



# THE MASS RATIO DISTRIBUTION OF SPECTROSCOPIC BINARY STARS

**Henri M.J. Boffin**

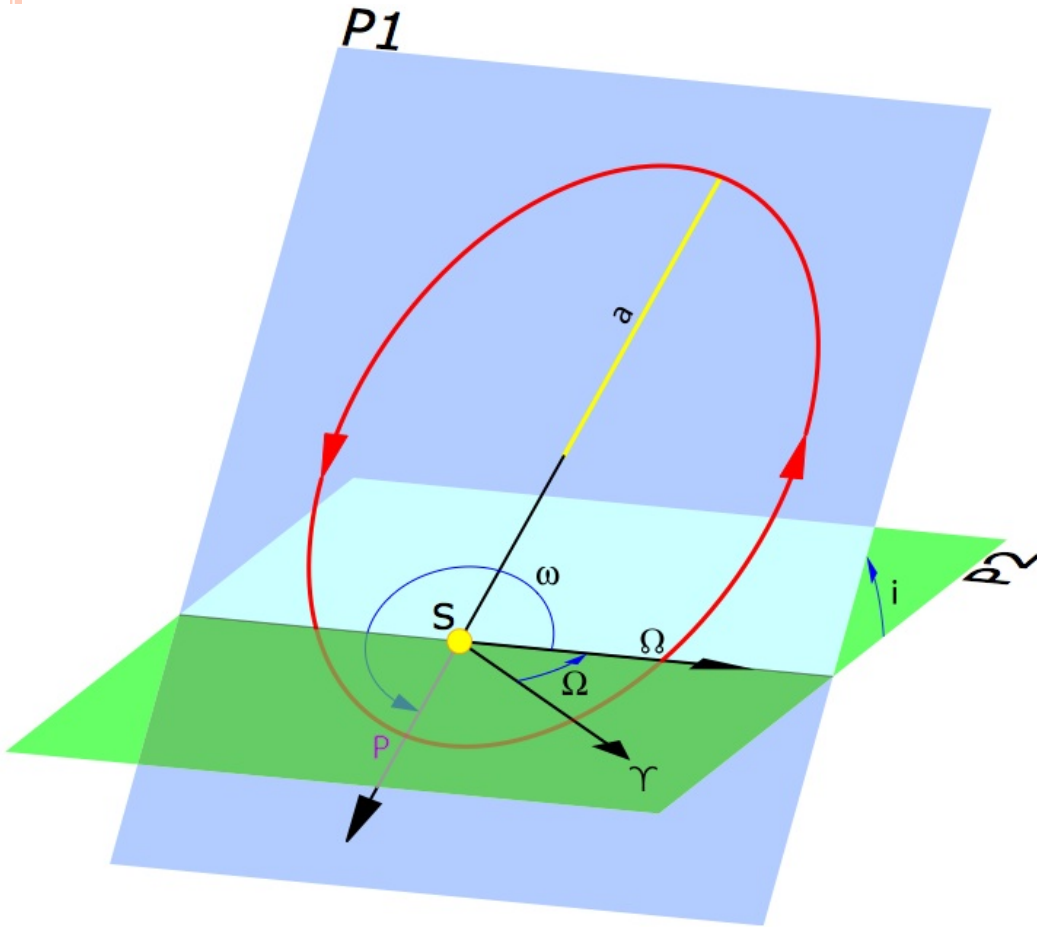
ESO, Chile



Orbital parameters  
 $i, \omega, \Omega, T_0$   
 $a, P, e$

Distribution of these elements important to understand binary formation and compare to models

# BINARY SYSTEM



# THE IMPORTANCE OF $f(q)$

- $M_A$  and  $M_B$  and the mass ratio  $q=M_B/M_A$  distributions
- Binary formation mechanisms? e.g. random pairing,  $f(q)$  constant,  $q$  depends on  $M_A$ ?
  - we already know that multiplicity is function of  $M_A$
  - and possibly  $f(q)$  also
- Evolution of binary systems? e.g. twins population?
- Comparison between populations or families of stars e.g. PRGs and normal G-K giants;  
long and small periods

# SPECTROSCOPIC BINARIES

- Many systems (and almost all exoplanets) are spectroscopic binaries → Only projection ( $\sin i$ )
- A few are eclipsing (or transiting) →  $i$  known
- SB: SB1 or SB2 ( $q$  known)
  - depends on  $\Delta m$  (generally  $\Delta m \approx 1.5 - 2.5$  mag)
  - For MS star, this means SB2 if  $q \geq \sim 0.65$
  - For giants, one needs  $q \approx 1$

# SPECTROSCOPIC BINARIES

$$f(M) = \frac{(M_B \sin i)^3}{(M_A + M_B)^2} = \frac{K_A^3 P}{2\pi G} (1 - e^2)^{3/2}$$

For exoplanet:

$$f(M) \approx \frac{M_B^3 \sin^3 i}{M_A^2} \quad (\text{since } M_B \ll M_A).$$

$M_A$  known  $\rightarrow$  one can obtain  $M_B \sin i$

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As we can assume  $i$  is randomly distributed, **if  $M_A$  is known**, one could thus use the distribution of

$$Y = f(m)/M_A = q^3 / (1+q)^2 \sin^3 i$$

to determine the distribution of  $f(q)$

# THE WRONG WAY

- *“There is always an easy solution to every human problem – neat, plausible, and **wrong**.”*

H.L. Mencken, 1917

- Simplest way: replace  $\sin^3 i$  by  $\langle \sin^3 i \rangle$   
e.g. Aitken 35; Trimble 90; Trimble 09 (!)

# THE WRONG WAY

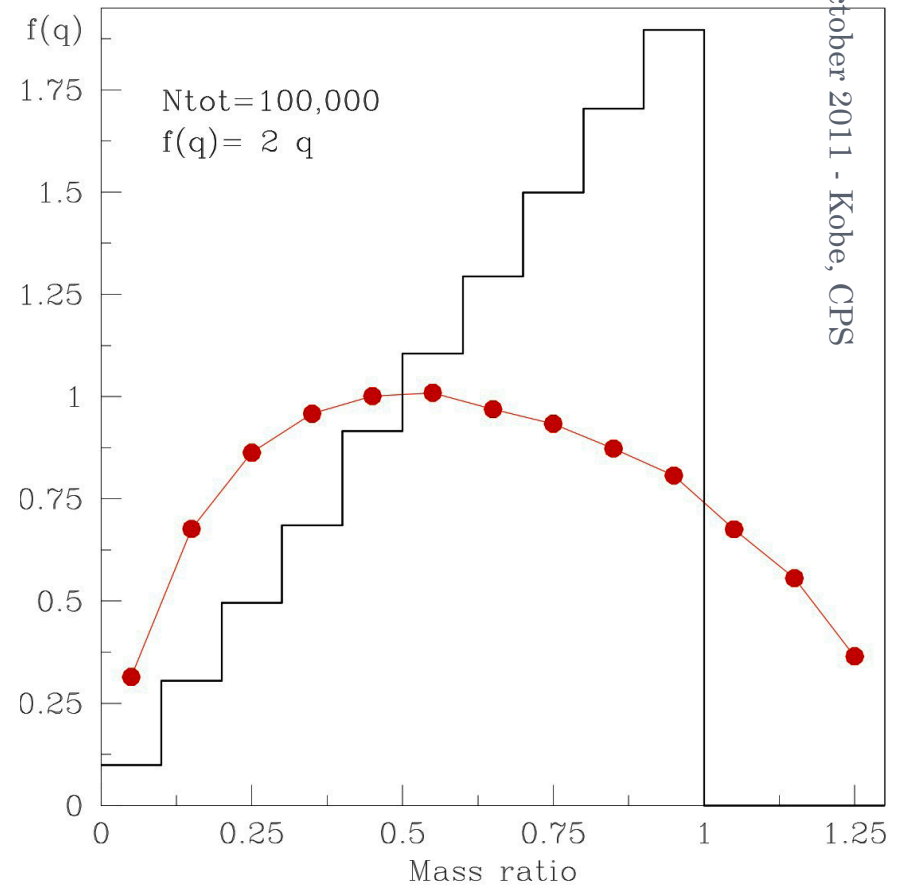
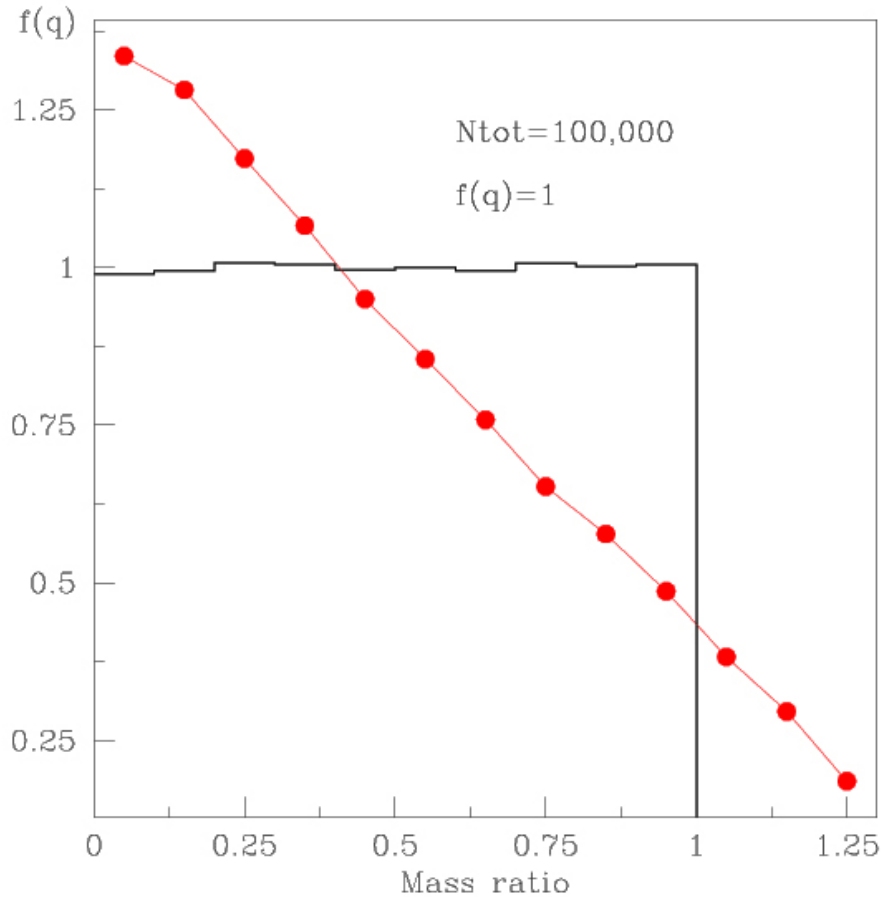
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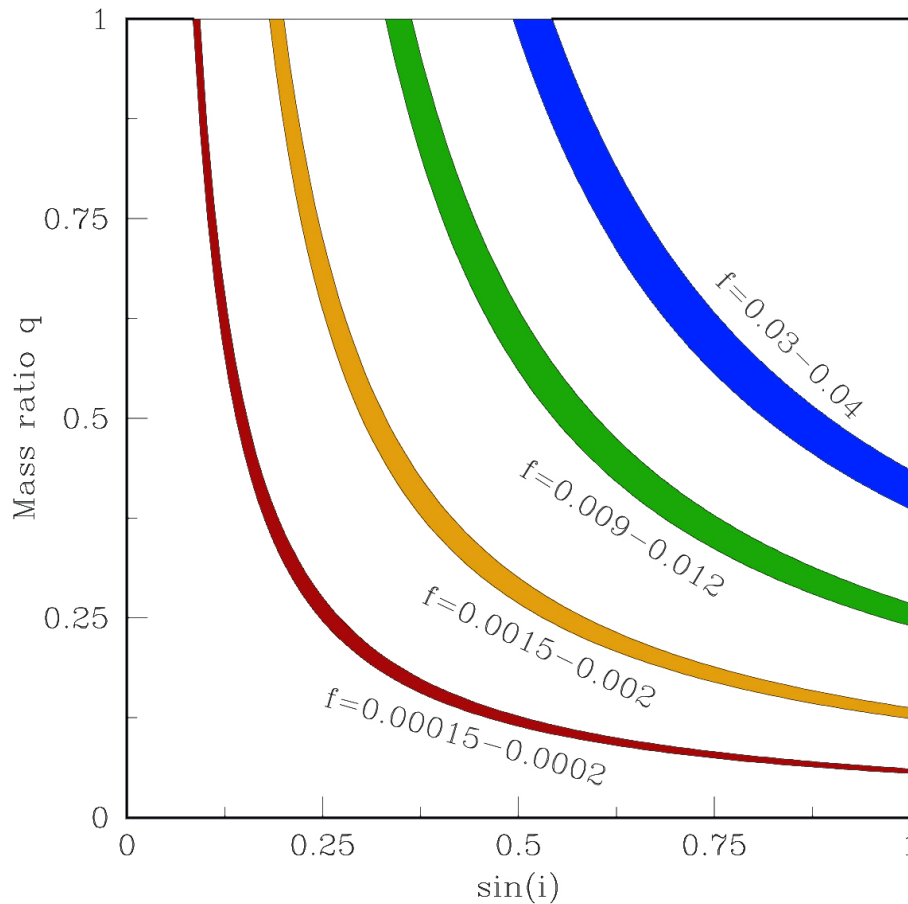
**THIS IS WRONG!**

# ASSUMING MEAN $\sin^3 i$ DOES NOT WORK!





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e.g. Mazeh & Goldberg 92

Errors arise because for a given  $f(m)$ ,  $i$  and  $q$  are not independent anymore and so the mean cannot be the same as when the full range of  $i$  is allowed

Similar for exoplanets:  $a$  *posteriori* distribution of  $\sin i$  is dependent on  $m_B$  distribution (see also Ho & Turner 11)

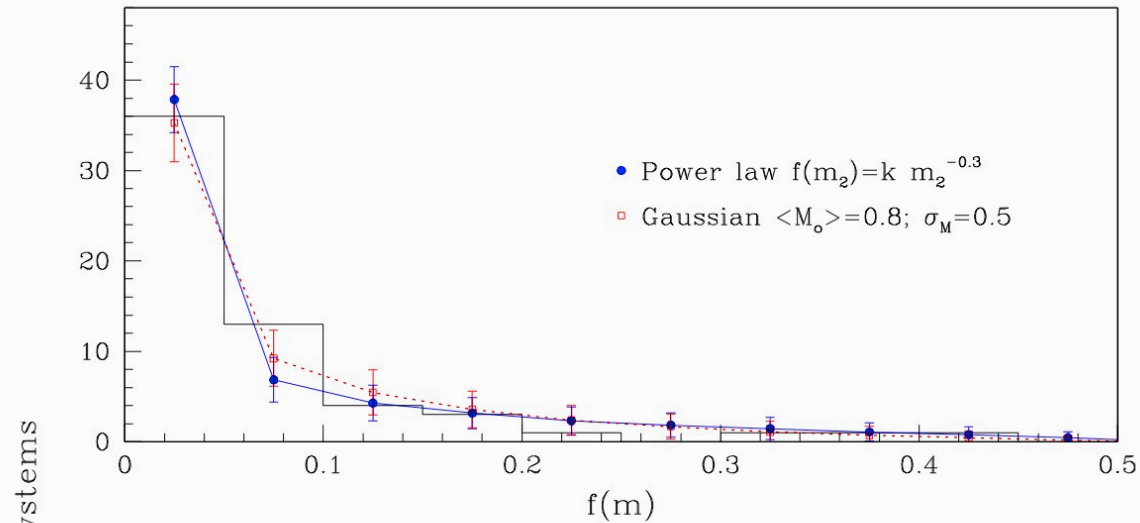
Error is also due to the shape of  $f(\sin^3 i)$ , cf. Halbwachs 87

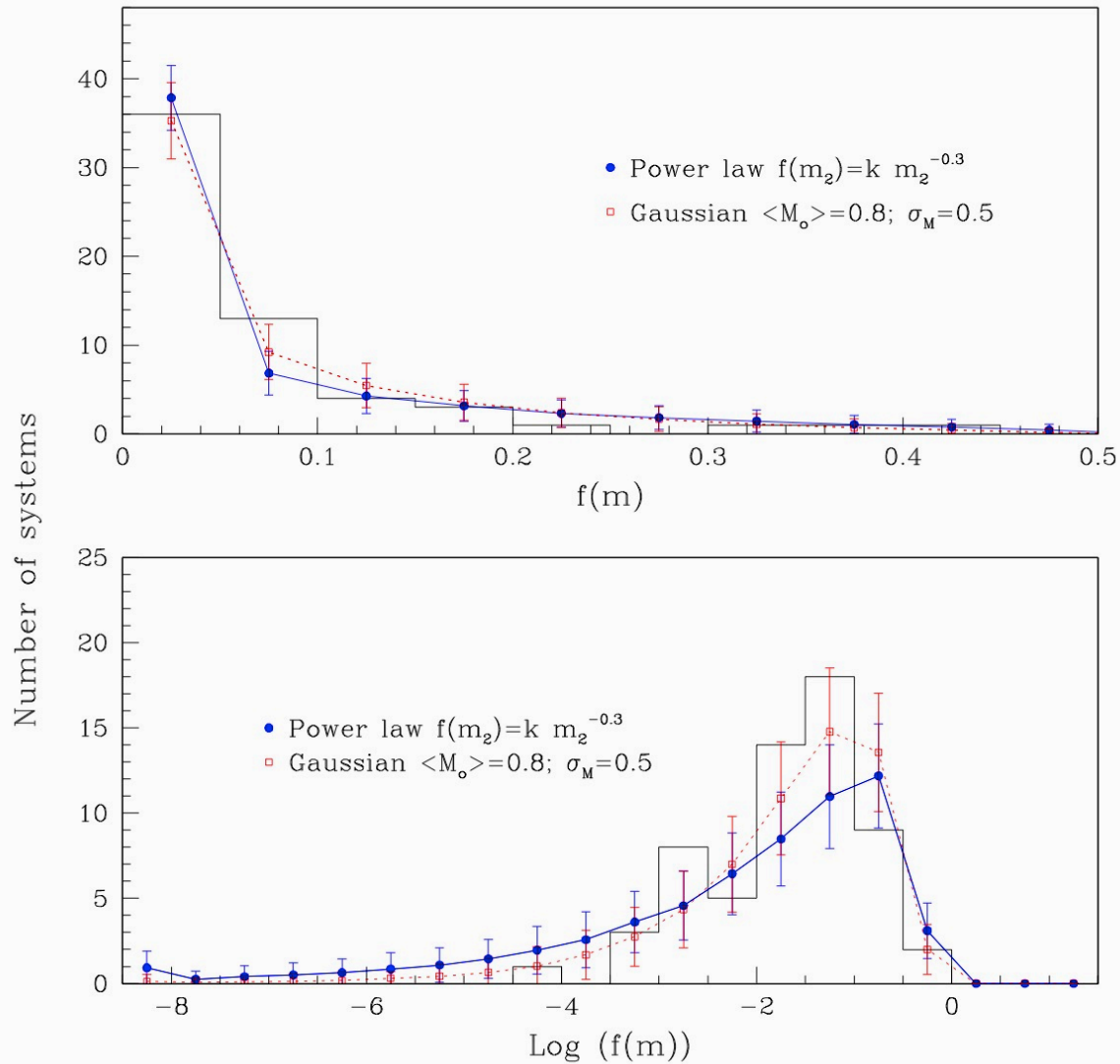
# FUNCTIONAL FORM FITTING

- Instead, one could assume a  $f(q)$  and then compute the  $f(Y)$  and compare to observed one – using a minimisation method  
(Jaschek & Ferrer 72; Halbwachs 87;  
see also Tabachnik & Tremaine 02 for exoplanets)
- Disadvantage: need to assume functional form and is thus very limited
- Advantage: not tempted to see spurious peaks
- **Important (although obvious) remark:** one should not compare to distribution of  $f(m)$  but distribution of  $\log f(m)$ , cf. wide dynamic range

## Am stars

- 60 orbits (for 53 Am systems)
- Fit distribution of mass function
- $M_1 = 2 M_\odot$
- Assume functional forms: gaussian and power law





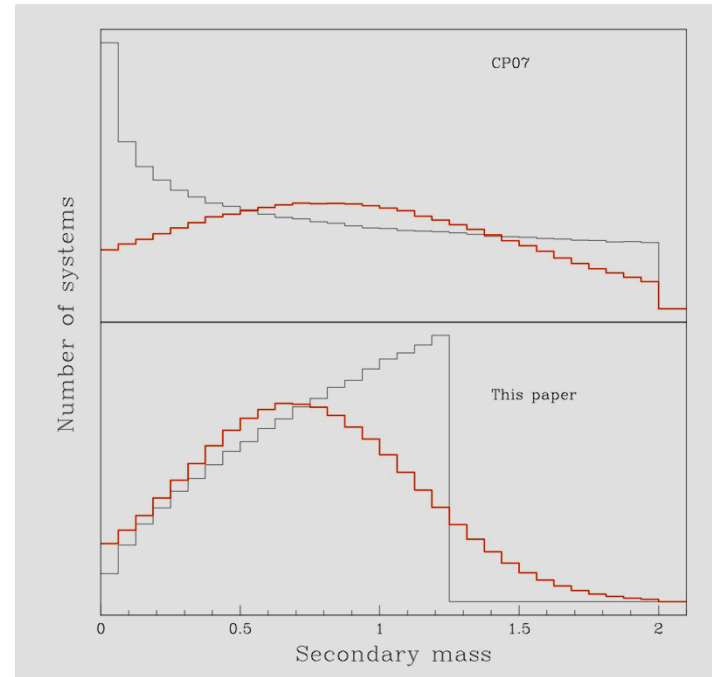
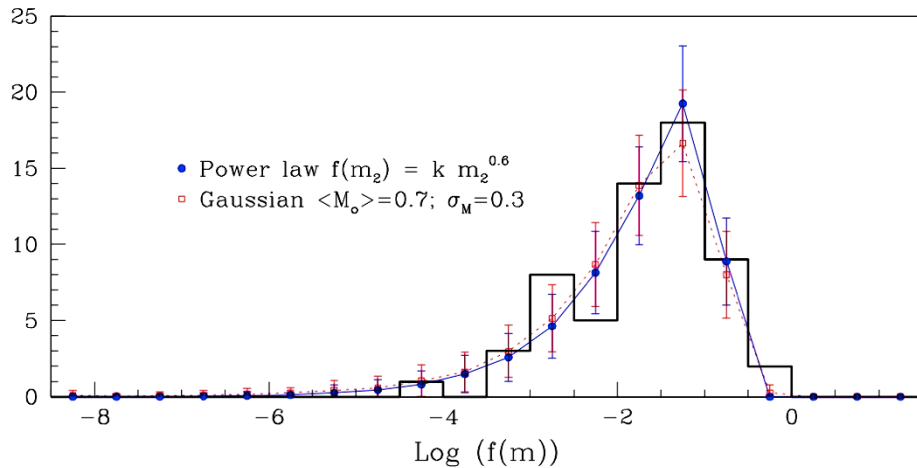
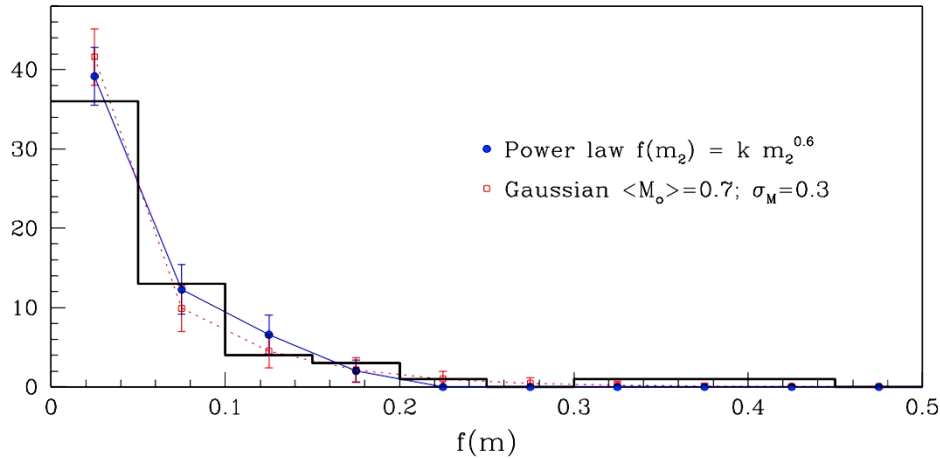
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**INCORRECT!**

**(as seen when compare to  $\log f(m)$  )**

# REVISITING CP07

- Fit the  $\log f(m)$  distribution
- Power law with positive index!
- Need to limit to  $M_2 < 1.25 M_\odot$
- Gaussian is narrower



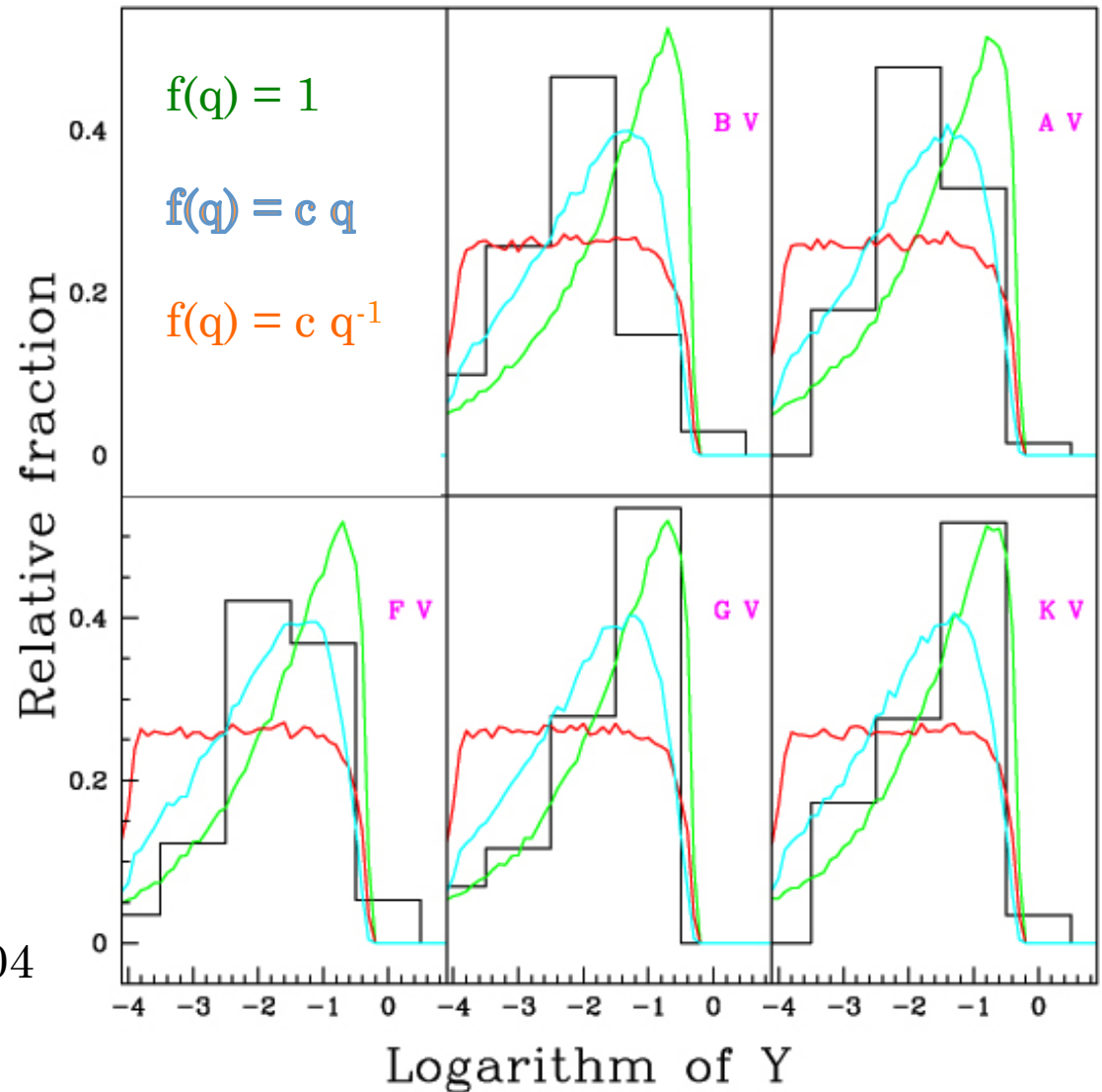
Boffin 10

# $S_B^9$ DATABASE

Spectral Type	Number of systems	$\langle q \rangle$
B V	101	0.3789
A V	67	0.4686
F V	57	0.6164
G V	43	0.5673
K V	29	0.6105

Pourbaix, Jancart & Boffin 04

## Distribution of Y



## INVERSION METHOD

From  $Q=q^3 / (1 + q)^2$ , we have  $f(m) = M_A Q^3 \sin^3 i$

Thus  $Y(Q) = Q \sin i$  is available from observations. The distribution  $\psi(Q)$  we are looking for is thus given by

$$\Phi(Y) = \int_0^{\infty} \Psi(Q) \Pi(Y|Q) dQ.$$

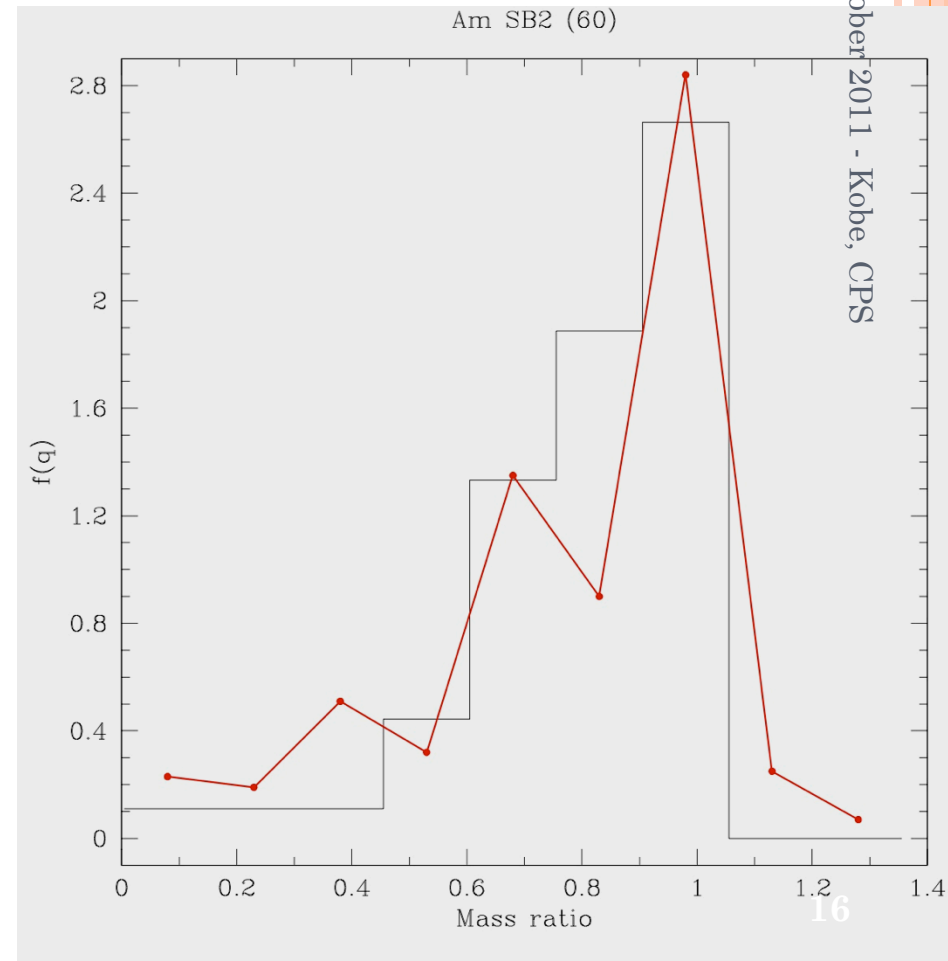
As  $f(i) = \sin i$ , this reduces to an Abel equation.

One can thus solve it, either by numerically computing it (need smoothing) or using the Lucy-Richardson inversion algorithm

Boffin+ 92, 93, Cerf & Boffin 94, Mazeh & Goldberg 92

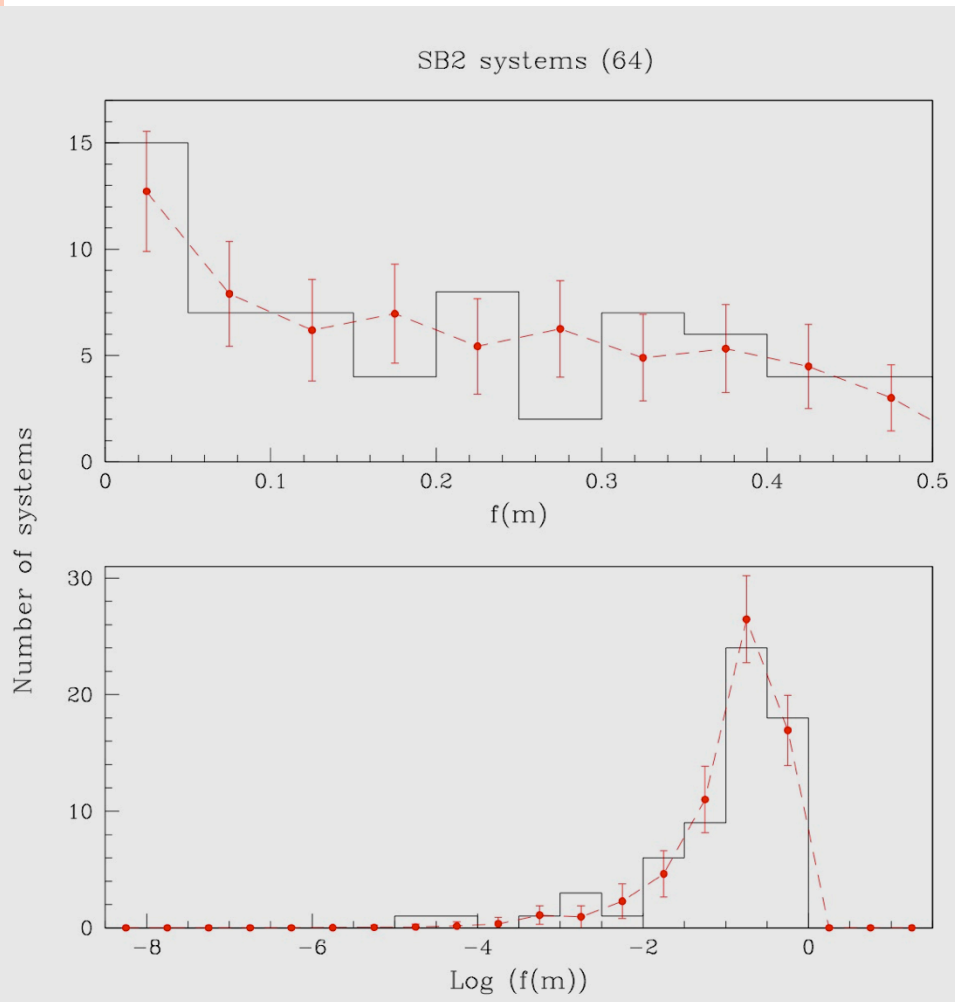
# BACK TO AM STARS: EXTENDING THE SAMPLE

- Literature search → created a new catalogue to have more orbits
- 162 orbits of Am stars : 98 SB1 and 64 SB2
- For SB2, we directly have  $q$
- I apply Richardson-Lucy inversion method – check with SB2





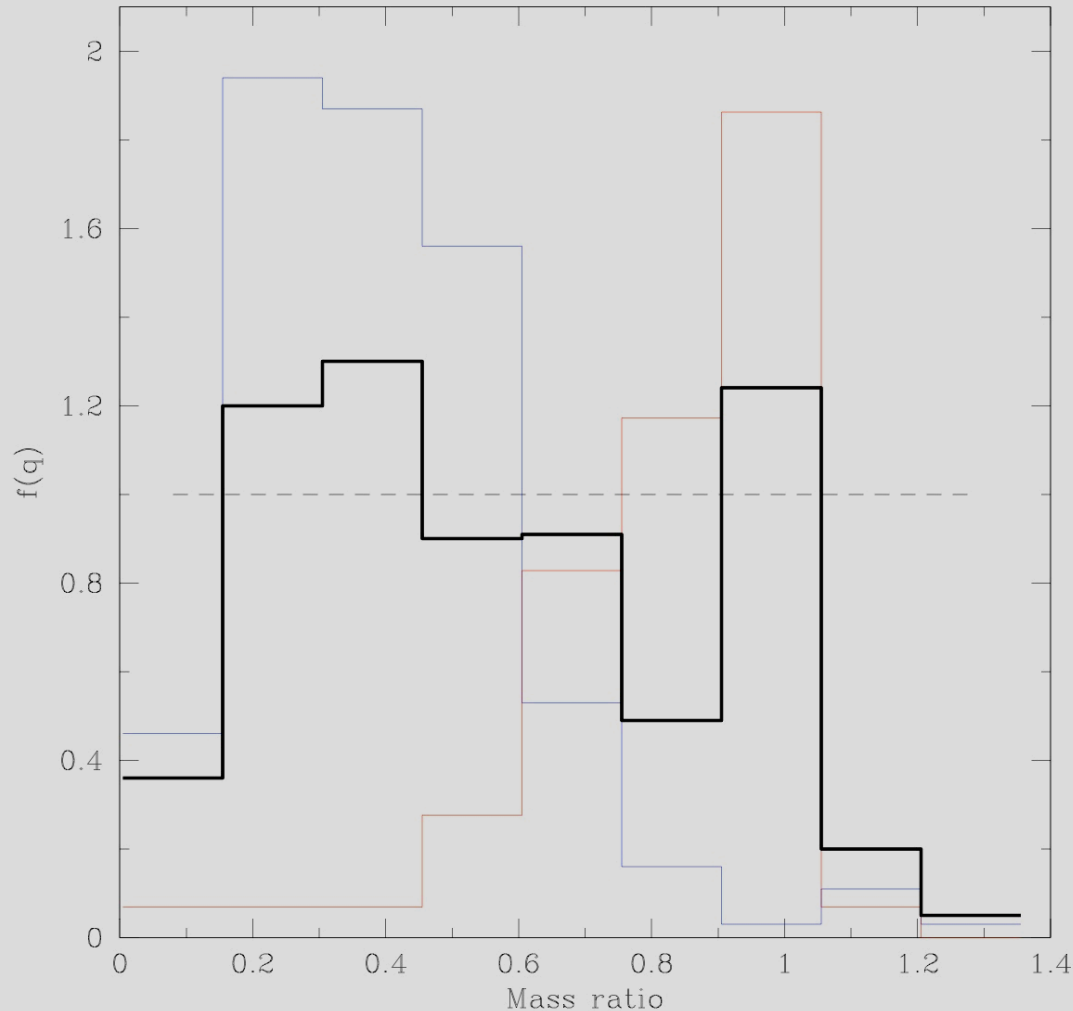
# EXTENDED AM STAR SAMPLE



- With SB2, I can also check the methodology, i.e.
    - random  $i$
    - $M_1$  constant ( $=2 M_{\odot}$ )
    - fitting  $\log f(m)$
  - works!
- can apply to the whole sample of SB1

# MASS RATIO DISTRIBUTION

Am SB1 (R-L inversion) + SB2 (observed)



- SB1 : R-L method
  - SB2 : direct
  - Or SB1 + SB2 : R-L method
- compatible
- Final  $f(q)$  will depend on ratio between SB1 and SB2
  - Observational biases difficult to assess!

# OBSERVATIONAL BIASES

- Magnitude selection (Öpik effect)
- Detection limit  $K_D$ , where  $K < K_D$  are not found
  - Typically  $K_D > 3-4 \sigma_{RV}$
- Orbits too long cannot be obtained (no solution but also  $K$  too small)
- One need to be aware of these and, if sure we understand them, correct them, or be sure we do not need to care about them.

# THE SOLAR-LIKE SAMPLE – NEARBY AND IN CLUSTERS

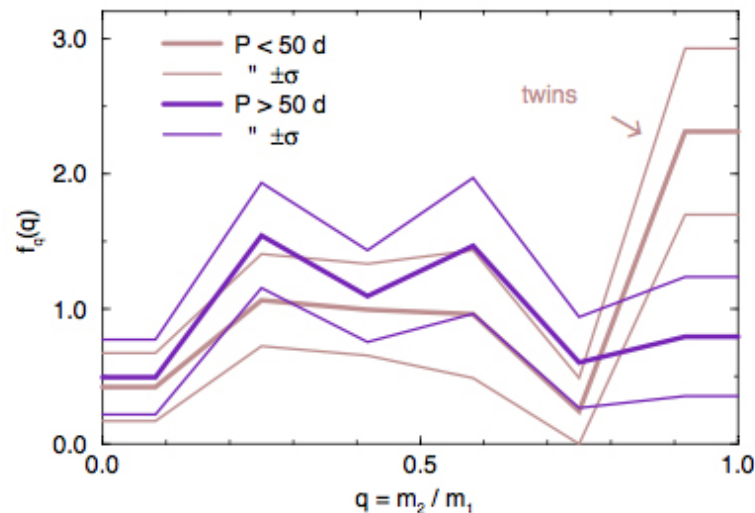
## *f(q) for binaries with $P < 10$ y*

*For G-K-M  
primaries, 2  
modes:*

*•  $0,1 < q < 0,7$ ,*

*With a "brown  
dwarf desert"  
( $q < 0,1$ ) vanishing  
for  $P > 2-3$  years*

*•  $q > 0.8$ , with a peak around  $q=1$  ("twins"). Vanishing  
when  $P$  increasing.*



Halbwachs+ 03

# BINARY FORMATION MECHANISMS: CONSTRAINTS FROM THE COMPANION MASS RATIO DISTRIBUTION

MADDALENA M. REGGIANI<sup>1</sup> AND MICHAEL R. MEYER<sup>1</sup>

2011, ApJ

27 October 2011 - Kobe, CPS

TABLE 1  
SAMPLE PROPERTIES

Sample	Ref <sup>a</sup>	Primary Type	No. Multiple Systems	Separation Range (AU)	$q_{lim}$
Field	1	M	27	1-2400	0.2
Field	2	F/G	30	28-1590	0.1
ScoOB2	3	A/late-B	60	29-1612	0.05
Pleiades	4	F/G	22	11-910	0.2
$\alpha$ Persei	5	F/G	18	26-581	0.25
Chamaeleon I	6	G/K <sup>b</sup>	13	20-800	0.1
Taurus	7	G/K <sup>c</sup>	40	5-5000	0.1

<sup>a</sup> References: (1) Fischer & Marcy (1992), (2) Metchev & Hillenbrand (2009), (3) Kouwenhoven et al. (2005), (4) Bouvier et al. (1997), (5) Patience et al. (2002), (6) Lafrenière et al. (2008), (7) Kraus et al. (2011).

<sup>b</sup> The mass range is 0.55 and 2.2  $M_{\odot}$ , comparable to MH09

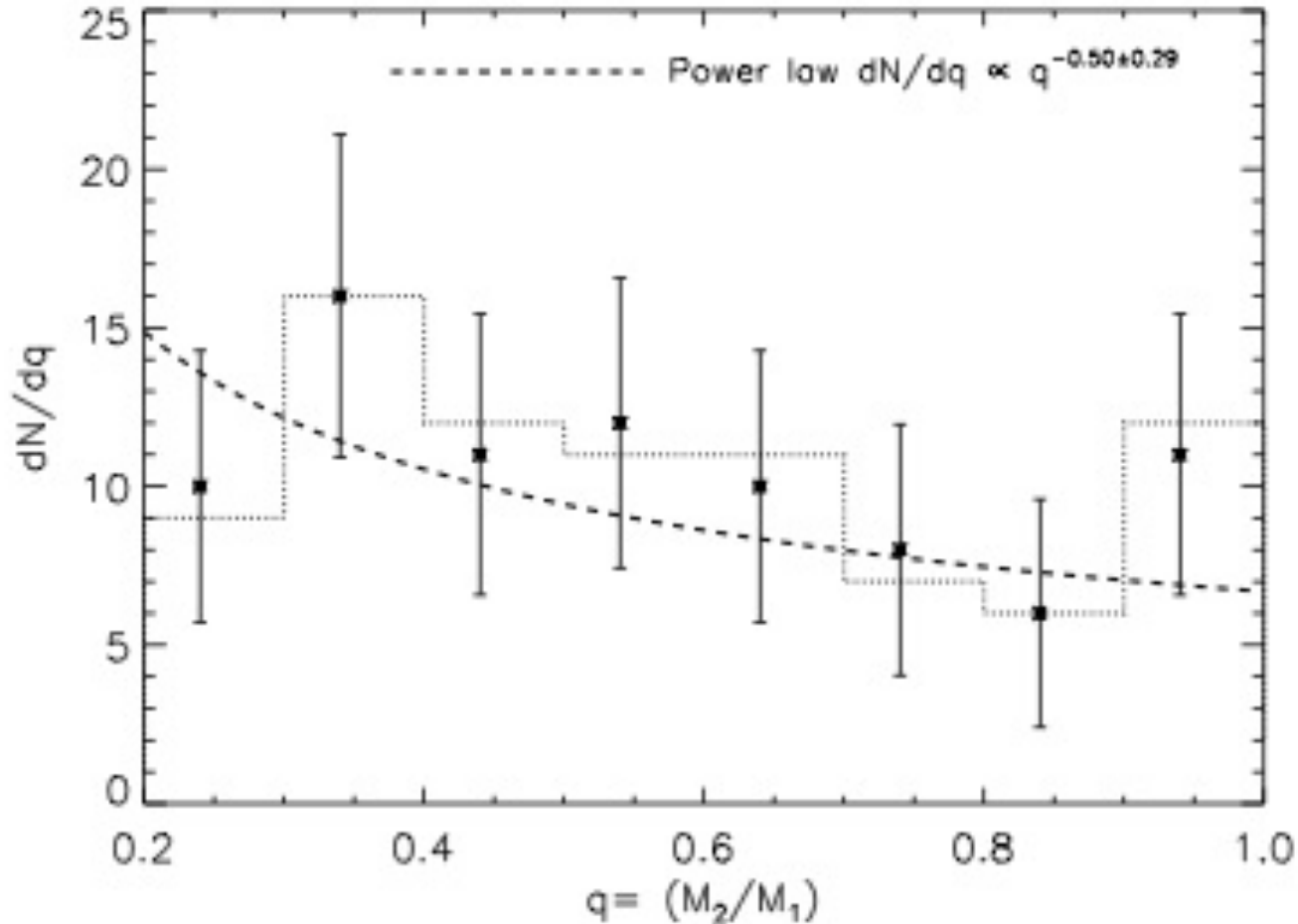
<sup>c</sup> The mass range is 0.7 and 2.7  $M_{\odot}$ , comparable to MH09

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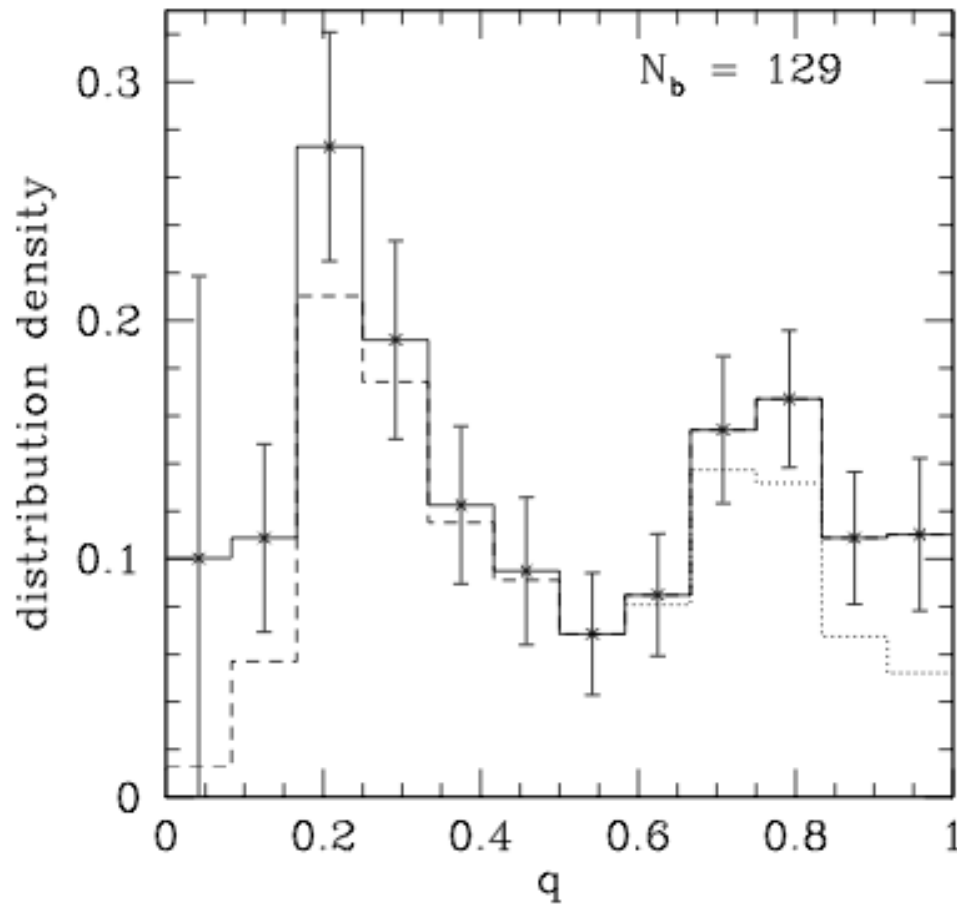
27 October 2011 - Kobe, CPS



Stars are not taken from an IMF (capture scenario)

Could be flat

# ANOTHER SAMPLE: LARGE PROPER MOTION SBs



129 SBs with  $P < 1250$  d  
 $K_{\min} = 2.5$  km/s  
Primary mass estimated  
Apply Öpik correction

distribution includes two populations, one with a high, asymmetric peak at  $q \sim 0.2$  and another with a smaller peak at  $q \sim 0.8$

# LARGE PROPER MOTION SBs

Halo subsample

Disc subsample

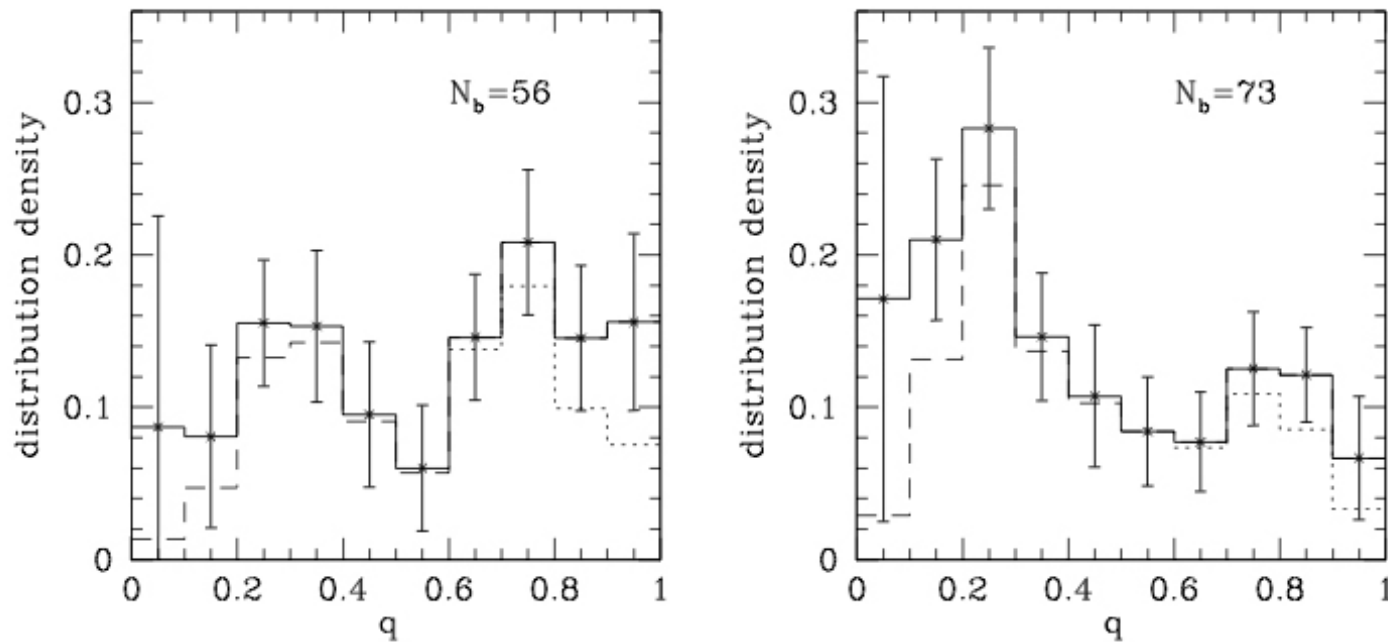


FIG. 6.—Mass-ratio distributions of the halo subsample (*left*) and of the disc subsample (*right*). See Fig. 3 for the meaning of the different line types.



# Large proper motion SBs

Split according to mass

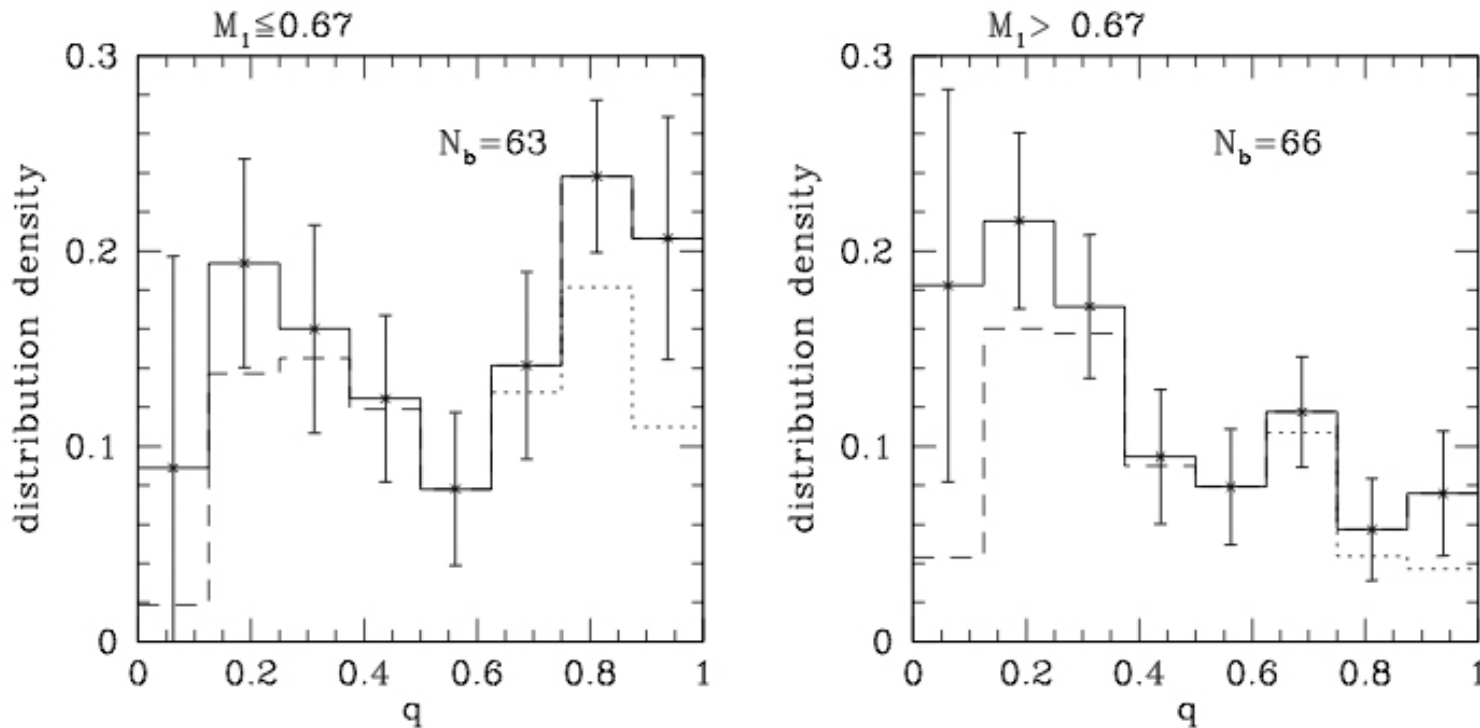
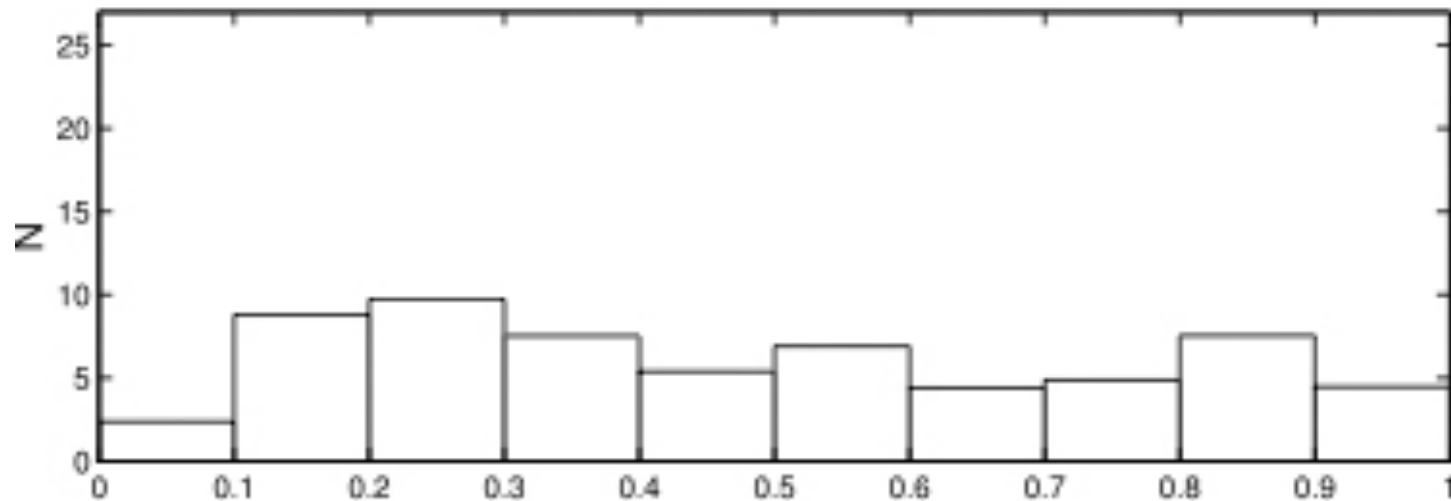


FIG. 7.—Same as Fig. 6, but for the subsample with low-mass primaries (*left*) and high-mass primaries (*right*)

## SAMPLE OF SB2 (USING IR LINES)

- The peak around  $q \sim 0.3-0.5$  disappears as distribution is flat...

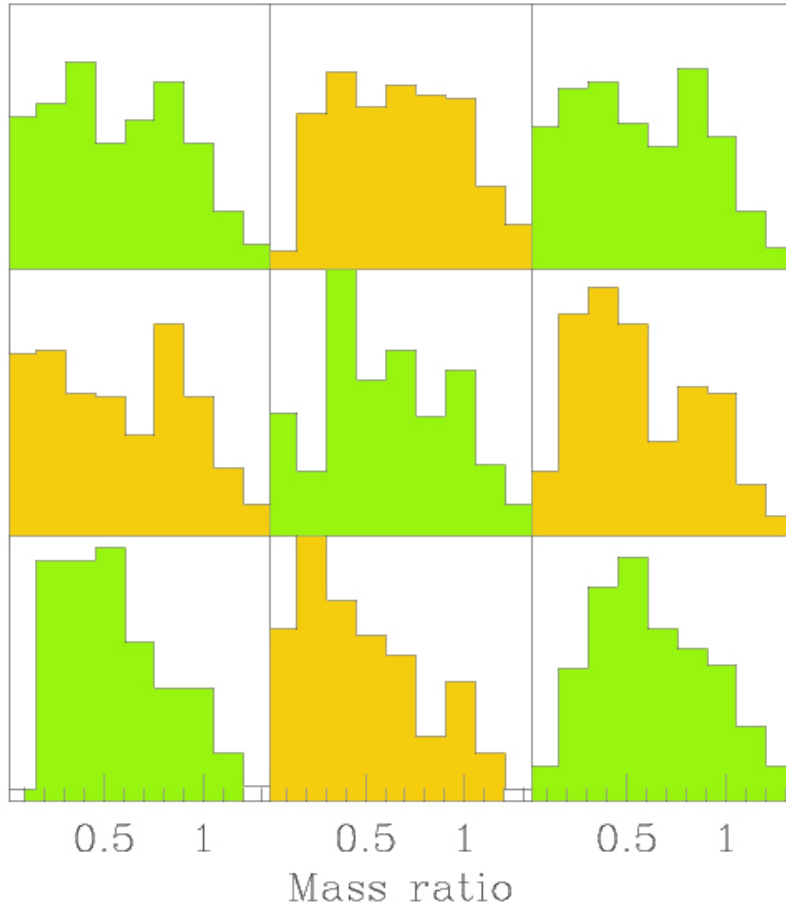


Mazeh+ 03

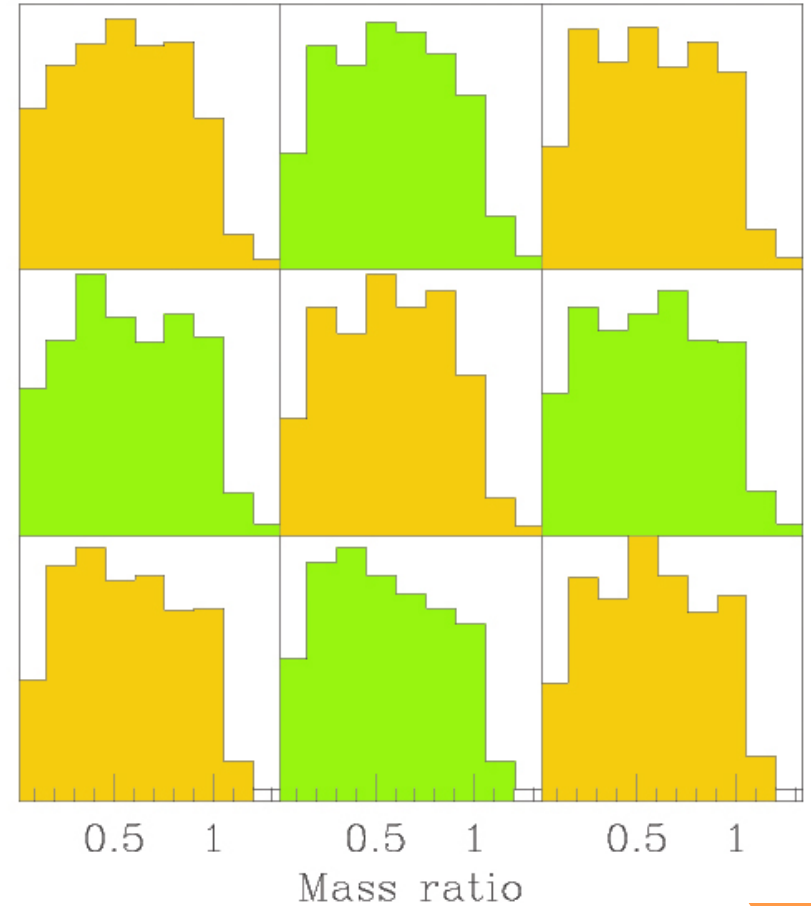
Perhaps, this should serve as a warning: do not overinterpret the data!

# EFFECT OF SAMPLE SIZE ( $f(q)$ UNIFORM)

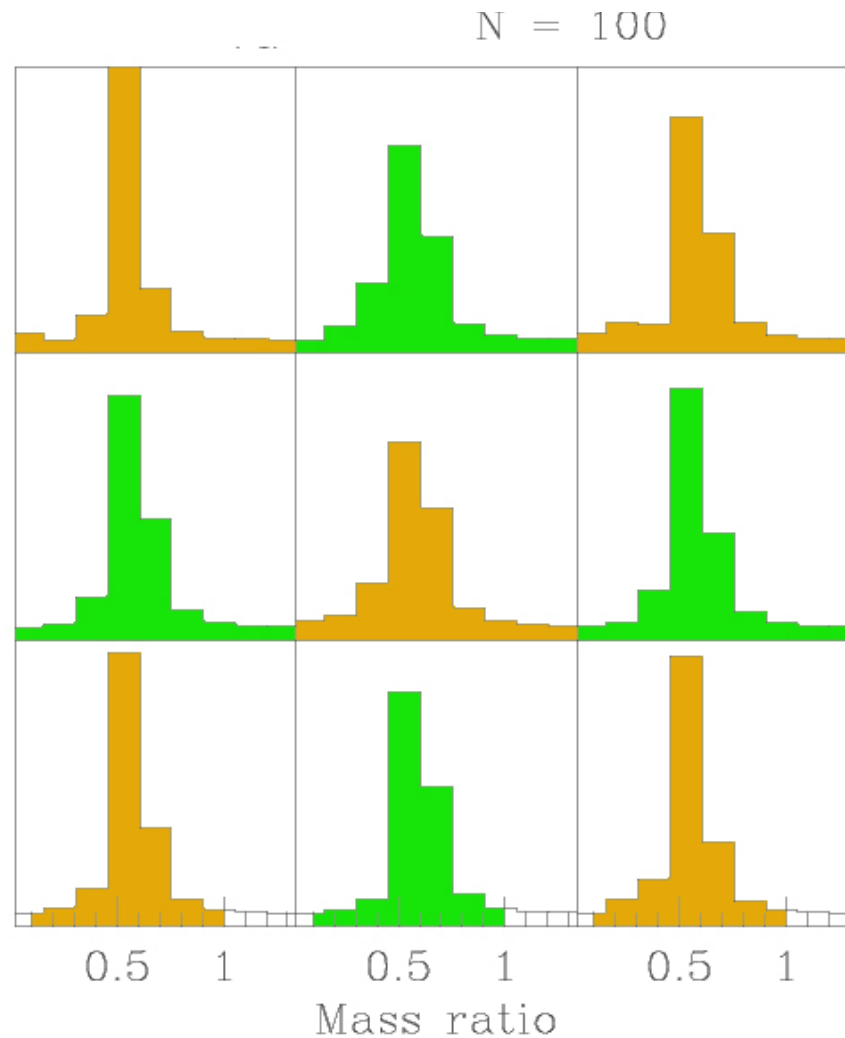
$f(q) = \text{cst}; N = 100$



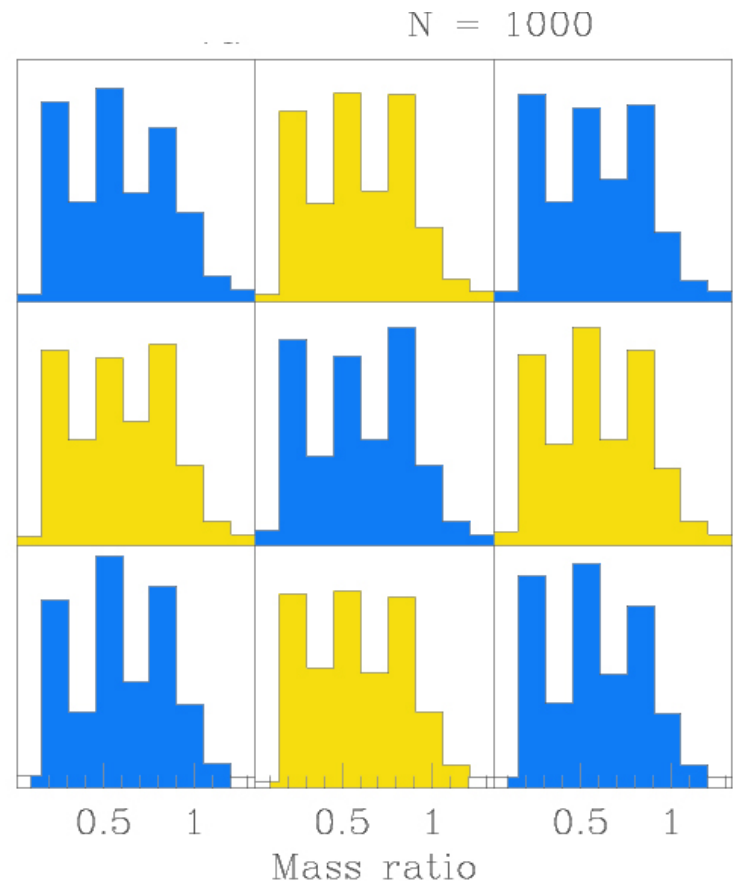
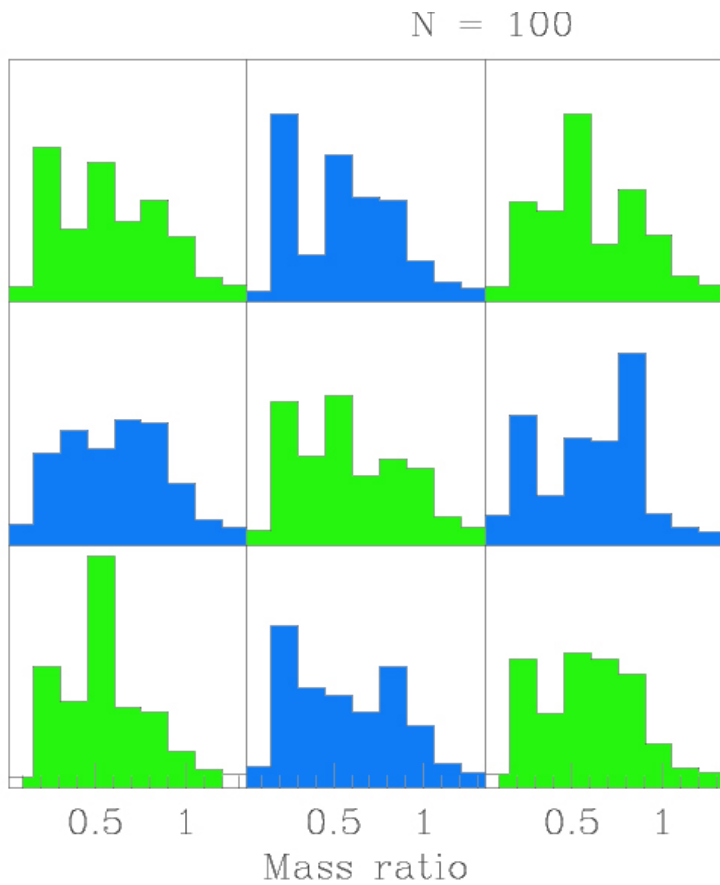
$f(q) = \text{cst}; N = 1000$



# SINGLE PEAK SEEMS MORE ROBUST

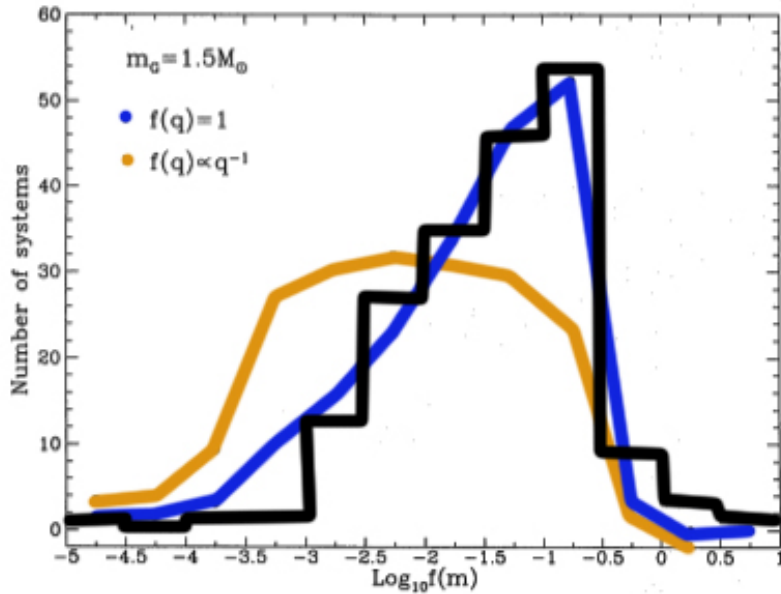


# 3 PEAKS

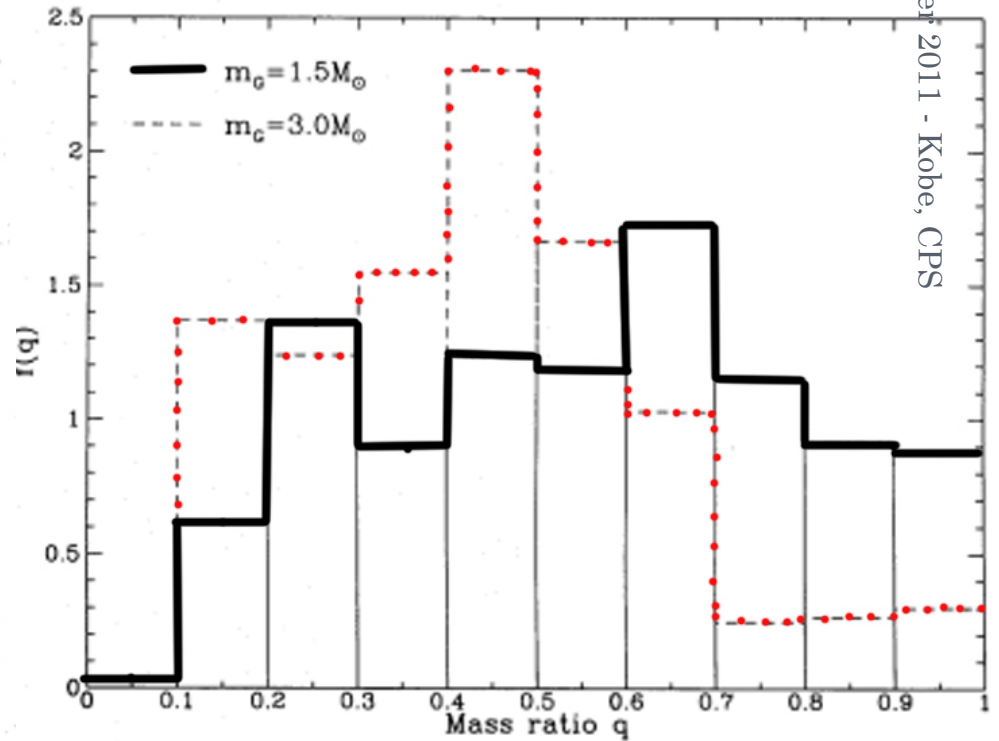


See also Brown 11, for application to exoplanets

# A CATALOGUE OF 213 G-K GIANTS

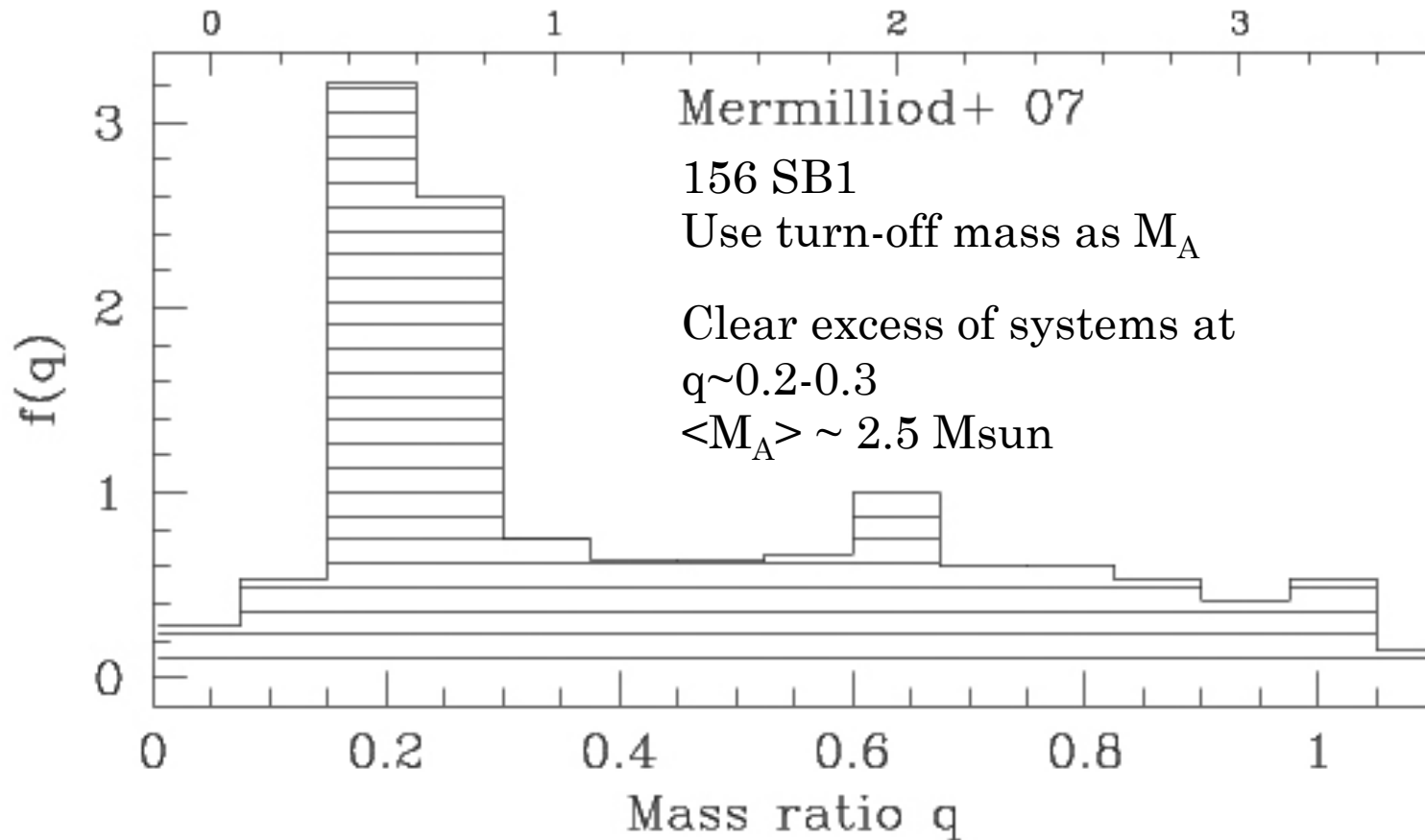


**Fig. 10.** Distribution of  $\log f(m)$  computed by the Monte Carlo technique using a giant mass  $m_G = 1.5 M_\odot$  and  $f(q) = 1$  (●) or  $f(q) \propto q^{-1}$  (■), compared to the observed one (solid line)



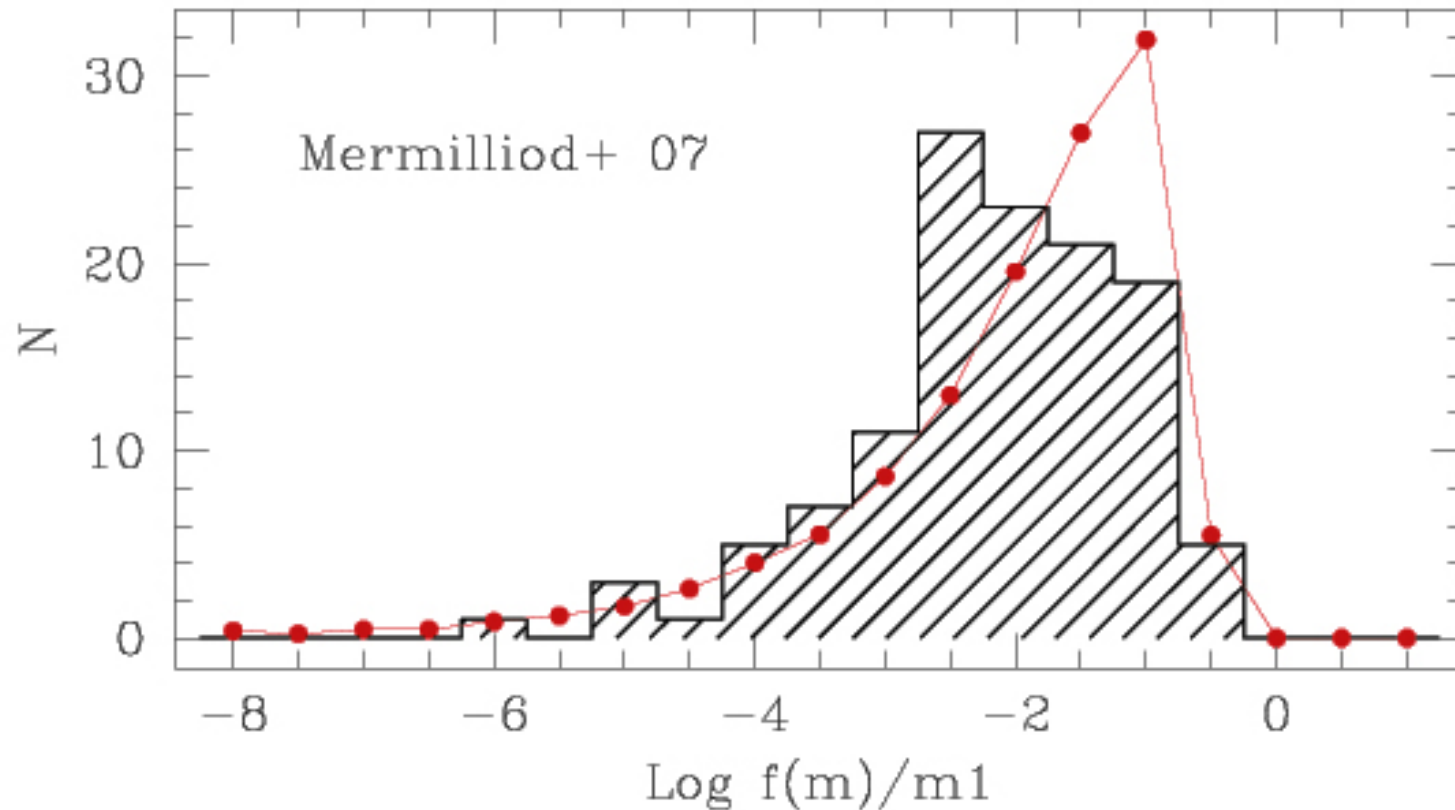
Boffin, Cerf & Paulus 93

# A MORE CONTROLLED SAMPLE: RED GIANTS IN OPEN CLUSTERS



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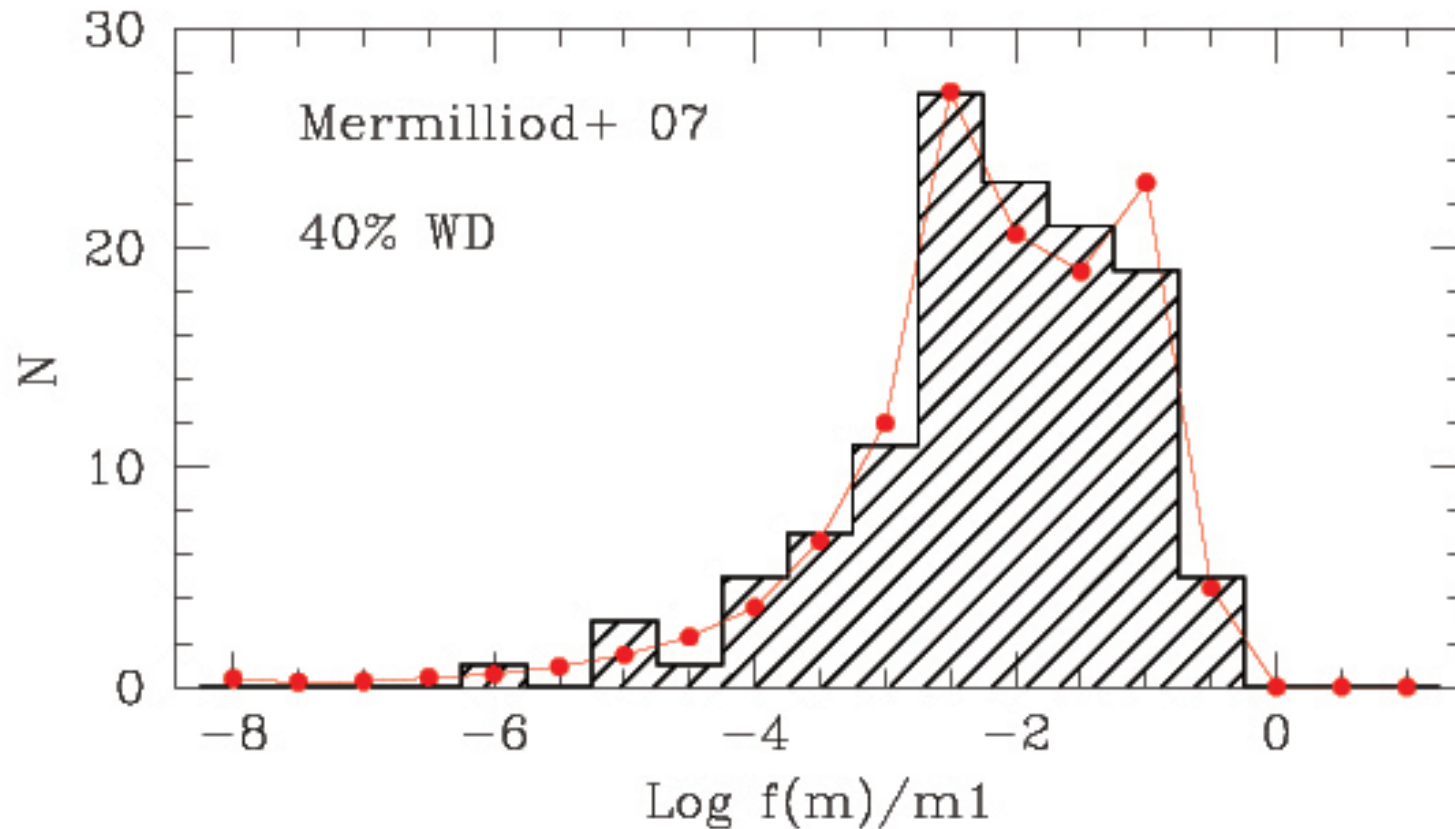
Checking that a uniform distribution does not do the trick





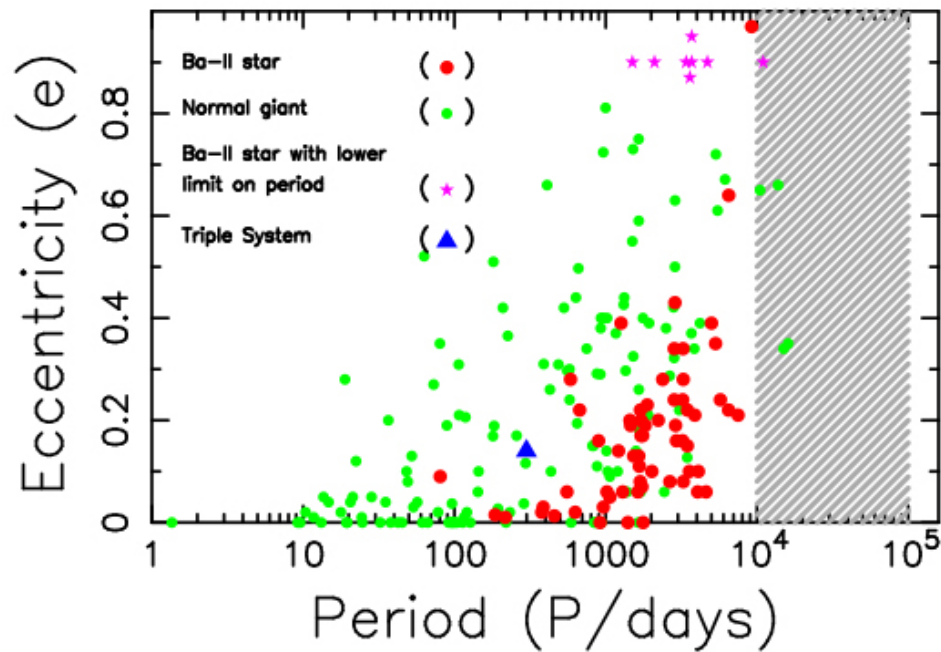
# A MORE CONTROLLED SAMPLE: RED GIANTS IN OPEN CLUSTERS

Adding low-mass objects is better – but not perfect



# BARIUM STARS

data from Jorissen et al (1998):



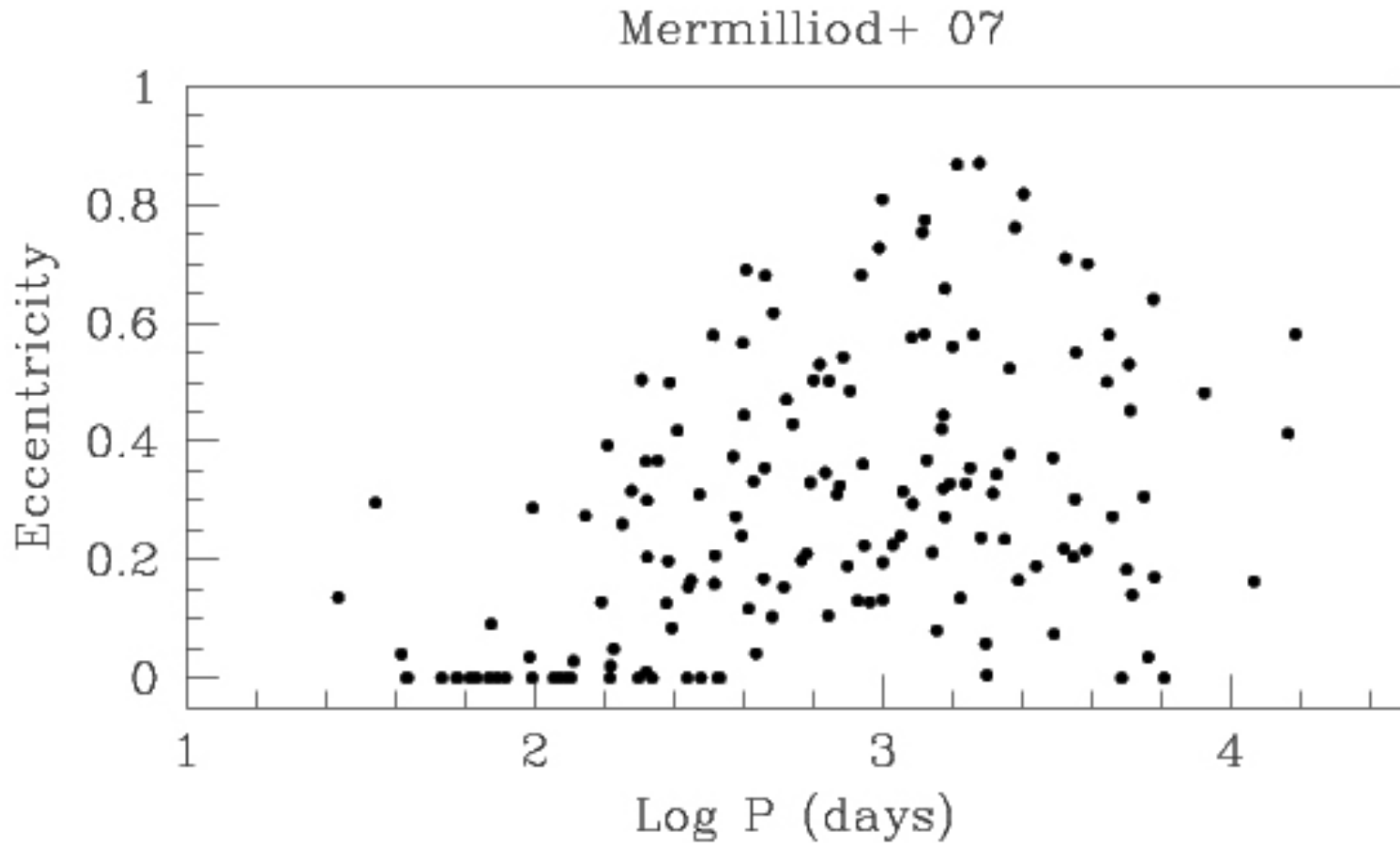
Mass transfer by wind

Predictions of model:

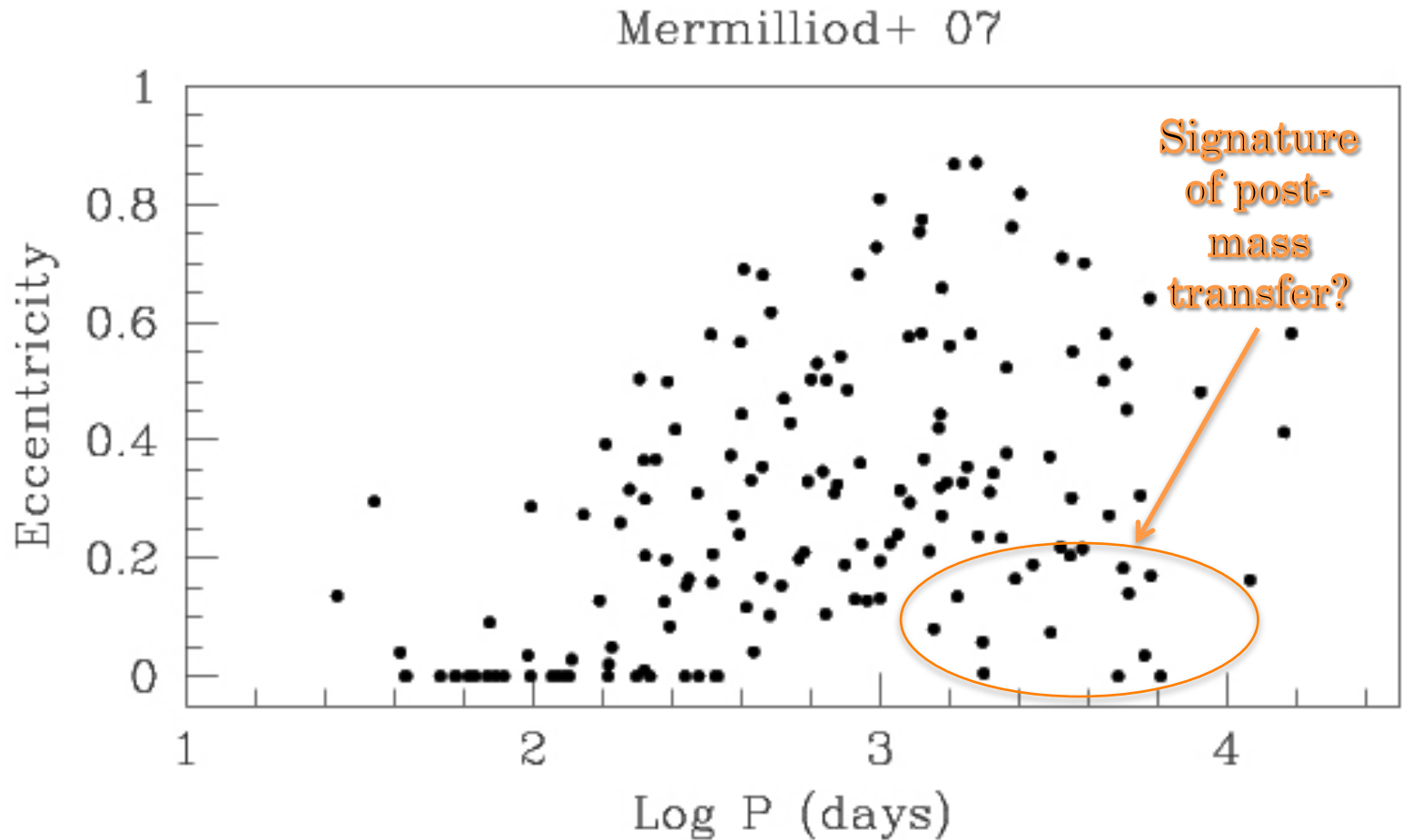
Barium stars have longer period than normal giant

Anomalies should be correlated with orbital period

# RED GIANTS IN OPEN CLUSTER

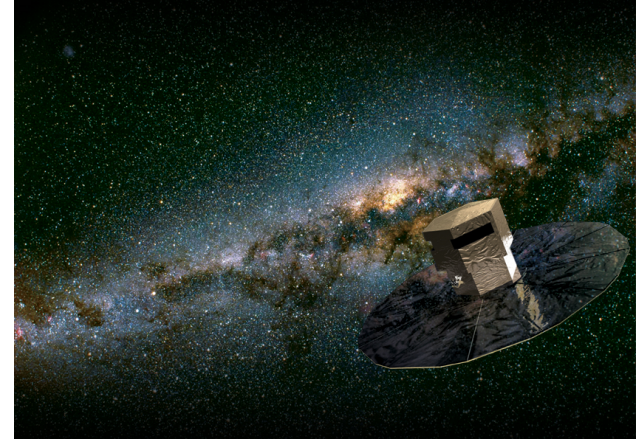


# RED GIANTS IN OPEN CLUSTER



# GAIA SATELLITE

- Launch in June 2013
- measure the positions of  $\sim 1$  billion stars both in our Galaxy and other members of the Local Group, with an accuracy down to  $20 \mu\text{as}$
- perform spectral and photometric measurements of all objects
- derive space velocities of the Galaxy's constituent stars using the stellar distances and motions
- create a three-dimensional structural map of the Galaxy



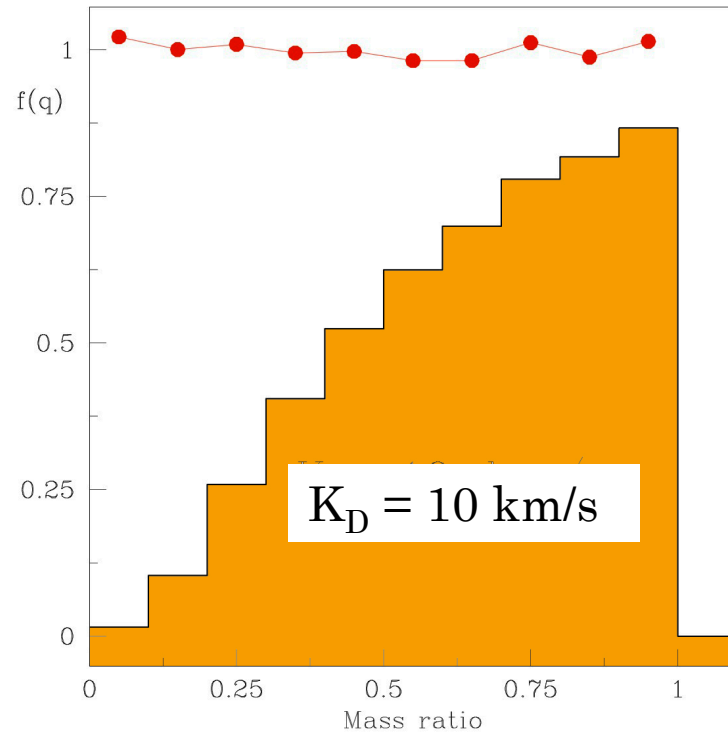
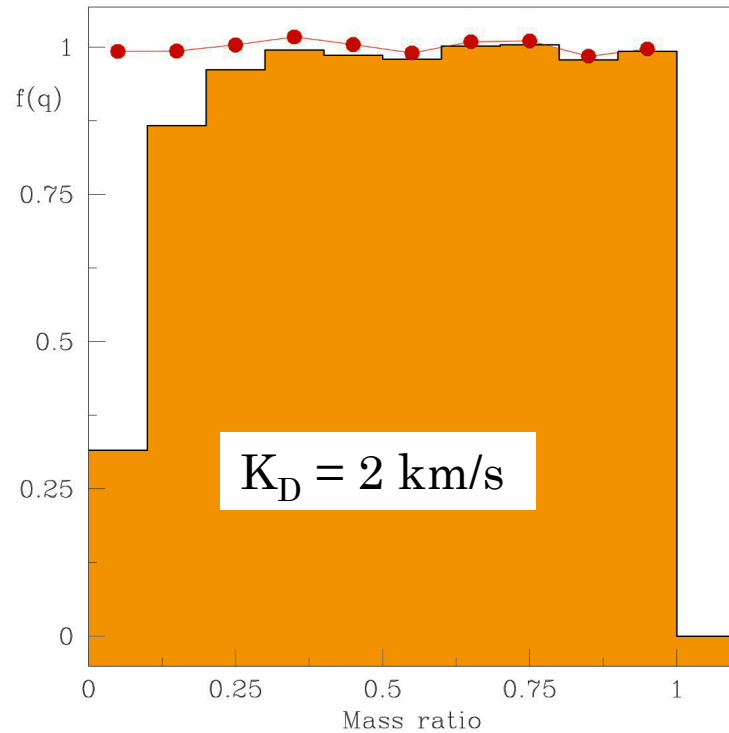
# WHAT ABOUT GAIA?

- GAIA will provide us with a flurry of new SBs
- Observe for 5 years
- $\sim 10^6$  orbits could be derived, finally making it possible to have huge samples for statistical analysis
- The survey will be homogeneous, so the bias should be quantifiable
- Simplest: look only at eclipsing binaries ( $i \sim 90$ )
- But why limit ourselves?

## GAIA (II)

- Problem:
  - RV accuracy degrades quickly with G and with spectral type
  - $\sigma_{RV} \sim$  a few km/s for relatively bright G-K star (single measurement)
  - $\sigma_{RV} \sim 10\text{--}20$  km/s for A-F stars (single measurement)

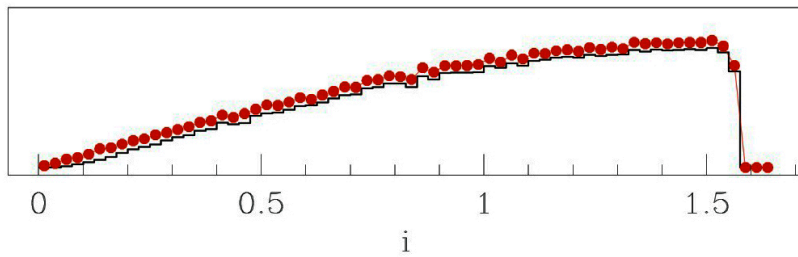
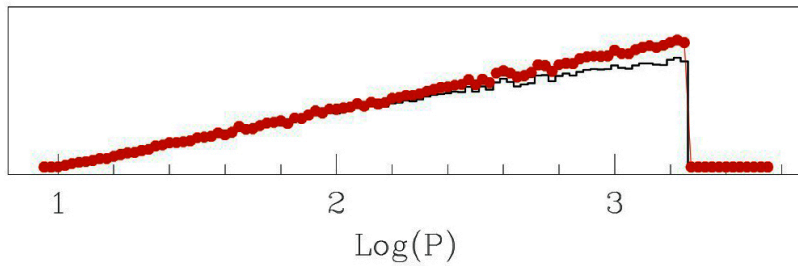
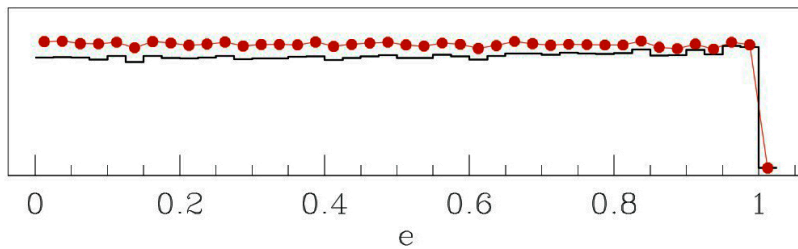
# EFFECT OF K ON DERIVED $f(q)$



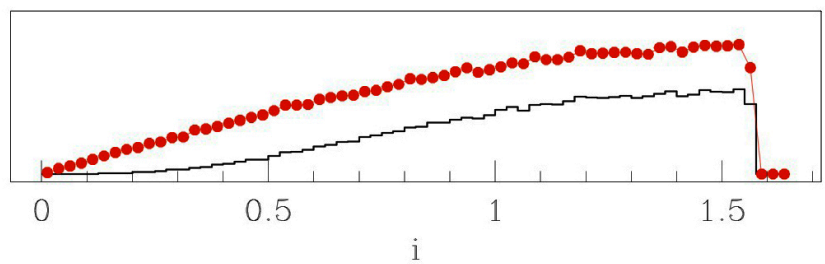
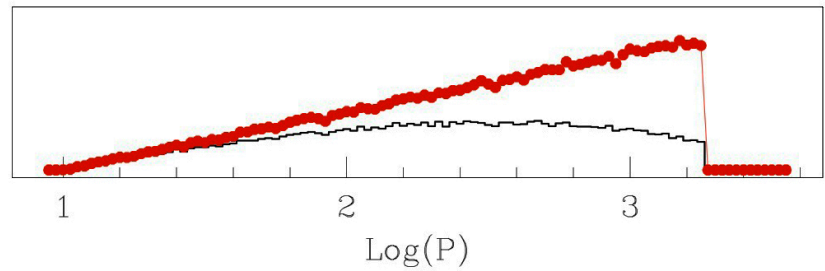
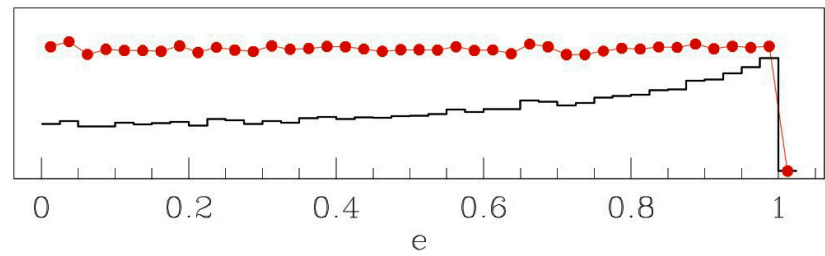


# EFFECT OF K

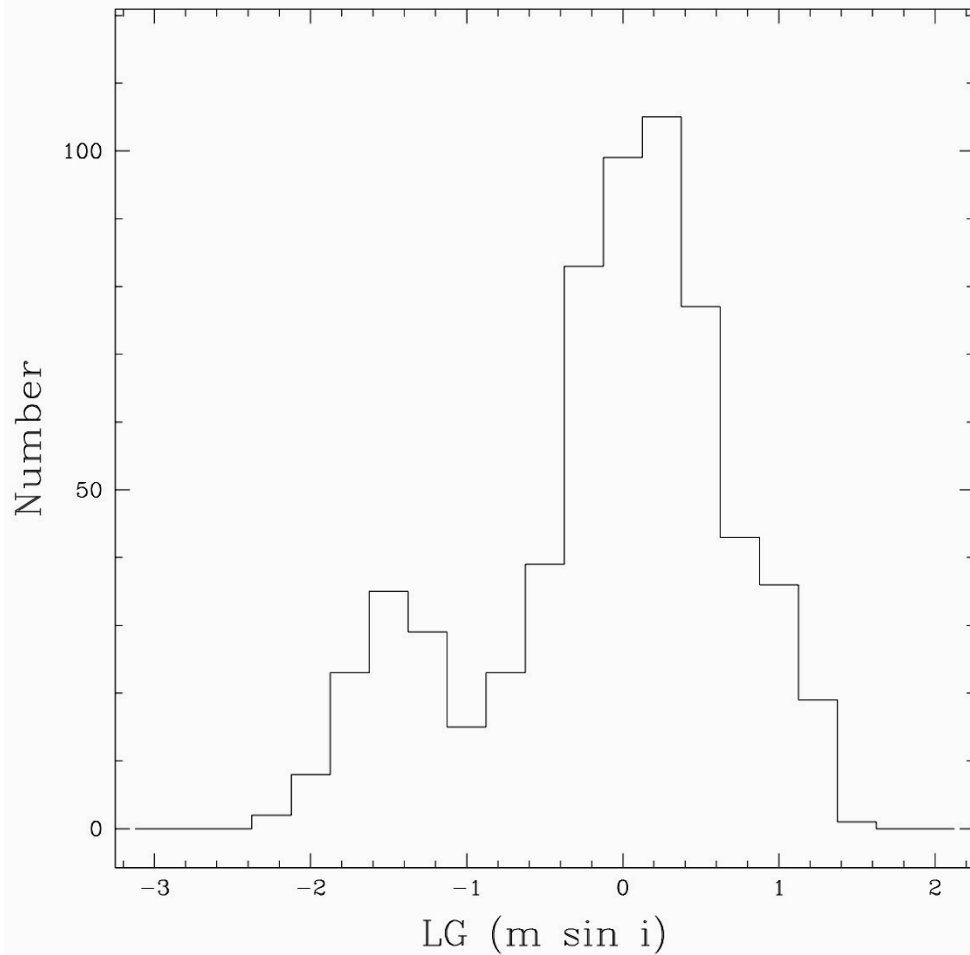
$K < 2 \text{ km/s}$



$K < 10 \text{ km/s}$



# EXOPLANETS MASS DISTRIBUTION

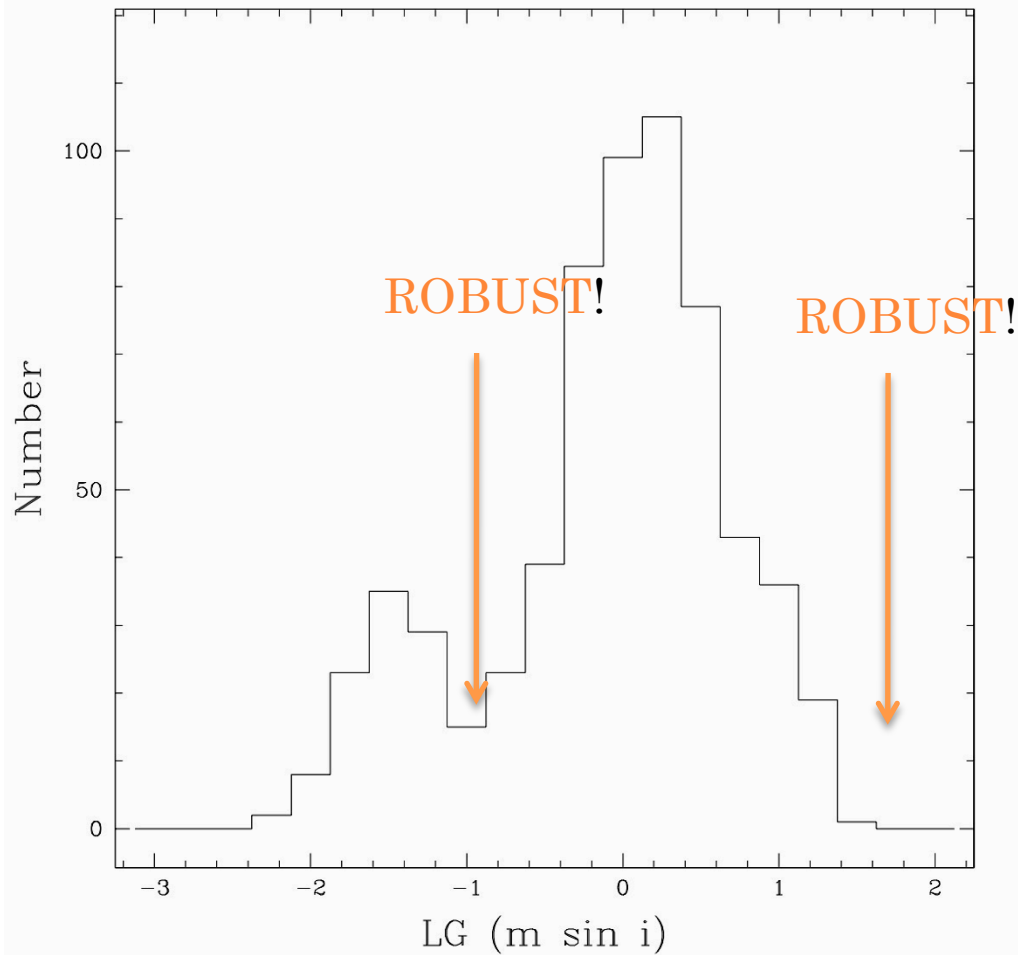


All RV systems from  
exoplanet.eu

Early October 2011

637 systems

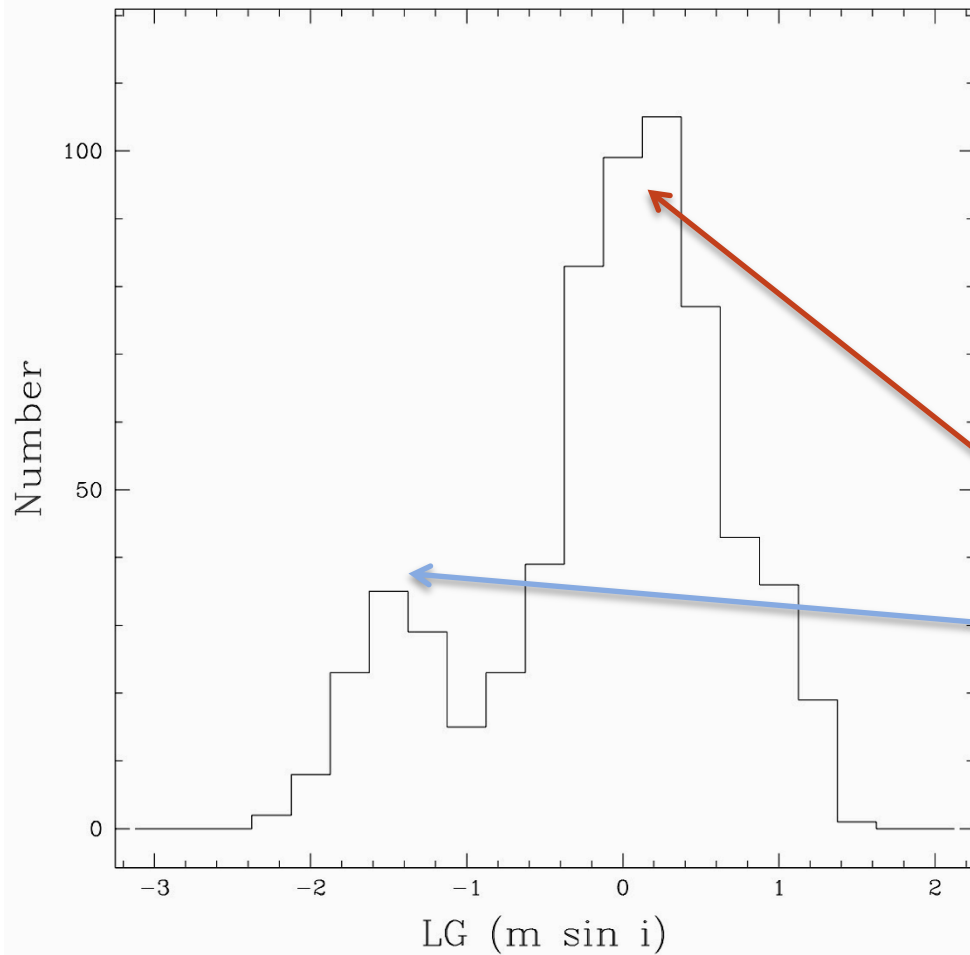
# EXOPLANETS



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637 systems

# EXOPLANETS



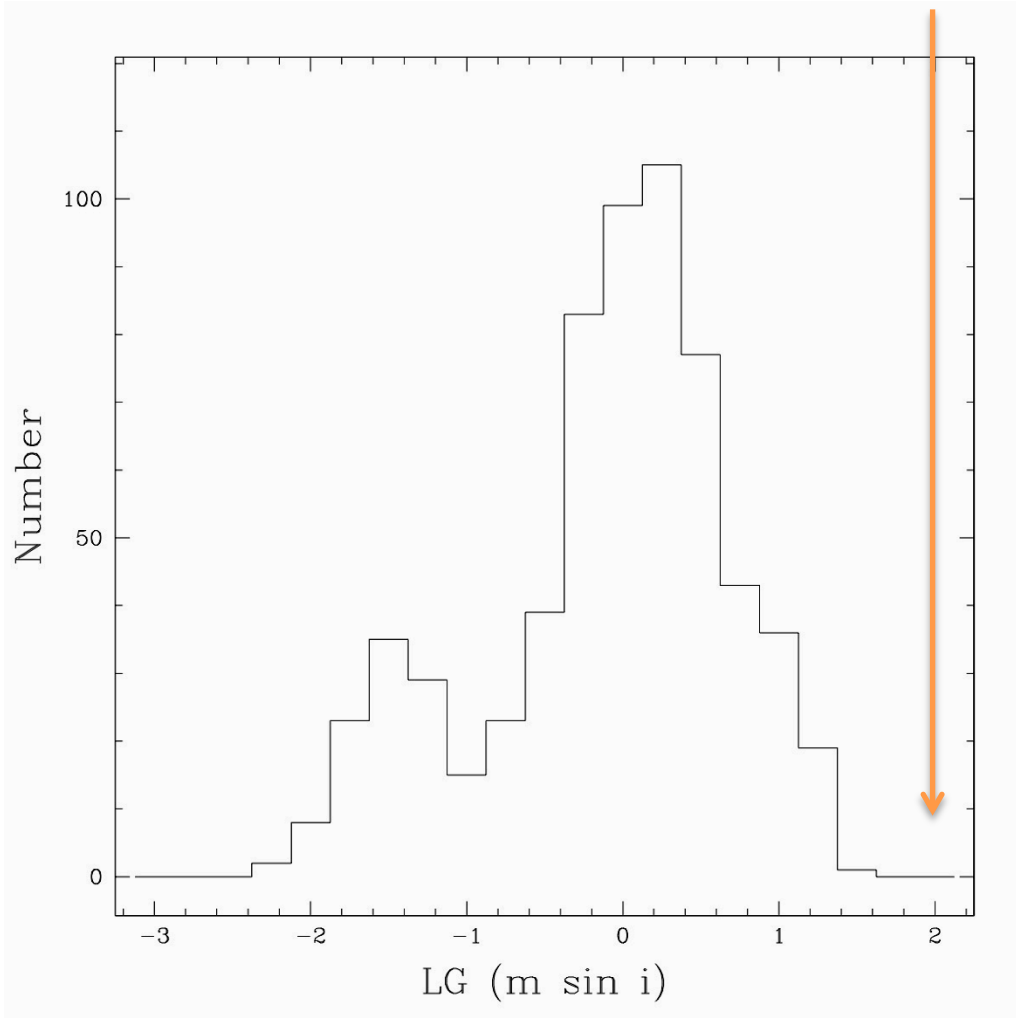
All RV systems from  
exoplanet.eu  
Early October 2011

637 systems

Functional fit:  
2 narrow gaussians  
centred on  
 $m \sim 0.6 M_{\text{Jup}}$   
 $m \sim 6 M_{\text{Earth}}$

# EXOPLANETS

Less than 25  
systems (6 %) may  
be brown dwarfs

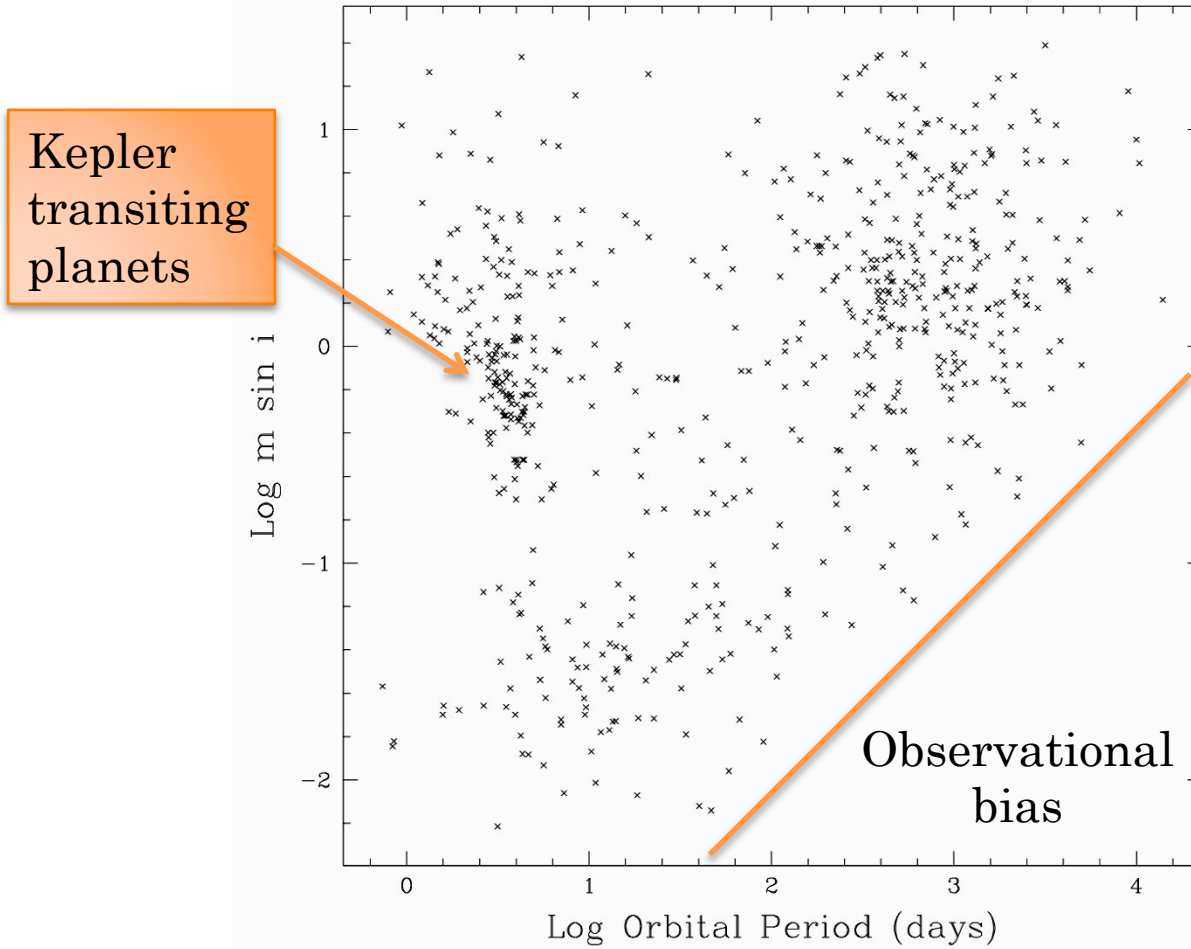


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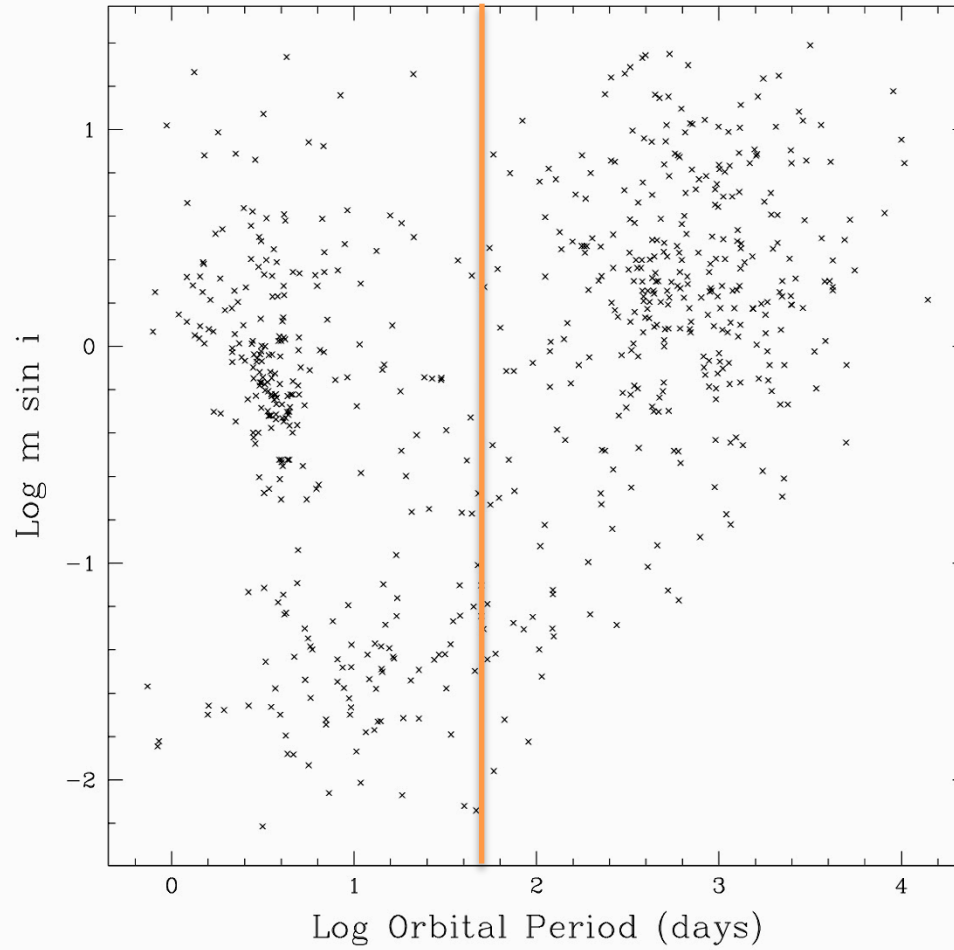
2 gaussians  
centred on  
 $m \sim 0.6 M_{\text{jup}}$   
 $M \sim 6 M_{\text{earth}}$

# EXOPLANETS

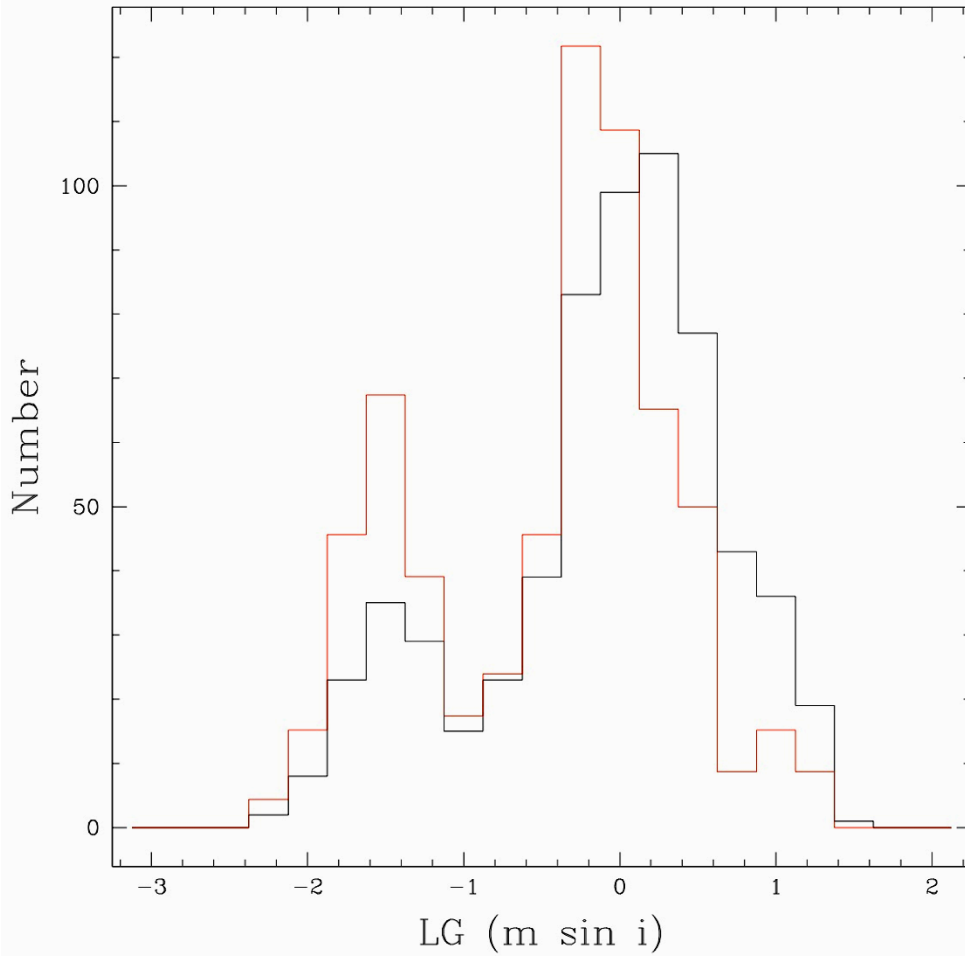


# EXOPLANETS

$P < 50$  days



# EXOPLANETS

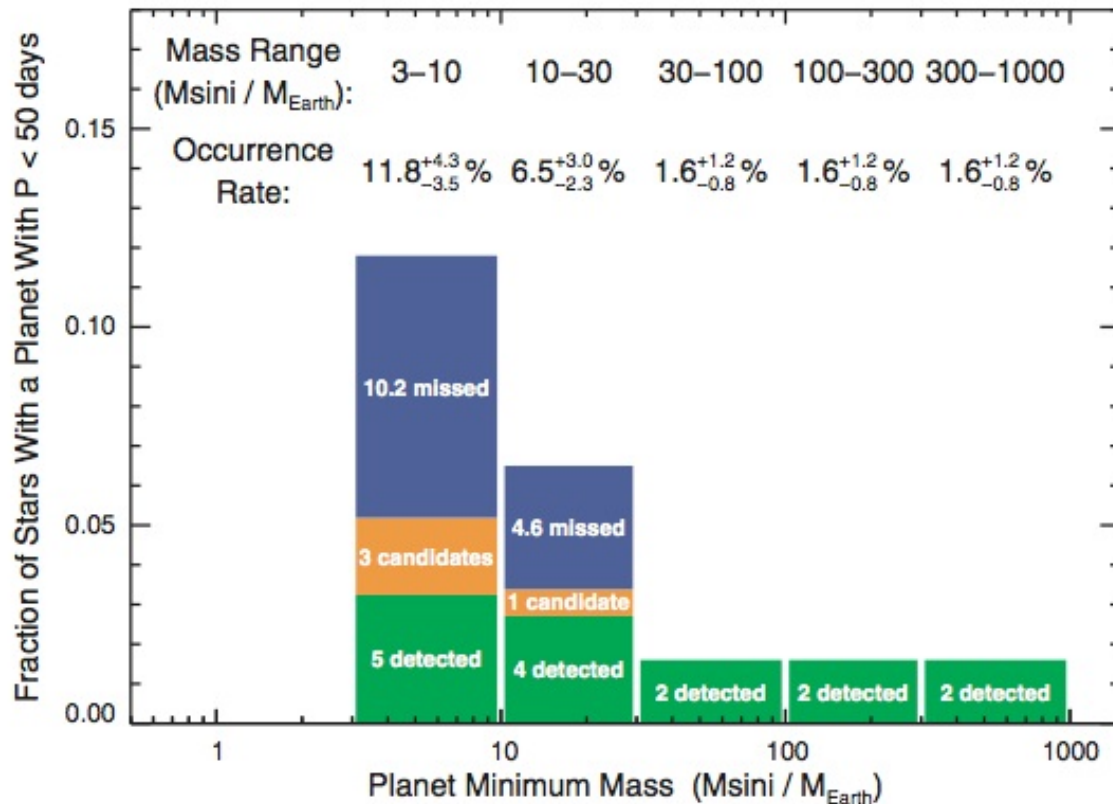


All systems (637)

Systems with  $P < 50$  d (293)



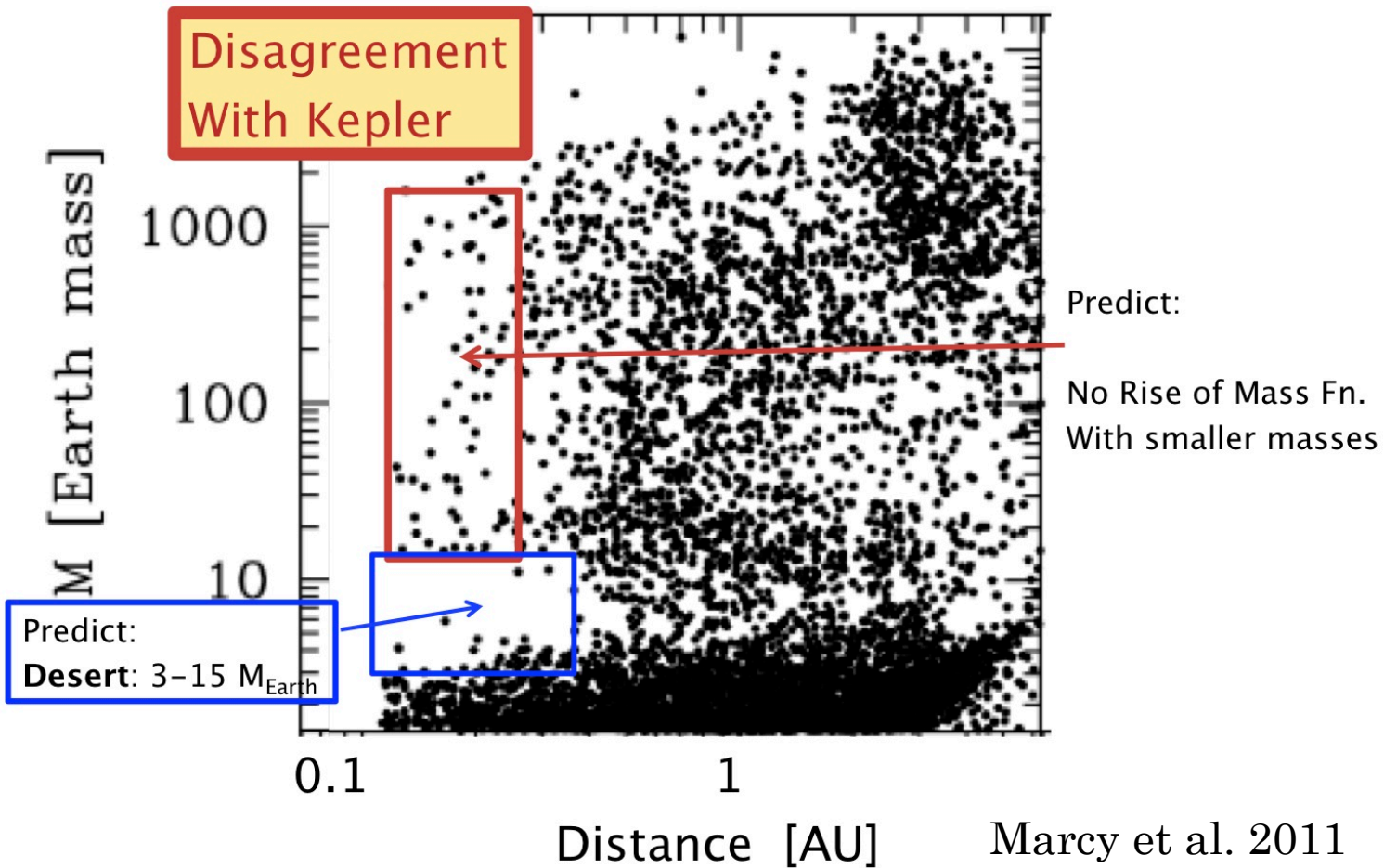
# KEPLER PLANETS $P < 50$ DAYS



**Fig. 2.** Histogram of the occurrence rate of stars hosting planets with orbital periods of less than 50 days in five mass ranges. Detected (green), candidate (orange), and missed (blue) planets are depicted separately. Missed planets statistically correct for planets that are detectable by measurements at  $1 \text{ m s}^{-1}$  but were missed because of nonuniform sensitivity.

# Extrasolar planet population synthesis

Lin & Ida (2010); Alibert, Mordasini, Benz (2011)



## CONCLUSION

- $f(q)$  important – and  $f(m_2)$  for exoplanets as well!
- There are some ways to retrieve it by statistical methods
- But be aware of the limitations and the rules of the game
  
- GAIA will revolutionise SB discoveries – even though it will also be limited in the binaries it can sample