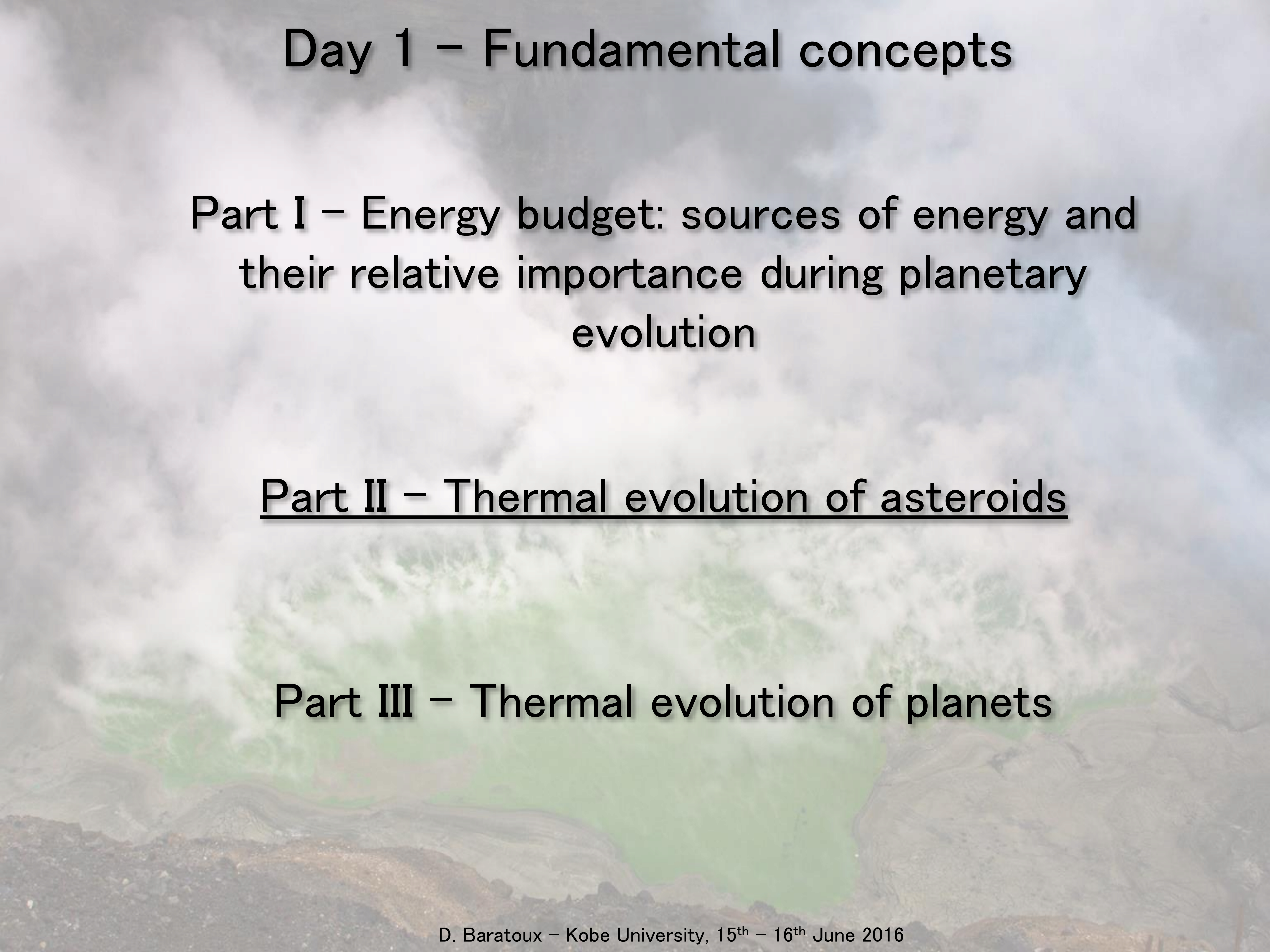
Day 1 – Fundamental concepts



Part I – Energy budget: sources of energy and their relative importance during planetary evolution

Part II – Thermal evolution of asteroids

Part III – Thermal evolution of planets

Part II – Thermal evolution of asteroids

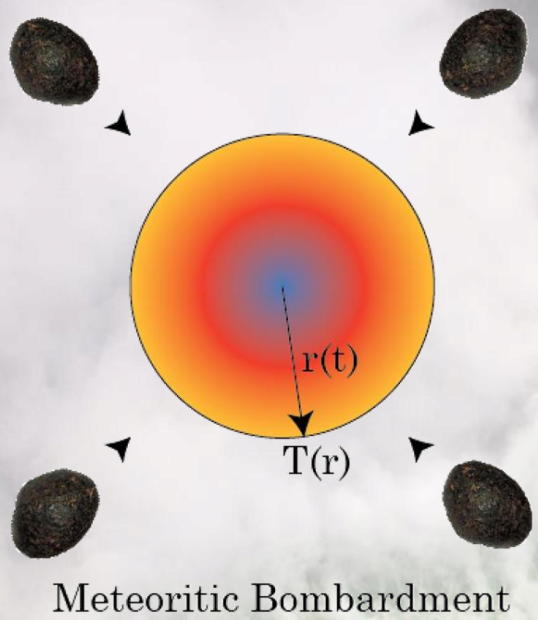
2.1 Thermal evolution from accretion energy

2.2 Thermal evolution from ^{26}Al

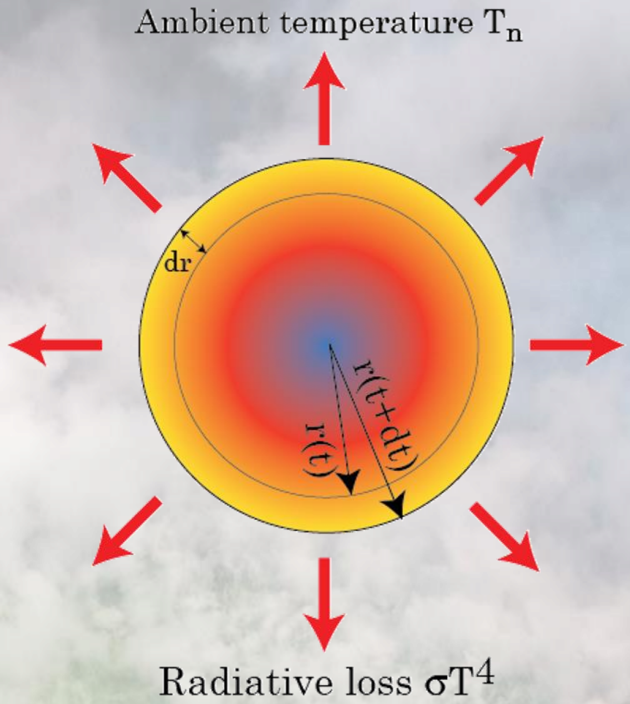
2.3 Application to the thermal evolution of the H-chondrites parent body

2.1 – Thermal evolution of asteroids

Accretion energy



Protoplanet at time t



Protoplanet at time t + dt

No heat transport

Source of energy: accretion

$$\frac{GM(R)}{R} * 4\pi R^2 dR\rho = 4\pi R^2 dR\rho C_p(T - T_n) + 4\pi R^2\sigma(T^4 - T_n^4)dt$$

2.1 – Thermal evolution of asteroids

Accretion energy

$$\rho \frac{GM(R)}{R}\frac{dR}{dt} = \rho C_p (T - T_n)\frac{dR}{dt} + \sigma (T^4 - T_n^4)$$

$$\rho C_p \frac{dR}{dt} T + \sigma T^4 = \sigma T_n^4 + \frac{4}{3}\pi R^2 G \rho^2 \frac{dR}{dt} + \rho C_p T_n \frac{dR}{dt}$$

Terms function of T

Terms function of R

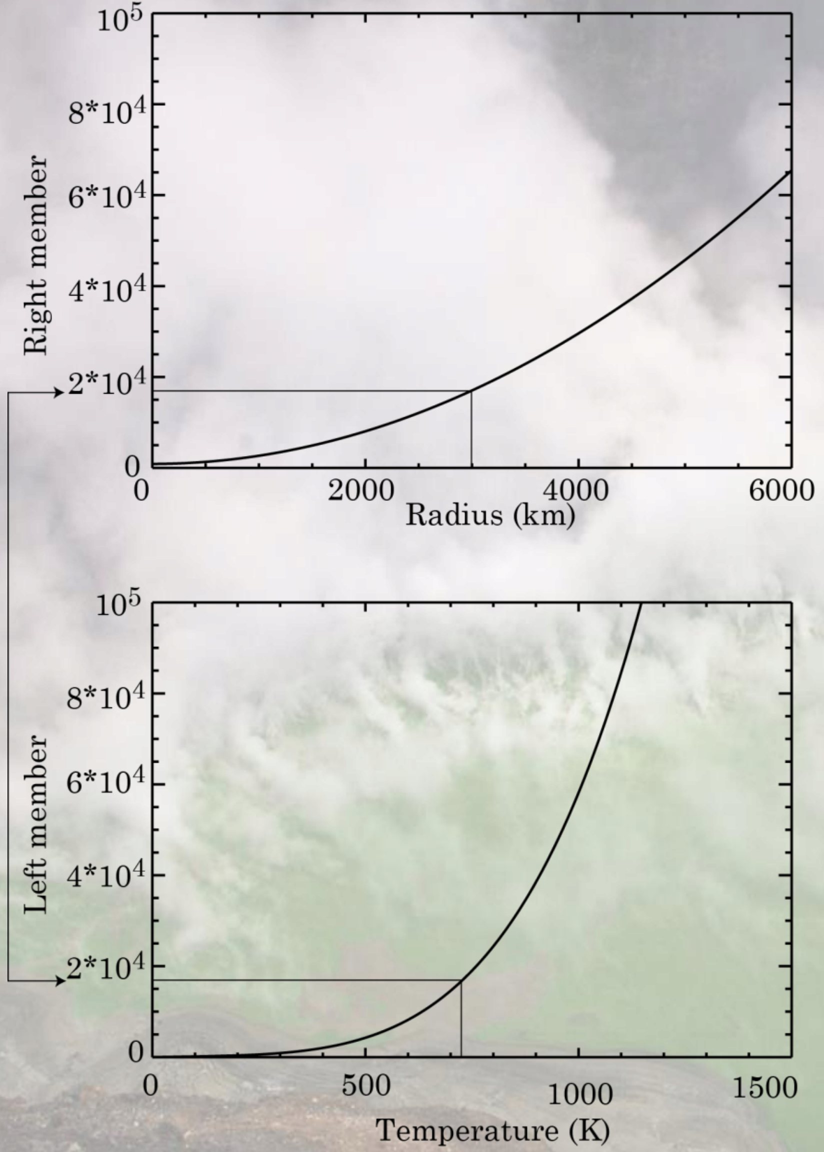
$$\rho C_p \frac{R_f}{t_{acc}} T + \sigma T_s^4 = \sigma T_n^4 + \frac{4}{3}\pi R_f^2 G \rho^2 \frac{R_f}{t_{acc}} + \frac{R_f}{t_{acc}} \rho C_p T_n$$

Equation for T as a function of final radius R

May be solved graphically

2.1 – Thermal evolution of asteroids

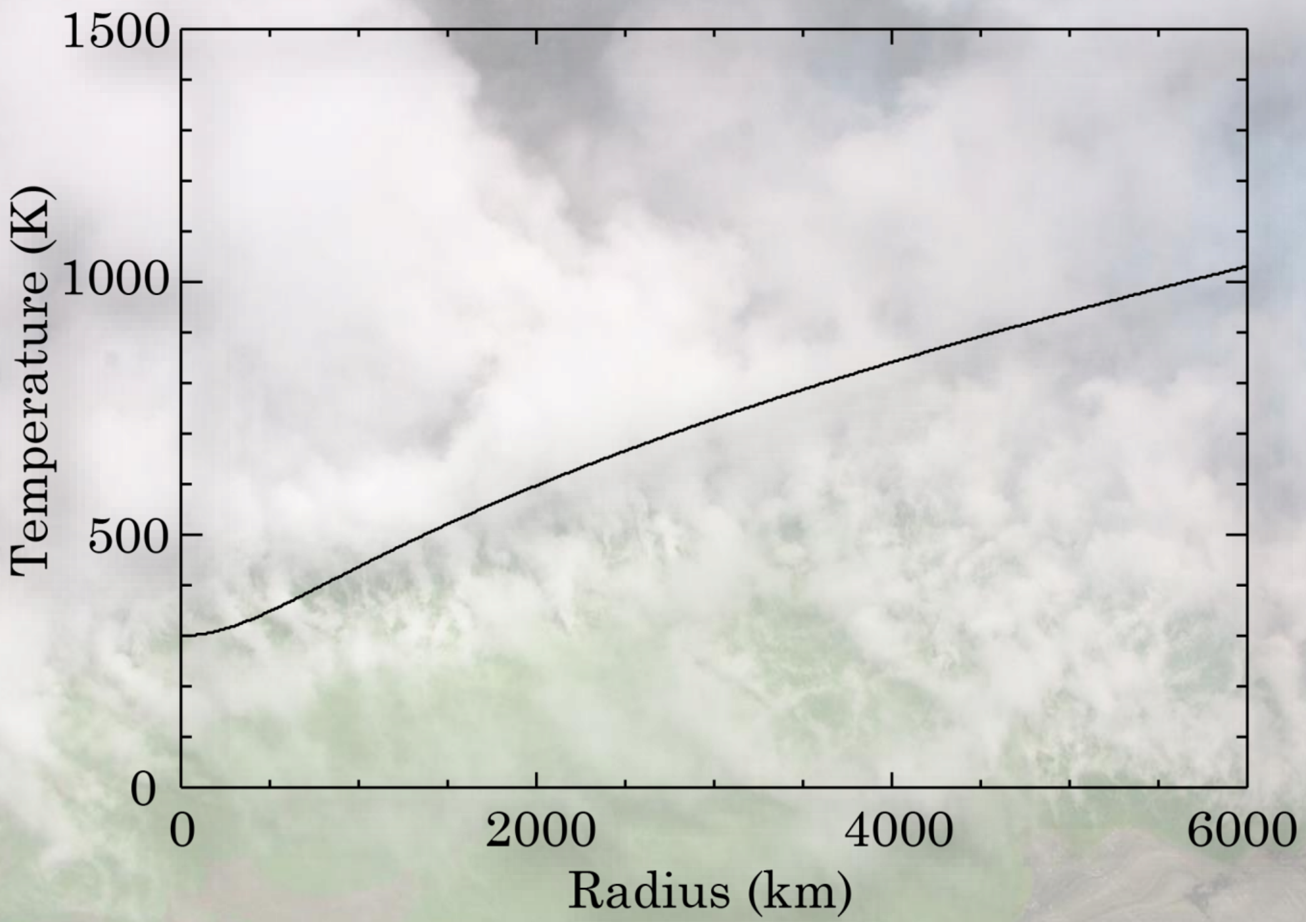
Accretion energy



Plots for $t_{acc} = 1$ Myr

2.1 – Thermal evolution of asteroids

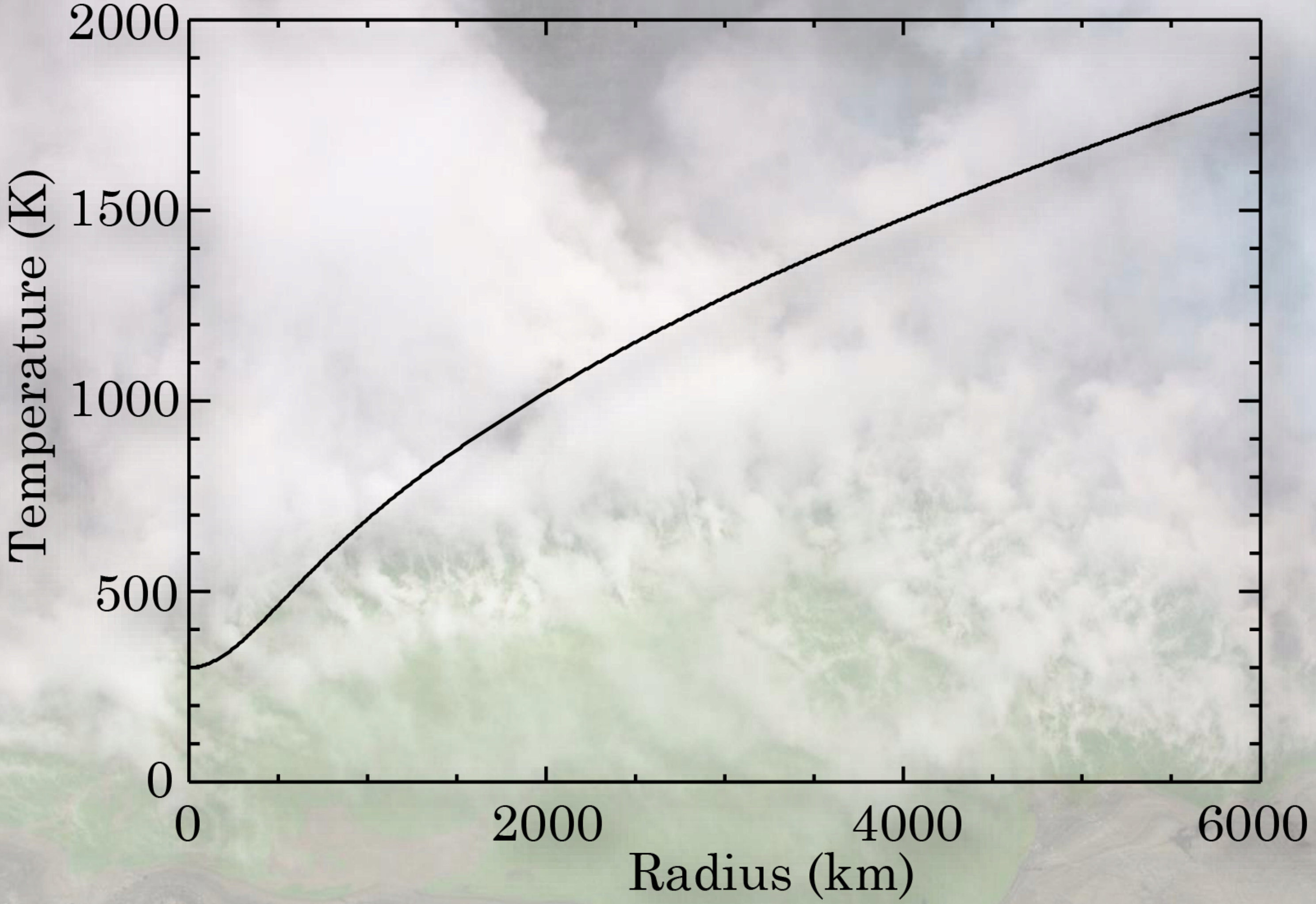
Accretion energy



Plots for $t_{acc} = 1$ Myr

2.1 – Thermal evolution of asteroids

Accretion energy



Plots for t_{acc} = 100 000 years

2.1 – Thermal evolution of asteroids

Accretion energy

$$\rho \frac{GM(R)}{R}\frac{dR}{dt} = \rho C_p (T - T_n)\frac{dR}{dt} + \sigma(T^4 - T_n^4)$$

$$\rho C_p \frac{dR}{dt} T + \sigma T^4 = \sigma T_n^4 + \frac{4}{3}\pi R^2 G\rho^2 \frac{dR}{dt} + \rho C_p T_n \frac{dR}{dt}$$

Terms function of T

Terms function of R

$$\rho C_p \frac{R_f}{t_{acc}} T + \sigma T_s^4 = \sigma T_n^4 + \frac{4}{3}\pi R_f^2 G\rho^2 \frac{R_f}{t_{acc}} + \frac{R_f}{t_{acc}}\rho C_p T_n$$

Equation for T as a function of final radius

Accretion time as a function of
Final (surface temperature)

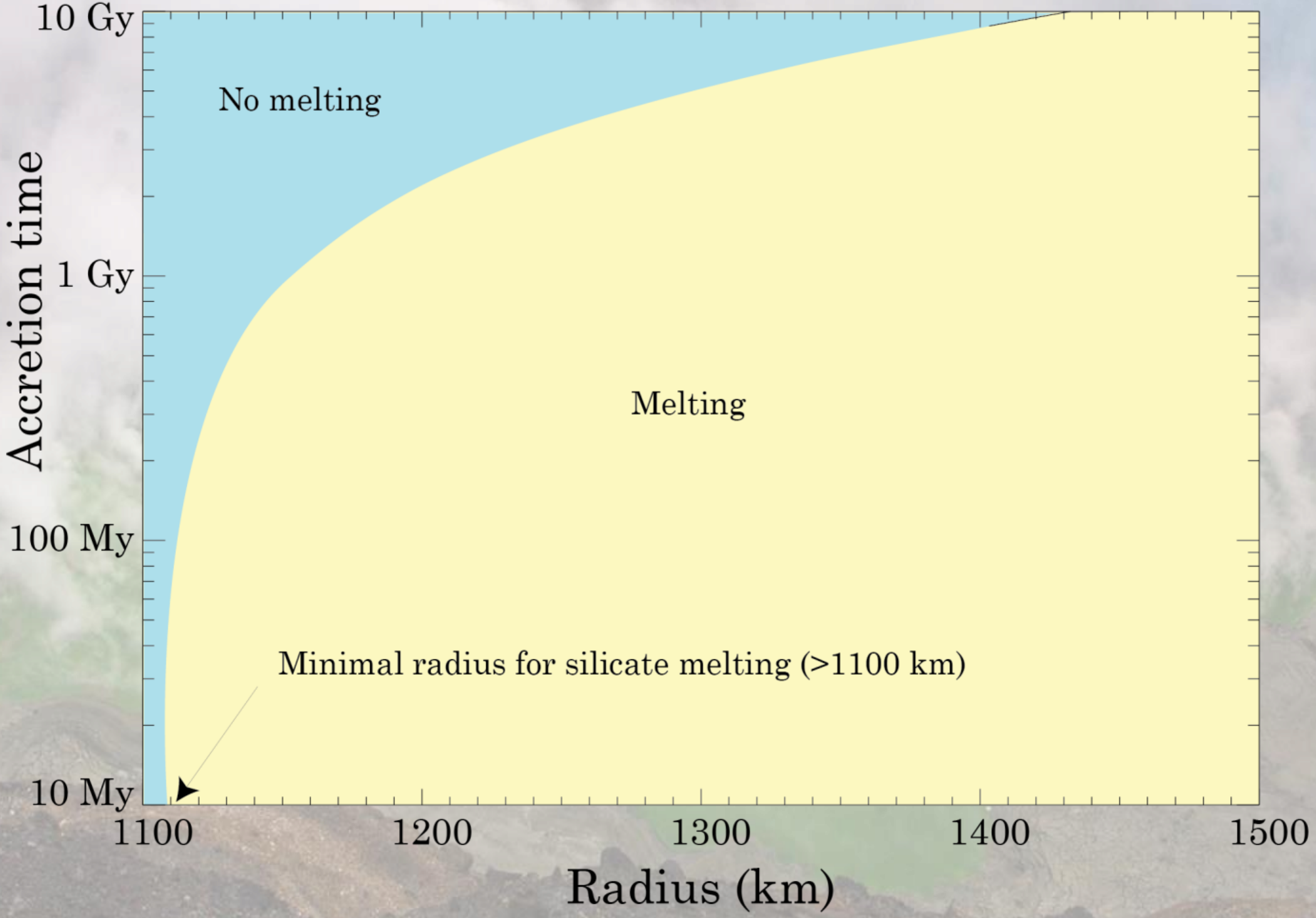
$$t_{acc} = \frac{\frac{4}{3}\pi R_f^3 G\rho^2 - \rho C_p (T_s - T_n) R_f}{\sigma(T_s^4 - T_n^4)}$$

2.1 – Thermal evolution of asteroids

Accretion energy

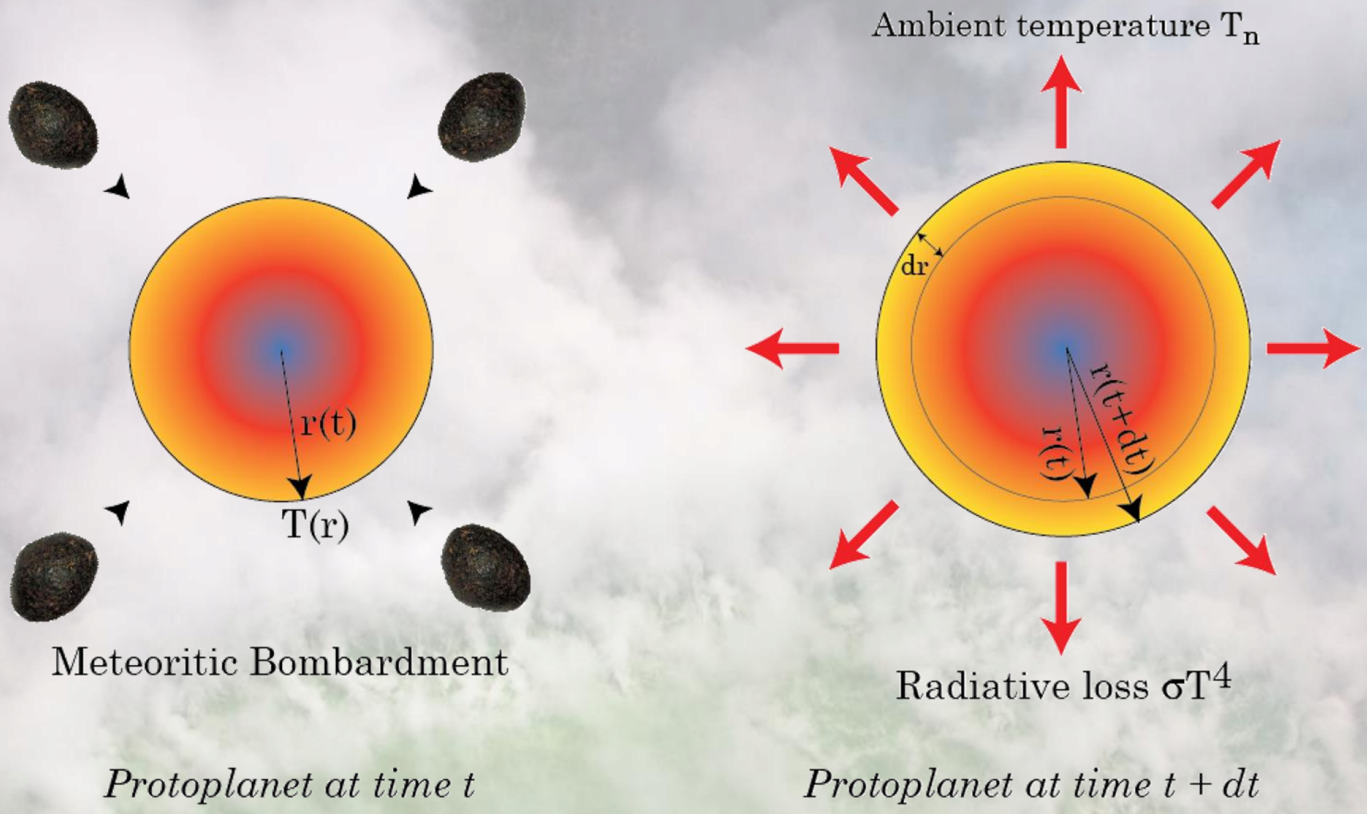
Melting is possible if $R > R_{fmin}$ (final critical radius)

$$R_{fmin} = \sqrt{\frac{\rho C_p (T_s - T_n)}{\frac{4}{3}\pi G \rho^2}}$$



2.1 – Thermal evolution of asteroids

Accretion energy + heat transport



Source of energy: accretion
Instantaneous heat transport

$$\frac{GM(R)}{R} * 4\pi R^2 dR\rho = d(\frac{4}{3}\pi R^3 \rho C_p(T - T_n)) + 4\pi R^2 \sigma(T^4 - T_n^4)dt$$

2.1 – Thermal evolution of asteroids

Accretion energy + heat transport

$$\frac{GM(R)}{R}\frac{dR}{dt}\rho = \rho C_p (T - T_n)\frac{dR}{dt} + \frac{1}{3}\rho R C_p \frac{dT}{dt} + \sigma(T^4 - T_n^4)$$

$$\downarrow$$

$$\rho C_p \tfrac{dR}{dt} T + \sigma T^4 + \tfrac{1}{3}\rho R C_p \tfrac{dT}{dt} = \sigma T_n^4 + \tfrac{4}{3}\pi R^2 G \rho^2 \tfrac{dR}{dt} + \rho C_p T_n \tfrac{dR}{dt}$$

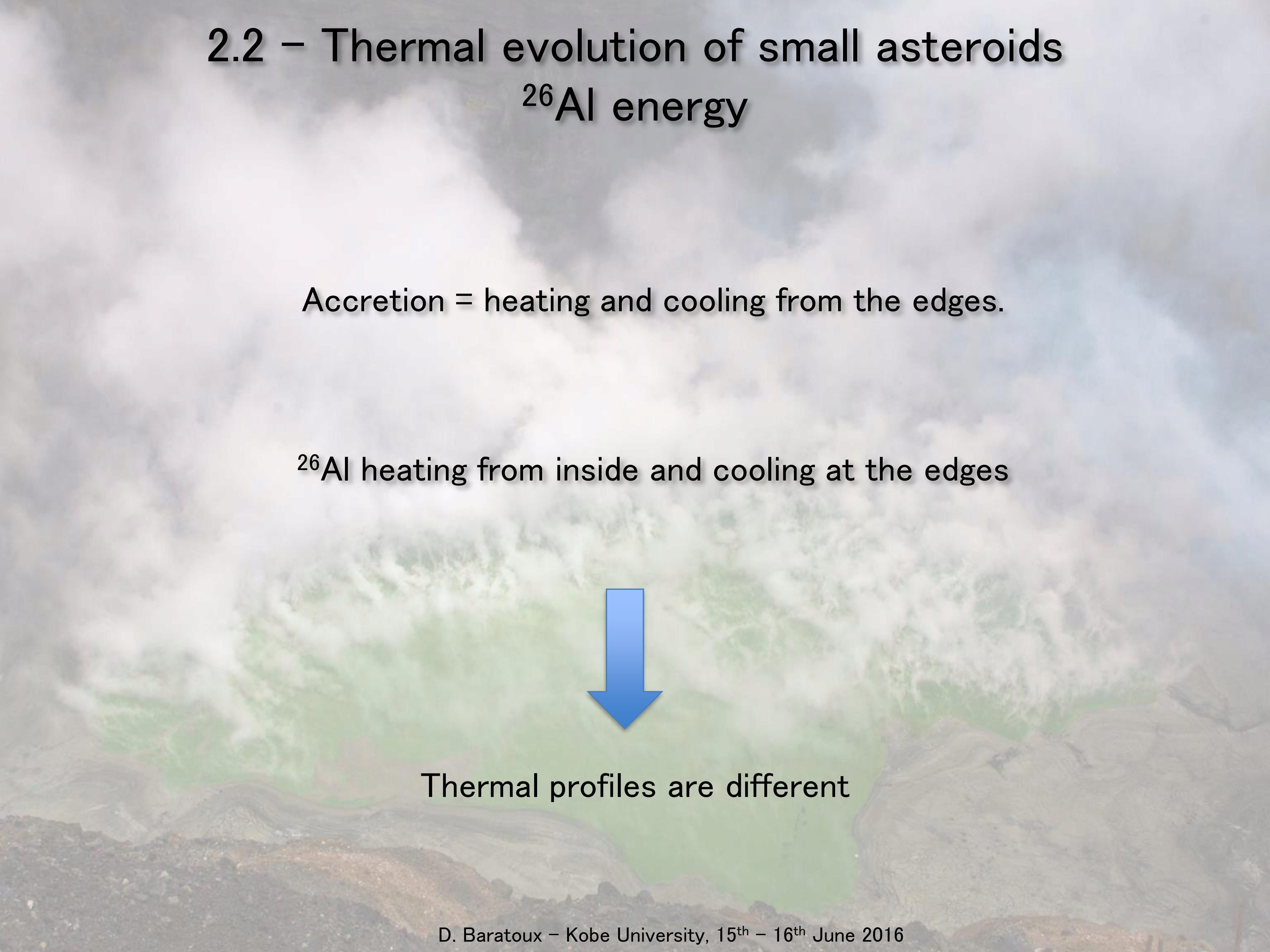
$$\downarrow$$

With a constant accretion rate:

$$\rho C_p \tfrac{R_f}{t_{acc}} T + \sigma T^4 + \tfrac{1}{3}\rho \tfrac{R_f}{t_{acc}} C_p \tfrac{dT}{dt} t = \sigma T_n^4 + \tfrac{4}{3}\pi G (\tfrac{R_f}{t_{acc}})^3 \rho^2 t^2 + \rho C_p T_n \tfrac{R_f}{t_{acc}}$$

↳ Numerical resolution

2.2 – Thermal evolution of small asteroids
^{26}Al energy



Accretion = heating and cooling from the edges.

^{26}Al heating from inside and cooling at the edges

↓

Thermal profiles are different

2.2 – Thermal evolution of small asteroids
^{26}Al energy

Energy conservation equation

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(K r^2 \frac{\partial T}{\partial r}) + A(r,t)$$

Source term (^{26}Al decay)
A_0 (W/m^3)

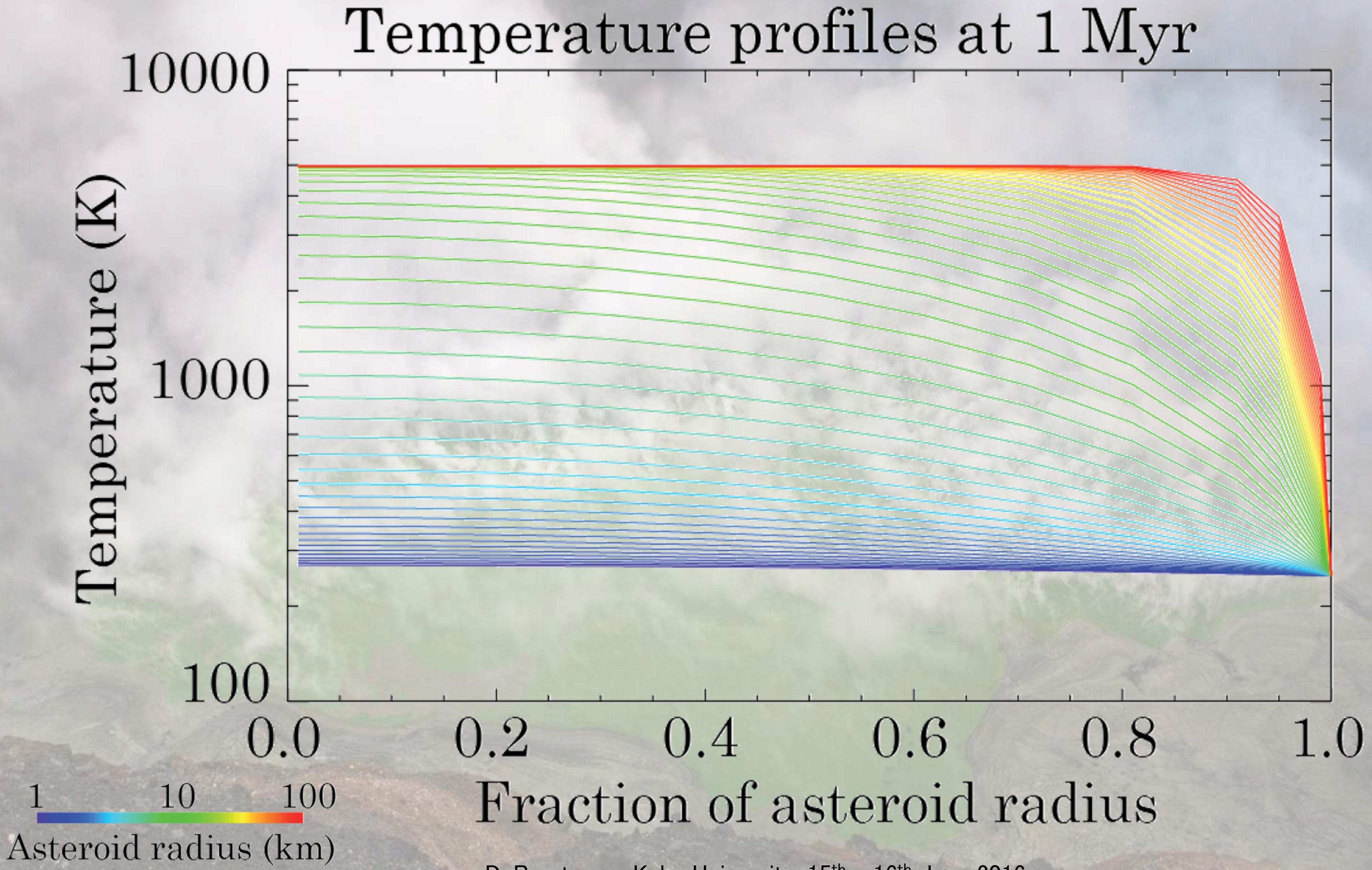
$$A(r,t) = A_0 e^{-\lambda t}$$

Analytical solution

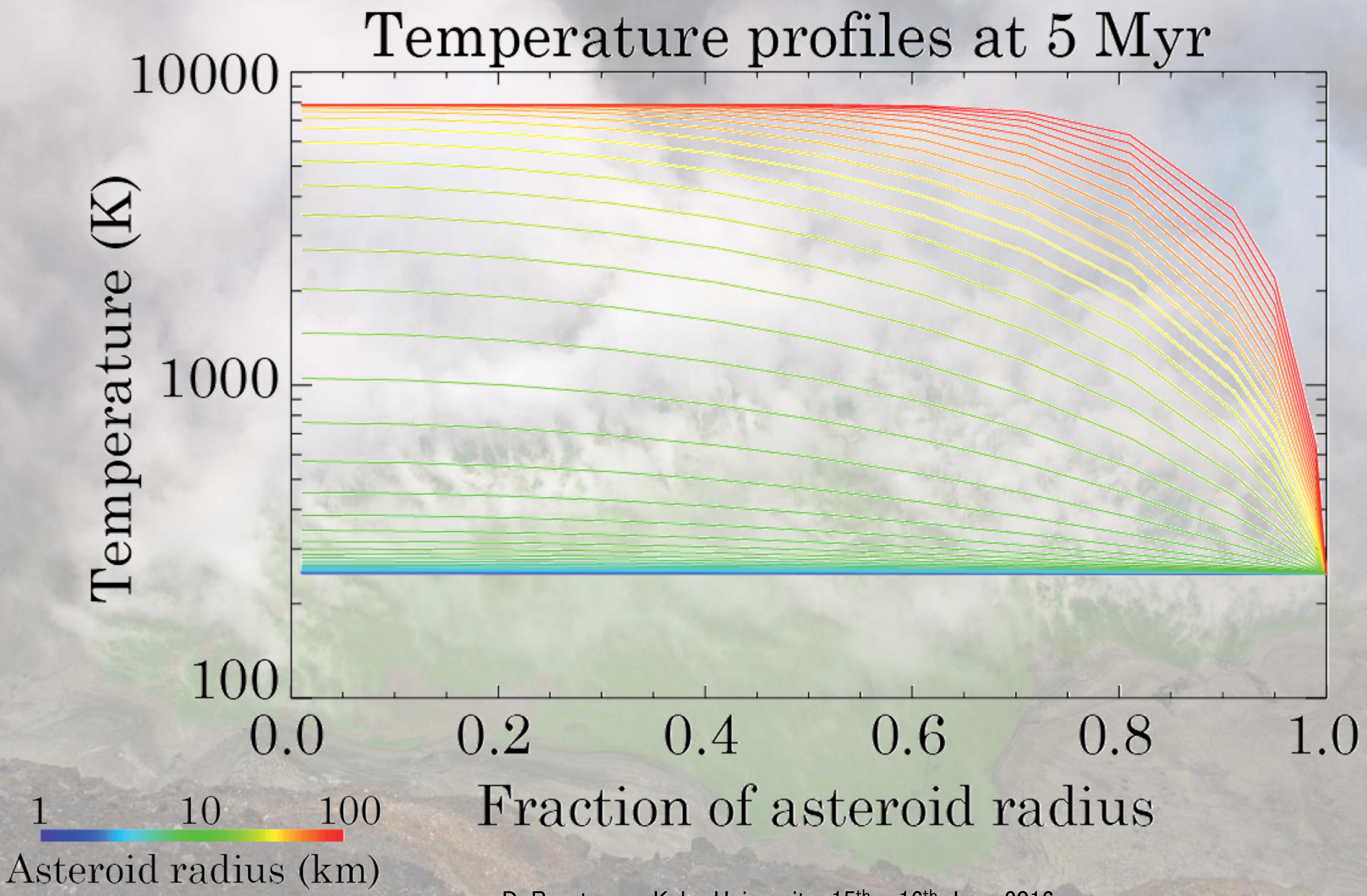
$$T(r,t) = \frac{\kappa A_0}{K\lambda} e^{-\lambda t}[\frac{R \sin(r\sqrt{\lambda/\kappa})}{r sin(R\sqrt{\lambda/\kappa})} - 1] + \frac{2R^3 A_0}{r\pi^3 K} \sum_{n=1}^{\infty} \frac{(-1)^n}{n * (n^2 - \frac{\lambda R^2}{\kappa \pi^2})} \sin \frac{n\pi r}{R} e^{-\frac{\kappa n^2 \pi^2 t}{R^2}}$$

2.2 – Thermal evolution of small asteroids

^{26}Al energy

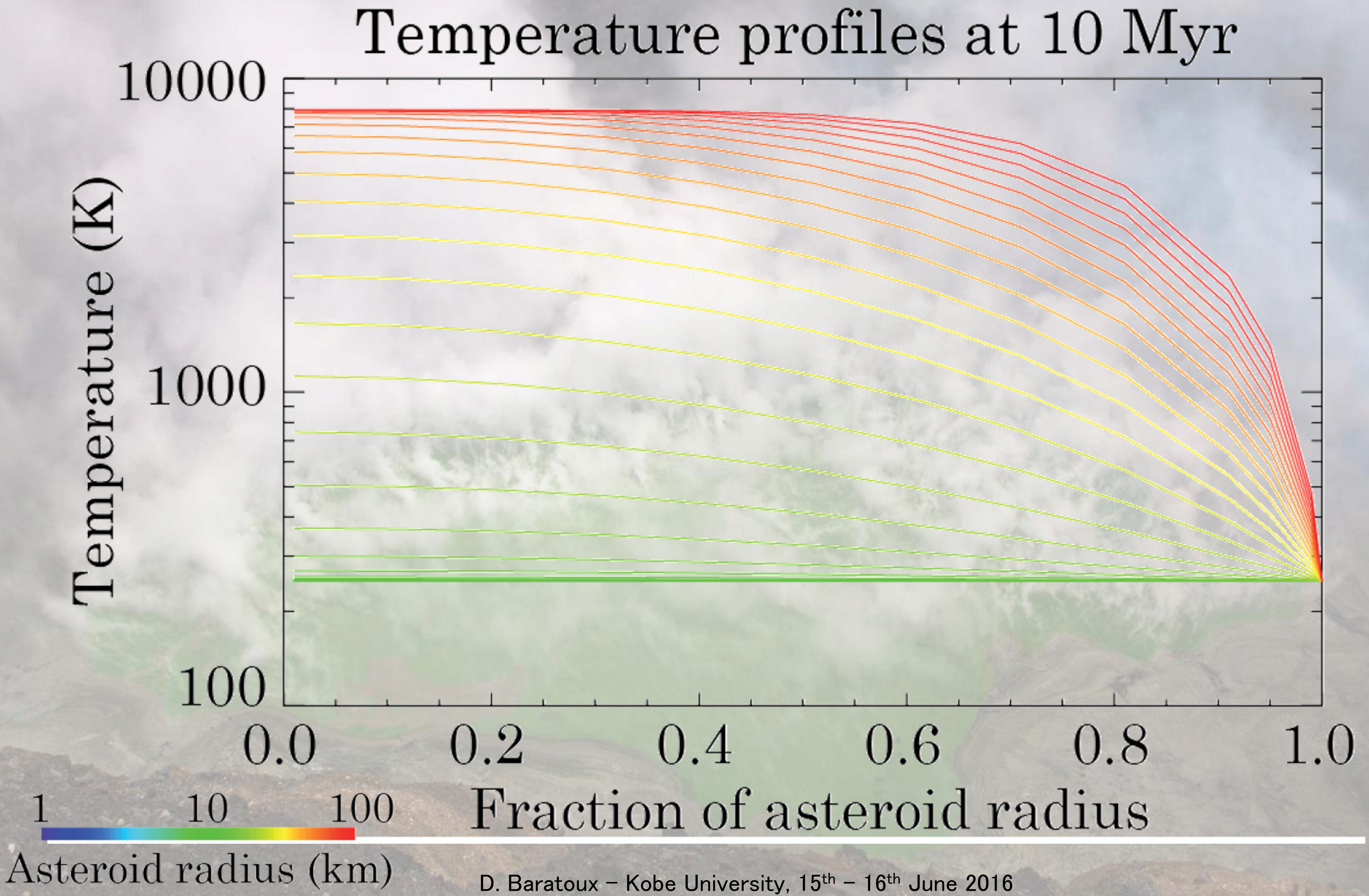


2.2 – Thermal evolution of small asteroids
^{26}Al energy



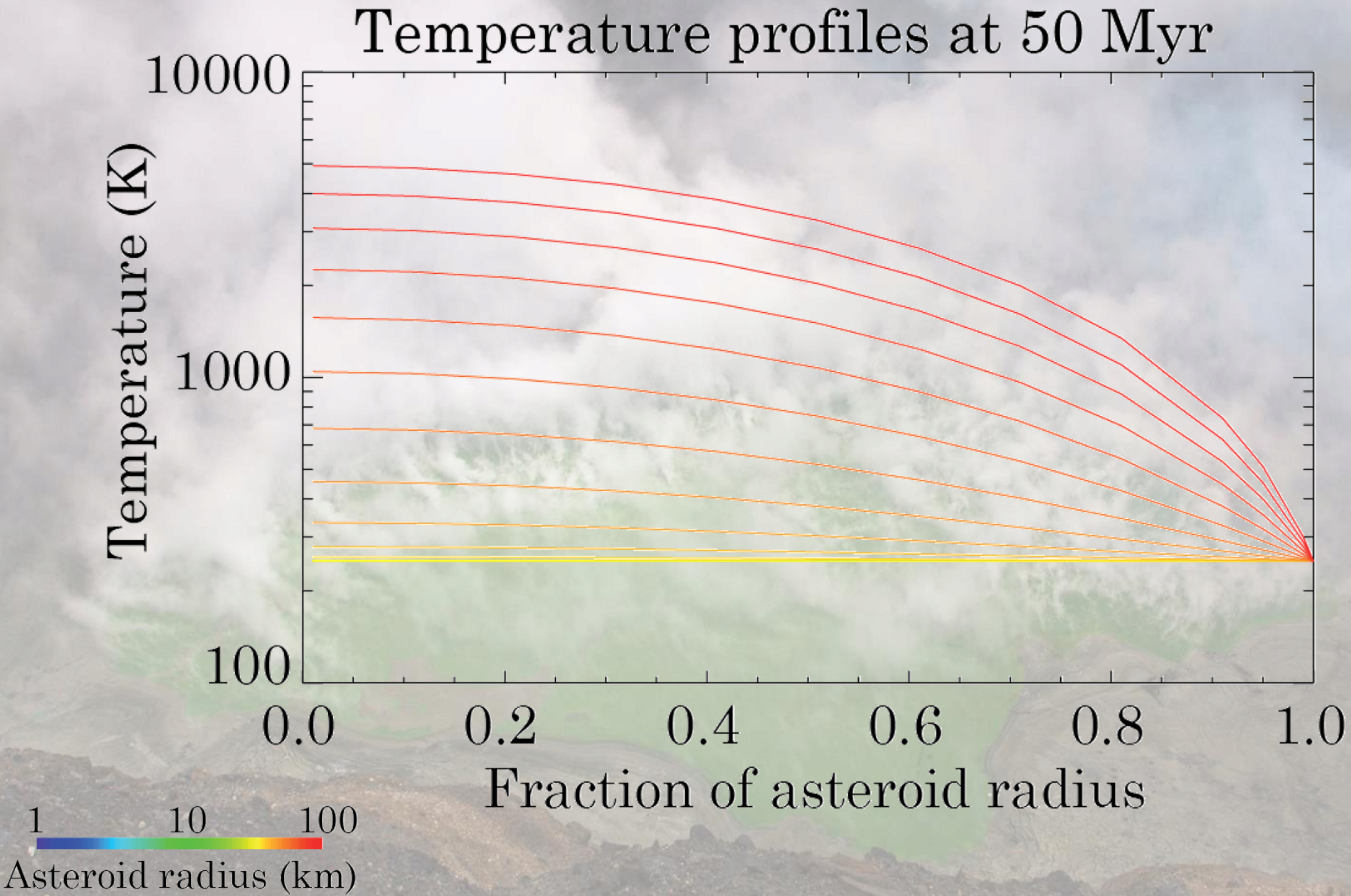
2.2 – Thermal evolution of small asteroids

^{26}Al energy



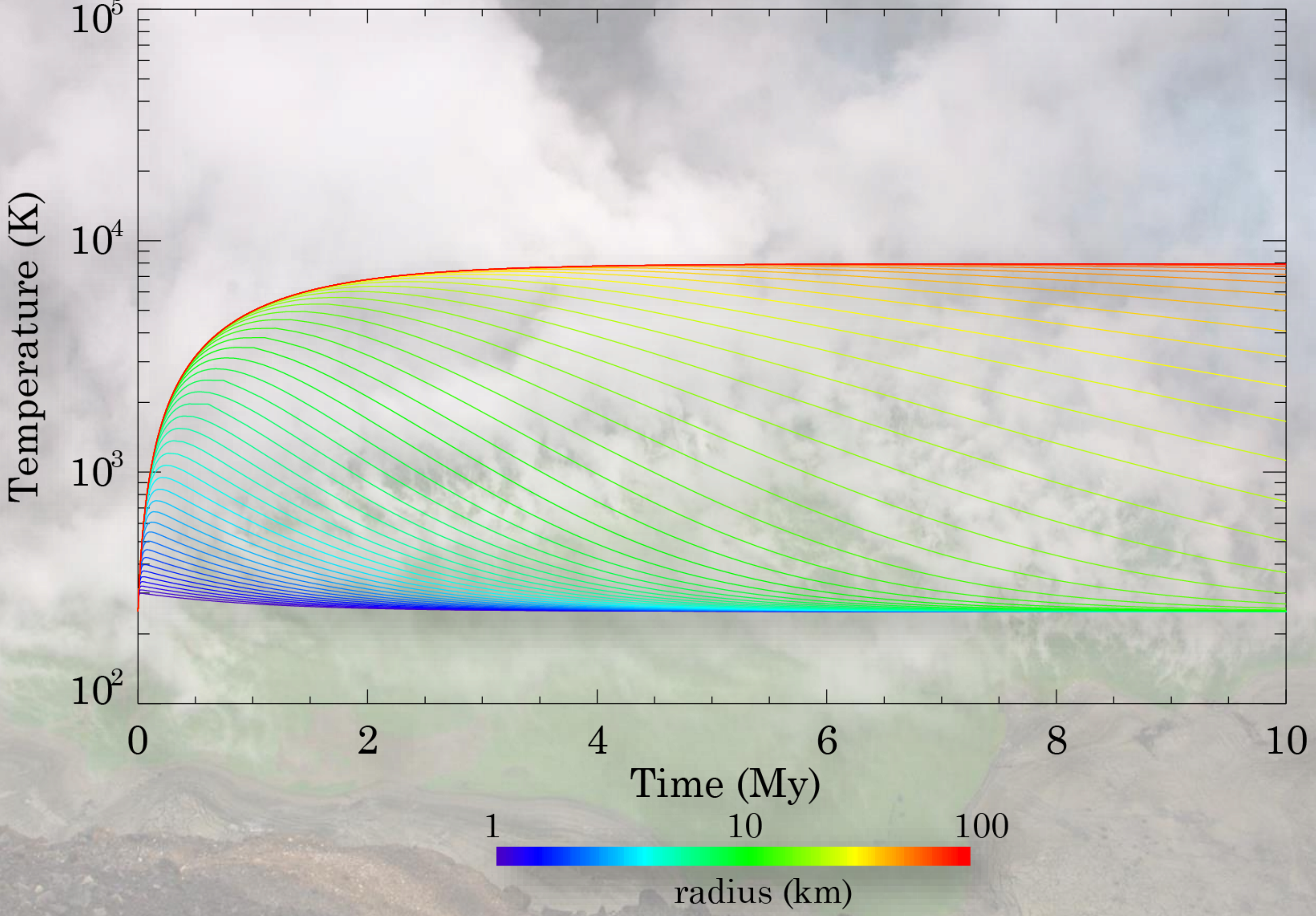
2.2 – Thermal evolution of small asteroids

^{26}Al energy



2.2 – Thermal evolution of small asteroids

^{26}Al energy



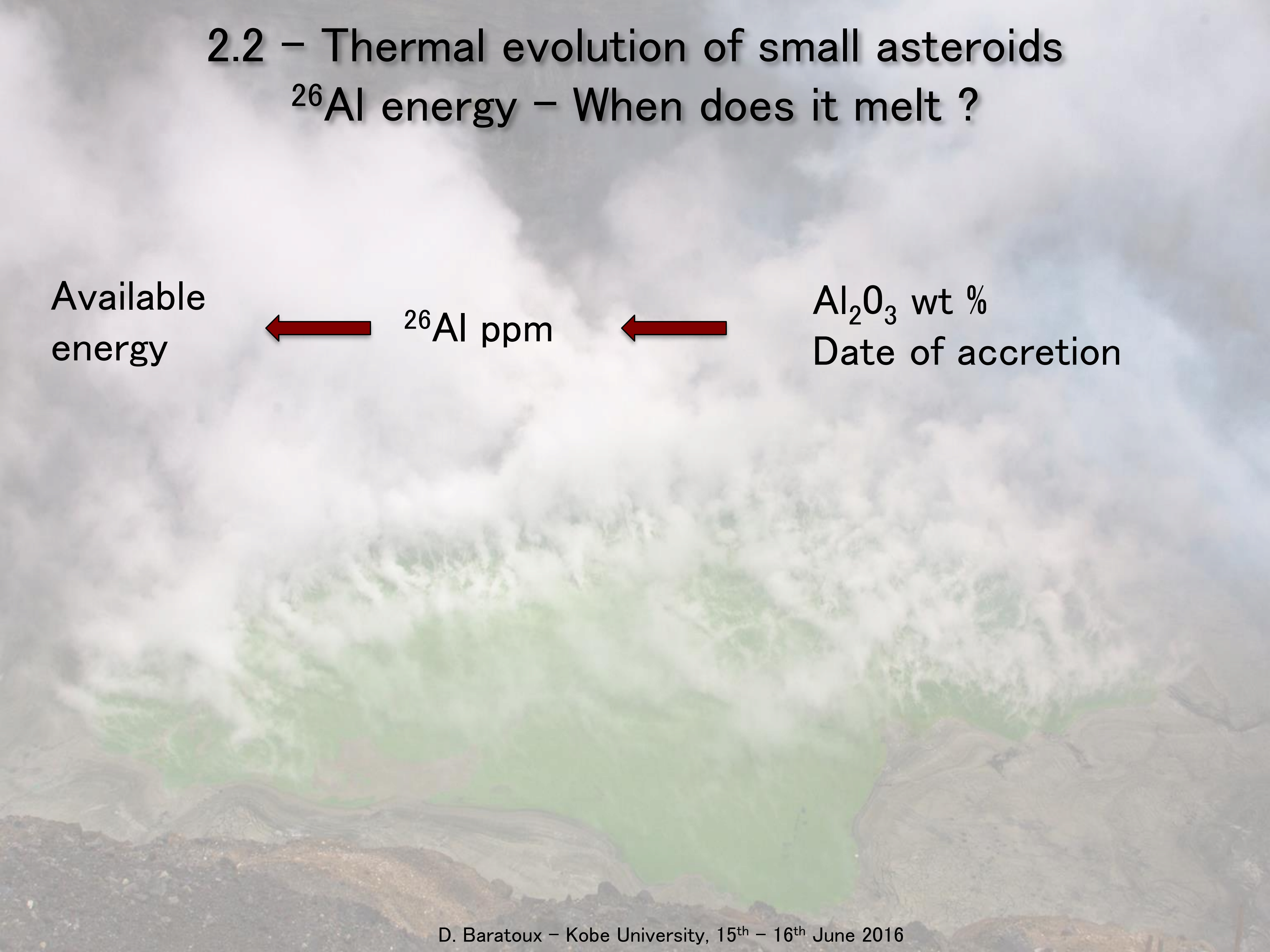
2.2 – Thermal evolution of small asteroids
^{26}Al energy

Two important factors
(instantaneous accretion)

Date of accretion
Canonical value t = 0 – $^{26}\mathrm{Al}/^{27}\mathrm{Al} = 5*10^{-5}$

→ **Controls the amount of available energy**

Size
→ **Controls the rate of energy loss**
(boundary condition T = Tn)

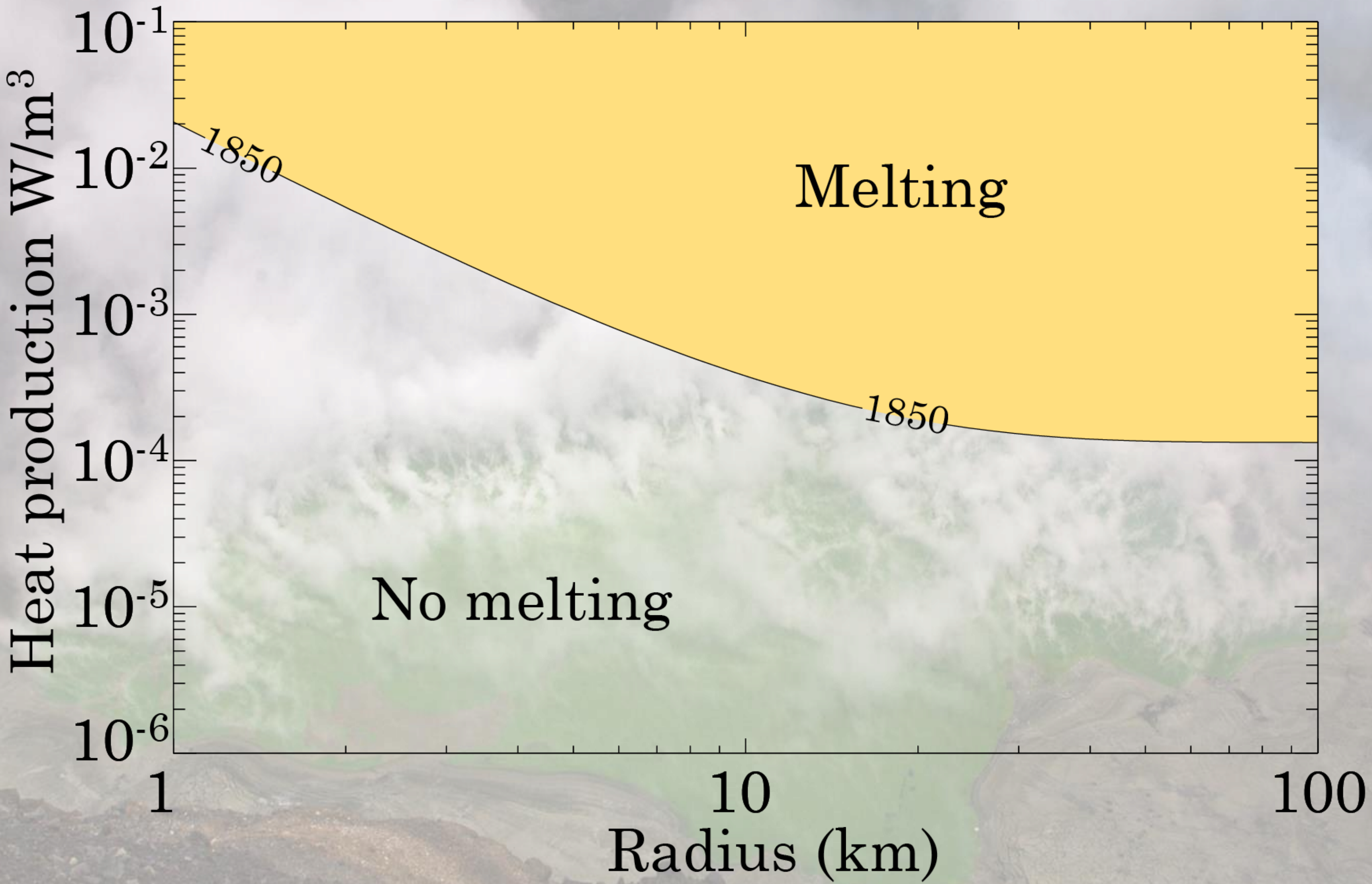


2.2 – Thermal evolution of small asteroids
^{26}Al energy – When does it melt ?

Available energy ← ^{26}Al ppm ← $Al_2 0_3$ wt % Date of accretion

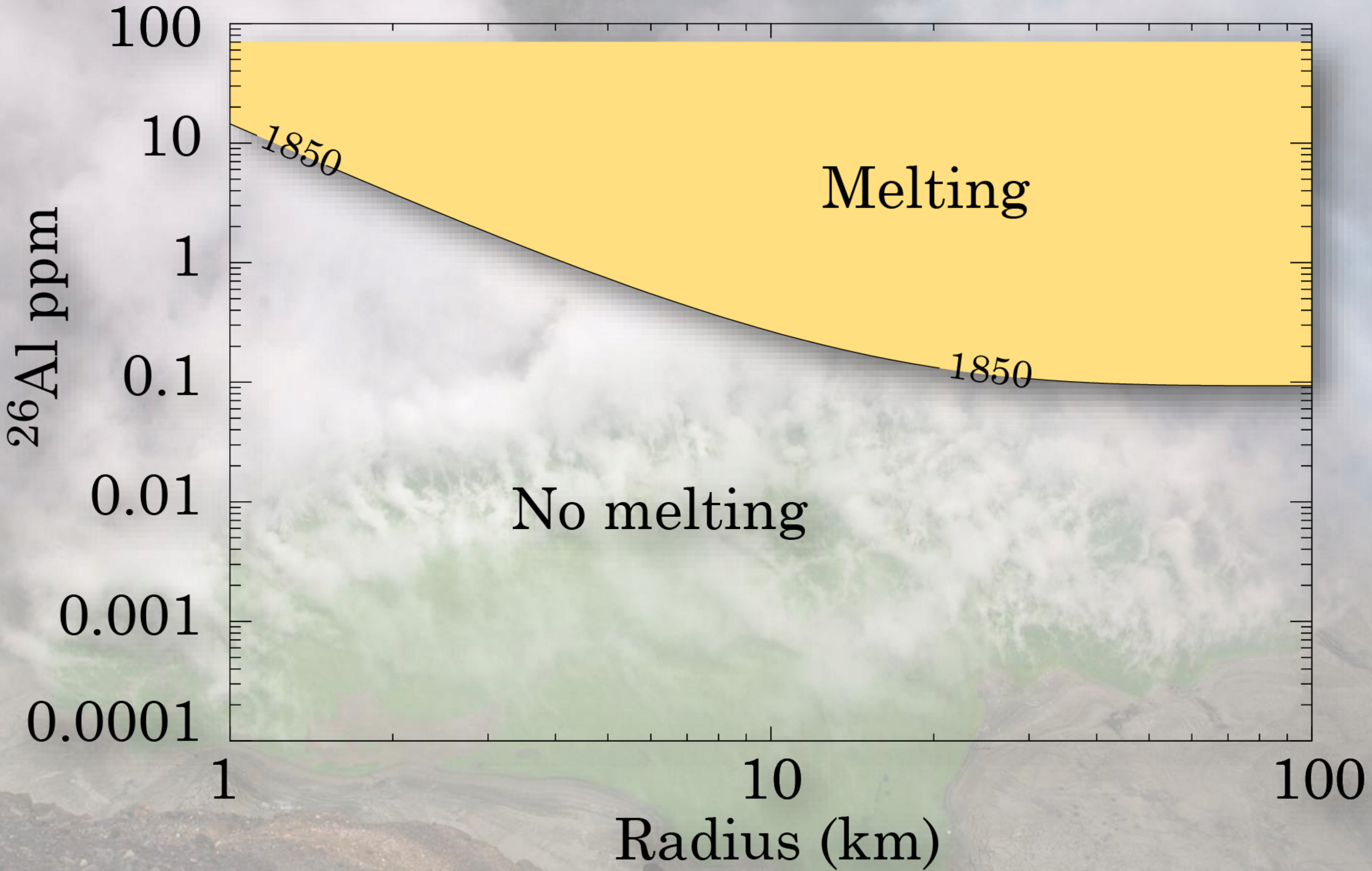
2.2 – Thermal evolution of small asteroids

^{26}Al energy – When does it melt ?

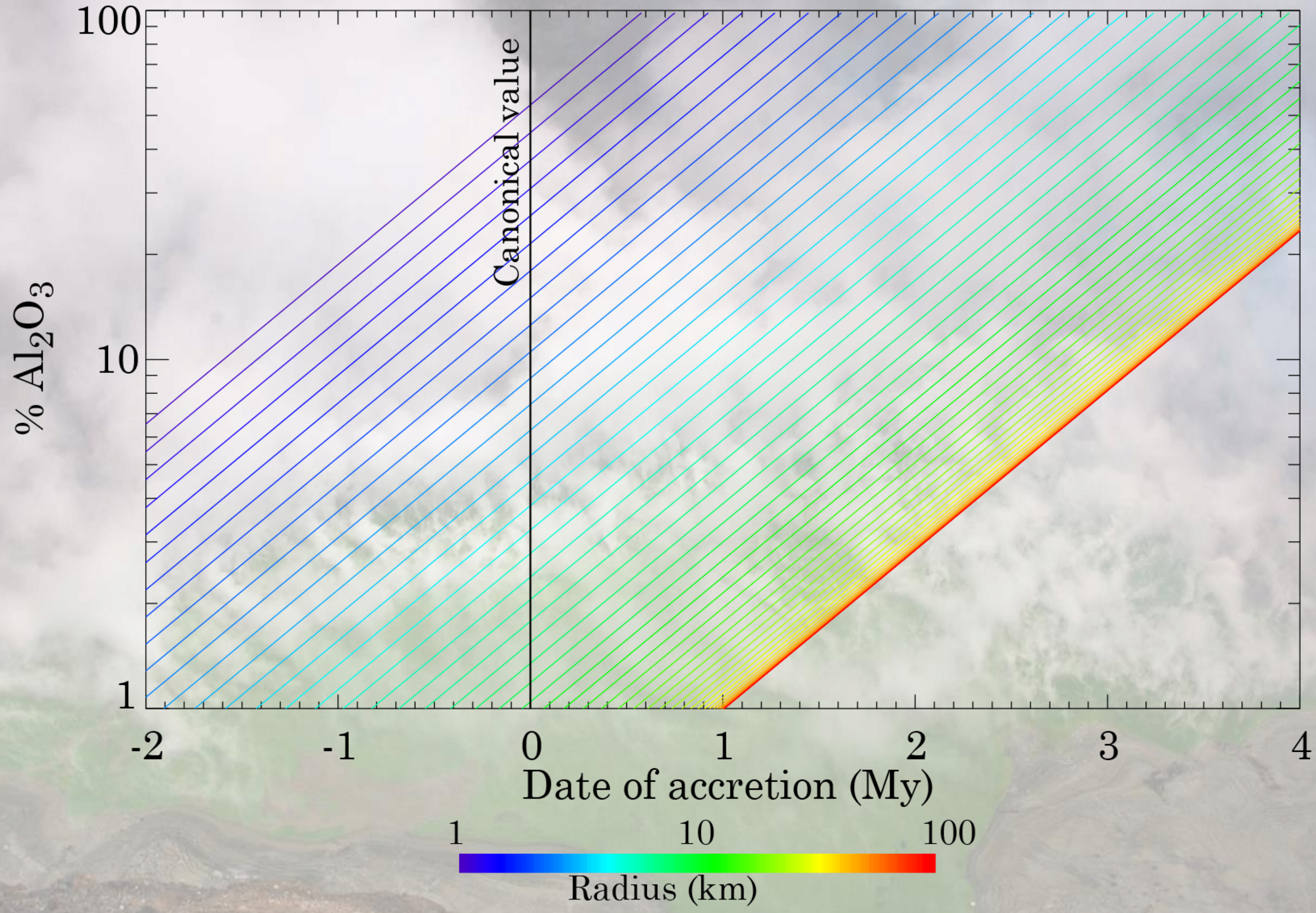


2.2 – Thermal evolution of small asteroids

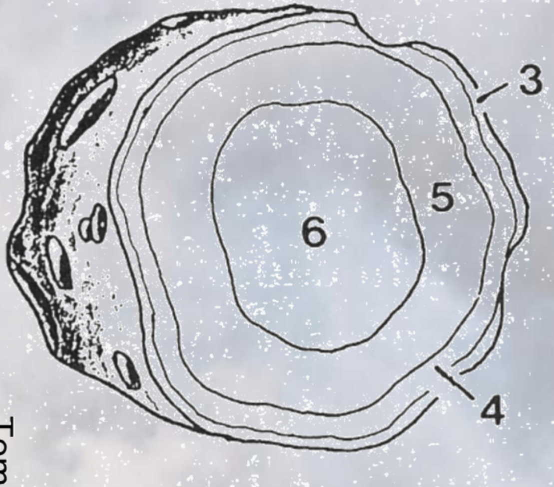
^{26}Al energy – When does it melt ?

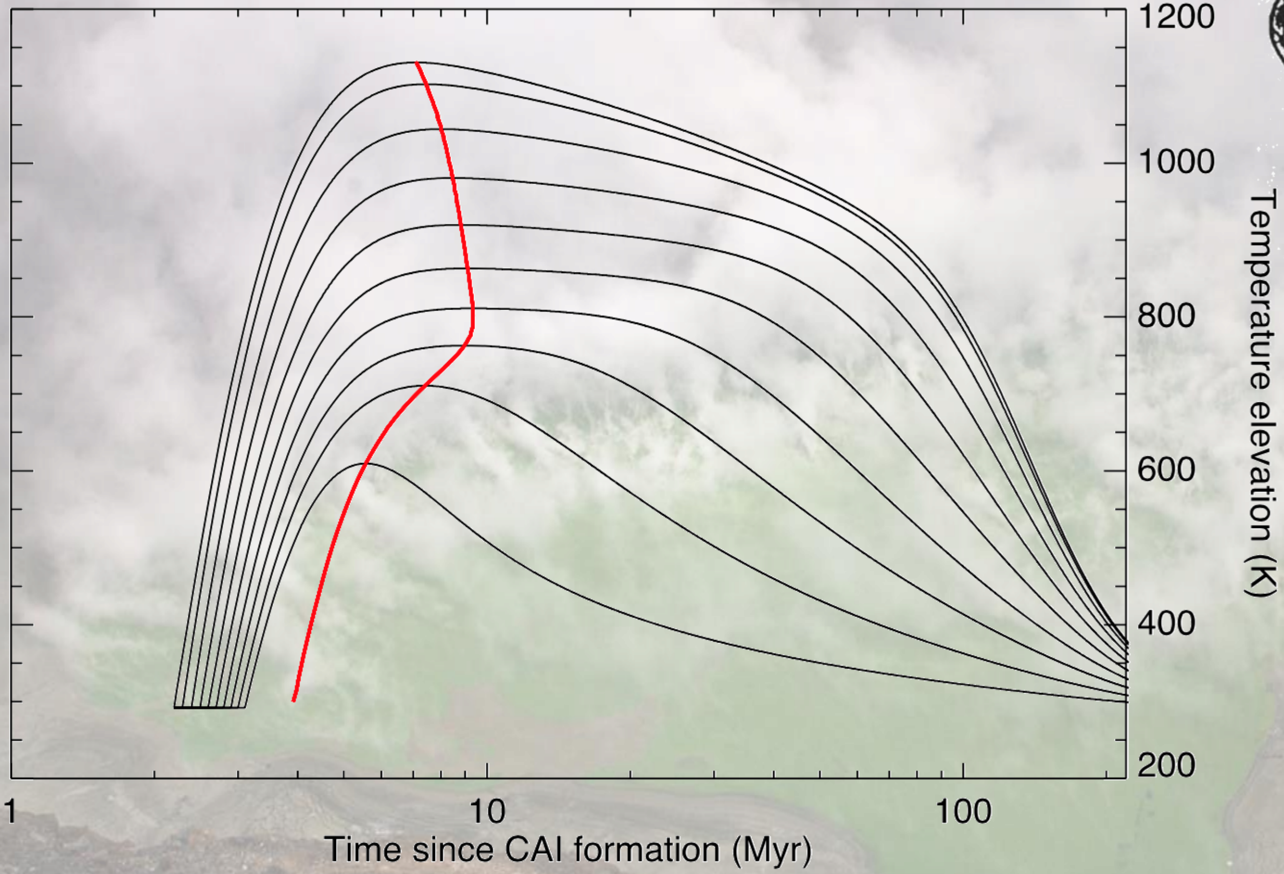


2.2 – Thermal evolution of small asteroids
^{26}Al energy – When does it melt ?



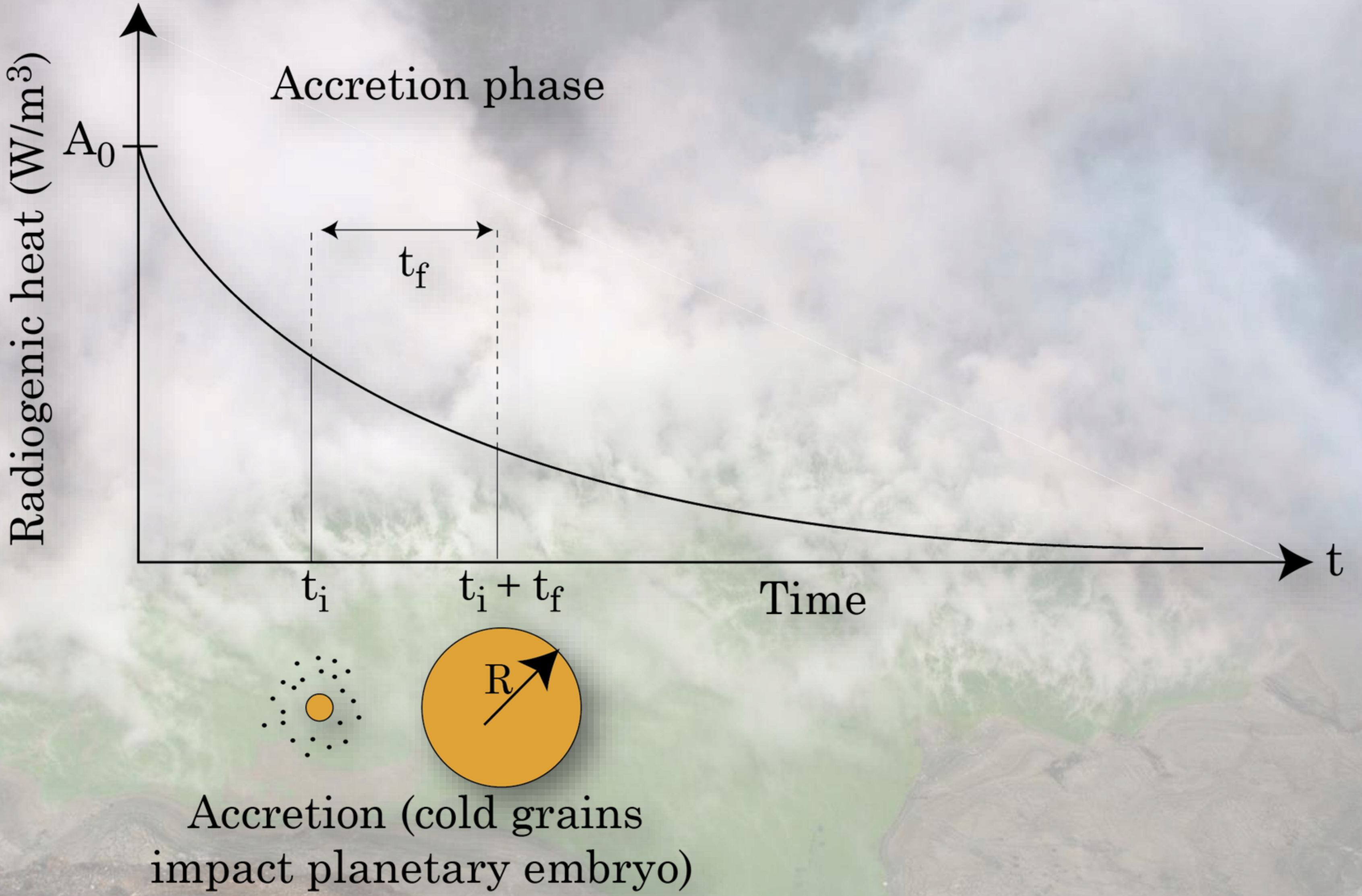
2.2 – Thermal evolution of small asteroids
^{26}Al energy – The onion shell structure





Exercises

Compare accretion energy with ^{26}Al energy



Accretion (cold grains impact planetary embryo)

2.3 – Application to the thermal history of the H-Chondrite parent body

Why H-Chondrites ?

Ordinary chondrites have experienced variable degrees of thermal metamorphism

Negative correlation exists between the Pb/Pb ages of phosphates and petrologic type => link between age and degree of metamorphism (T).

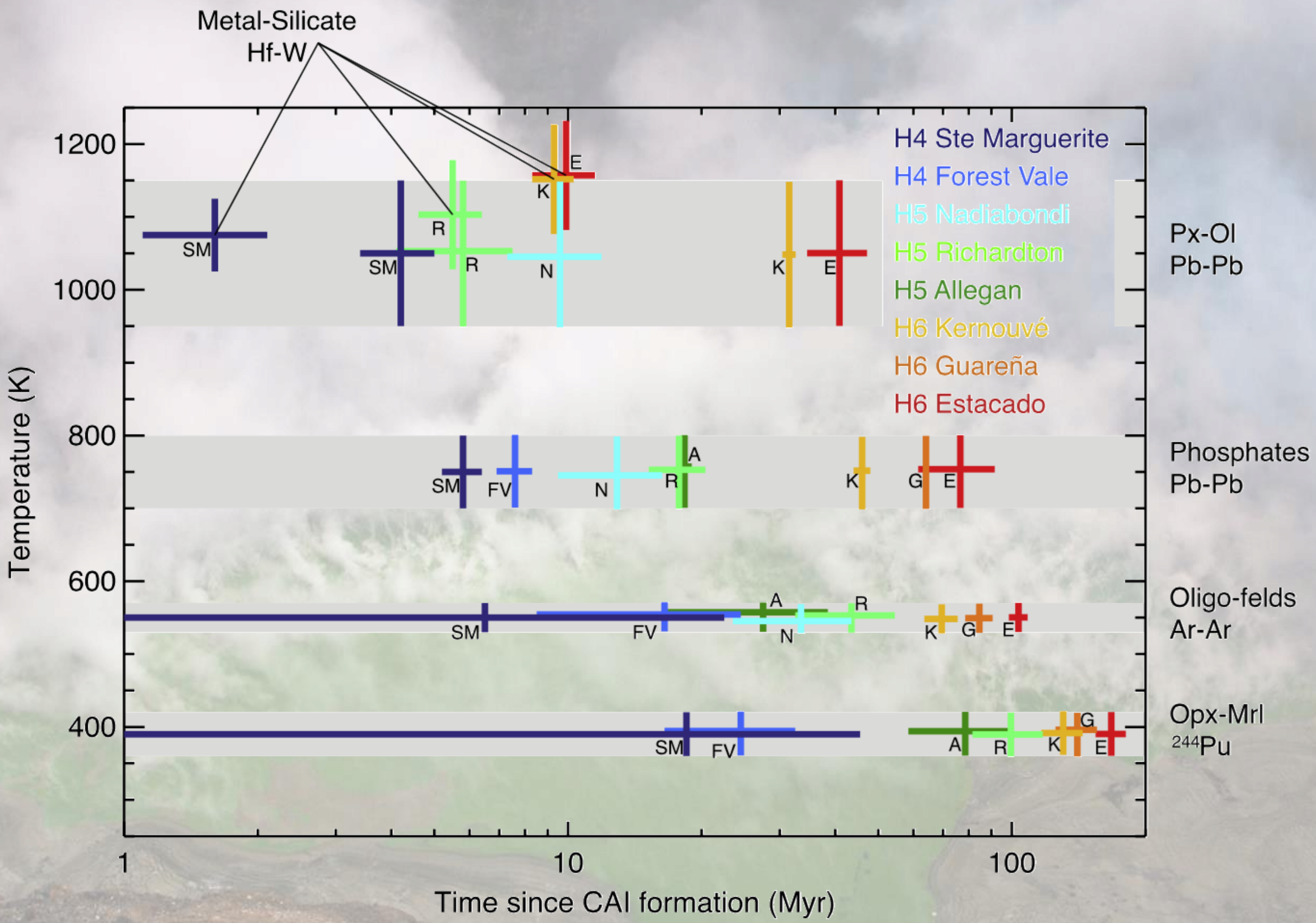
Furthermore, a smooth variation of these ages was found as a function of metallographic cooling rates obtained on the same samples

H-chondrites may belong to the same parent body.
Onion-shell structure of the parent body.

2.3 – Application to the thermal history of the H-Chondrite parent body

Type	Meteorite	Closure date (Ma)	ΔCAI[a] (Myr)	Temperature (K)	Minerals	System	Reference
H4							
	Ste Marguerite	4566.9 ± 0.5[b]	1.6 ± 0.5	1075 ± 50	Metal–Silicate	Hf–W	Kleine et al. (2008)
		4564.3 ± 0.8[b]	4.2 ± 0.8	1050 ± 100	Px–Ol	Pb–Pb	Bouvier et al. (2007)
		4562.7 ± 0.6	5.8 ± 0.6	750 ± 50	Phosphates	Pb–Pb	Göpel et al. (1994)
		4532 ± 16	36.5 ± 16	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
		4520 ± 27[c]	48.5 ± 27	390 ± 30	Opx-Mrl	^{244}Pu	Trieloff et al. (2003)
	Forest Vale	4560.9 ± 0.7	7.6 ± 0.7	750 ± 50	Phosphates	Pb–Pb	Göpel et al. (1994)
		4522 ± 8	46.5 ± 8	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
		4514 ± 8[c]	54.5 ± 8	390 ± 30	Opx-Mrl	^{244}Pu	Trieloff et al. (2003)
H5							
	Nadiabondi	4558.9 ± 2.3	9.6 ± 0.5	1050 ± 100	Px–Ol	Pb–Pb	Bouvier et al. (2007)
		4555.6 ± 3.4	12.9 ± 3.4	750 ± 50	Phosphates	Pb–Pb	Göpel et al. (1994)
		4505 ± 10	62.5 ± 10	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
	Richardton	4563.0 ± 0.9	5.5 ± 0.9	1100 ± 75	Metal–Silicate	Hf–W	Kleine et al. (2008)
		4562.7 ± 1.7	5.8 ± 1.7	1050 ± 100	Chondrule	Pb–Pb	Amelin et al. (2005)
		4550.7 ± 2.6	17.8 ± 2.6	750 ± 50	Phosphates	Pb–Pb	Göpel et al. (1994)
		4495 ± 11	73.5 ± 11	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
		4439 ± 18[c]	134.5 ± 10	390 ± 30	Opx–Mrl	^{244}Pu	Trieloff et al. (2003)
	Allegan	4550.2 ± 0.7	18.3 ± 0.7	750 ± 50	Phosphates	Pb–Pb	Göpel et al. (1994)
		4511 ± 11	57.5 ± 11	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
		4460 ± 20[c]	108.5 ± 20	390 ± 30	Opx–Mrl	^{244}Pu	Trieloff et al. (2003)
H6							
	Kernouvé	4559.2 ± 1.0	9.3 ± 1.0	1150 ± 75	Metal–Silicate	Hf–W	Kleine et al. (2008)
		4537.0 ± 1.1	31.5 ± 1.1	1050 ± 100	Px–Ol	Pb–Pb	Bouvier et al. (2007)
		4522.5 ± 2.0	46 ± 2.0	750 ± 50	Phosphates	Pb–Pb	Göpel et al. (1994)
		4469 ± 6	99.5 ± 6	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
		4408 ± 14[c]	160.5 ± 14	390 ± 30	Opx–Mrl	^{244}Pu	Trieloff et al. (2003)
	Guareña	4504.4 ± 0.5	64.1 ± 0.5	750 ± 50	Phosphates	Pb–Pb	Göpel et al. (1994)
		4454 ± 6	114.5 ± 6	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
		4398 ± 15[c]	170.5 ± 15	390 ± 30	Opx–Mrl	^{244}Pu	Trieloff et al. (2003)
	Estacado	4558.6 ± 1.6	9.9 ± 1.6	1150 ± 75	Metal–Silicate	Hf–W	Kleine et al. (2008)
		4527.6 ± 6.3	40.9 ± 6.3	1050 ± 100	Chondrule	Pb–Pb	Blinova et al. (2007)
		4492 ± 15	76.5 ± 15	750 ± 50	Phosphates	Pb–Pb	Blinova et al. (2007)
		4435 ± 5	133.5 ± 5	550 ± 20	Oligo. Felds	Ar–Ar	Trieloff et al. (2003)
		4371 ± 13[c]	197.5 ± 13	390 ± 30	Opx-Mrl	^{244}Pu	Trieloff et al. (2003)

2.3 – Application to the thermal history of the H-Chondrite parent body



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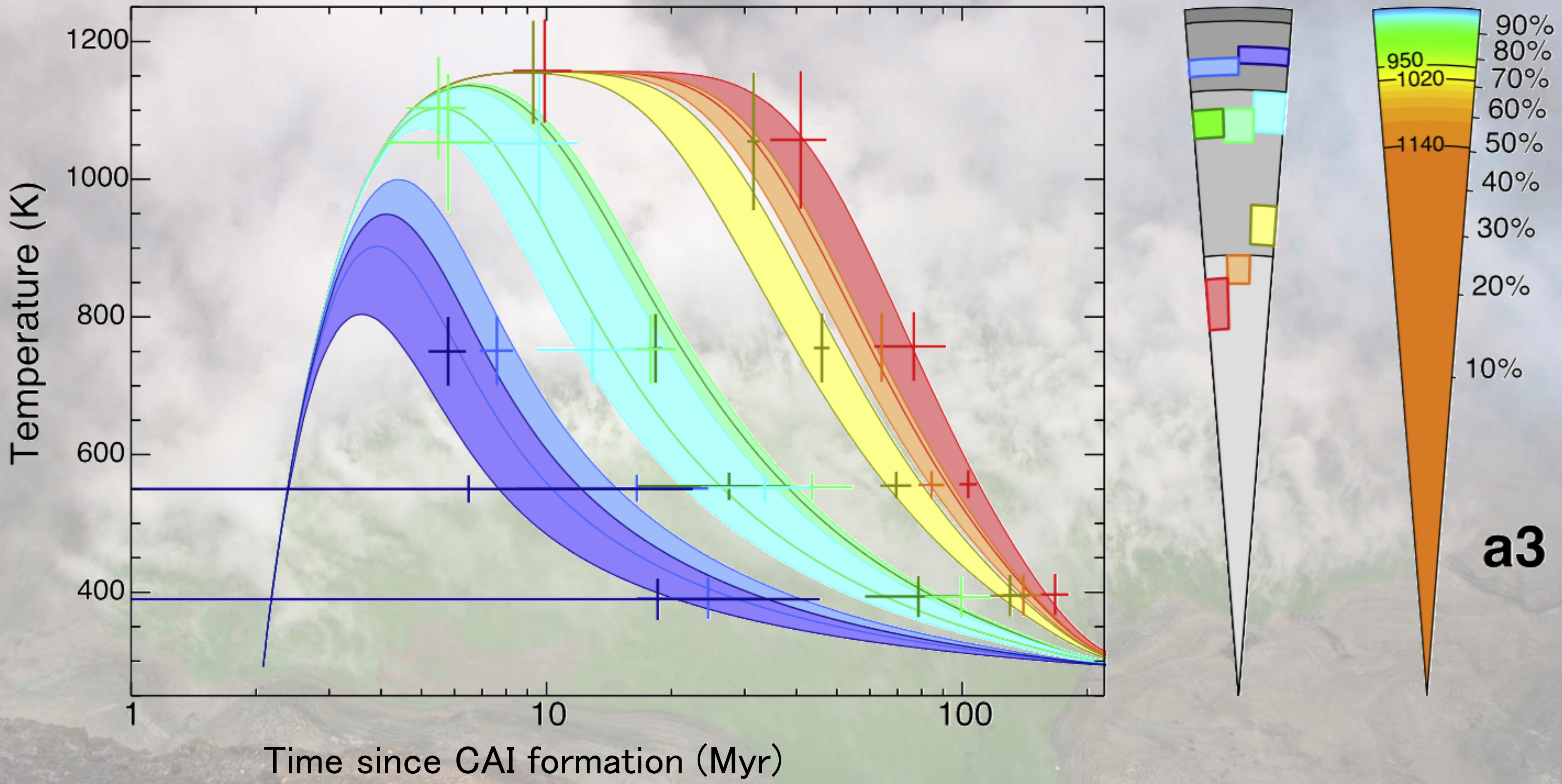
2.3 – Application to the thermal history of the H-Chondrite parent body

Duration of accretion 0.001 Myr



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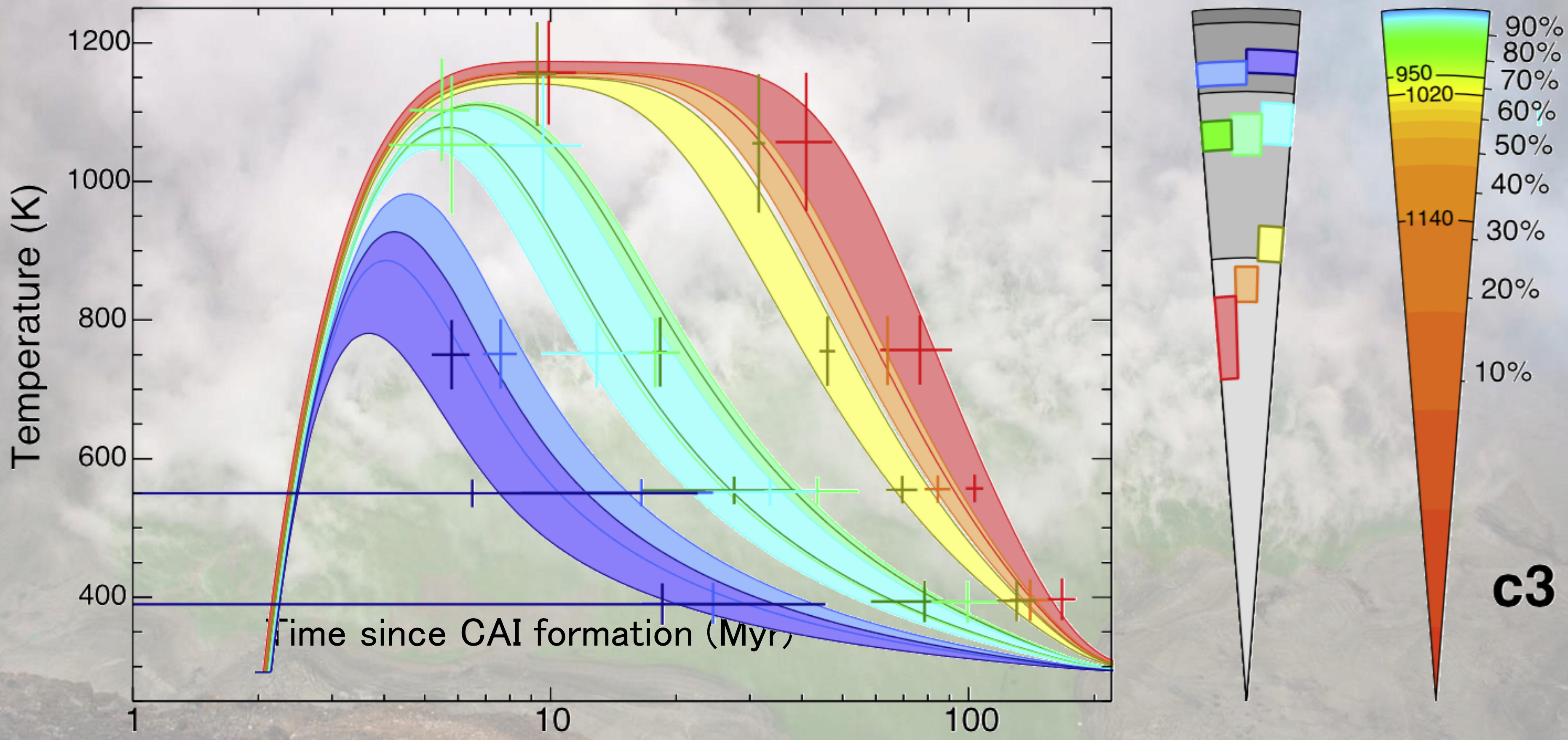
2.3 – Application to the thermal history of the H-Chondrite parent body

Duration of accretion 0.2 Myr



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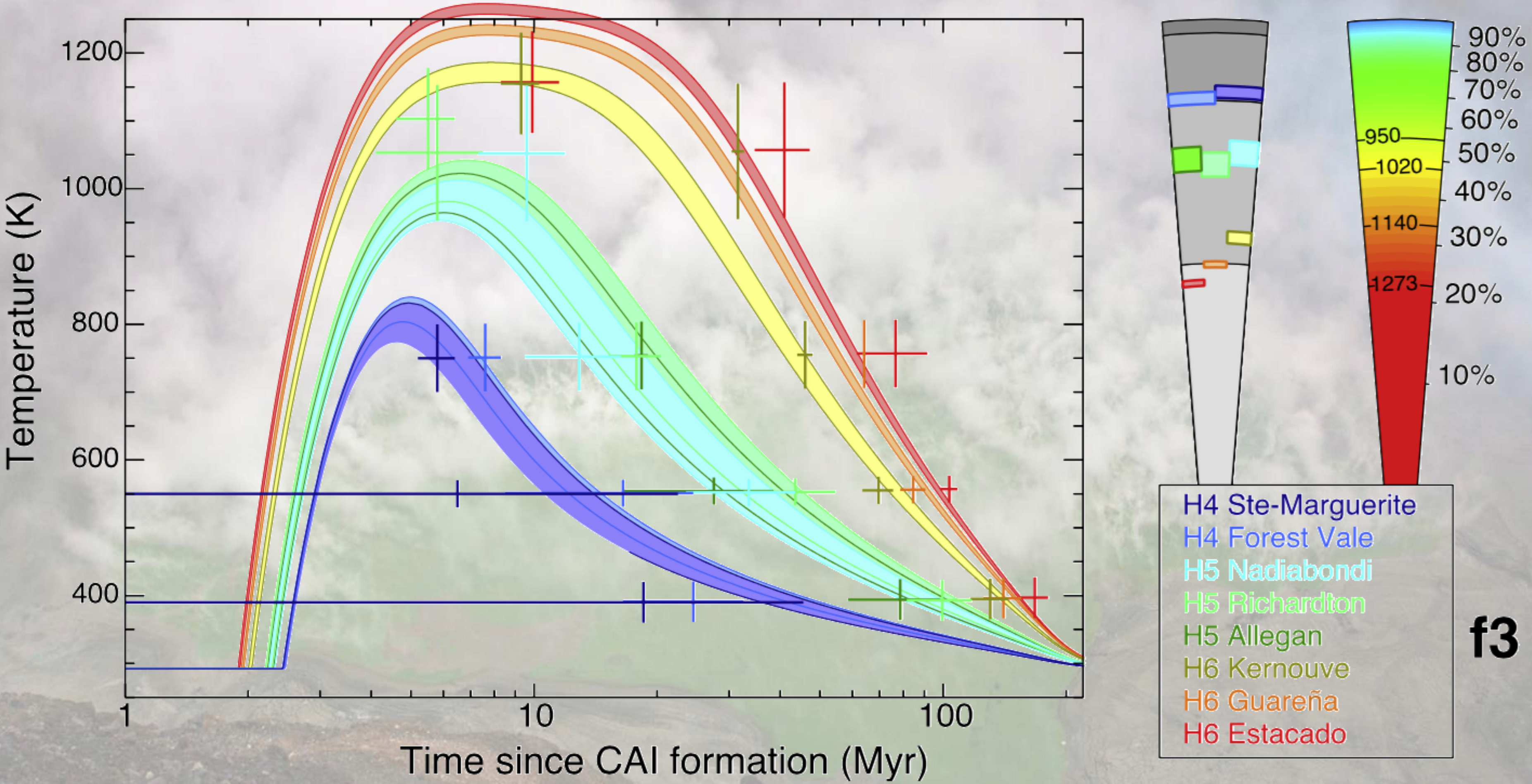
2.3 – Application to the thermal history of the H-Chondrite parent body

Duration of accretion 2 Myr



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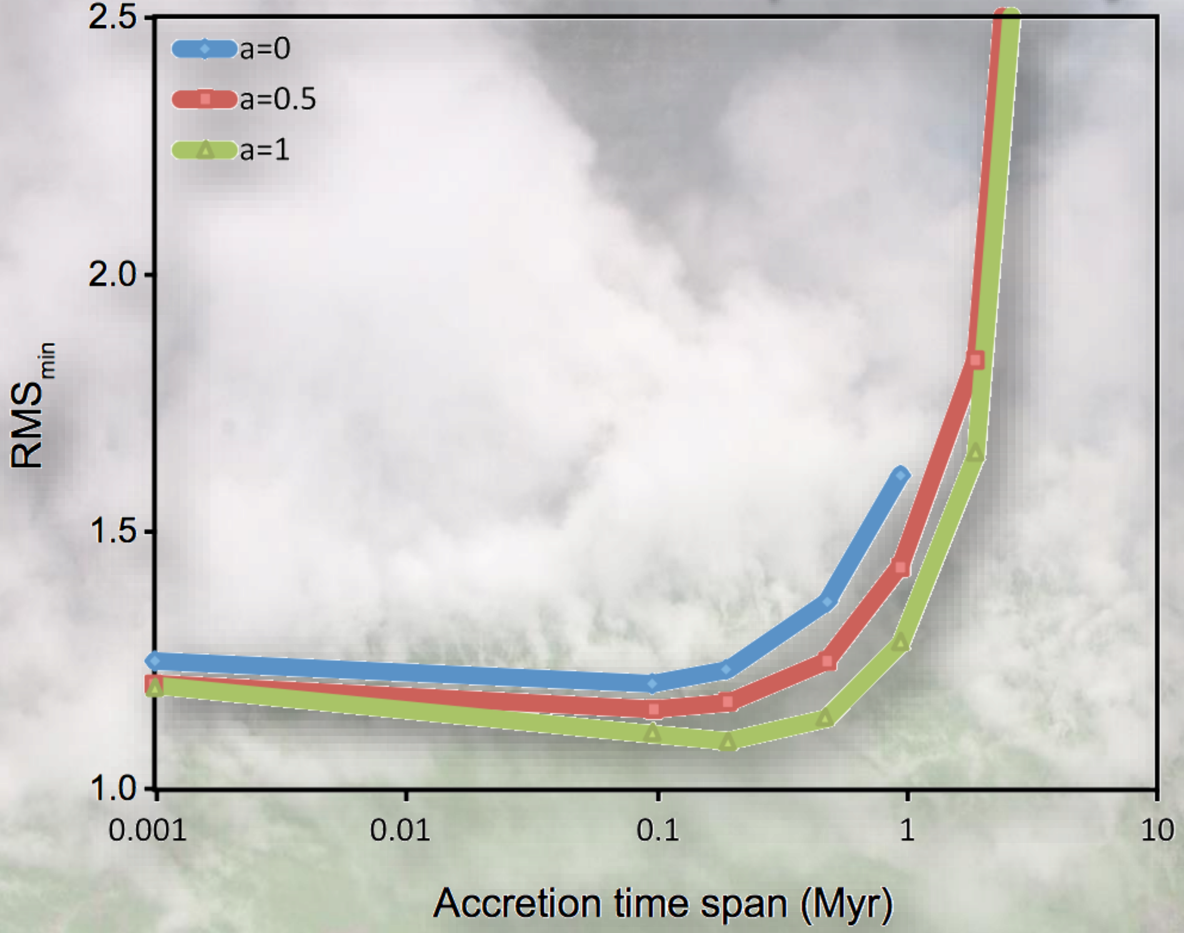
2.3 – Application to the thermal history of the H-Chondrite parent body



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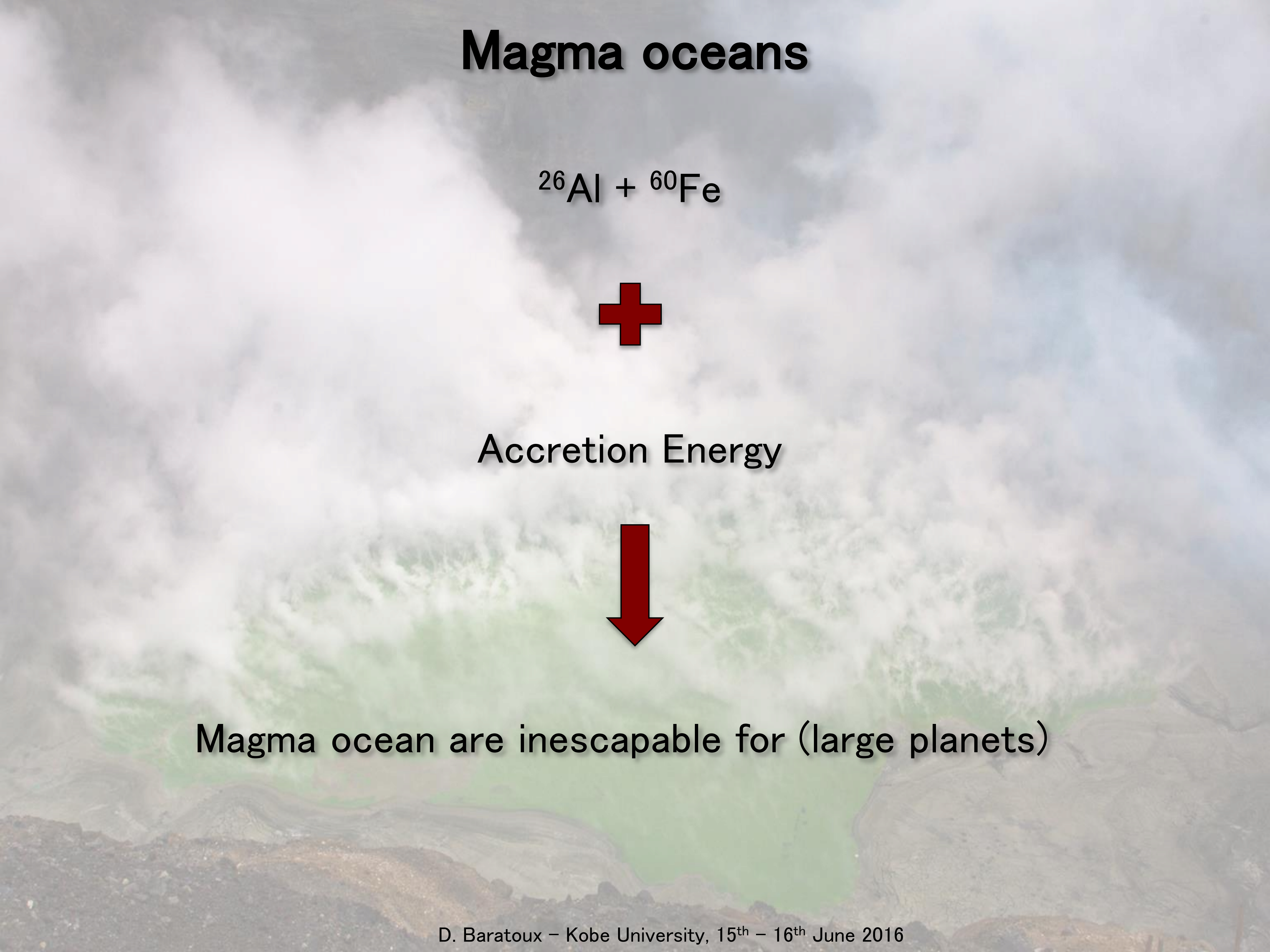
The time interval of accretion is unlikely to have been more than 0.5 Myr supporting evidence in favour of rapid accretion

Magma oceans



$^{26}Al + ^{60}Fe$

+

Accretion Energy

↓

Magma ocean are inescapable for (large planets)