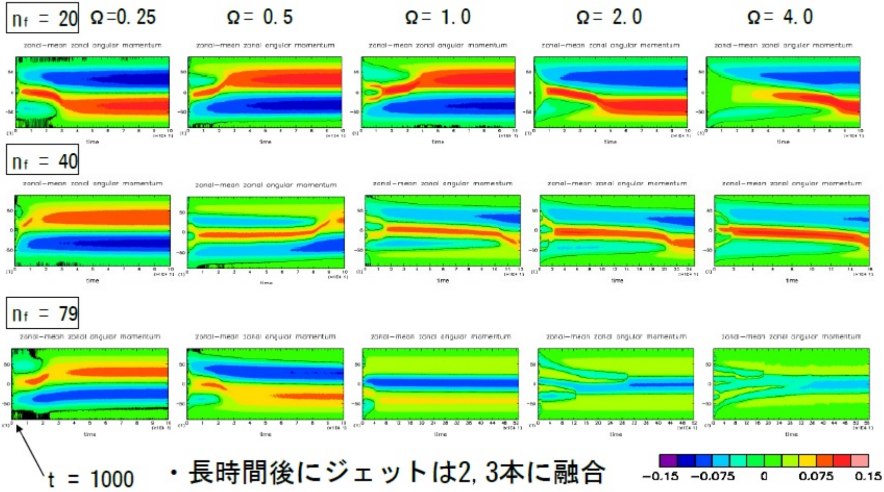
- Obuse et al. (2010)
  - Merger and disappearance of jets ⇒ banded structure disappears



Time development of latitudinal distribution of angular momentum (left) and final zonal wind profile (right) (Obuse et al. 2010)

Forced 2D barotropic turbulence on a rotating sphere

- Obuse et al.(2010)



Model setup

- Boussinesq fluid in a rotating spherical shell.
  - Scaling: the shell thickness, rotation period, temperature difference.

$$\nabla\cdot\boldsymbol{u} = 0,$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \mathrm{E}\nabla^2\boldsymbol{u} + 2\boldsymbol{k}\times\boldsymbol{u} + \nabla p = \frac{\mathrm{Ra}\mathrm{E}^2}{\mathrm{Pr}}\frac{\boldsymbol{r}}{r_o}T,$$

$$\frac{\partial T}{\partial t} + (\boldsymbol{u}\cdot\nabla)T = \frac{\mathrm{E}}{\mathrm{Pr}}\nabla^2 T.$$

- Parameters:
  - Prandtl number: $\mathrm{Pr} = \frac{\nu}{\kappa}$
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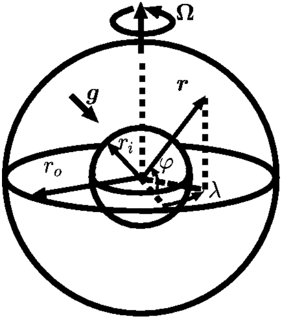


Experimental setup

- Boundary condition: Isothermal, Impermeable and Stress free.
- Input parameters:

Prandtl number: Pr	0.1
Radius ratio: η	0.85
Ekman number: E	3×10^{-6}
Modified Rayleigh number: Ra^*	0.05

  - the definition of modified Rayleigh number: $\mathrm{Ra}^* = \frac{\mathrm{RaE}^2}{\mathrm{Pr}} = \frac{\alpha g \Delta T}{\Omega^2 D}$
    - Ratio of Coriolis term and buoyancy term

Numerical methods

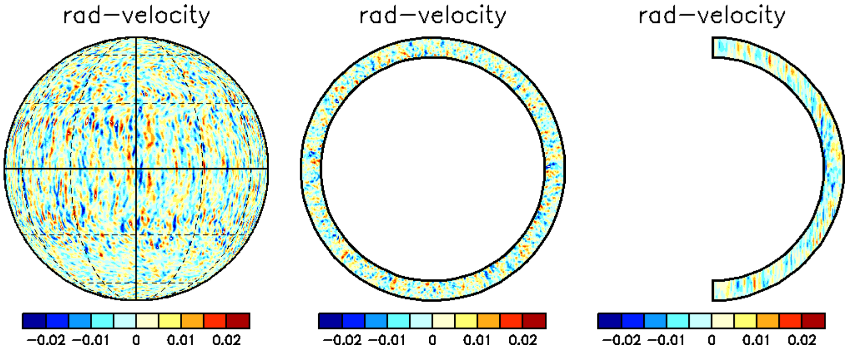
- Traditional spectral method.
  - Toroidal and Poloidal potentials of velocity are introduced.
  - The total wave number of spherical harmonics is truncated at 341, and the Chebychev polynomials are calculated up to the 48th degree.
    - The numbers of grid points:1024, 512, and 65 in the longitudinal, latitudinal, and radial directions, respectively.
- In order to save computational resources, we use hyperdiffusion with the same functional form as the previous studies

$$\nu = \begin{cases} \nu_0, & \text{for } \; l \leq l_0, \\ \nu[1 + \varepsilon(l - l_0)^2], & \text{for } \; l > l_0. \end{cases}$$

  - we choose $l_0 = 21, 42, 85, 170, \varepsilon = 10^{-2}$
    (l_0 is gradually increased)
- The time integration is performed using the Crank-Nicolson scheme for the diffusion terms and the second-order Adams-Bashforth scheme for the other terms.

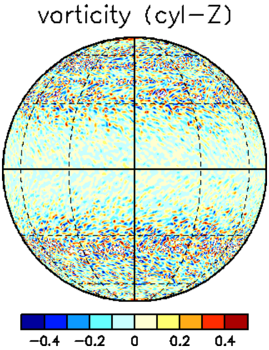
Convective activity

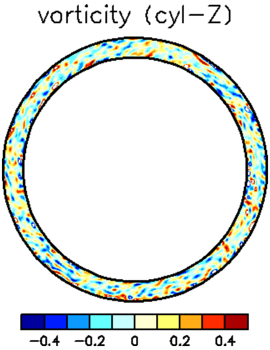
- Radial velocity at $t = 64030$ (about 10000 rotation).

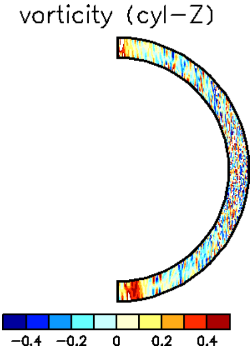


Vorticity

- Axial vorticity at $t = 64030$ (zonal mean component removed)

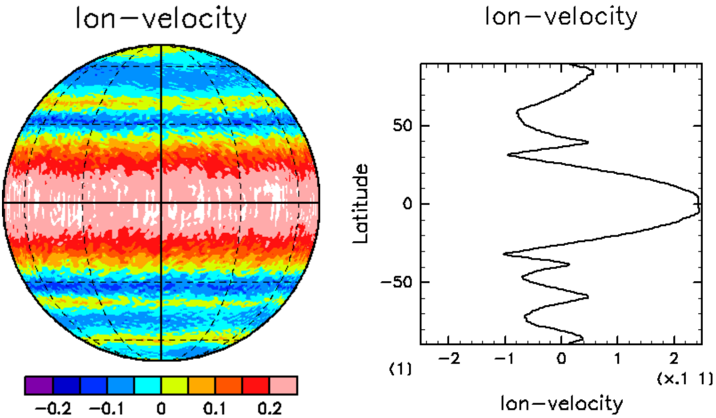






Mean zonal flows

- Mean zonal flow at $t = 47630$ (about 7500 rotation).
  - Banded structure appears in mid- and high-latitudes.

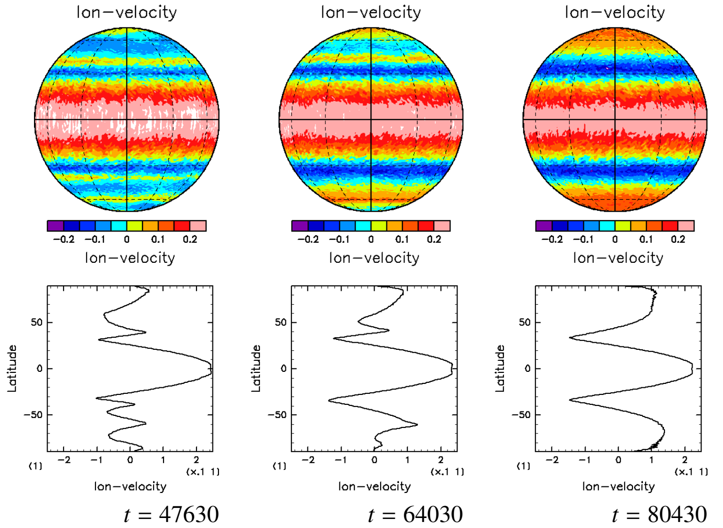


Meridional plane

Outer surface

Development of zonal flows

- Further time integration.
  - Number of jets decreases



Outline

Summary and discussion

- Disappearance of alternating jets in high latitudes
  - Prograde acceleration in high latitudes
  - Retrograde acceleration around the tangential cylinder
- What is acceleration mechanism? One possible story is...
  - Thermal convection occurs in high latitudes
  - It excites 2-dim. axial vortices
  - The vortices propagate as Rossby waves outward and are absorbed near the tangent cylinder
  - Associated with the propagation of Rossby waves, negative angular momentum is transferred outward
- Deep models may by difficult to explain the banded structure of Jupiter and Saturn?

Acknowledgement

The numerical computation of thermal convection in a rotating thin spherical shell was carried out on the Earth Simulator (ES2) at the Japan Agency for Marine Earth Science and Technology.

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