The Problem – Modelling Dense Stellar Systems

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47 Tucanae

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The Globular Star Clusters of the Milky Way

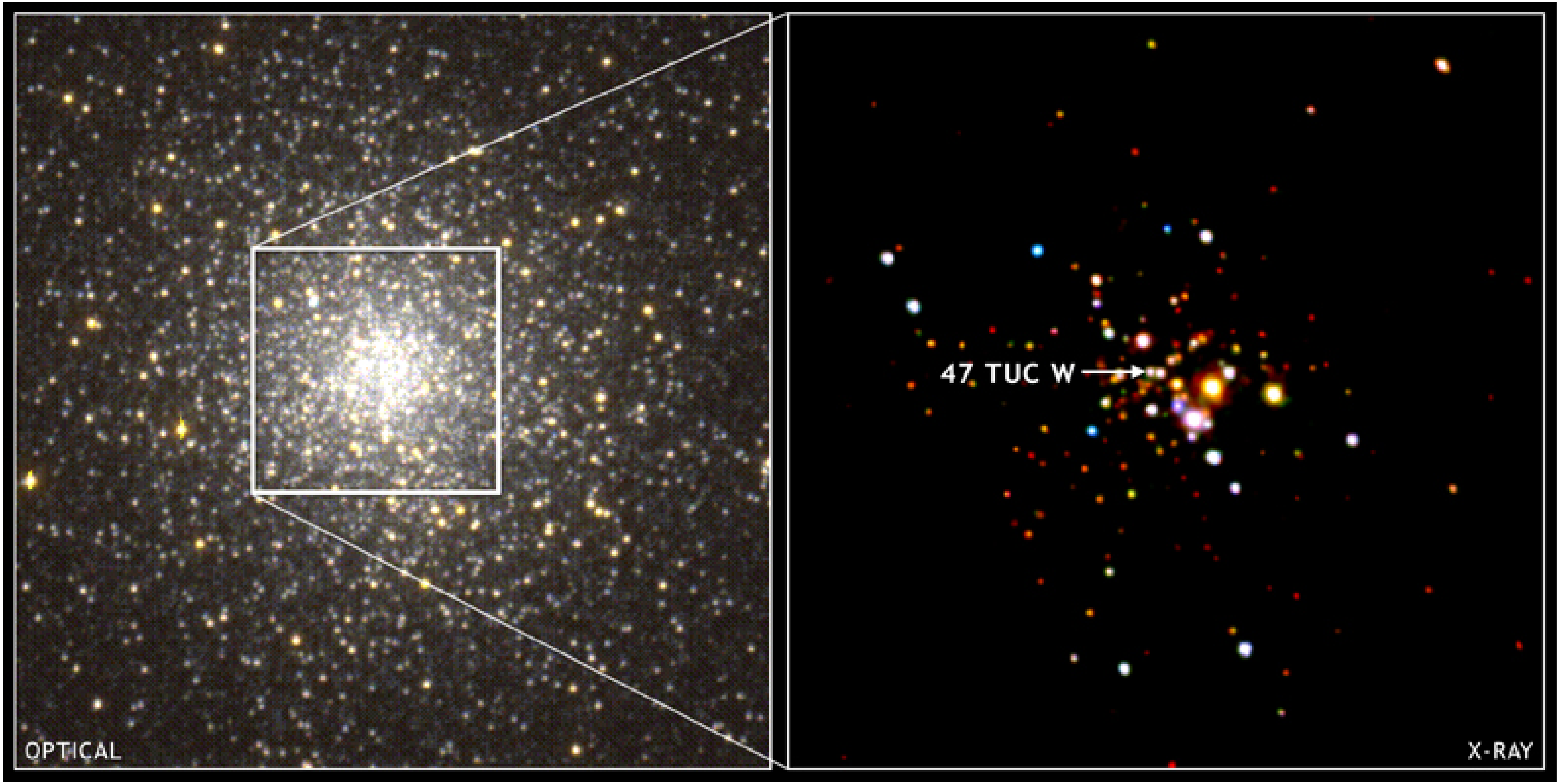
- Number known: about 150
- Median number of stars: about 3×10^5
- Typical age: about 12×10^9 years

Why they are important

They give unique information on

- stellar evolution
- star formation
- the formation of the Milky Way
- exotic kinds of stars:
  - *blue stragglers*
  - *X-ray sources*
  - *millisecond pulsars*

47 Tucanae in visible light (left) and in X-rays (right)



- X-ray sources about 100x more common in globular star clusters than elsewhere
- high stellar densities promote numerous non-gravitational interactions between stars

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Computer simulation of a globular star cluster
The essential ingredients

Gravitational interactions:

- Two-body interactions
- Few-body interactions

Stellar evolution

Gas dynamics

- Expulsion of residual gas
- Stellar winds
- Mass transfer in binary stars

Range of time scales

- Neutron star binary period < 1sec
- Age: $> 3\times10^{17}$ sec

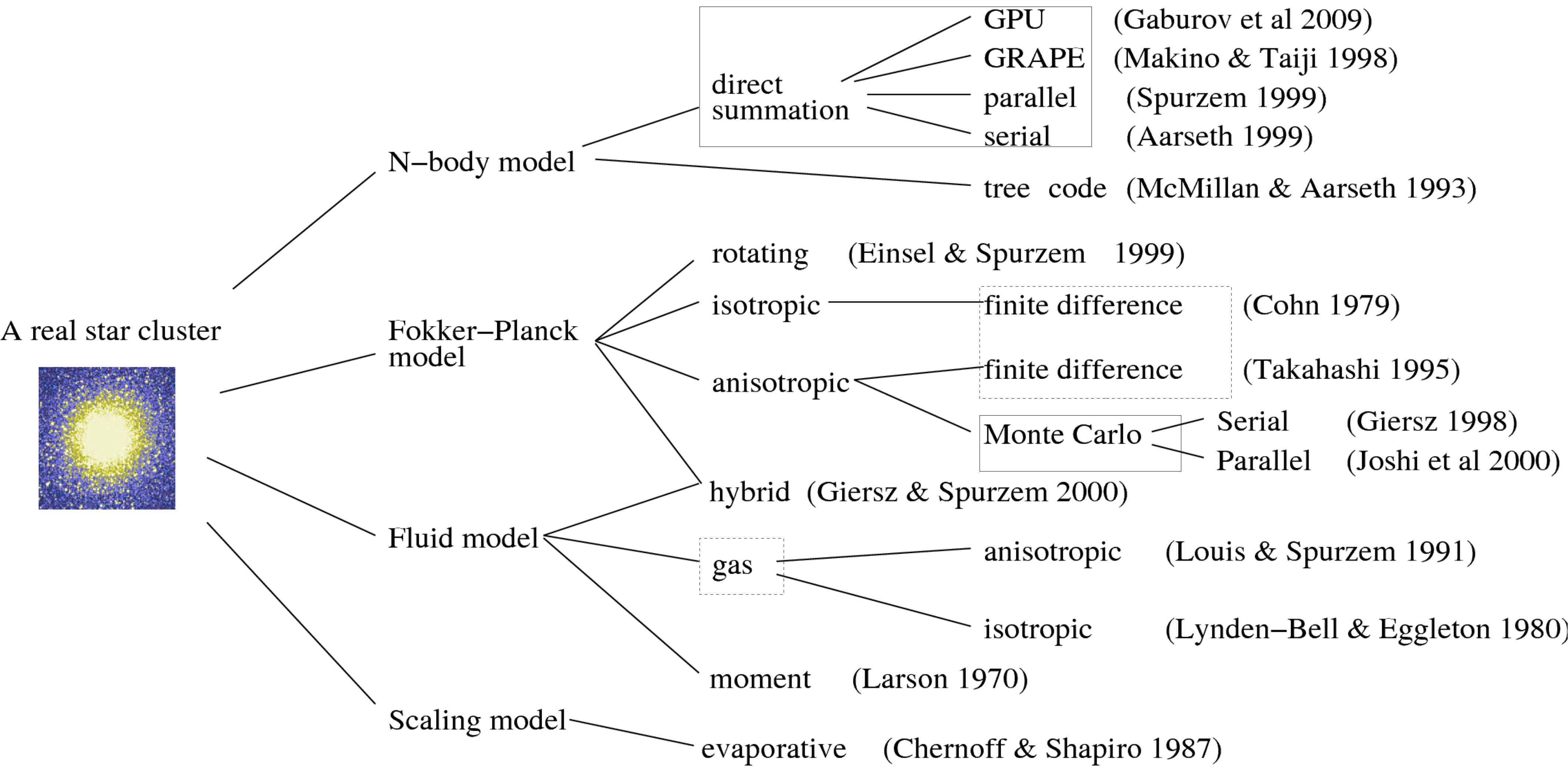
Range of length scales

- Neutron star binary radius $< 10^{3}$ km
- Cluster size ~ 10^{15} km

A challenging multi-scale, multi-physics problem

The most promising comprehensive approach: see http://amusecode.org/

Simulating the classical gravitational *N*-body problem



After Heggie & Hut (2003)

"...we give what we judge to be the most *informative* introductory reference..."

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Direct summation codes for the *N*-body problem

Equations of motion: $\ddot{r}_i \equiv a_i = -G \sum m_i \frac{r_i - r_j}{\left| r_i - r_j \right|^3}$

where

- r_i is the position vector of the *i*th star in space
- m_i is its mass
- G is the universal constant of gravitation
- the sum is over all stars $1 \leq j \leq N$ $^{\dot{r}_i}$; $j \neq i$

Hermite integration routine

Prediction: $r_i^{t+\Delta t} = r_i^t + \Delta t . \dot{r}_i^{\,t} + \frac{1}{2} \Delta t^2 . a_i^t + \frac{1}{6} \Delta t^3 . \dot{a}_i^{\,t}$

Correction: higher-order terms based on values of $a_i^{t+\Delta t}, \dot{a}_i^{t+\Delta t}$

Similar expressions for $\dot{r}_i$

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Examples of a star cluster simulation

Crossing time t_{cr} is time taken for a typical star to cross the cluster.

Typically 10^6 years.

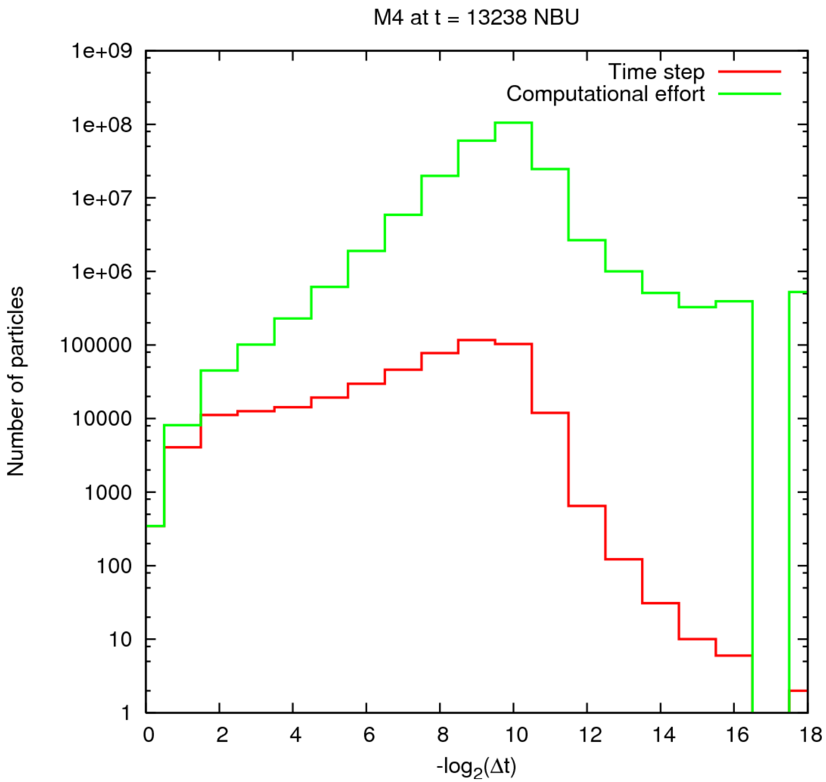
Age of clusters ~ $10^4\, t_{cr}$

ftp://www.ari.uni-heidelberg.de/staff/berczik/douglas_mirek/

http://www.sns.ias.edu/~starlab/animations/king12_1k_mass.gif

Time step

- Typically $\Delta t \sim t_{cr}/N^{1/3}$ (travel time to nearest neighbour)
- Widely different Δt are appropriate for different particles ("individual time steps")
- Time steps organised in powers of 2 ("block time steps")
- Example: $N \sim 5\times10^5$, $t_{cr}/N^{1/3} \sim .04 \sim 2^{-5}$



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Small time steps

Associated with binary stars and other few-body events or systems

Binaries

- If unperturbed, no need to integrate
- If lightly perturbed, can use "slow-down" (allows longer Δt while correctly modelling perturbations)
- Perturbations due to small number of neighbours, requiring keeping of *neighbour list* (also used for all particles, to expedite force calculation)
- Highly eccentric binaries require some form of *regularisation* (use of relative coordinates, possibly KS)

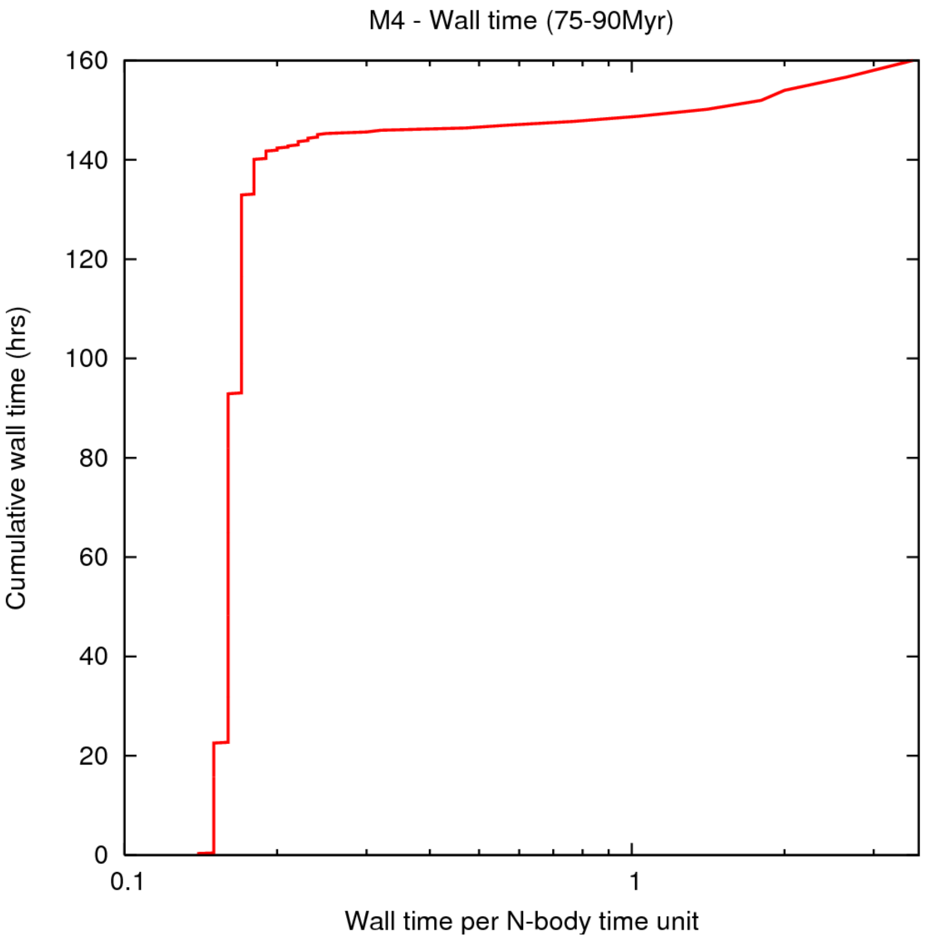
Triples,quadruples,...

- If unperturbed and stable, no need to integrate
- Slow-down, neighbour perturbations, regularisation all applicable

Significance of particles with smallest Δt

In this simulation

- only 10% wall time spent on "difficult events"
- binary fraction of 7% slows down simulation by factor ~5



Reference for treatment of binaries, triples, etc:
S.J. Aarseth, *Gravitational N-Body Simulations* (CUP)

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The force calculation

Neighbour scheme

•Evaluate neighbour forces frequently

•Evaluate non-neighbour force (far field) infrequently

- *effectiveness depends on order of integrator*

•Requires neighbour list

Hierarchical schemes (tree codes)

•Insufficient accuracy (in regime where they are efficient)

•Do not simulate mass segregation accurately

Hardware acceleration

•GRAPEx

•GPU – in M4 simulation 12% wall time on force calculation

•etc

Accuracy of simulations

Chaotic system: errors grow with *e*-folding time ~ $0.1t_{cr}$

Typical simulation $10^4\ t_{cr}$

⇒ Positions and velocities of particles are wrong

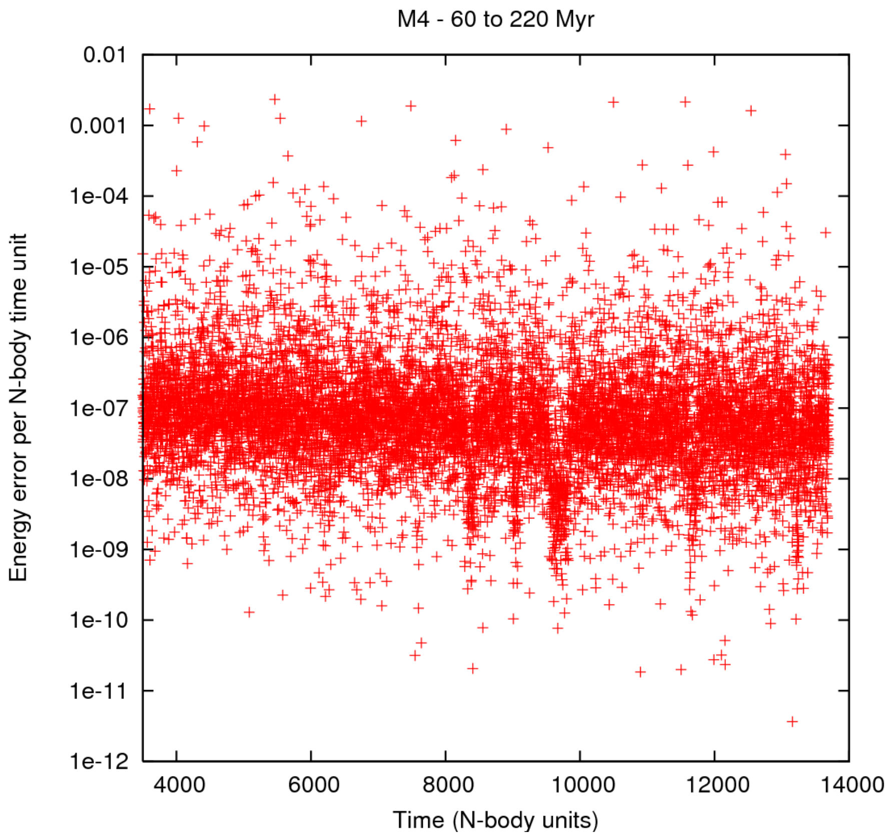
We *hope* that statistical properties are correct

We *assume* that satisfactory energy conservation is sufficient

What is "satisfactory" is determined by *custom.* And do we really need 10^{-9} - 10^{-10}?

These issues have no rigorous foundation.

Example:



Cost of simulations with a direct N-body code

Number of particles = N

Number of time steps per $t_{cr} \sim N^{1/3}$

Cost of one force calculation $\propto N$

$\Rightarrow$ Cost to time $t \propto N^{7/3}\,(t/t_{cr})$

Two consequences

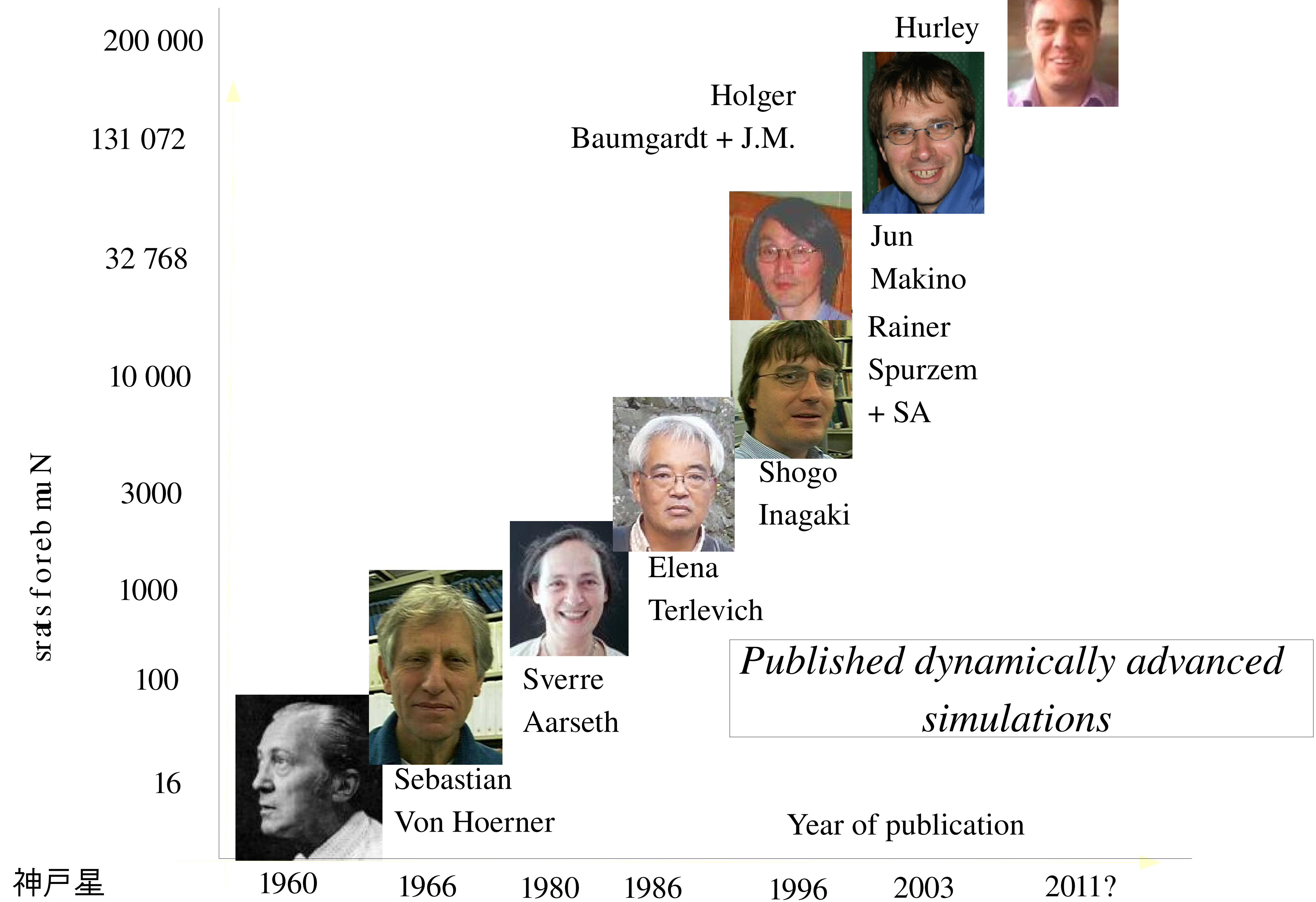
- Since theory implies that the time scale of evolution $\propto N\, t_{cr}$ (approximately)

  the cost on the time of dynamical evolution $\propto N^{10/3}$ (approximately)

- Since theory implies that $t_{cr} \propto R^{3/2}/\,N^{1/2}$ (where R is the radius of the cluster)

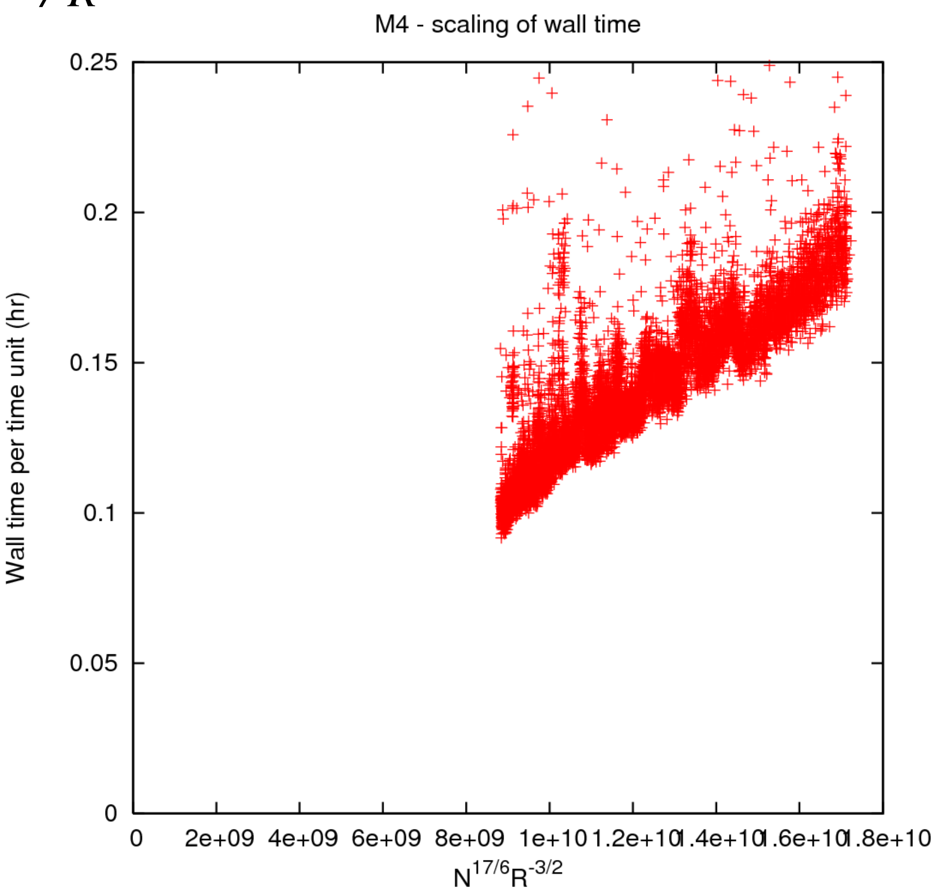
  the cost per time unit is $\propto N^{17/6}/\,R^{3/2}$

Towards the million-body problem

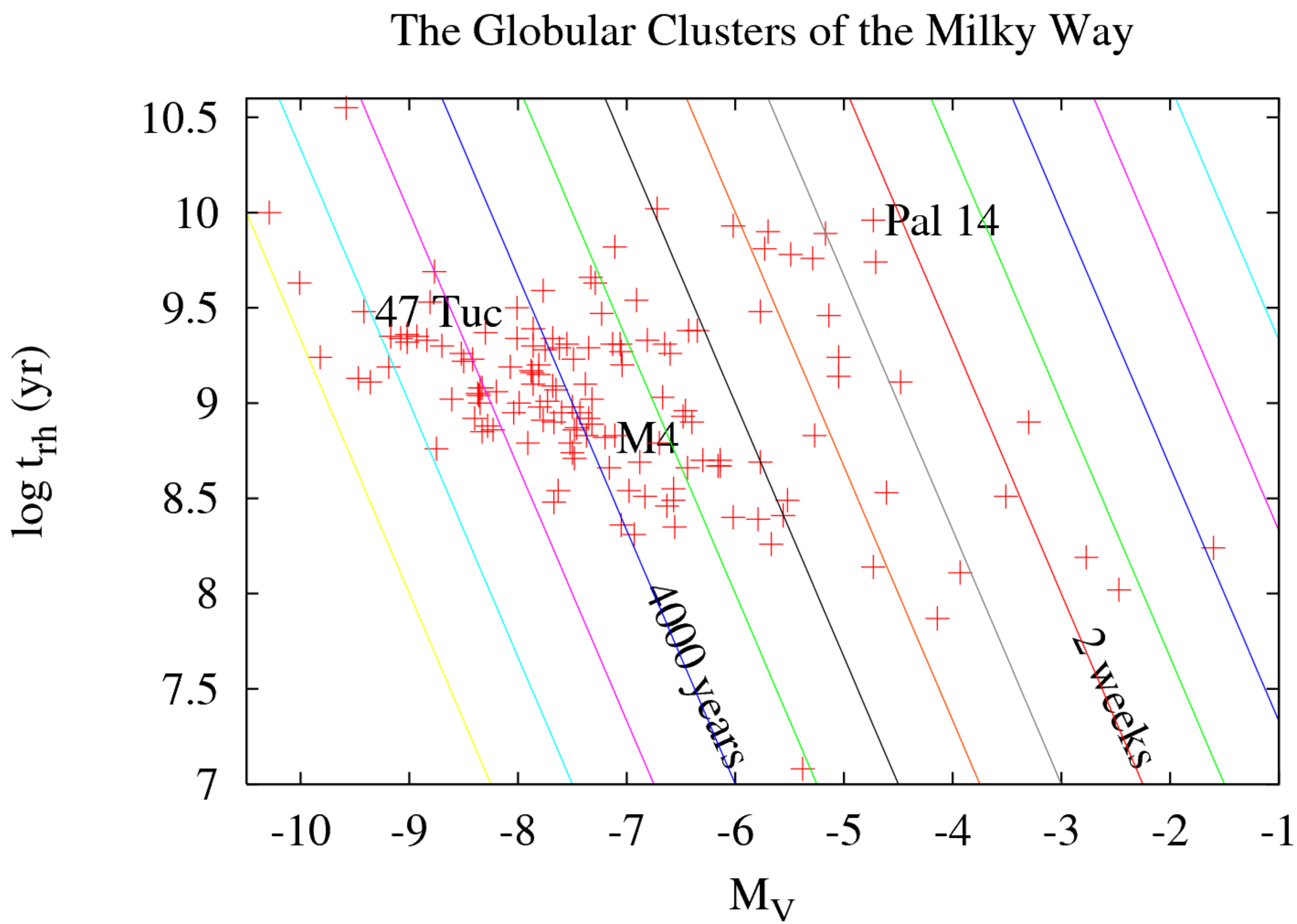


Scaling with radius and *N* – Simulation of M4

- If force calculation dominates, then cost per t_{cr} is $\propto N^{17/6}/R^{3/2}$
- Simulation of M4
  - $N(0) = 484709$
  - $R(0) = 0.59$pc
  - $t_{rh}(0) = 0.047$Myr
  - NBODY6
  - 2xTesla C2050
  - 8xIntel Xeon CPU X5650 @ 2.67GHz
- Now at t = 290 Myr after 10 months ⇒ 34 yrs
- Initial rate of progress ⇒ 250 yrs
- Current rate of progress ⇒ 9 yrs
- Extrapolation using the above scaling and an independent approximate model for the evolution ⇒ 0.8 yrs



N-body models:
the globular clusters of the Milky Way



$t_{rh} \propto N^{1/2} R^{3/2}$

Pal 14: see Zonoozi et al (2011)

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$$M_V \simeq \text{const} - 2.5 \log_{10} N$$

Data: Harris

N-body models: scaling down

The idea: model a cluster with N stars by a model with N*<<N stars

The principle: get the time scale of the major evolutionary effects correct

Stellar evolution: set by stellar evolution models

Two-body relaxation (secular evolution): time scale $t_r \propto N^{1/2}R^{3/2}$

Interaction of binaries: interface between hard and soft binaries at binaryseparation $R/N \propto N^{-4/3}$

Internal evolution of binaries: need to scale stellar radii $R_* \propto N^{-4/3}$

But then other processes do not scale properly:

Collision time scale:

$1/(n\sigma v) \propto R^3/(N^{3/2}R_*) \propto \mathbf{N^{-7/6}}$

Escape time scale:

$N^{-1/4}\, t_r \propto \mathbf{N^{-1/4}}$

Sampling effects of upper mass function

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Monte Carlo Simulation of Star Clusters

Codes

- M. Giersz (Warsaw) – available (soon?) on AMUSE
- J. Fregeau (ex Northwestern)

The idea

- gravity treated as a "smooth" spherical potential plus
  the statistical effect of encounters
- few-body interactions treated by cross-sections or numerical integration
- stellar and binary evolution as in *N*-body code

Outline of the Monte Carlo algorithm

Each star has a radius r, energy E, angular momentum J, mass m, and may be binary or single

1. Order stars by radius and calculate potential
2. For each pair
   - if both are single, alter E, J corresponding to (conditional) average effect of a encounters in time Δt
   - if one or more is binary, compute probablity of an encounter in time Δt; if so, compute outcome
3. Update binary and stellar evolution as necessary
4. Compute new radii for all objects, given individual E,J
5. Repeat

Performance

- 1 and 4 can be done in $N \log N$
- $\Delta t \sim N\, t_{cr} / \log N$
- Cost to time $t \propto (t/t_{cr})$ (recall $N^{7/3}\,(t/t_{cr})$ for direct N-body)
- < 1 day for 47 Tuc (almost $2\text{x}10^6$ stars)

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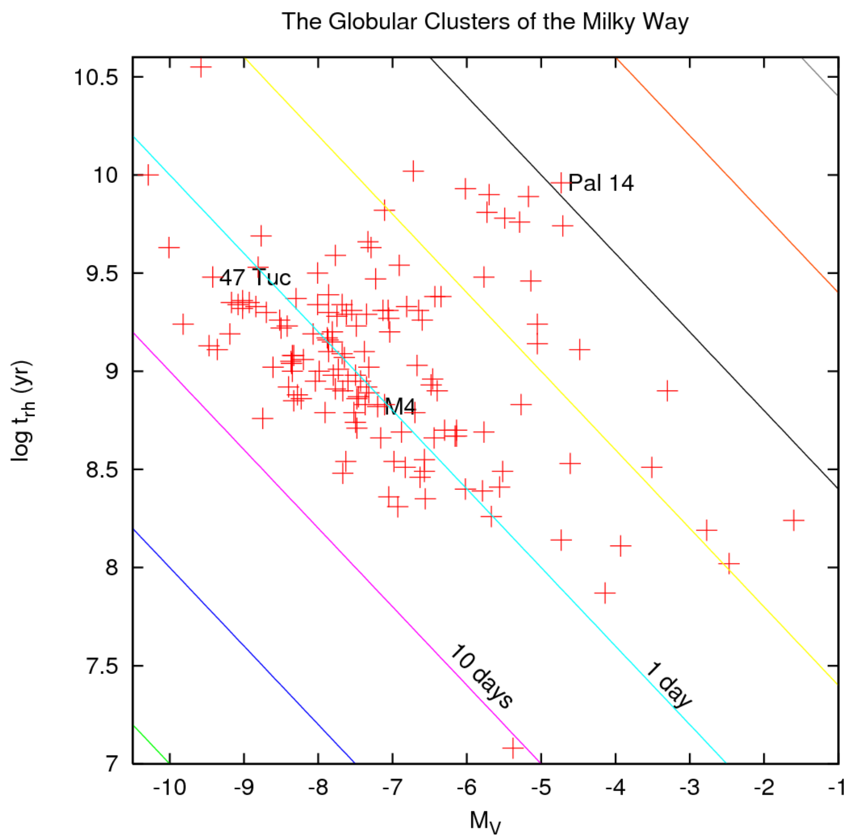
Monte Carlo models:
the globular clusters of the Milky Way



No parallelism, OMP, GPU, etc...

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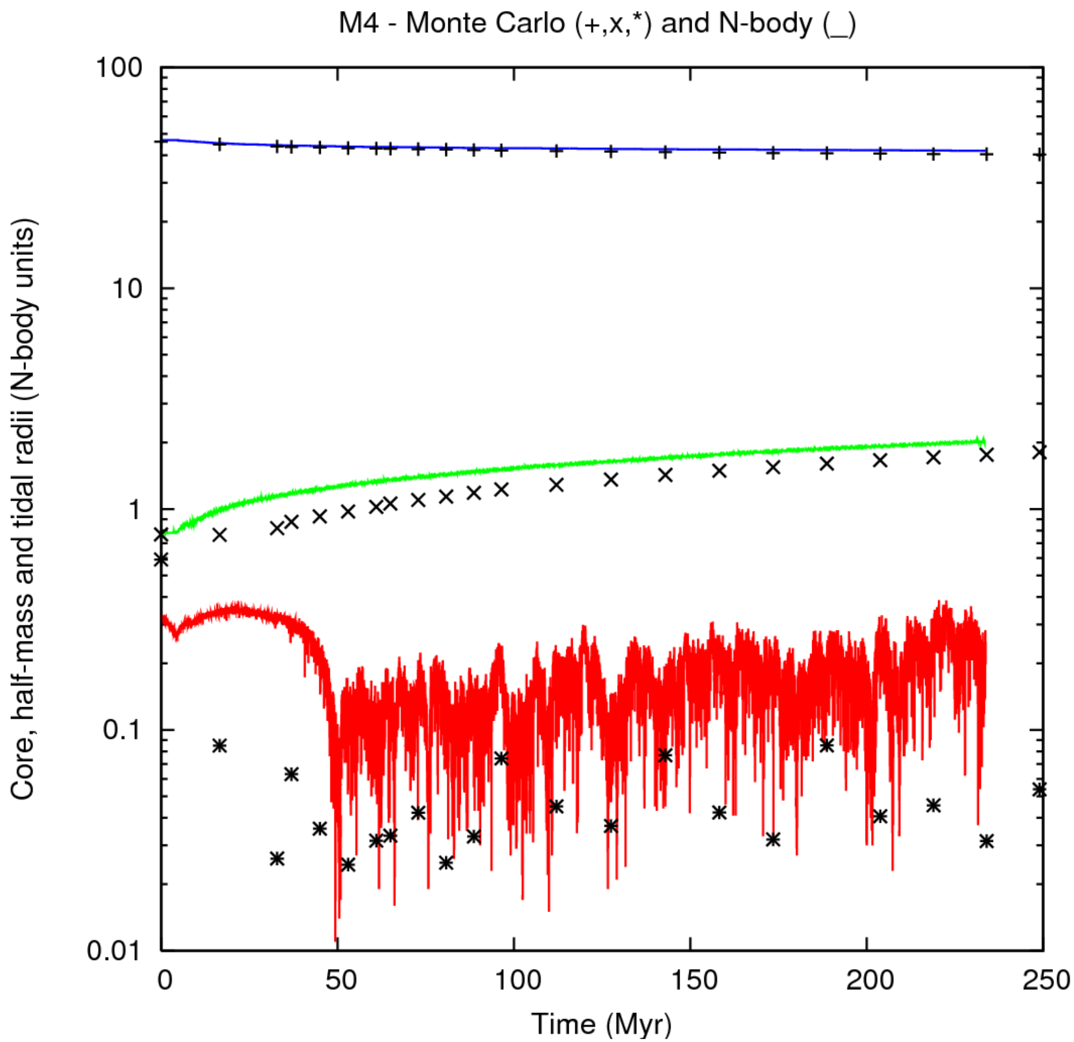
Monte Carlo – is it any good?



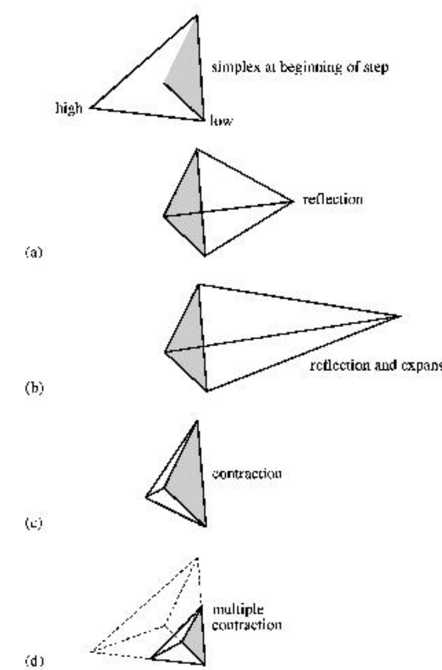
Finding initial conditions

The problem:

choose initial mass, radius, mass function so that, after 12Gyr of evolution, the match of the model to observations (surface brightness profile, velocity dispersion profile, luminosity function, etc.) is optimal.

Solutions:

- grid search
- automatic search (example: downhill simplex **->**)
- bayesian approach (Markov Chain Monte Carlo;
  in progress)



From: Numerical Recipes

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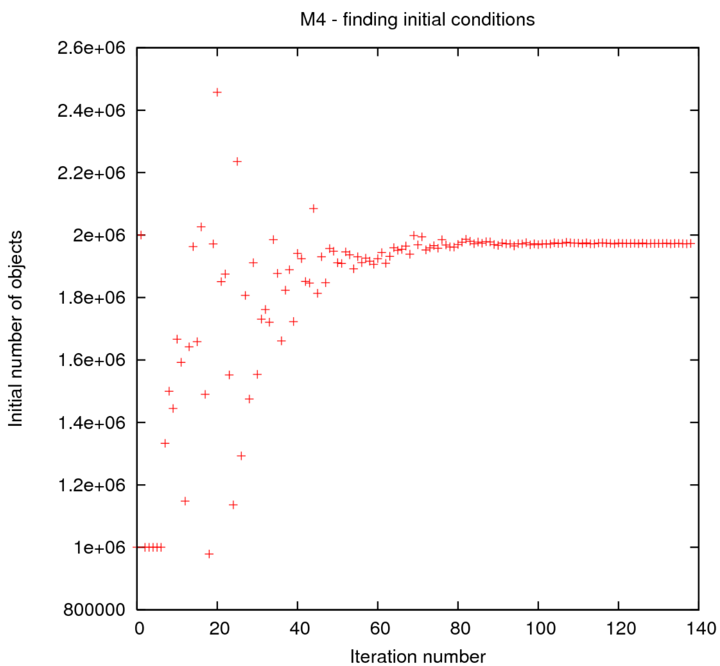
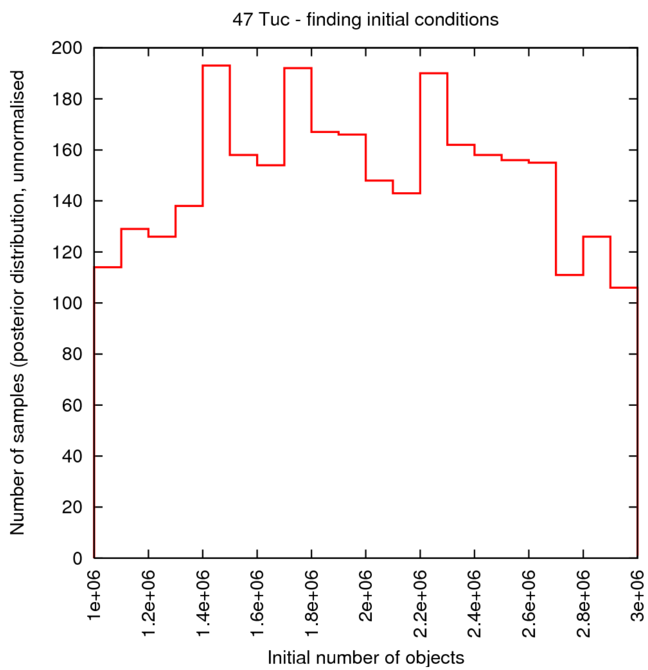
Finding initial conditions with downhill simplex

Downhill simplex



MCMC (in progress)



- Heggie & Giersz found $2x10^6$ by "hand"-searching
- These results use scaled MC models with $N = 10^4$
- Downhill simplex needs 10s of models, MCMC needs

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GPU Hardware Issues

Typical hardware for N-body work in this talk

?12*Intel Xeon CPU X5650 @ 2.67GHz
4(2)*Tesla C2050

Possible hardware for such work:
Cluster at NAOC, Beijing (since 2009)
170* Tesla

Image: P. Berczik

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Software for GPU clusters

NBODY6++:

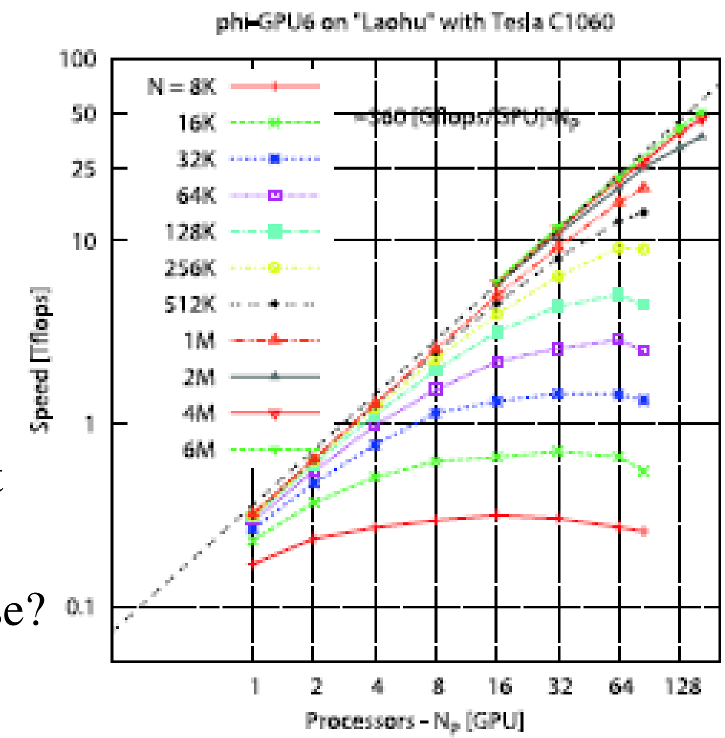
- (Still) under construction
- Will it scale well?
- Can the cluster be devoted to one problem for many months?

ϕ-GRAPE/GPU:

- Excellent scaling
- Suggests M4 might take months
- But softened

Steve's hybrid

- Combine ϕ-GRAPE/GPU with separate treatment of close encounters (dynamical and physical)
- How will it scale if wall time is dominated by these?
- Under development



ing. *Right*: NAOC GPU cluster in Beijing, speed in Teraflop/s reached as a function of number of processes, each process with one GPU, 51.2 Tflop/s sustained were reached with 164 GPU's (3 nodes with 6 GPU's were down at the time of testing)

From Spurzem et al 2011

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Work for the future

1. Get NBODY6++ and/or “Steve’s Hybrid” working
2. Get computation time for M4 down to
    1 month (for single simulation)
    1 day (for determination of initial conditions)
3. Need better understanding of scaling: dependence on
    N, R_h
    Binary parameters (fraction and hardness)
    Regularization and neighbour-list parameters
4. How accurate is “accurate enough”?
    another job for Alf?
5. Improvements of the Monte Carlo code: OMP, MPI, GPU
6. Improvement of MCMC determination of initial conditions

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