2015-04-25 Akatsuki Workshop on
Cloud Tracking for Venus at ISAS

On possible use of data assimilation for Venusian cloud tracking

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Introduction

- We should use all the available data; the more data, the more accurate (if we do that adequately)
- Ikegawa & Horinouchi (2015) (IH15) is one way to utilize all of sequential images, but it is not the only way…

Cross-correlation (CC) method revisited

- Underlying dynamical model:

$$D_t f' \equiv (\partial_t + u\,\partial_x + v\,\partial_y) f' = \varepsilon$$

  where f' is the brightness deviation normalized in a small area; (u,v): horizontal wind averaged over the area & a period Δt; ε: small departure)

- Method: Find (u,v) that maximizes $\langle f'(x+u\Delta t, y+v\Delta t, t+\Delta t) f'(x,y,t)\rangle$ (area mean)
- Assumption: $\nabla u\,\Delta t$ and $\nabla v\,\Delta t$ are small enough so that deformation is negligible (or is nearly linear for tracking with the Affine transformation).
- Demerit: the results can violate the assumption → post-processing needed
- Merit: error (of each flow vector) can be inferred with no *a priori* assumption (IH15)

Method of IH15

- [ST type] Same as the one-pair method but to minimize

$$r(x,y,t) \equiv \sum_{(t_1, t_2)} \langle f' (x+u t_1, y+v t_1, t+t_1) f' (x+u t_2, y+v t_2, t+t_2) \rangle$$

- [STS type] Like ST but further apply running mean to CC surface: $\sum_{(p,q)} r(x+p, y+q, t)$

Data assimilation

- Dynamical model: any
- Method: to minimize the sum of the estimated error (global optimization). --- Constraints can be introduced flexibly.
- Long tradition in the field of "optical flow" (motion tracking from image sequence) since Horn & Schunck (1981). (also some in PIV)
- Merit: no inconsistency → necessity for post-processing is smaller than in the CC methods
- Error estimate (of each *u*,*v*): available but depends on the assumed errors in the first guess and observation

Further remark on the comparison

- CC method: local&global minimization of the evaluation function (residuals).
  - At the expense of consistency
  - Error correction by Koyama et al (2012) etc is a remedy. To chose secondary CC peaks is to loosen the minimization and to increase consistency (but not explicitly formulated as such).
  - It is the case for IH15 (superposition decreases the correlation peaks)

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Example

Papadakis and Memin (2008)

A Variational Technique for Time Consistent Tracking of Curves and Motion

N. Papadakis · E. Mémin

J Math Imaging Vis (2008) 31: 81–103

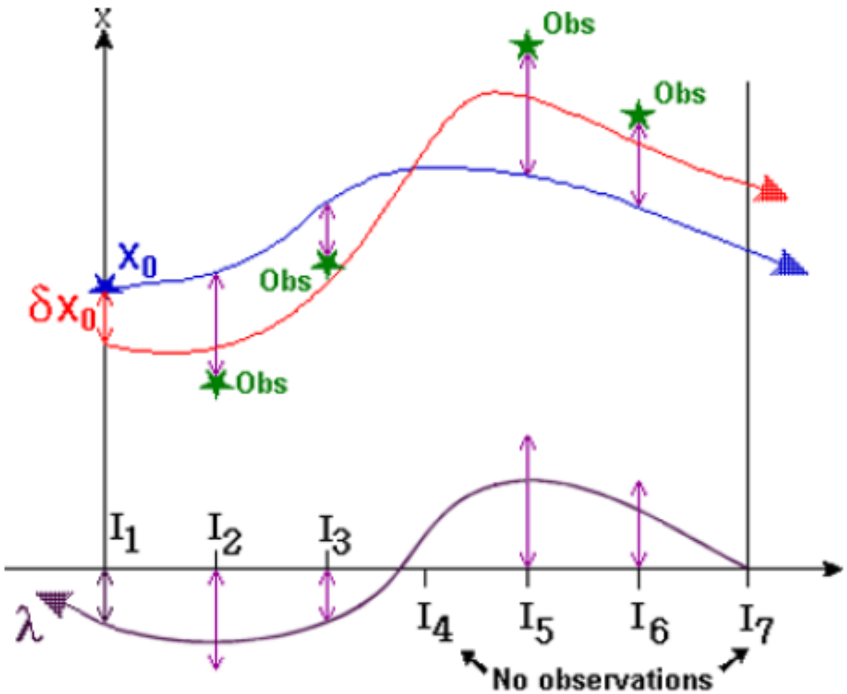


Fig. 1 Assimilation algorithm principle. This figure gives a synoptic of the overall principle of the method. After an integration of the initial condition X_0, a backward integration of the adjoint variable relying on a measurement discrepancy enables to compute a forward incremental correction trajectory and so on...

Papadakis and Memin (2008)

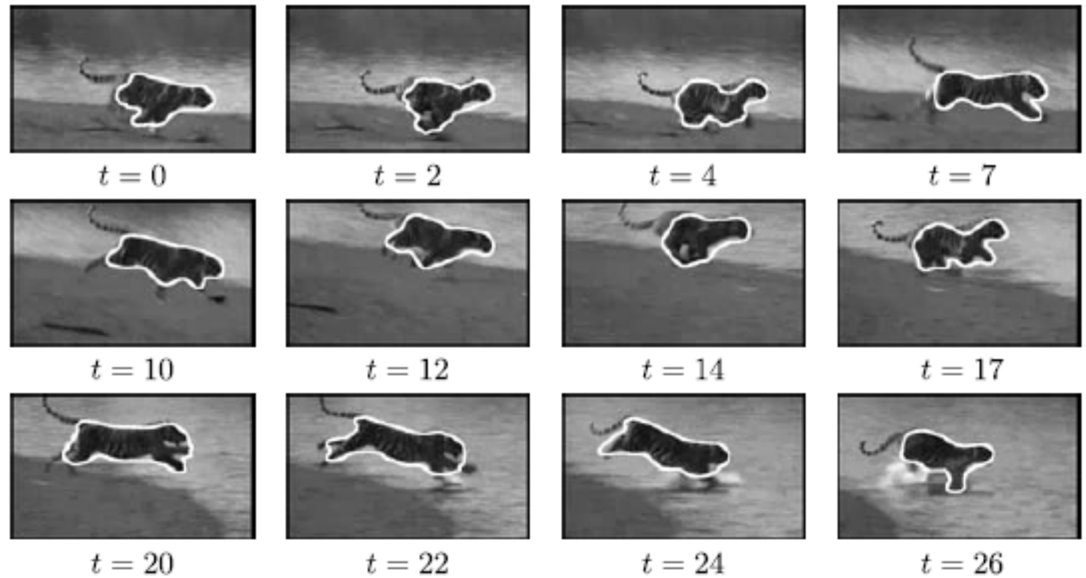


Fig. 5 Tiger sequence. Result of the assimilation technique with the photometric measurement model based on the local probability densities

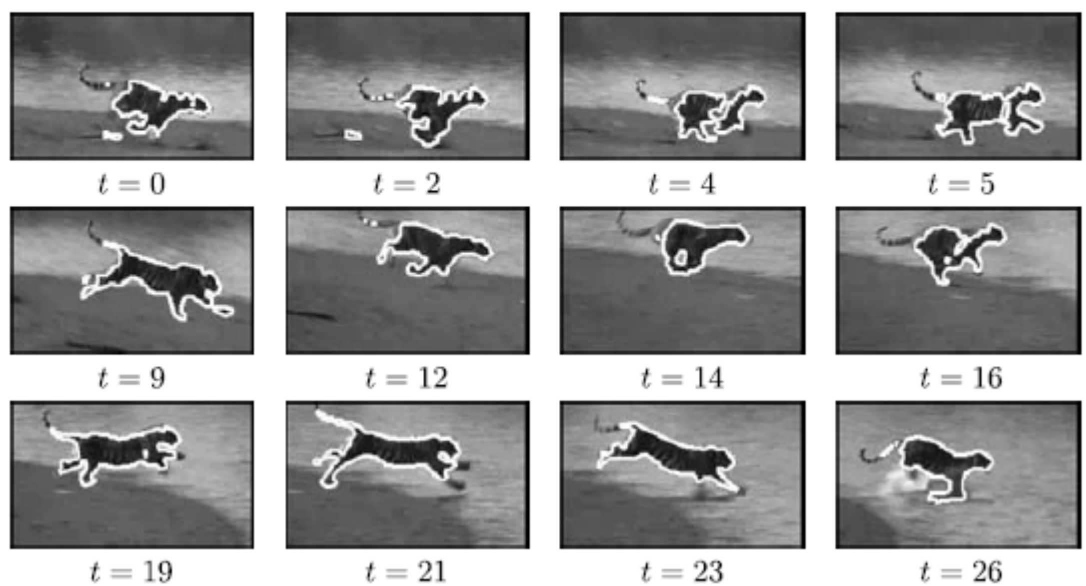


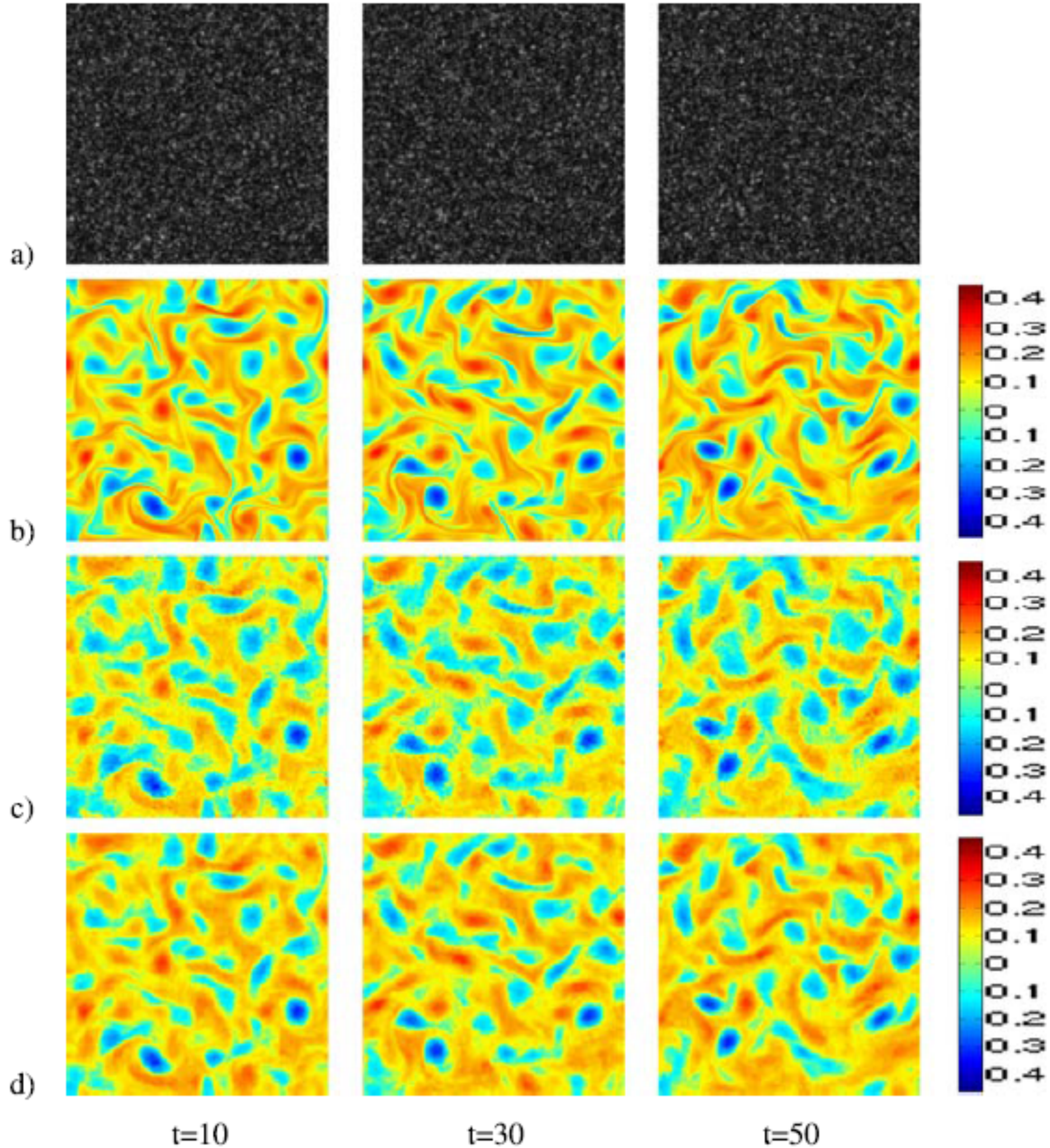
Fig. 6 Tiger sequence. Successive segmentations obtained through a Chan and Vese level-set techniques with a data model based on the local probability densities measurement and a Bhattacharya distance (see (36–37))

Papadakis and Memin (2008)

98 J Math Imaging Vis (2008) 31: 81–103

Fig. 9 2D direct numerical simulation. (**a**) Particle images sequence. (**b**) True vorticity. (**c**) Vorticity observed by the optical flow estimator. (**d**) Assimilated vorticity

Demonstration using numerically simulated flow data



Assimilation methods

- Variational method
- Kalman filter

Both have been used in the numerical weather prediction (4D Var; Ensemble Kalman filter)

- 4D Var is used at JMA (気象庁), ECMWF, etc

If CC method is mimicked in data assimilation

- Dynamical model:

$$D_t I \equiv (\partial_t + u\partial_x + v\partial_y) I = \alpha + \beta I$$

$$D_t u = \varepsilon_u, \ D_t v = \varepsilon_v$$

  α, β: smooth functions to let I vary gradually. ε_u, ε_v: small diffusion to avoid turnover (zero in CC).

- Method: Find $\boldsymbol{x}$ (entire grid-point values of I, u, v, α, β treated as vectors) that maximizes $J \equiv 1/2\,(\boldsymbol{x}_0 - \boldsymbol{x}_0^f)^T \mathrm{P}^{-1} (\boldsymbol{x}_0 - \boldsymbol{x}_0^f) + 1/2\,(H(\boldsymbol{x}) - \boldsymbol{y}^o)^T \mathrm{R}^{-1} (H(\boldsymbol{x}) - \boldsymbol{y}^o)$

  $\boldsymbol{x}_0$: $\boldsymbol{x}$ at $t=0$. $\boldsymbol{x}_0^f$: first guess. $\boldsymbol{y}^o$: observation (here, $\boldsymbol{I}^o$). H: observation function (here, a simple 0,1 matrix).

  P, R: covariance matrices (subject to tuning). Smoothness of u, v, α, β can be required through them. (Or you can reduce the grid for them. Also you can add a term to J; e.g., Weickert&Schnorr 2001)

Possible extension of dynamical models

- $D_t I = -c\,\mathrm{div}(u,v) + \dots$: upwelling increases the absorption (In general, $D_t I = F(I,u,v;\,\alpha,\beta,..)$)
- $D_t f = -c\,\mathrm{div}(u,v)$, $I = H(f)$: separating the advected scalar and brightness
- 2-layer model: $(\partial_t + u_1 \partial_x + v_1 \partial_y) f_1 = -c_1\ \mathrm{div}(u_1, v_1)$, $(\partial_t + u_2 \partial_x + v_2 \partial_y) f_2 = -c_2\ \mathrm{div}(u_2, v_2)$,

$I = H(f_1, f_2)$: simplified radiation transfer model (e.g., $I \propto f_1 + a e^{-b f_1} f_2$: upper cloud mask the lower).

  – If you like, you can constrain through I like $\mathrm{div}(u_1, v_1) + \mathrm{div}(u_2, v_2) \cong 0$

What's P and R? (roughly speaking)

- $\mathrm{P} = \langle (\boldsymbol{x}^f - \boldsymbol{x})(\boldsymbol{x}^f - \boldsymbol{x})^T \rangle$: forecast (first guess) error covariance ($\boldsymbol{x}$ here is true $\boldsymbol{x}$, and $\langle \ \rangle$ is computation of climatologically expected value).
- $\mathrm{R} = \langle (\boldsymbol{y} - H(\boldsymbol{x}))(\boldsymbol{y} - H(\boldsymbol{x}))^T \rangle$: observational error covariance. (note: in the variational method, error can be non-Gaussian, so wrong reports (e.g., bad pixels) can be considered; ref: Tsuyuki)

Combination with the CC method

- We do not have a forecast, but we may use the **IH15** method to derive $\boldsymbol{x}_0^f$ and its error estimates (perhaps the worth of $\mathcal{E}$ and σ in IH15) to derive the diagonal components of P; assume correlation over some distance to derive the non-diagonal

Variational method (from Lecture 5 at Kobe Univ. by **T. Tsuyuki**; http://wtk.gfd-dennou.org/2012-09-24/tsuyuki/pub/)

4次元変分法(1)

時刻 t_0 から時刻 t_K までの同化ウィンドウ内の観測データ $\{\mathbf{y}_1, \cdots, \mathbf{y}_K\}$ から、同化ウィンドウ内の状態変数 $\{\mathbf{x}_0, \cdots, \mathbf{x}_K\}$ を一括推定する。

同化ウィンドウの長さが、数値モデルのランダム誤差の影響が無視できる程度に短ければ

$$\mathbf{x}_k = M_k(\mathbf{x}_0) \qquad (k = 1, \cdots, K)$$

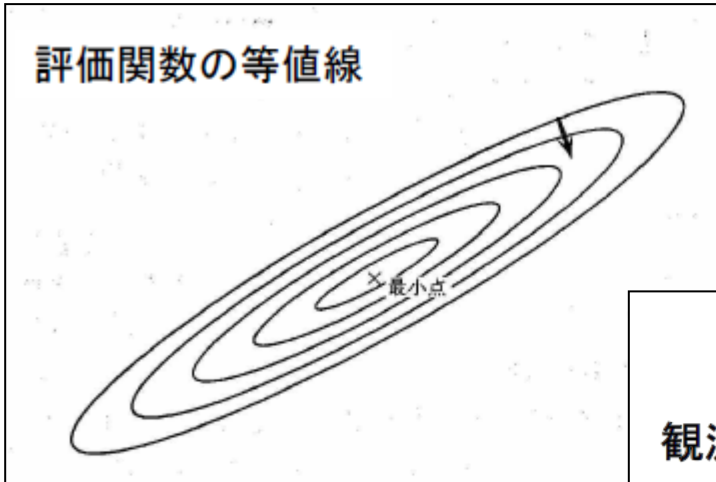
したがって、$\mathbf{x}_0$ を推定すればよい。異なる時刻の観測データが互いに独立とすれば、Bayesの定理より

$$p(\mathbf{x}_0 \mid \mathbf{y}_1, \cdots, \mathbf{y}_K) = \frac{p(\mathbf{y}_1 \mid \mathbf{x}_1) \cdots p(\mathbf{y}_K \mid \mathbf{x}_K) p(\mathbf{x}_0)}{\int p(\mathbf{y}_1 \mid \mathbf{x}_1) \cdots p(\mathbf{y}_K \mid \mathbf{x}_K) p(\mathbf{x}_0) d^n \mathbf{x}}$$

評価関数($\mathbf{x}_0$ に依存しない項を省略する)

$$J(\mathbf{x}_0) := -\log p(\mathbf{x}_0 \mid \mathbf{y}_1, \cdots, \mathbf{y}_K)$$

$$= \frac{1}{2}\left(\mathbf{x}_0 - \mathbf{x}_0^f\right)^{\mathrm{T}} \left(\mathbf{P}_0^f\right)^{-1} \left(\mathbf{x}_0 - \mathbf{x}_0^f\right) + \sum_{k=1}^{K} \frac{1}{2}\left(H_k(\mathbf{x}_k) - \mathbf{y}_k\right)^{\mathrm{T}} \mathbf{R}_k^{-1} \left(H_k(\mathbf{x}_k) - \mathbf{y}_k\right)$$



変分法 QC

観測データの品質管理(QC)

- 測器の故障などによる誤った観測データをあらかじめ除去する。
- 方法：気候学的チェック、内的整合性チェック、外的整合性チェック
（他の観測データや予報値との比較）

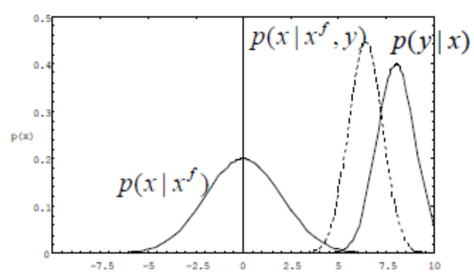
変分法では、外的整合性チェックをデータ同化の中で行える。

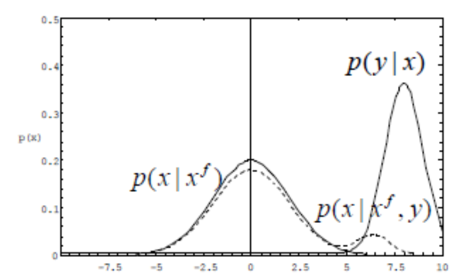
$$p(y\,|\,x) = \frac{1}{\sqrt{2\pi}\,\sigma^o}\exp\left[-\frac{(y-x)^2}{2(\sigma^o)^2}\right]$$

$$p(x\,|\,x^f) = \frac{1}{\sqrt{2\pi}\,\sigma^f}\exp\left[-\frac{(x-x^f)^2}{2(\sigma^f)^2}\right]$$

$$p(y\,|\,x) = \frac{a}{x_2 - x_1} + \frac{1-a}{\sqrt{2\pi}\,\sigma^o}\exp\left[-\frac{(y-x)^2}{2(\sigma^o)^2}\right]$$

（a ：誤データ発生確率）





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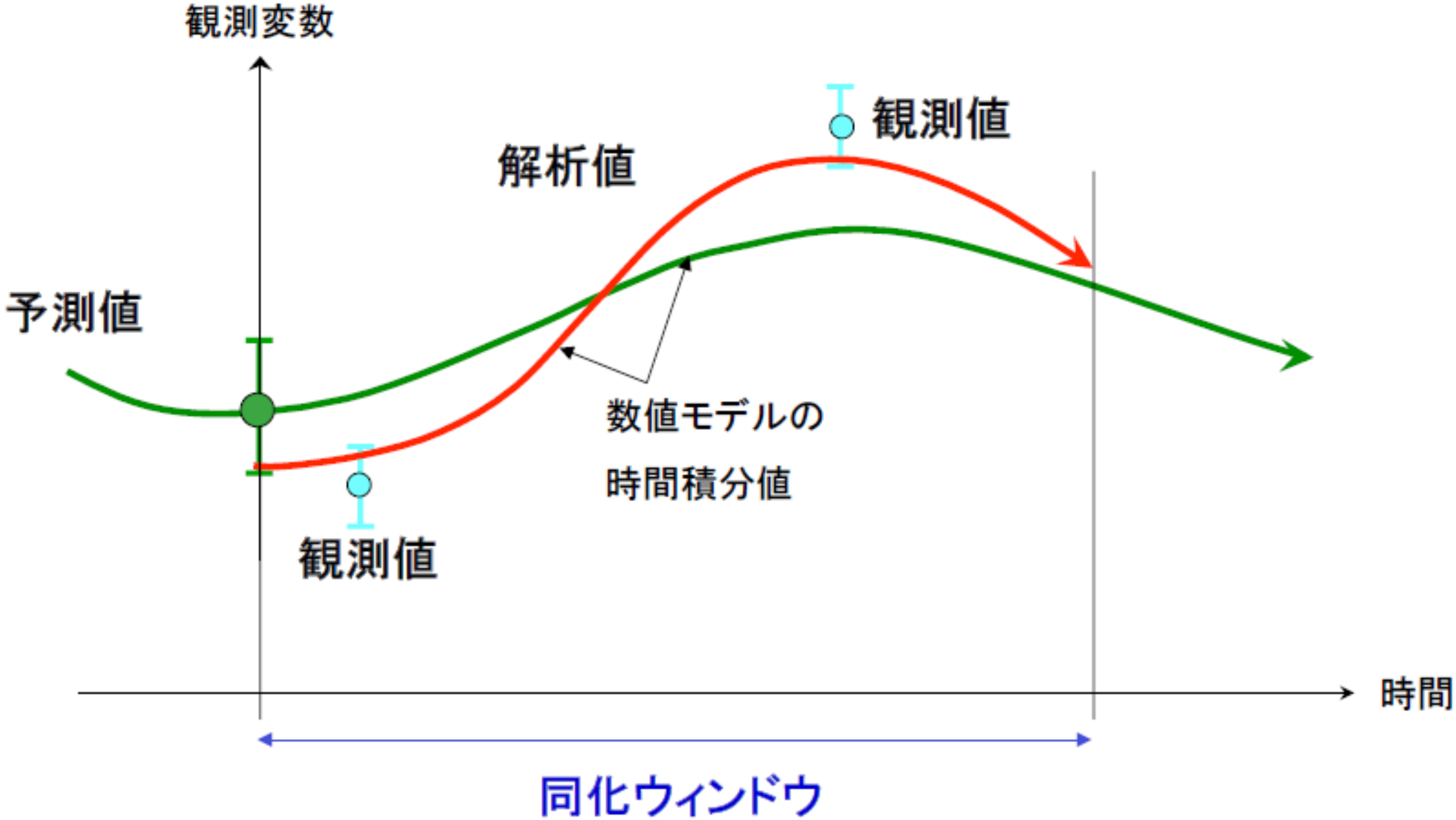
4次元変分法(2)

4次元変分法では、同化ウィンドウ内の予測値と観測値の両方に最も近い数値シミュレーションの結果を解析値とする。



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数値モデル

$$\mathbf{x}_k = M_k(\mathbf{x}_k) \qquad (k = 1, \cdots, K)$$

should be k-1

線形モデル

$$\delta\mathbf{x}_k = \mathbf{M}_k \delta\mathbf{x}_{k-1} \qquad (k = 1, \cdots, K)$$

評価関数の勾配ベクトル

$$\nabla J(\mathbf{x}_0) = \left(\mathbf{P}_0^f\right)^{-1}\left(\mathbf{x}_0 - \mathbf{x}_0^f\right) + \sum_{k=1}^{K} \mathbf{M}_k{}^{\mathrm{T}} \mathbf{H}_k{}^{\mathrm{T}} \mathbf{R}_k{}^{-1} \left(H_k(\mathbf{x}_k) - \mathbf{y}_k\right)$$

$$\left(\mathbf{M}_k\right)_{ij} := \left.\frac{\partial\left(M_k(\mathbf{x})\right)_i}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}_0}$$

Adjoint 方程式

$$\mathbf{p}_k = \mathbf{M}_{k+1}{}^{\mathrm{T}} \mathbf{p}_{k+1} \qquad (k = 0, \cdots, K-1)$$

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アジョイントコードの書き方(1)

線形モデル: $\delta\mathbf{x}_{k+1} = \mathbf{M}_k \delta\mathbf{x}_k$

アジョイントモデル: $\mathbf{p}_k = \mathbf{M}_k^{\mathrm{T}} \mathbf{p}_{k+1}$

$$\mathbf{A} = \mathbf{A}_r \cdots \mathbf{A}_2 \mathbf{A}_1 \quad \rightarrow \quad \mathbf{A}^{\mathrm{T}} = \mathbf{A}_1^{\mathrm{T}} \mathbf{A}_2^{\mathrm{T}} \cdots \mathbf{A}_r^{\mathrm{T}}$$

1. 数値モデルのプログラムを任意の基本場の周りで線形化することによって、線形モデルのプログラムを書く。その際、条件分岐文の分岐条件には摂動を考慮しない。

2. 線形モデルの摂動変数の処理部分を、後ろから機械的に変換すれば、アジョイントモデルのプログラムが得られる。

3. 線形かつ可逆な定型処理のアジョイントコードは、元のプログラムをそのまま利用して作成できる。

TAMC: アジョイントコード自動生成ソフトウェア

http://www.autodiff.com/tamc/

http://wtk.gfd-dennou.org/2012-09-24/tsuyuki/pub/

Kalman filter (from Lecture 4 at Kobe Univ. by **T. Tsuyuki**; http://wtk.gfd-dennou.org/2012-09-24/tsuyuki/pub/)

解析誤差分散を最小にする $\mathbf{K}_k$ を求めるために、$\mathbf{K}_k$ に関する第1変分をとると

$$\begin{aligned}\delta\mathbf{P}_k{}^a &= \delta\mathbf{K}_k\left(\mathbf{R}_k+\mathbf{P}_k{}^{HH}\right)\mathbf{K}_k{}^{\mathrm{T}}+\mathbf{K}_k\left(\mathbf{R}_k+\mathbf{P}_k{}^{HH}\right)\delta\mathbf{K}_k{}^{\mathrm{T}}\\ &\quad-\mathbf{P}_k{}^{xH}\delta\mathbf{K}_k{}^{\mathrm{T}}-\delta\mathbf{K}_k\mathbf{P}_k{}^{Hx}\\ &=\delta\mathbf{K}_k\left[\left(\mathbf{R}_k+\mathbf{P}_k{}^{HH}\right)\mathbf{K}_k{}^{\mathrm{T}}-\mathbf{P}_k{}^{Hx}\right]+\left[\mathbf{K}_k\left(\mathbf{R}_k+\mathbf{P}_k{}^{HH}\right)-\mathbf{P}_k{}^{xH}\right]\delta\mathbf{K}_k{}^{\mathrm{T}}\end{aligned}$$

これが0になる条件から

$$\mathbf{K}_k=\mathbf{P}_k{}^{xH}\left(\mathbf{R}_k+\mathbf{P}_k{}^{HH}\right)^{-1}$$

このときの解析値と解析誤差共分散行列は

$$\mathbf{x}_k{}^a=\mathbf{x}_k{}^f+\mathbf{P}_k{}^{xH}\left(\mathbf{R}_k+\mathbf{P}_k{}^{HH}\right)^{-1}\left(\mathbf{y}_k-H_k(\mathbf{x}_k{}^f)\right)$$

$$\mathbf{P}_k{}^a=\mathbf{P}_k{}^{xx}-\mathbf{P}_k{}^{xH}\left(\mathbf{R}_k+\mathbf{P}_k{}^{HH}\right)^{-1}\left(\mathbf{P}_k{}^{xH}\right)^{\mathrm{T}}$$

第2式の右辺第2項の行列は非負定値行列なので、解析誤差分散は一般に予測誤差分散より小さい。

4次元アンサンブルカルマンフィルタ(1)

時刻 t_0 における解析値と解析誤差共分散行列が得られているとして、時刻 t_1 から時刻 t_K までの同化ウィンドウ内の観測データを一括処理して、その期間内の状態変数を推定する。

$$\begin{pmatrix} \mathbf{x}_1^a \\ \vdots \\ \mathbf{x}_K^a \end{pmatrix} = \begin{pmatrix} \mathbf{x}_1^f \\ \vdots \\ \mathbf{x}_K^f \end{pmatrix} + \widetilde{\mathbf{K}} \begin{pmatrix} \mathbf{y}_1 - H_1(\mathbf{x}_1^f) \\ \vdots \\ \mathbf{y}_K - H_K(\mathbf{x}_K^f) \end{pmatrix}$$

カルマンゲイン

$$\widetilde{\mathbf{K}} = \widetilde{\mathbf{P}}^f \widetilde{\mathbf{H}}^{\mathrm{T}} \left(\widetilde{\mathbf{R}} + \widetilde{\mathbf{H}} \widetilde{\mathbf{P}}^f \widetilde{\mathbf{H}}^{\mathrm{T}} \right)^{-1}$$

$$\widetilde{\mathbf{P}}^f := \begin{pmatrix} \mathbf{P}_{11}^f & \cdots & \mathbf{P}_{1K}^f \\ \vdots & & \vdots \\ \mathbf{P}_{K1}^f & \cdots & \mathbf{P}_{KK}^f \end{pmatrix}, \quad \widetilde{\mathbf{H}} := \begin{pmatrix} \mathbf{H}_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{H}_K \end{pmatrix}, \quad \widetilde{\mathbf{R}} := \begin{pmatrix} \mathbf{R}_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{R}_K \end{pmatrix}$$

異なる時刻の予測値の間の誤差共分散行列 $\mathbf{P}_{kl}^f$ は、予測値のアンサンブルから近似計算できる(サンプリングエラーへの対策が必要)。

http://wtk.gfd-dennou.org/2012-09-24/tsuyuki/pub/

Slide by T. Miyoshi (on EnKF)

http://www.dpac.dpri.kyoto-u.ac.jp/thorpex/activities/20051119/slides/miyoshi.pdf