Asteroseismology: a tool to unveil stellar interiors

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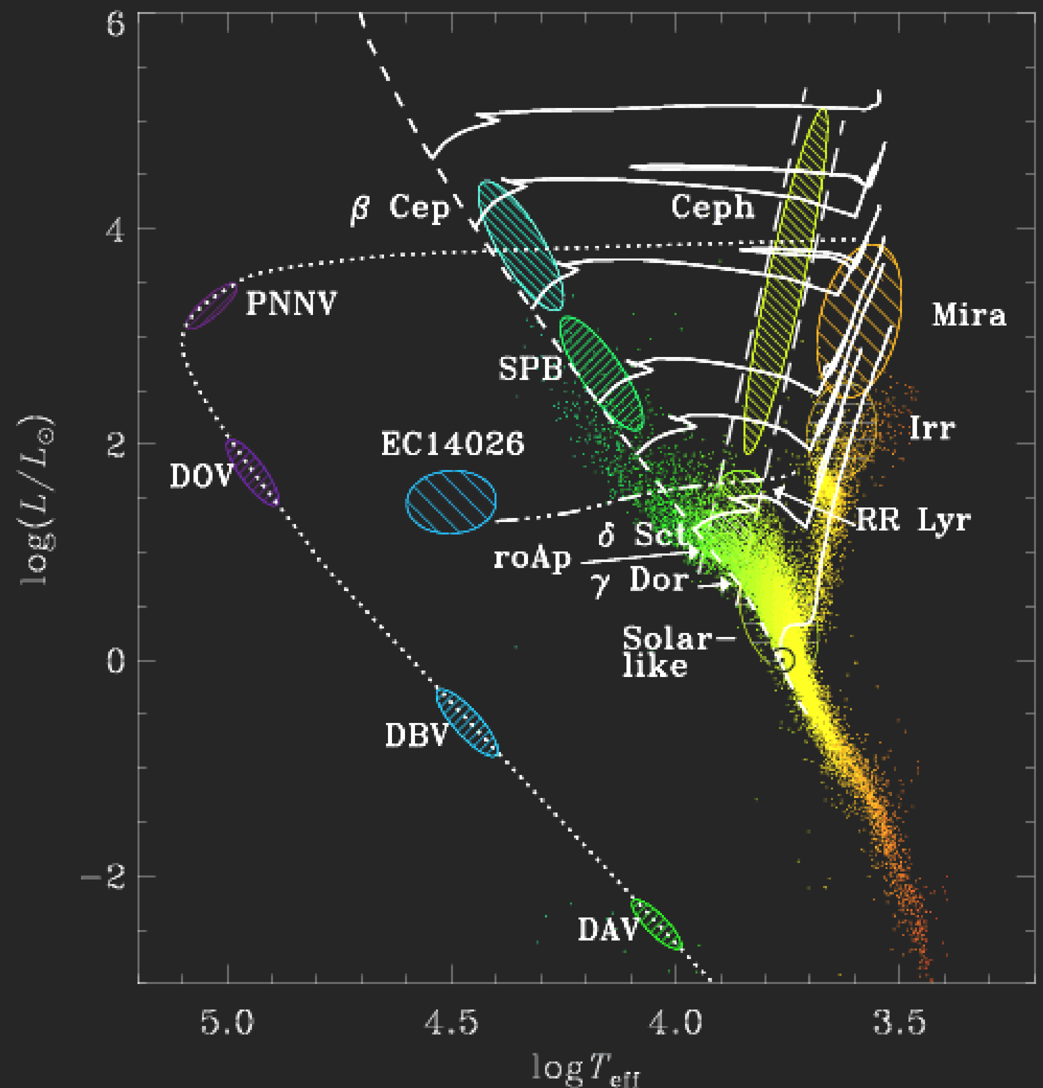
Part 1

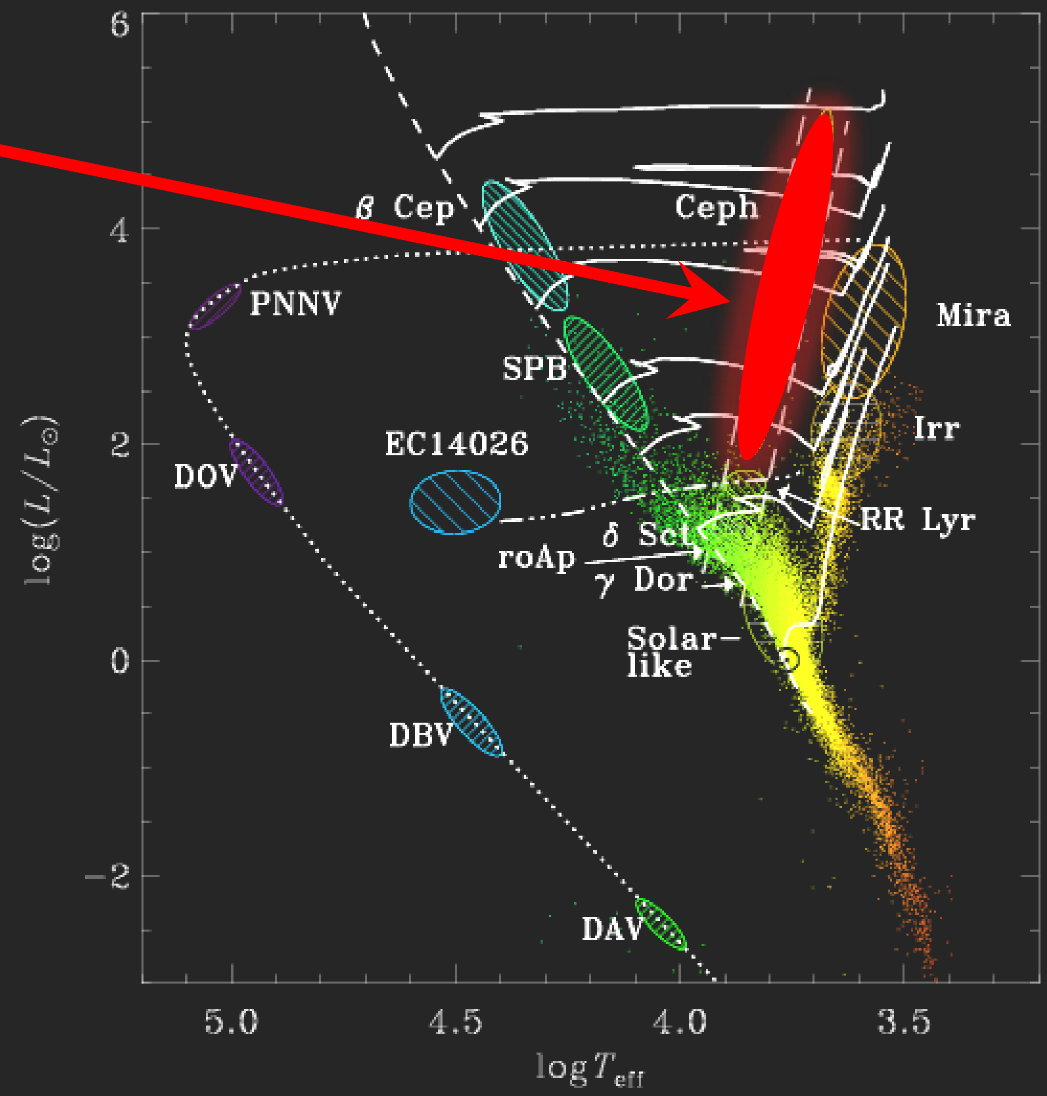
1. Stellar oscillations across the HR diagram



β Cep
Ceph
PNNV
Mira
SPB
Irr
EC14026
DOV
RR Lyr
δ Sct
roAp
γ Dor
Solar-
like
DBV
DAV
log(L/L☉)
log T_eff
6
4
2
0
−2
5.0
4.5
4.0
3.5

Cepheids

Periods : 1 – 100 d

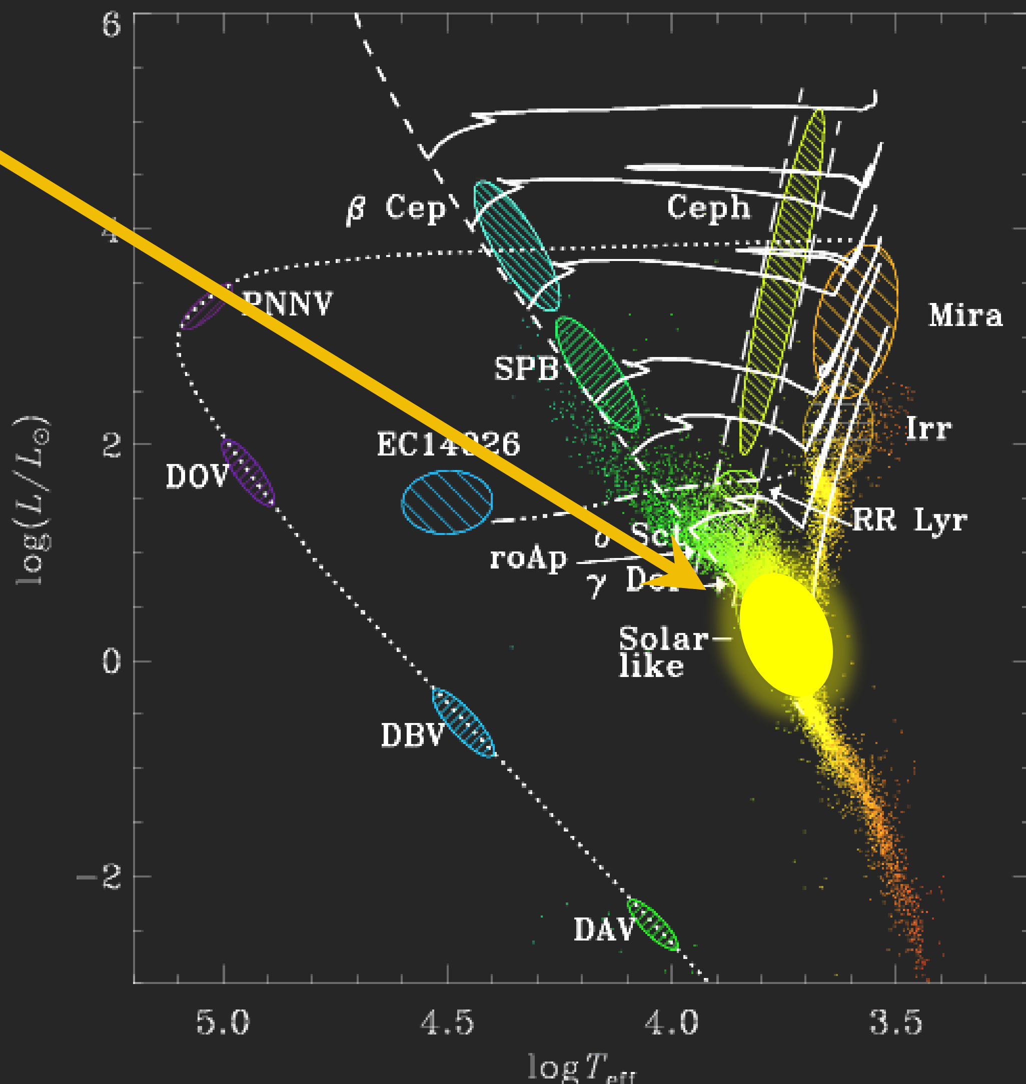


Solar-type stars

Periods : 3 – 8 min

- *pressure modes*

  periods : 3 – 8 min

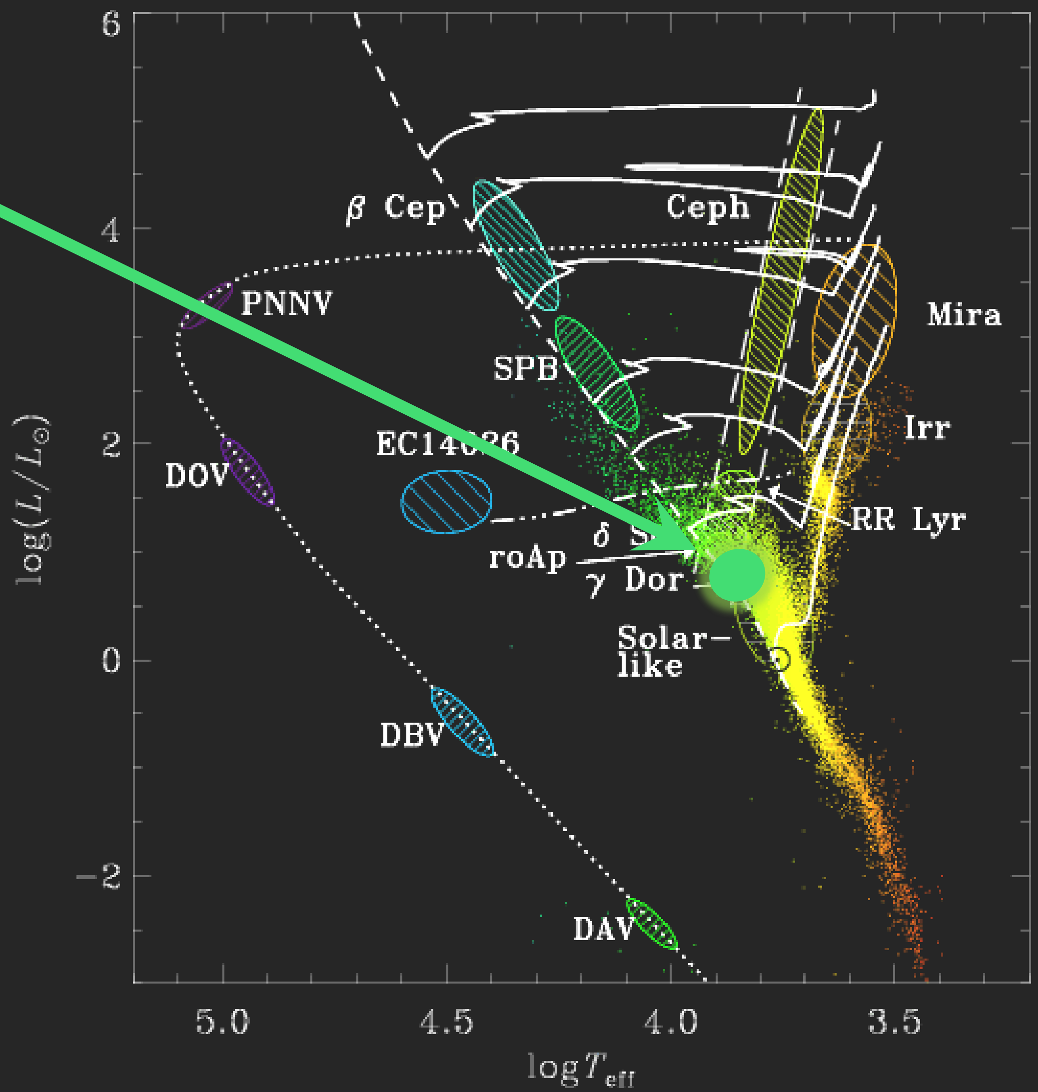


γ Doradus

Periods : 0.3 – 3 d

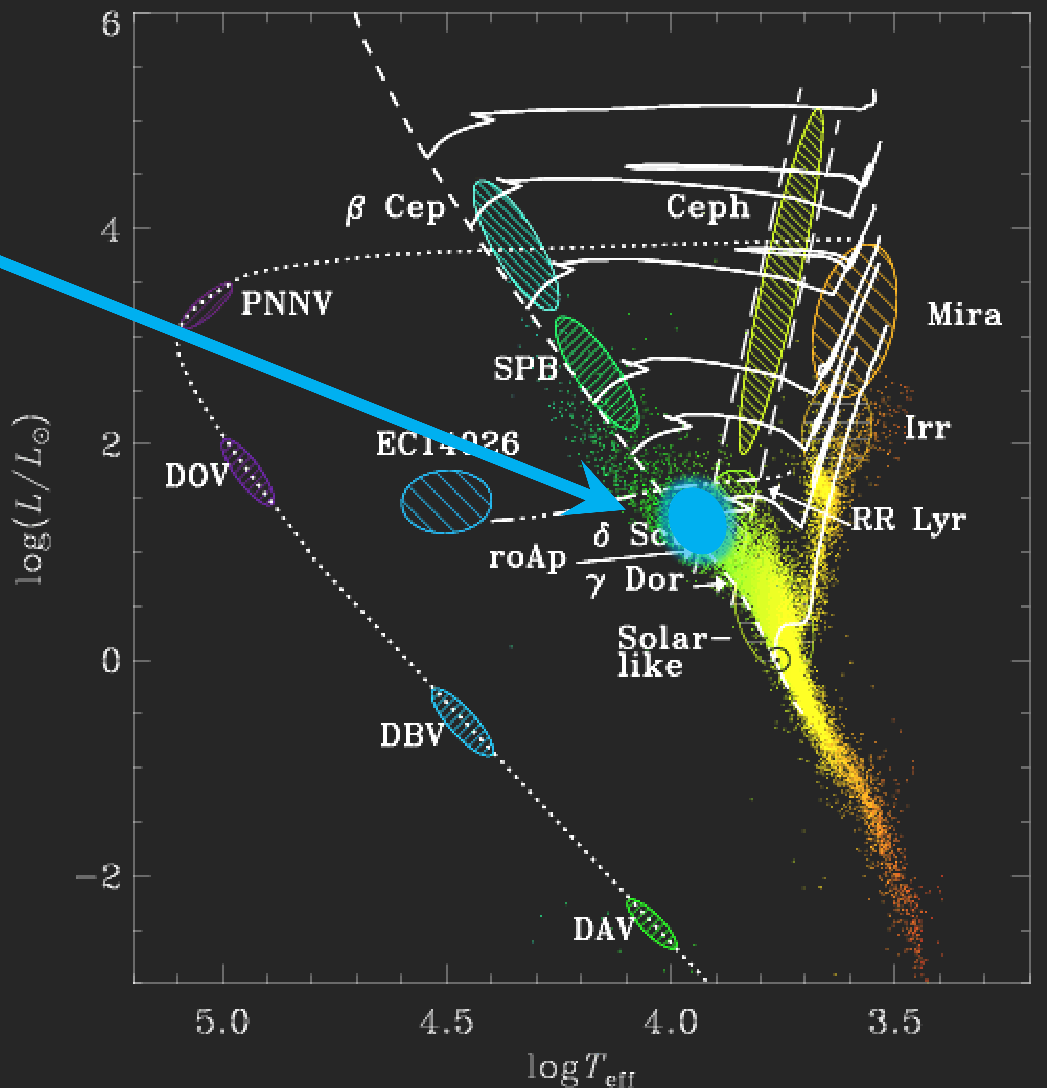
- *gravity modes*

  periods : 0.3 – 3 d



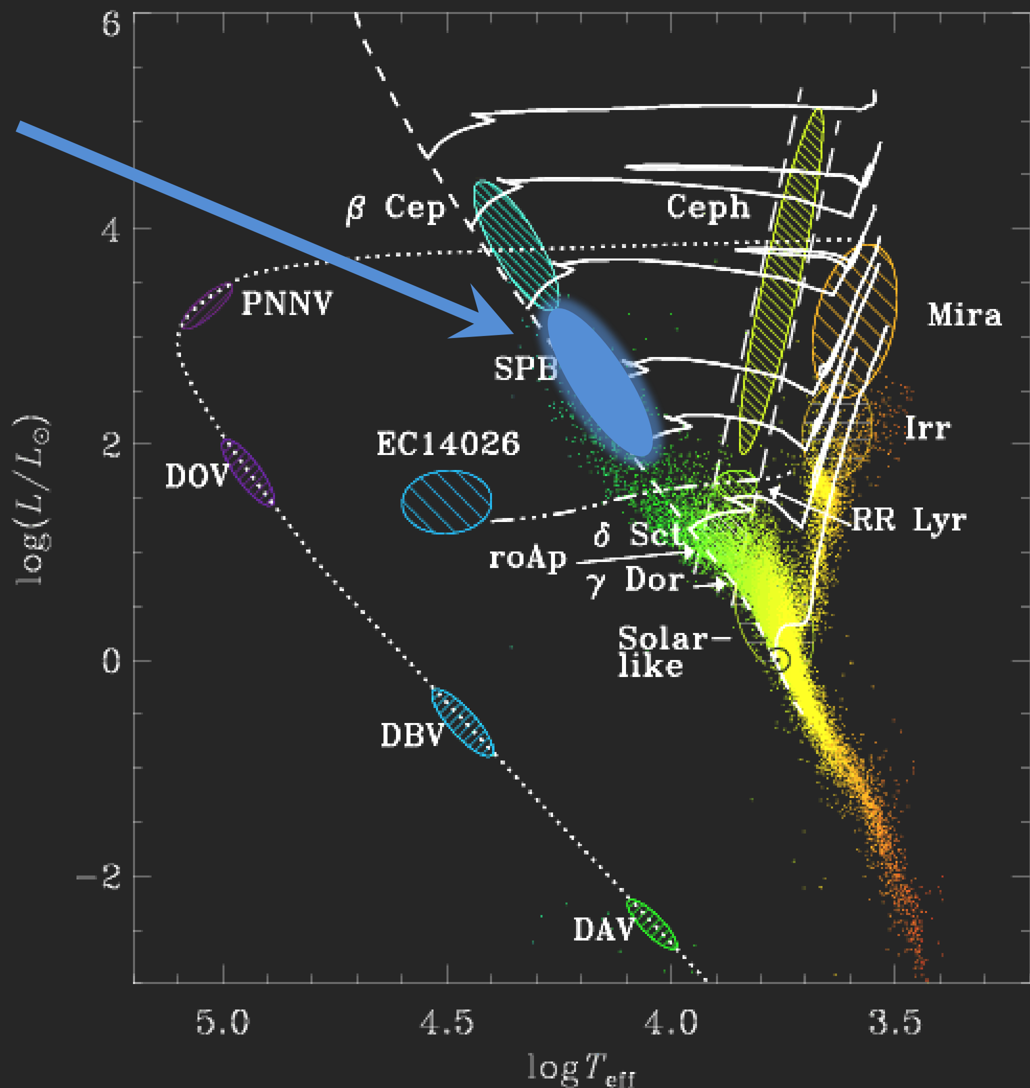
δ Scuti

Periods : 0.5 – 6 h



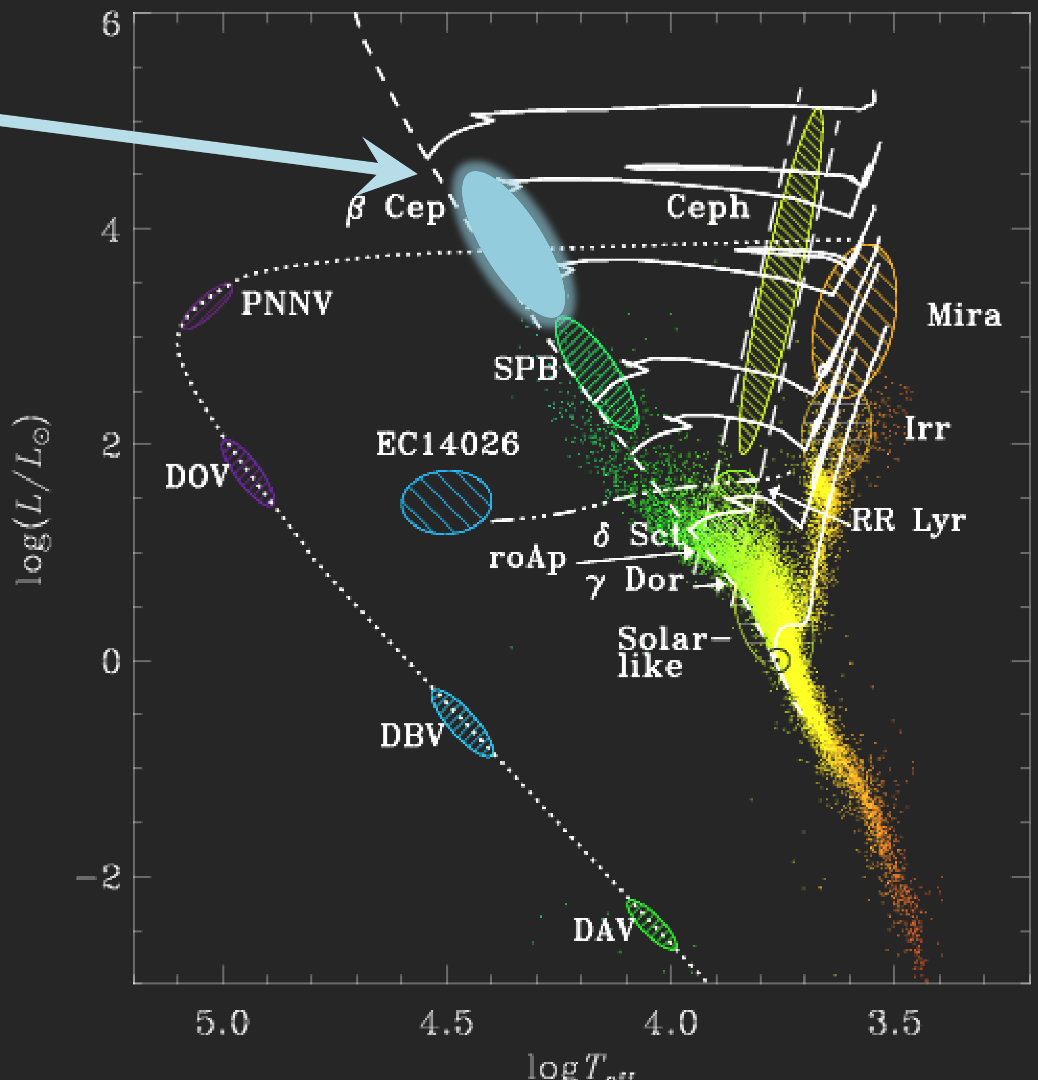
Slowly Pulsating B

Periods : 1 – 4 d



β Cephei stars

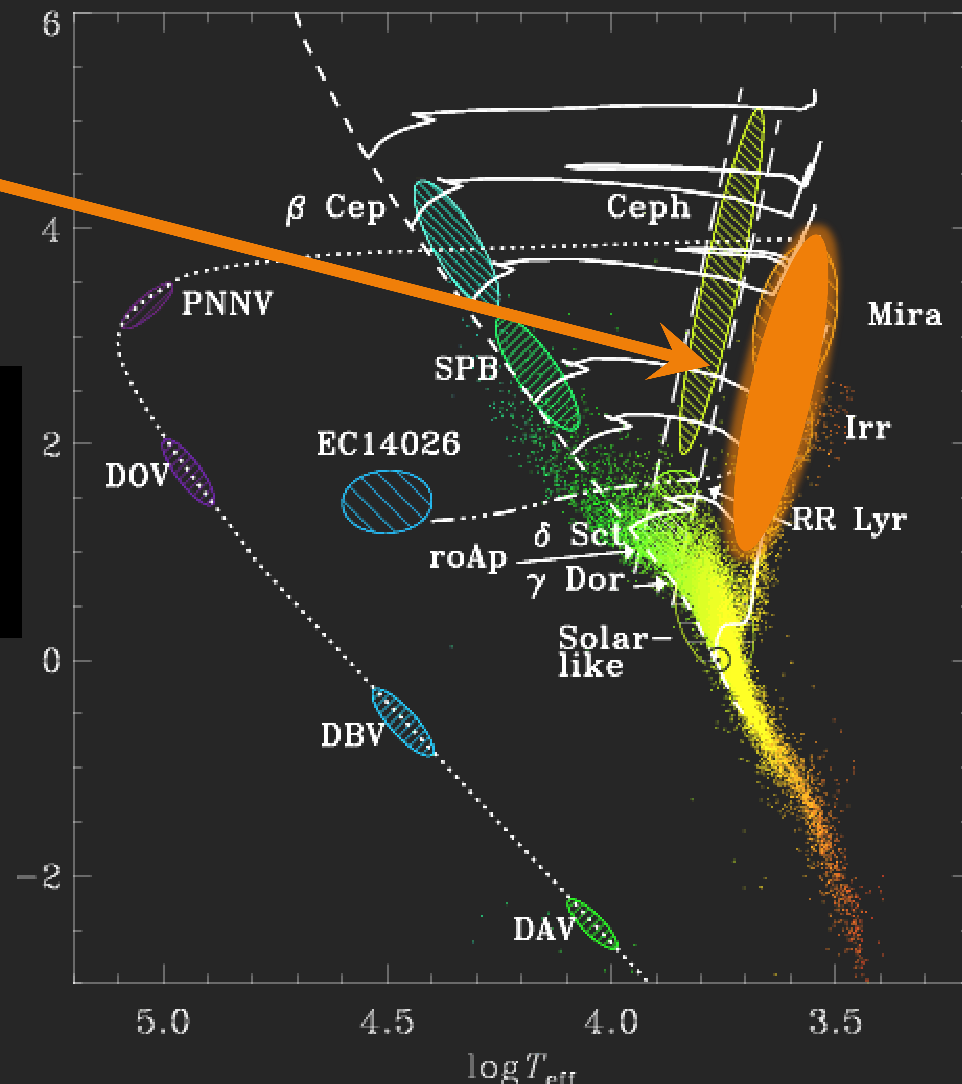
Periods : 3 – 8 h



Red giants

Radial and non radial pulsations

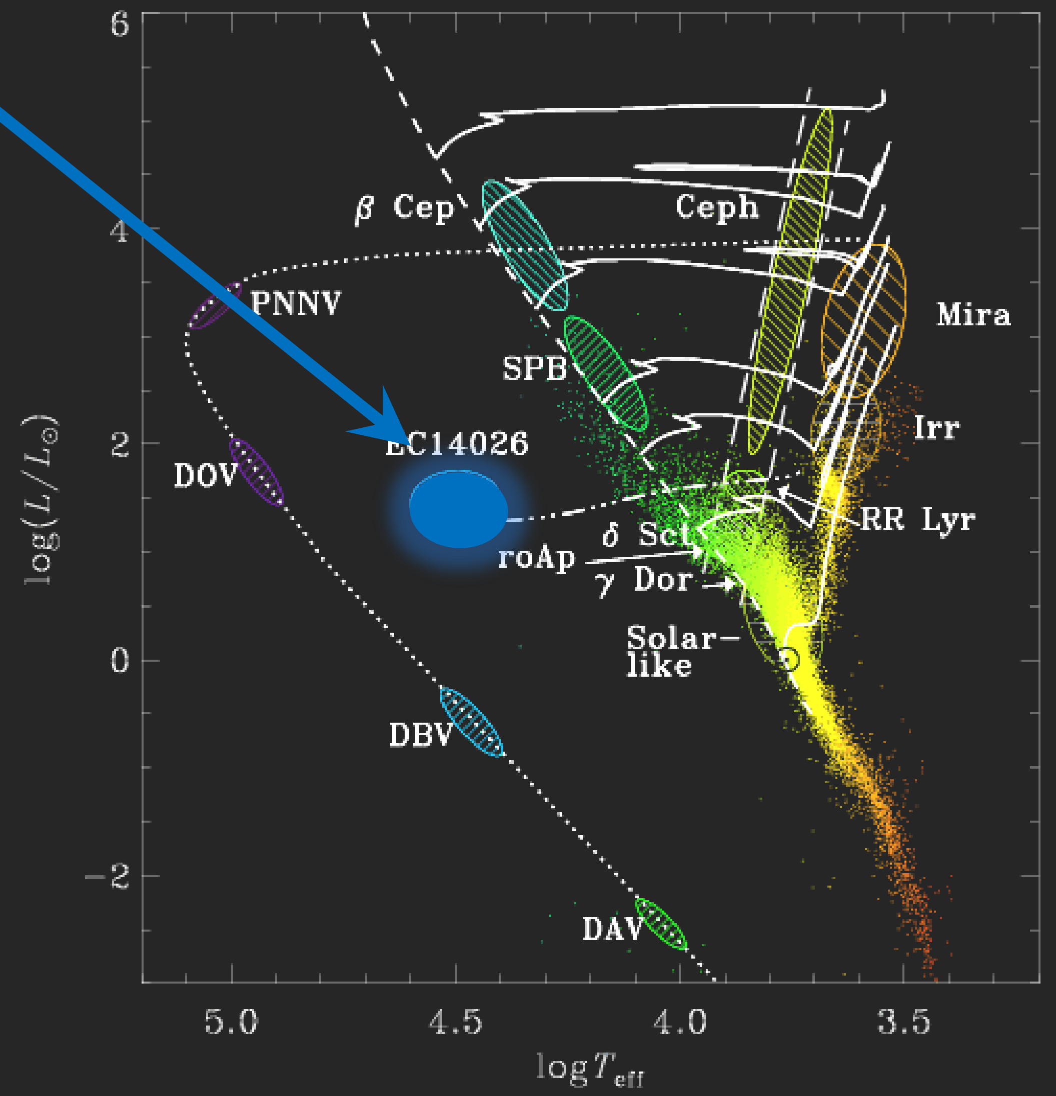
[redacted]



Subdwarf B stars

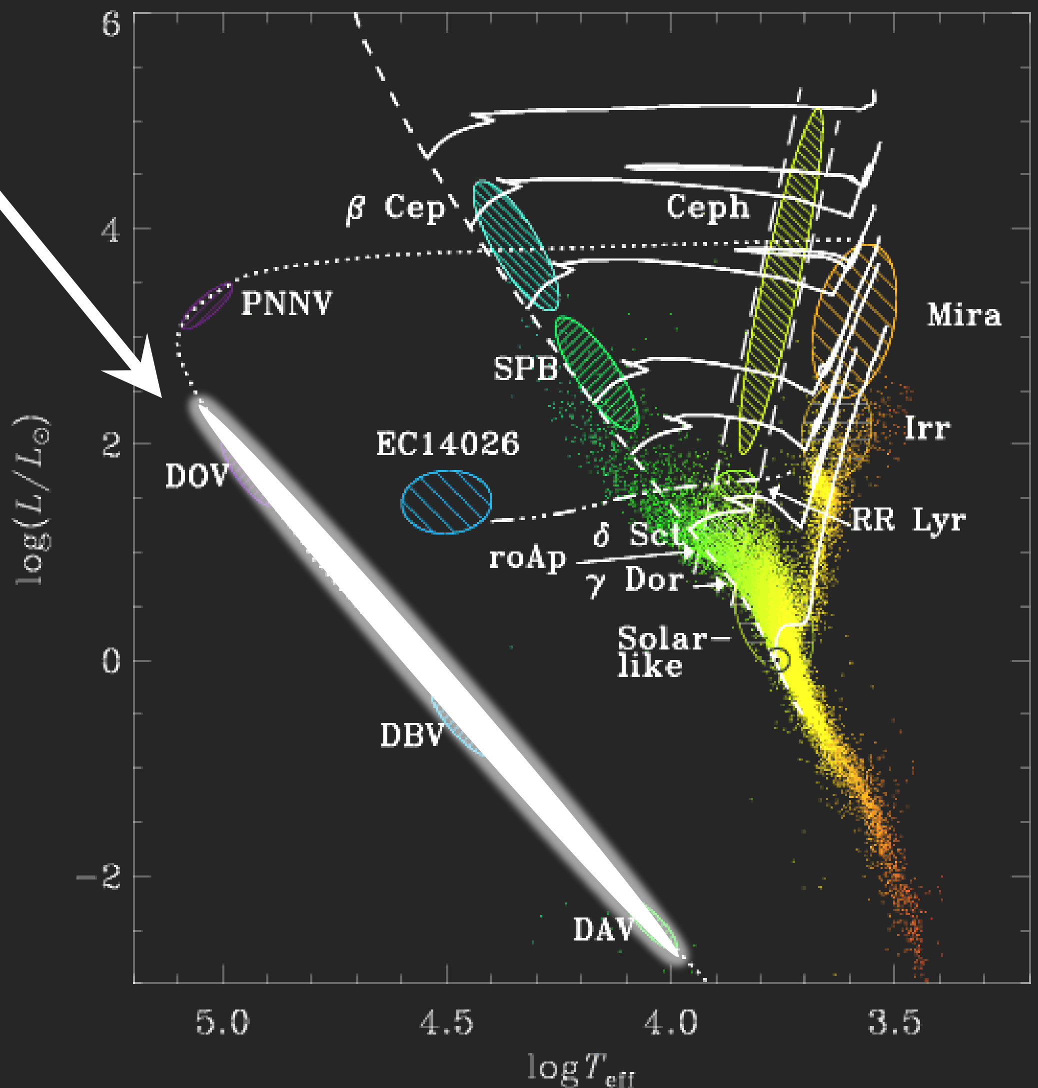
Periods : 1 – 6 min

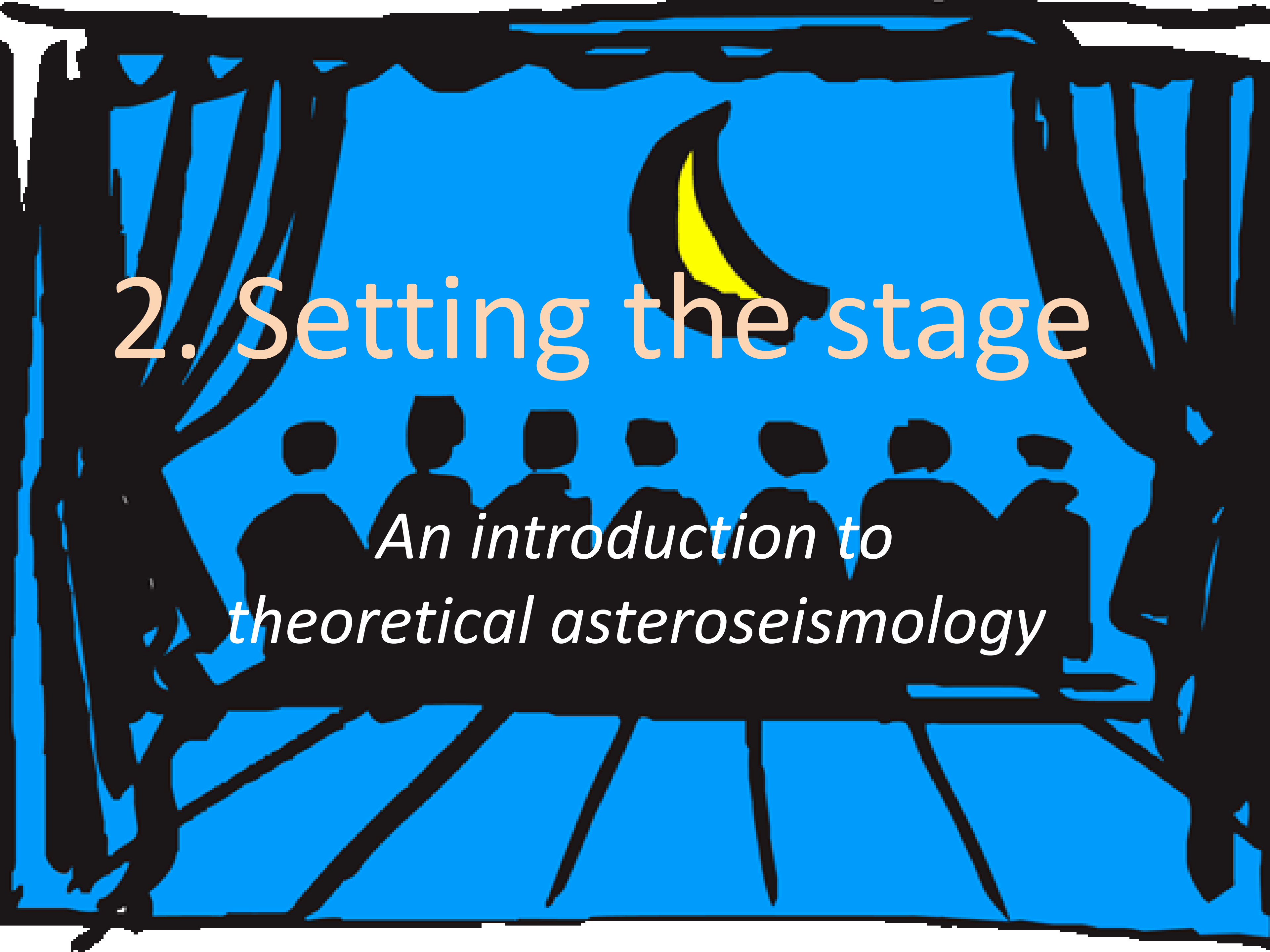
~1 h



White dwarfs

Periods : 2 – 15 min





2. Setting the stage

An introduction to theoretical asteroseismology

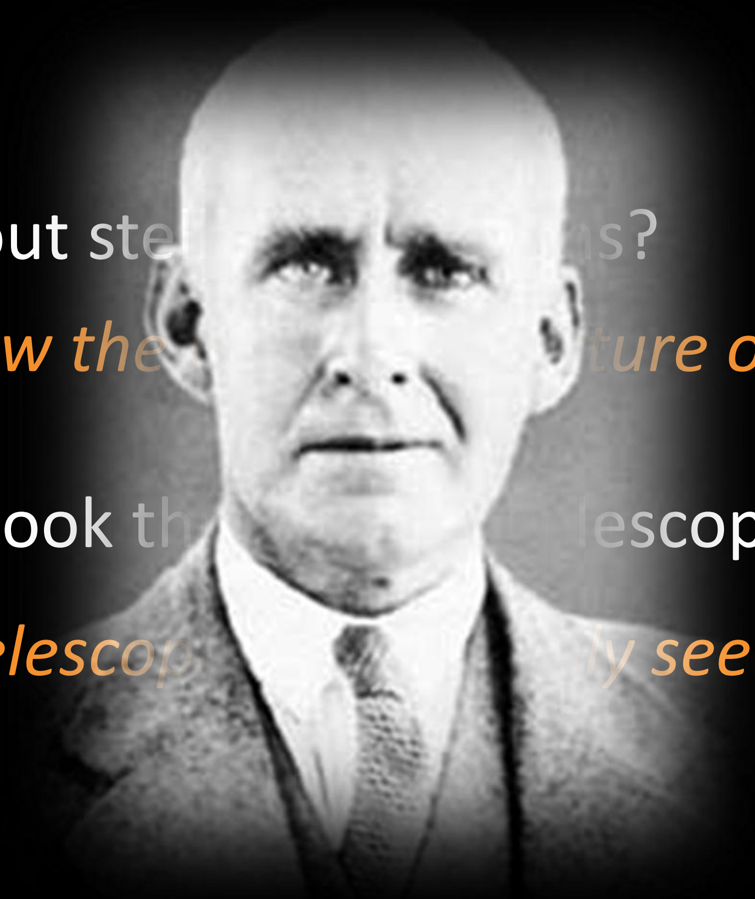
Questions:

- Why do we worry about ste… s?

because we want to know the … ture of stars…

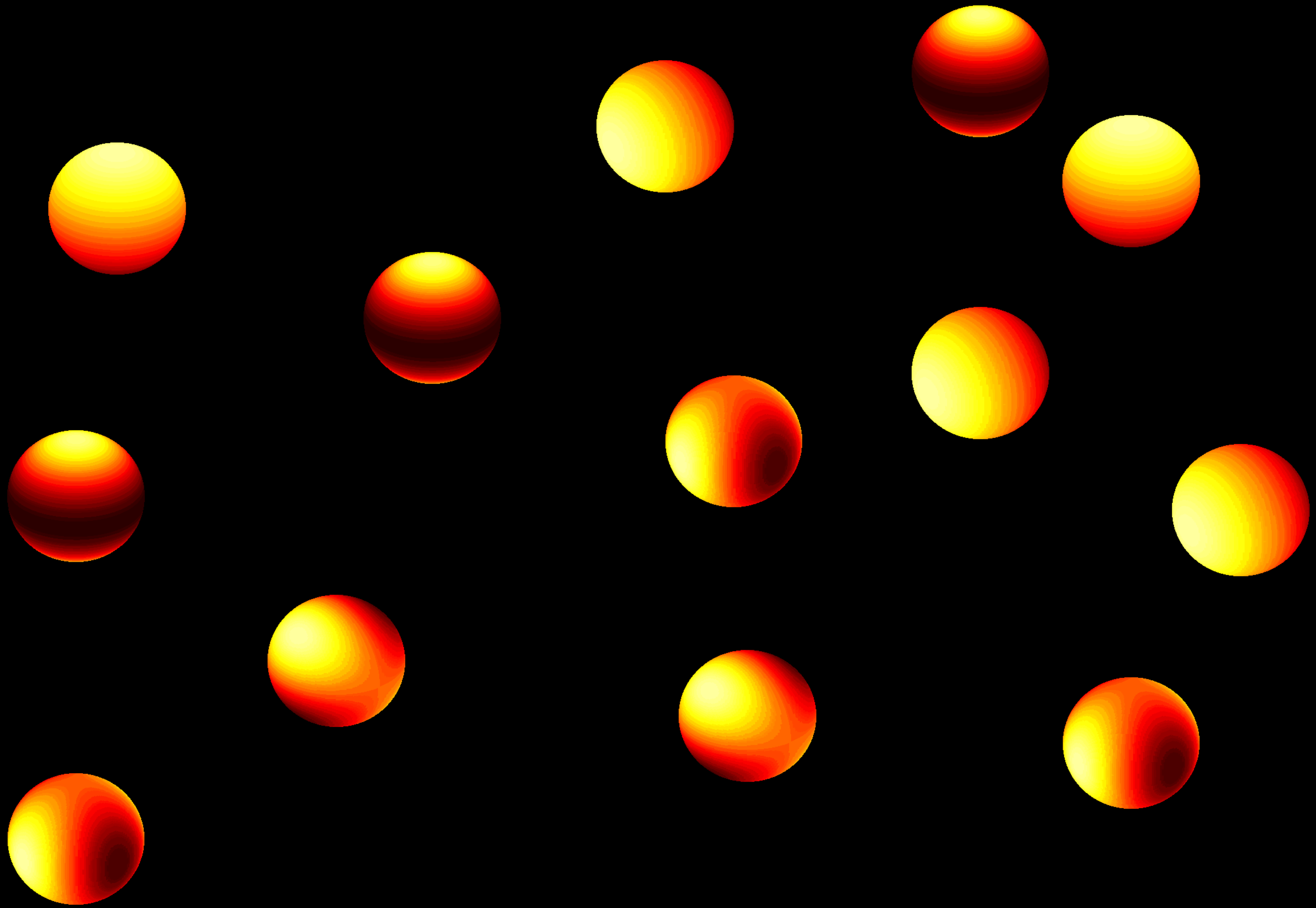
- Why don't we go and look th… escopes?

because with classical telesco… y see the surface layers…



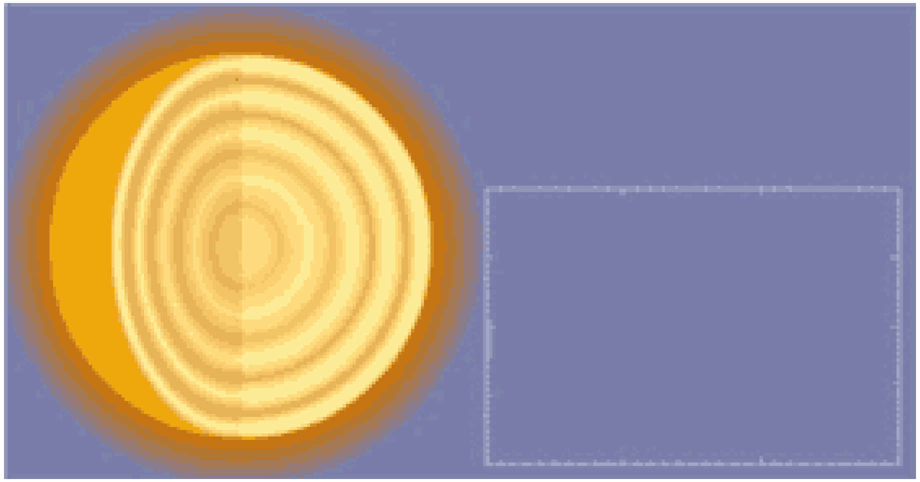
Stars are essentially opaque and we cannot access the internal layers through classical observations

But … stars pulsate

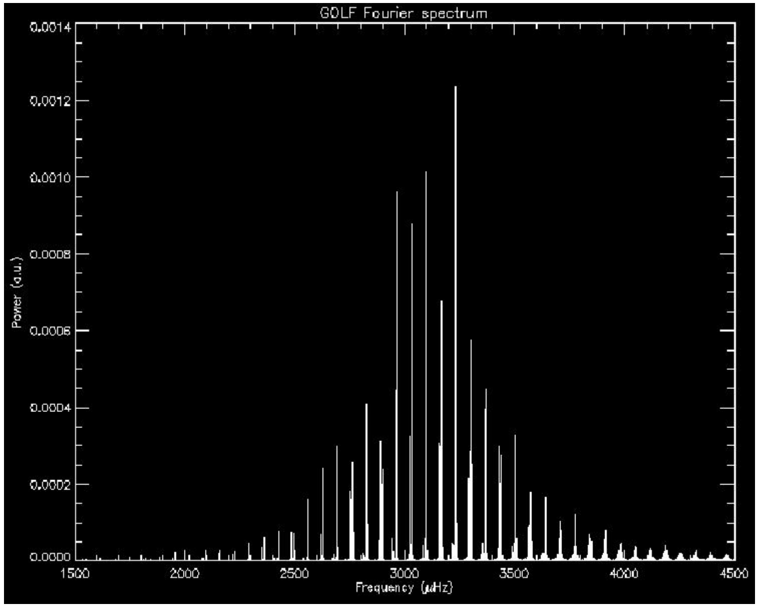


Bases of asteroseismology

Measure of temporal - stellar flux variations
- radial velocity variations



Fourier space
➔ frequencies



frequencies

Frequencies are characteristics of the source ➔ stellar structure

2.1 Adiabatic stellar pulsations

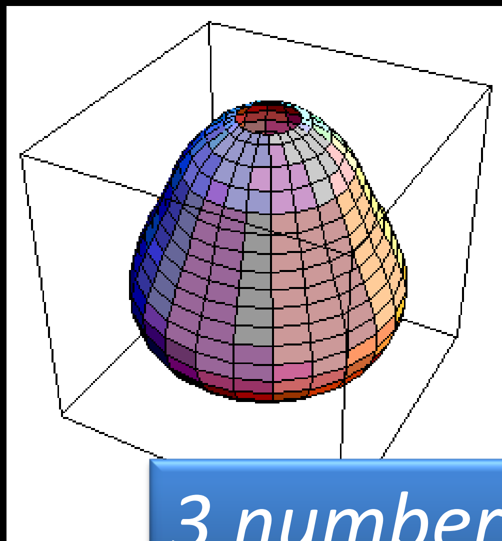
The specific entropy is conserved during the oscillation

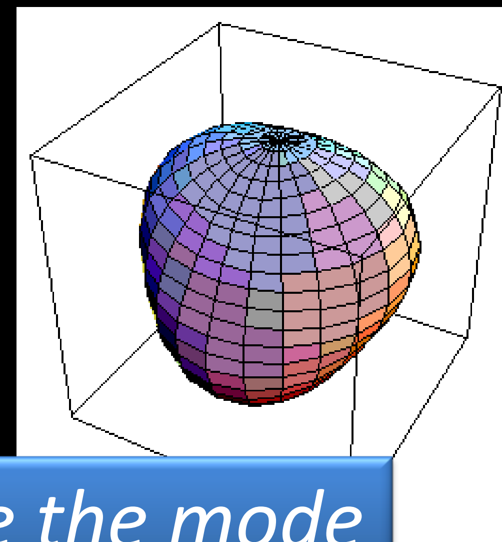
$\delta S = 0$

No energy exchange between mass elements

- **standing waves**
- **strictly periodic** ➔ **exp(iσt)** ***with σ real***
- **no amplitude growth**
- **no amplitude damping**

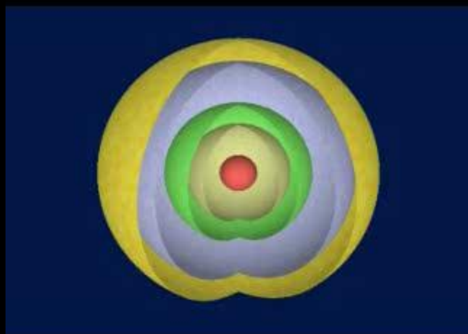
3D standing waves – a star

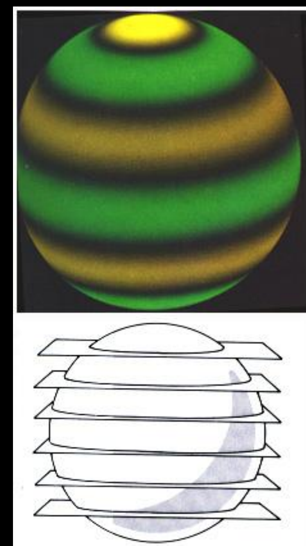


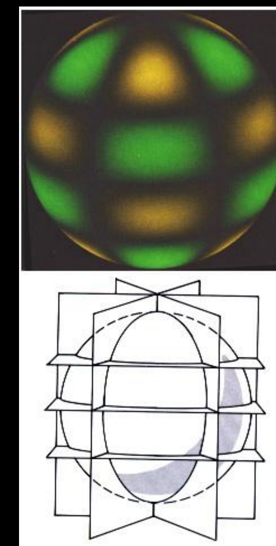


3 numbers to describe the mode

3D standing waves – a star







n : radial order	*radial nodes*	*n = 0, ..., ∞*
l : degree	*nodal planes*	l = 0, ..., ∞
m : azimuthal order	*meridian nodal planes*	m = – l, ..., + l

2.1.1 Adiabatic pulsation modes

Perturbation and linearization of the equations of

- **mass conservation**
- **momentum conservation**
- **energy conservation $\delta S = 0$**

$$\delta X(r,\theta,\varphi,t) = \delta X(r)\ Y_\ell^m(\theta,\varphi)\ \exp(i\sigma t)$$

Spherical harmonics

$Y_\ell^m(\theta, \phi)$

Cowling approximation : Φ' = 0

mass and momentum equations → 2nd order eigenvalue problem

$$\frac{dP'}{dr}$$

$$\frac{P'}{\rho\, c^2}$$

- ***N^2 = 0 in a convective zone***
- ***N^2 develops a peak in a region of r***
  varying mean molecular weight

N : Brunt-Väisälä frequency $N^2 = \frac{G\,m}{r^2}\left(\frac{1}{\Gamma_1}\frac{d\ln P}{dr} - \frac{d\ln\rho}{dr}\right)$

$$N^2 = \frac{g}{r}\frac{d\ln P}{d\ln r}\left[\left(\frac{\partial\ln\varrho}{\partial\ln T}\right)_{P,\mu}(\nabla_{ad}-\nabla) - \nabla_\mu\left(\frac{\partial\ln\rho}{\partial\ln\mu}\right)_{T,P}\right]$$

Unno, Osaki, Ando, Saio, Shibahashi 1989 *Non radial oscillations of stars*

Cowling approximation : $\Phi' = 0$

mass and momentum equations $\rightarrow$ 2nd order eigenvalue problem

$$\frac{dP'}{dr} + \frac{g}{c^2} P' = (\sigma^2 - N^2)\, \rho\, \delta r$$

$$\frac{P'}{\rho c^2}\left(1 - \frac{L_\ell^2}{\sigma^2}\right) - \frac{g}{c^2}\delta r + \frac{1}{r^2}\frac{d}{dr}\left(r^2 \delta r\right) = 0$$

$c^2 = \Gamma_1 P/\rho$
$\Gamma_1 = (\partial \ln P/\partial \ln \rho)_S$
$\nabla = (\partial \ln T/\partial \ln P)$
$\nabla_\mu = (\partial \ln \mu/\partial \ln P)$

L_ℓ: Lamb frequency

L_ℓ^2 **depends on l**

Unno, Osaki, Ando, Saio, Shibahashi 1989 *Non radial oscillations of stars*

Local analysis

Assumption:

- constant coefficients *(equilibrium values)*

→ *Plane wave equation*

$$\frac{d^2\bar{\delta r}}{dr^2} + k^2(r)\,\bar{\delta r} \simeq 0$$

k : wavenumber

k_r : radial component

k_h: horizontal component

$$\bar{\delta r} \equiv \frac{\rho^{1/2} r c}{\left|L_\ell^2/\sigma^2 - 1\right|^{1/2}}\,\delta r$$

$\delta r \sim \exp(ik_r r)$

$$k_r^2 = (1/\sigma c)^2\,(\sigma^2 - L_\ell^2)\,(\sigma^2 - N^2)$$

Dispersion relation *relating* ***wave number*** *and* ***frequency***

Local analysis

$\delta r(r) \sim exp(ik_r r)$

$$k_r^2 = (1/\sigma c)^2 (\sigma^2 - L_\ell^2)(\sigma^2 - N^2)$$

$k_r^2 > 0$: propagation

$k_r^2 < 0$: evanescence

$N^2 < \sigma^2 < L_l^2$

$L_l^2 < \sigma^2 < N^2$

$\sigma^2 > N^2, L_\ell^2$

pressure mode

Restoring force: pressure

p_1 p_2

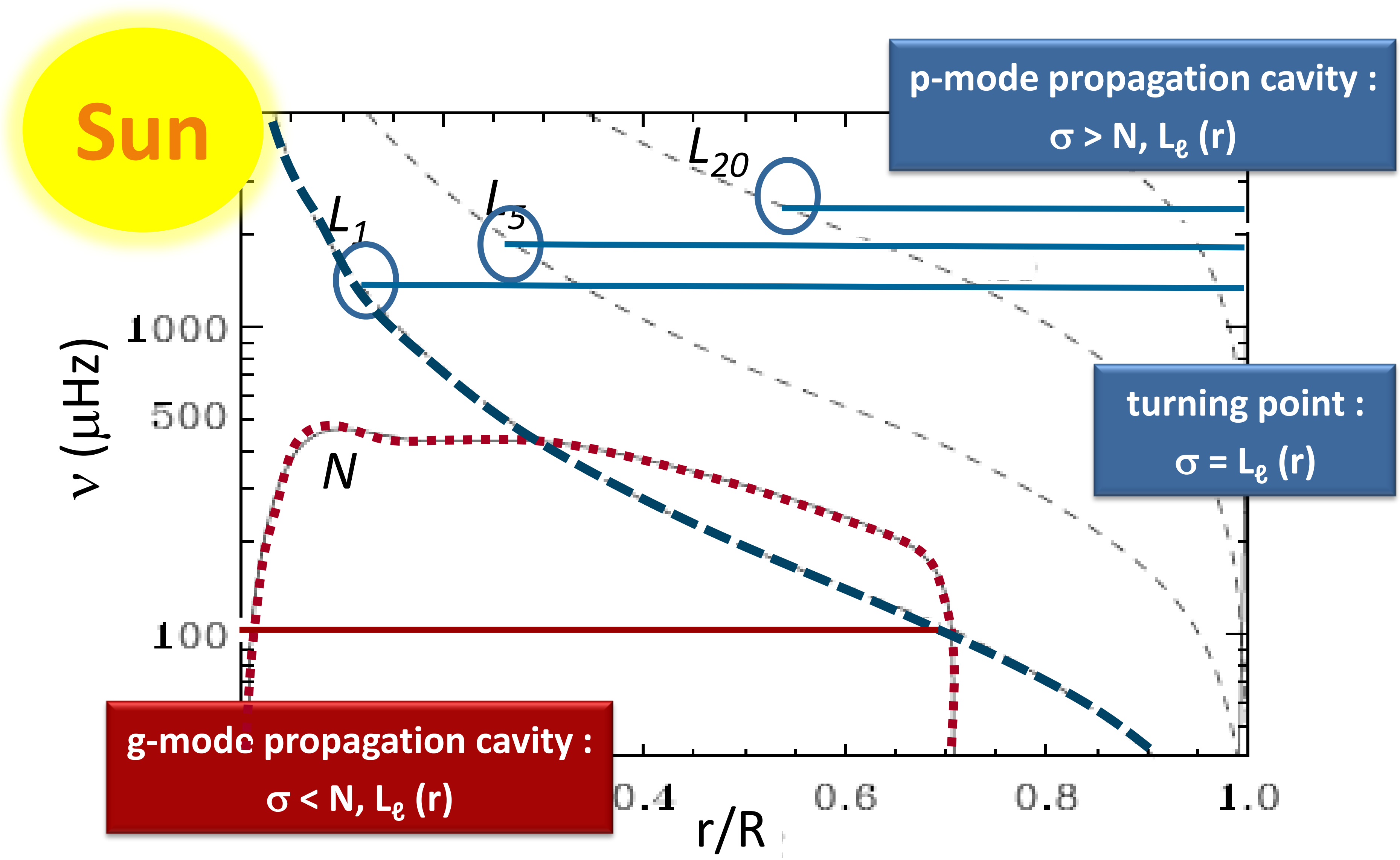
$\sigma^2 < N^2, L_\ell^2$

gravity mode

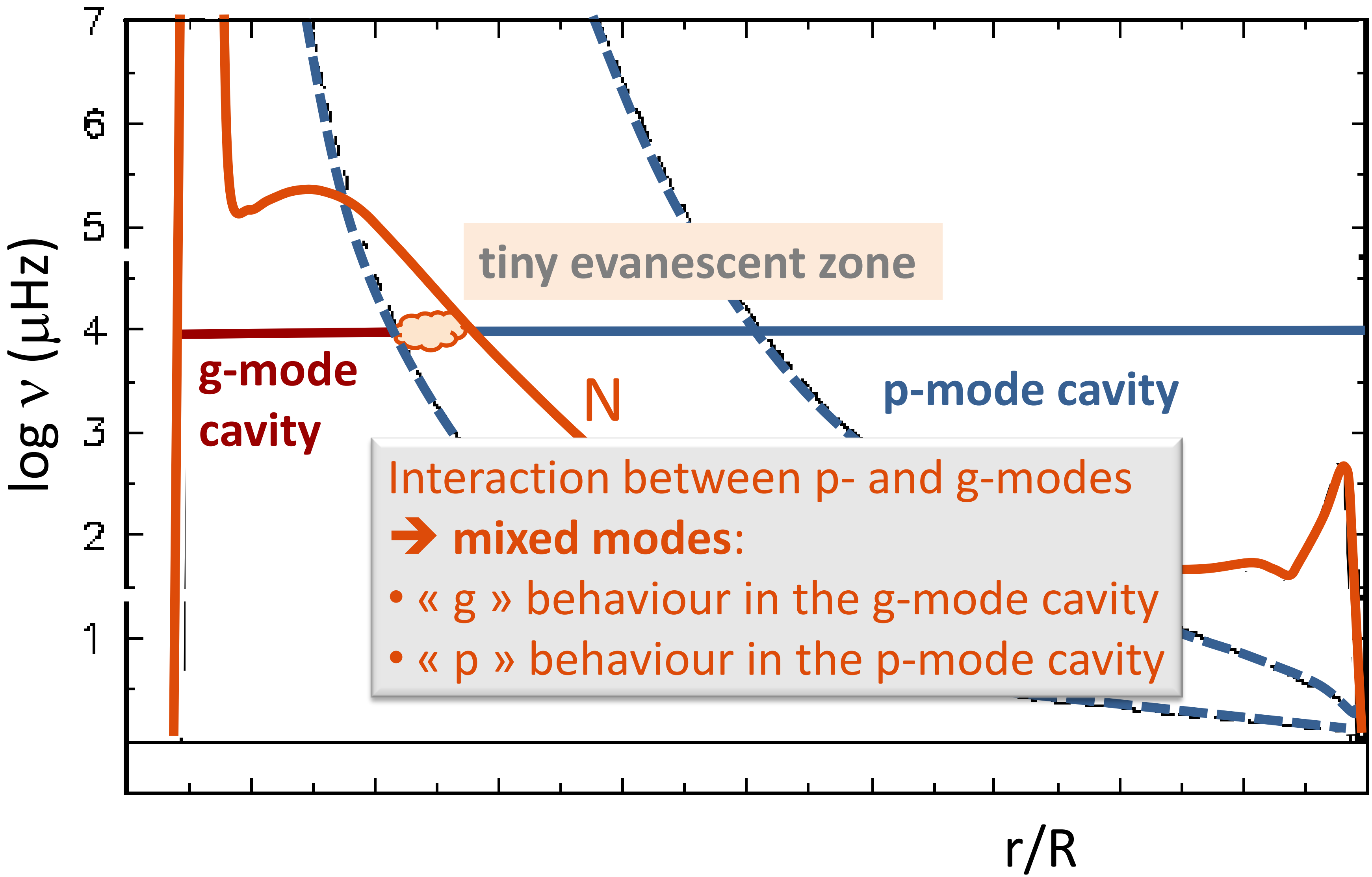
Restoring force: gravity

g_1 g_2

Propagation diagram



As the star evolves, N ➚ as a result of the formation of a μ-gradient



Effect of rotation

Sperical symmetry ➔ no terms f(φ) in the equations

➔ *modes of given l and different m have the same frequency*

With a rotation Ω, degeneracy is lifted

$$\nu = \nu + mC\Omega - m\Omega$$

Coriolis term C <<

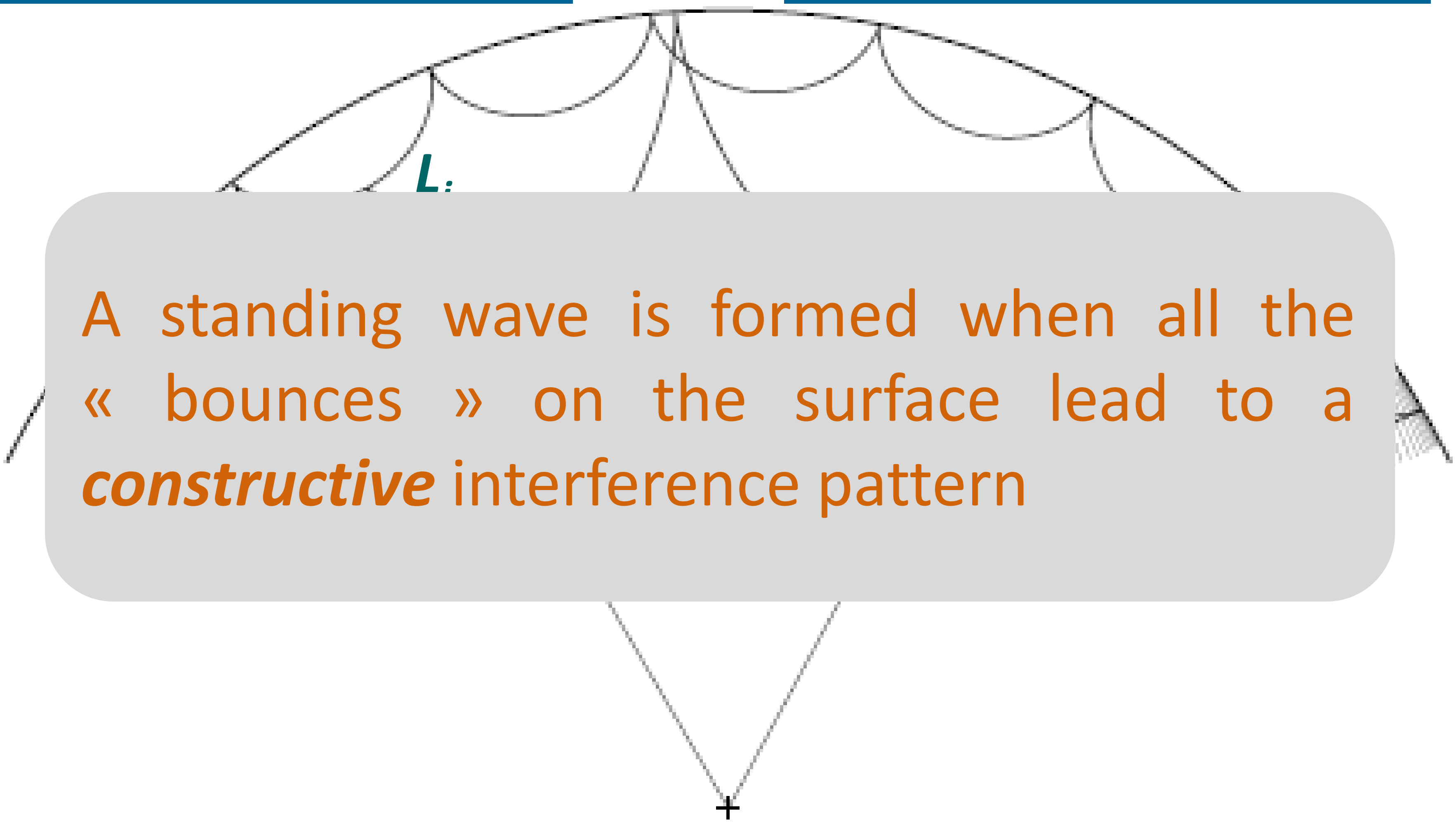
l=2

*Since modes of different l probes layers located at different depths, **rotational splittings** gives **Ω (r)***

m=–2 m=–1 m=0 m=1 m=2

2.1.2 Pressure modes $\sigma^2 > N^2, L_\ell^2$

as **l** increases, the p cavity becomes smaller → modes with different (**l**,ν) probe different layers



A standing wave is formed when all the « bounces » on the surface lead to a ***constructive*** interference pattern

Pressure modes $\sigma^2 > N^2, L_\ell^2$

$k_r^2 = \sigma^2/c^2 \, (L_\ell^2/\sigma^2 - 1) \, (N^2/\sigma^2 - 1)$

$\sigma^2 \gg N^2$

$k_r^2 \sim \sigma^2/c^2 \, (1 - L_\ell^2/\sigma^2)$

$k_h^2 = l\,(l+1)/r^2 = L_l^2/c^2$

$k^2 = k_r^2 + k_h^2 \sim \sigma^2/c^2$ → *constraint on the **sound speed** in the outer layers*

$$c^2 = \frac{P\,\Gamma_1}{\rho} \propto \frac{\Gamma_1\, T}{\mu}$$

T → temperature distribution, convective envelope limit

Γ_1 → partial ionization, equation of state

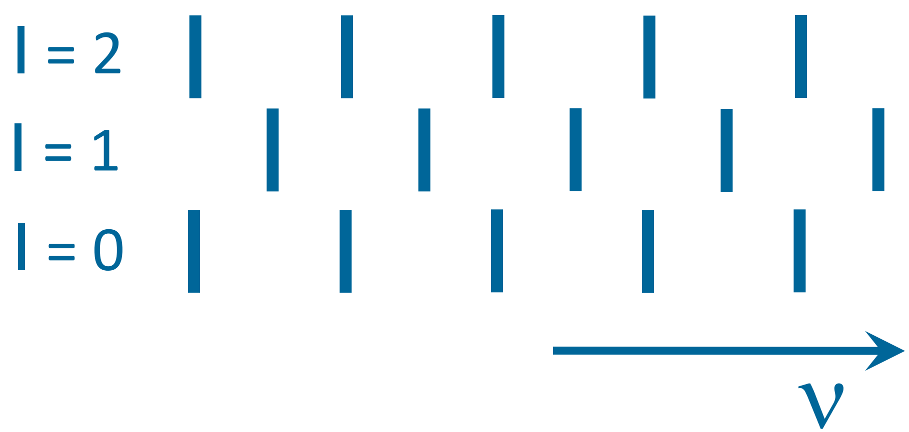
μ → mean molecular weight, chemical composition

Pressure modes – asymptotic regime n >>

Seismic indicators: 1. Large separation

$$\Delta\nu_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell} \approx \left(2\int_{r_i}^{r_s}\frac{\mathrm{d}r}{c}\right)^{-1} \propto \sqrt{\langle\rho\rangle}$$

$$\nu_{n,l} \sim (n + l/2 + 1/4 + \alpha)\,\Delta\nu$$



Tassoul 1980, Smeyers & Tassoul 1988

Pressure modes – asymptotic regime n >>

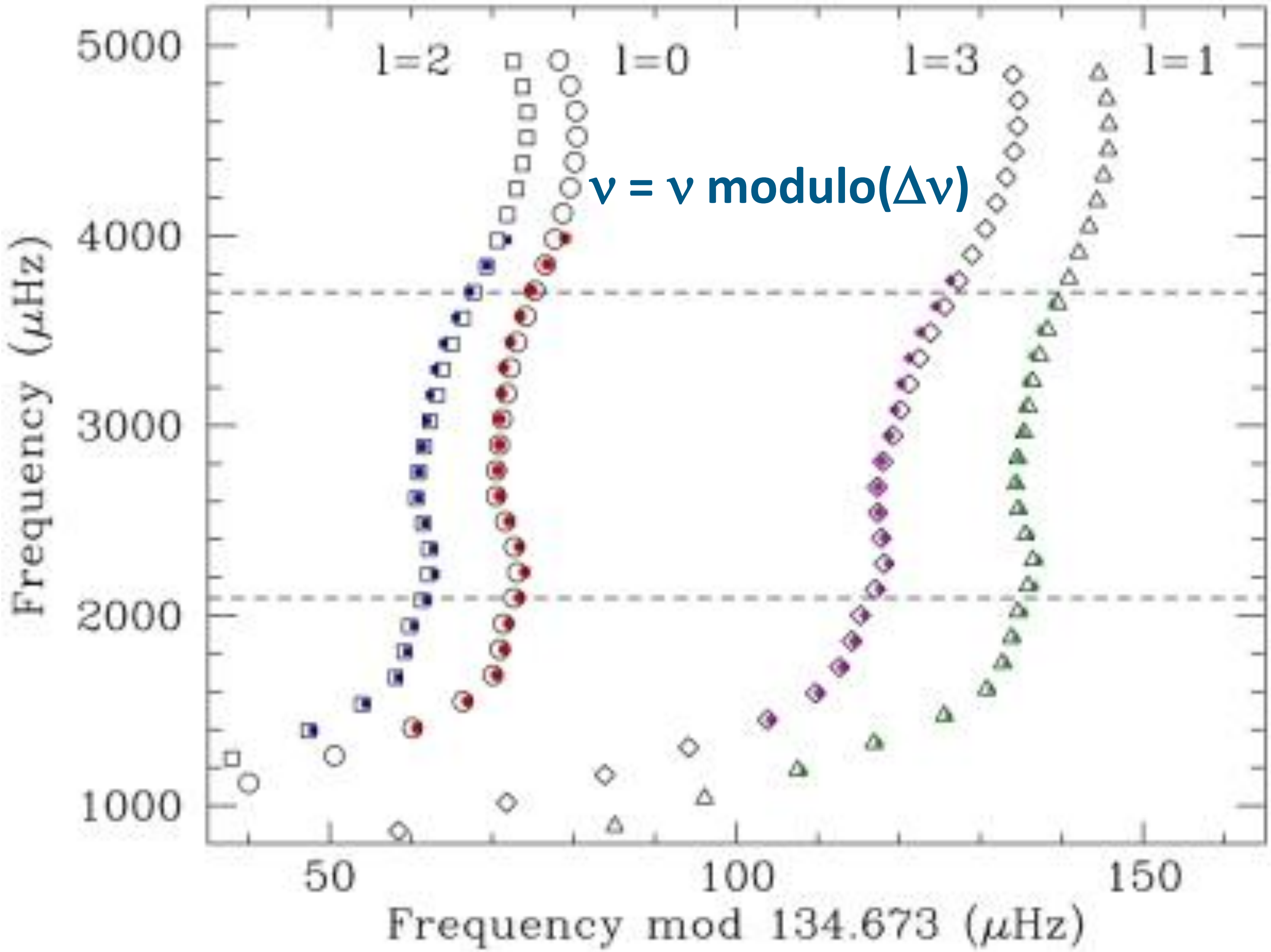
Seismic indicators: 1. Large separation

$$\Delta\nu_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell} \approx \left(2\int_{r_i}^{r_s} \frac{\mathrm{d}r}{c}\right)^{-1} \propto \sqrt{\frac{GM}{R^3}}$$

Constraint on the distribution of the sound speed in the p-mode cavity

Tassoul 1980, Smeyers & Tassoul 1988

Large separation - Echelle diagram



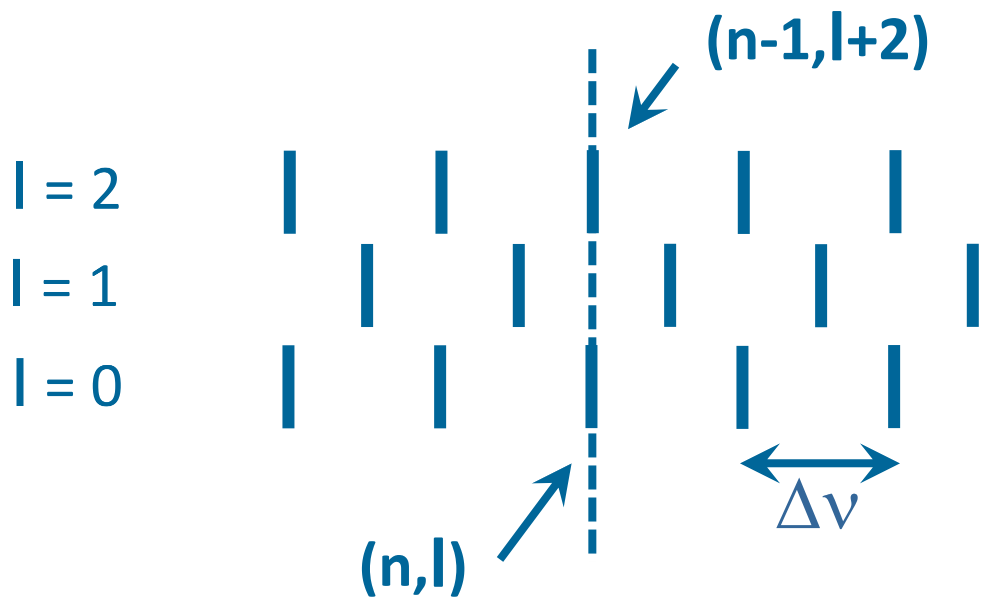
Smeyers et al. 1988: In ESA, Seismology of the Sun and Sun-Like Stars p.623-627

High Altitude Observatory

Pressure modes – asymptotic regime n>>

Seismic indicators: 2. Small separation

$$\nu_{n,l} = \Delta\nu\,(n + l/2 + \varepsilon) + \delta_{n,l}$$

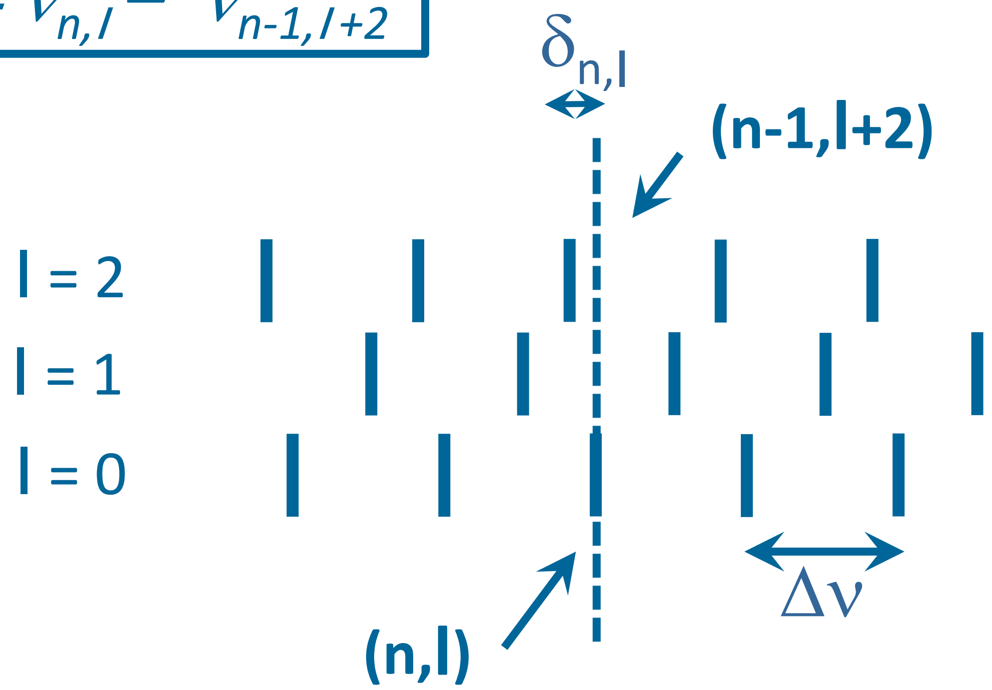


Pressure modes – asymptotic regime n >>

Seismic indicators

$$\nu_{n,l} = \Delta\nu\,(n + l/2 + \varepsilon) + \delta_{n,l}$$

$$\delta_{n,l} = \nu_{n,l} - \nu_{n-1,l+2}$$



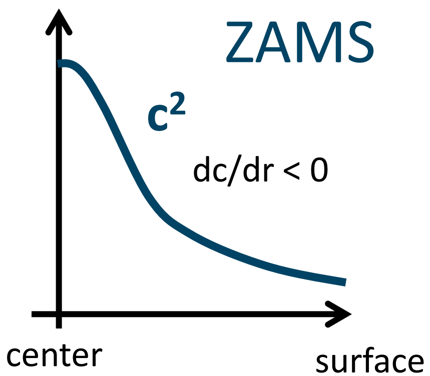
Small separation → Evolutionary state

$$\delta_{n,\ell} = \nu_{n,l} - \nu_{n-1,l+2}$$

$$\simeq -(4\ell+6)\frac{\Delta\nu}{4\pi^2\nu_{n,l}}\int_0^R \frac{dc}{dr}\frac{dr}{r}$$

$$c^2 = \frac{P\,\Gamma_1}{\rho} \simeq \frac{\Gamma_1 R\, T}{\mu}$$

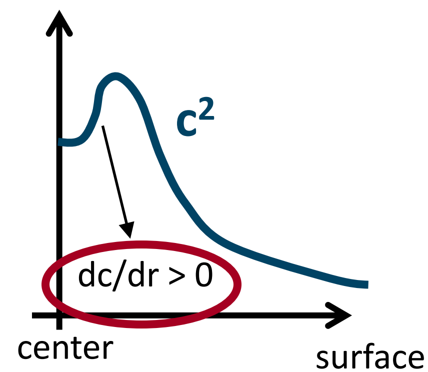
ZAMS



As H transforms into He

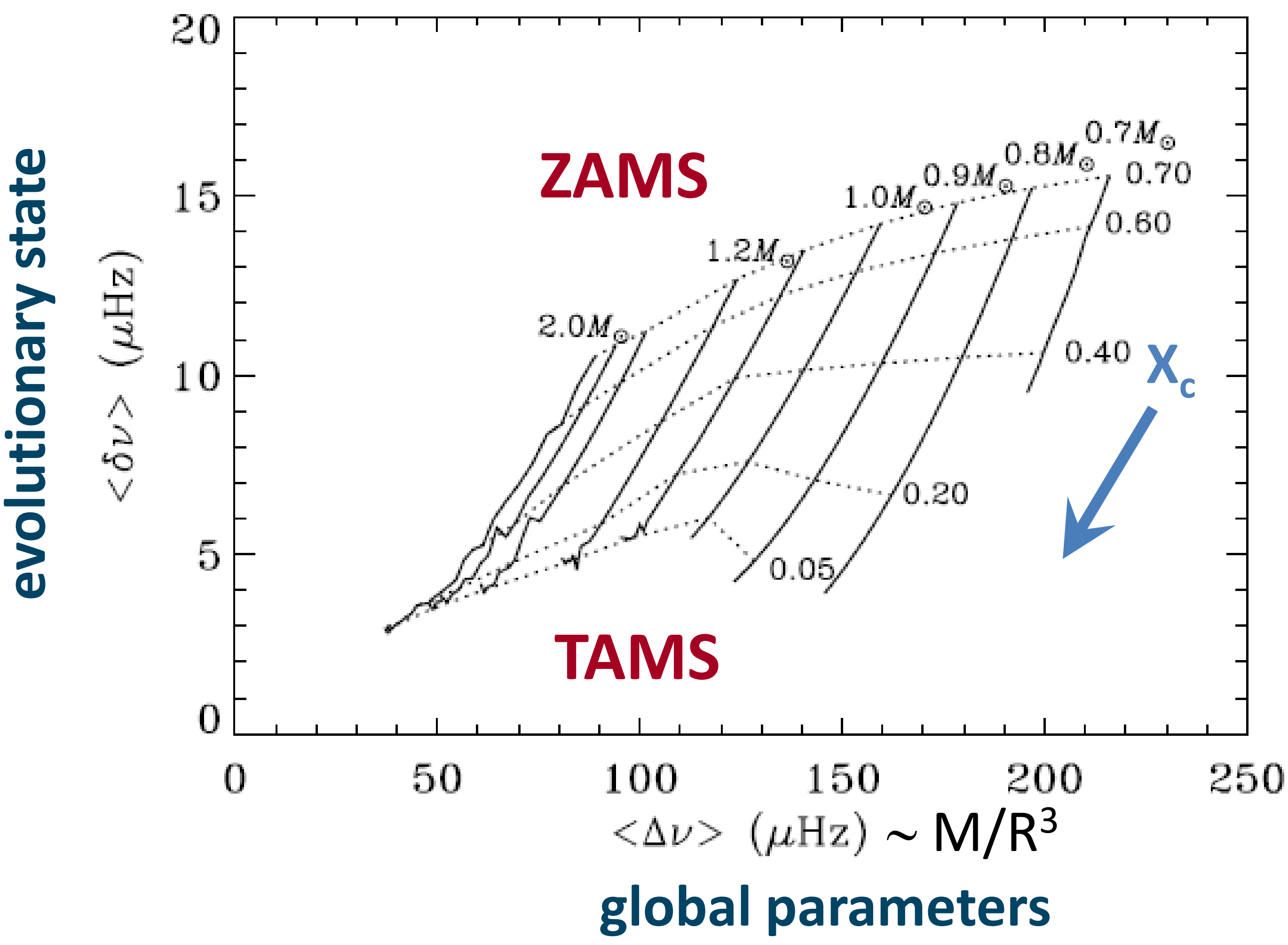
μ ↗ near the center

→ c^2 ↘



→ $\delta_{n,\ell}$ ↘

Main Sequence Asteroseismic Diagram



Christensen-Dalsgaard 1982

Pressure modes – asymptotic regime n>>

Seismic indicators

large separation $\Delta_{n,\ell} = \nu_{n,\ell} - \nu_{n-1,\ell}$ — *depends on c(r), on global effects, <ρ>*

small separation $\delta_{n,l} = \nu_{n,l} - \nu_{n-1,l+2}$ — *depends on dc/dr*

small spacing $\delta_{01,n} = \nu_{n,0} - 2\nu_{n,1} + \nu_{n+1,0}$ — *discontinuities in dc/dr*

2.1.3 Gravity modes $\sigma^2 < N^2, L_\ell^2$

$k_r^2 \sim l\,(l+1)\,N^2/(r^2\sigma^2)$

→ *constraints on the **Brunt-Väisälä frequency** in the inner layers*

$$N^2 \simeq \frac{\rho g^2}{P}\left(\nabla_{\rm ad} - \nabla + \nabla_\mu\right)$$

super-adiabatic T gradient
convective zone boundaries
overshooting

gradient of mean molecular weight
chemical composition profile

Gravity modes – asymptotic regime n >>

Seismic indicator: Period spacing

→ *constant ΔP instead of constant $\Delta\nu$*

$$P_{n,\ell} - P_{n-1,\ell} \approx \frac{2\pi^2}{\sqrt{\ell(\ell+1)}} \frac{1}{\int_{g-cav} \frac{N}{r}\,\mathrm{d}r}$$

Constraint on the distribution of the Brunt-Väisälä frequency in the g-mode cavity with a weight in 1/r

2.2 Non adiabatic stellar pulsations

In order to be observed, an oscillation must be excited

There must be energy exchanges between mass elements within the star during the pulsation

Oscillations are damped if $\eta < 0$
Oscillations are excited if $\eta > 0$

Energy conse

$$T \frac{\partial S}{\partial t} = \left(\varepsilon - \frac{\partial L}{\partial m} \right)$$

exp(iσt) ➔ term ~ i σ ➔ complex eigenvalue ➔ σ – iη

$$\delta X(r,t) = \delta X(r) \sin(\sigma t + \varphi(r)) \exp(\eta t)$$

2.2 Non adiabatic stellar pulsations

Energy conservation equation

$$T\frac{\partial S}{\partial t} = \left(\varepsilon - \frac{\partial L}{\partial m}\right)$$

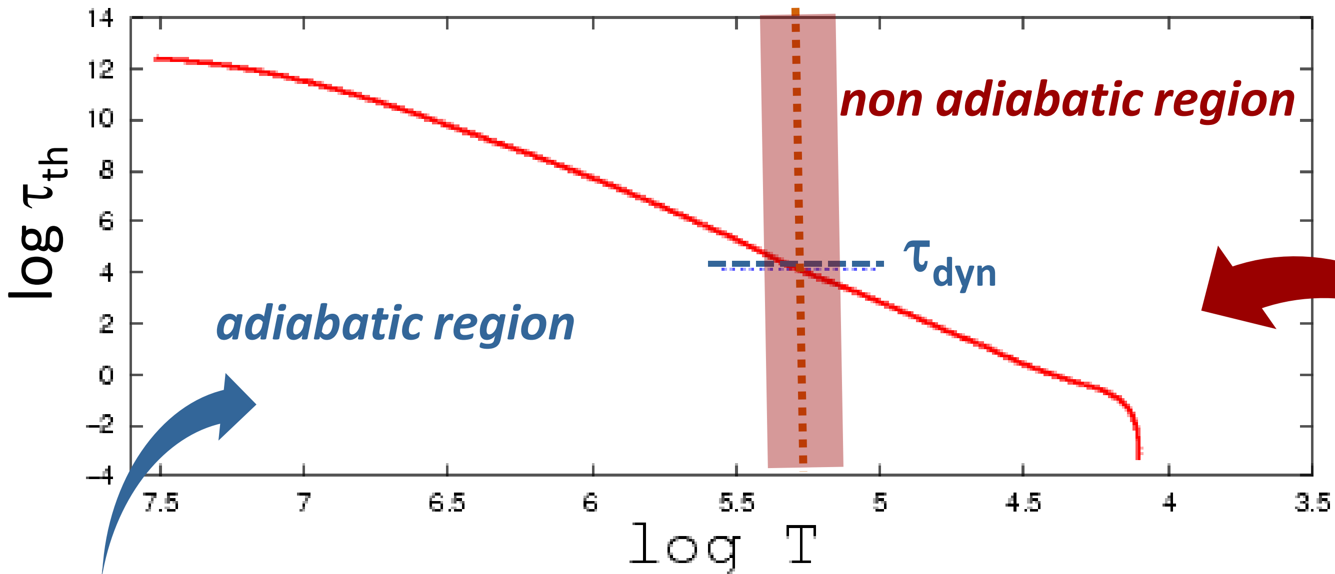
Perturbation ➔

$$\frac{\tau_{th}}{\tau_{dyn}} i\,\omega \frac{\delta S}{c_P} = \left(\delta\varepsilon - \frac{\partial \delta L}{\partial m}\right)\frac{4\pi r^3 \rho}{L}$$

$\omega = \sigma\, \tau_{dyn}$
$\tau_{dyn} = (R^3/GM)^{1/2}$
$\tau_{th} = (4\pi r^3 \rho c_P T)/L$

δS becomes large if τ_{th}/τ_{dyn} decreases

Non adiabatic region



transition region $\tau_{th} \sim \tau_{dyn}$

main driving region

coupling between the dynamical and the thermal equations

structure of the star
➔ oscillation frequency

Excitation mechanisms

Heat engine

Self excited oscillations

Amplitude ↗ until a limit cycle is reached

- κ mechanism
- ε mechanism

Cepheids, δ Scuti, SPB, β Cep …

Stochastic excitation

Damped oscillations

Excitation by turbulent motions in a convective zone « drum excitation »

Solar-like oscillations

Excitation mechanism – heat engine

$\delta X(r,t) = \delta X(r) \sin(\sigma t+\varphi (r)) \exp(\eta t)$

$$\eta = \frac{1}{2\sigma^2} \frac{\int_0^M -\frac{\delta T}{T} \frac{\partial \delta L}{\partial m} dm}{\int_0^M \delta r^2 dm} = \frac{W}{2E}$$

work

energy

hot phase:
heat is blocked

cold phase:
heat is released

→

η > o
positive work

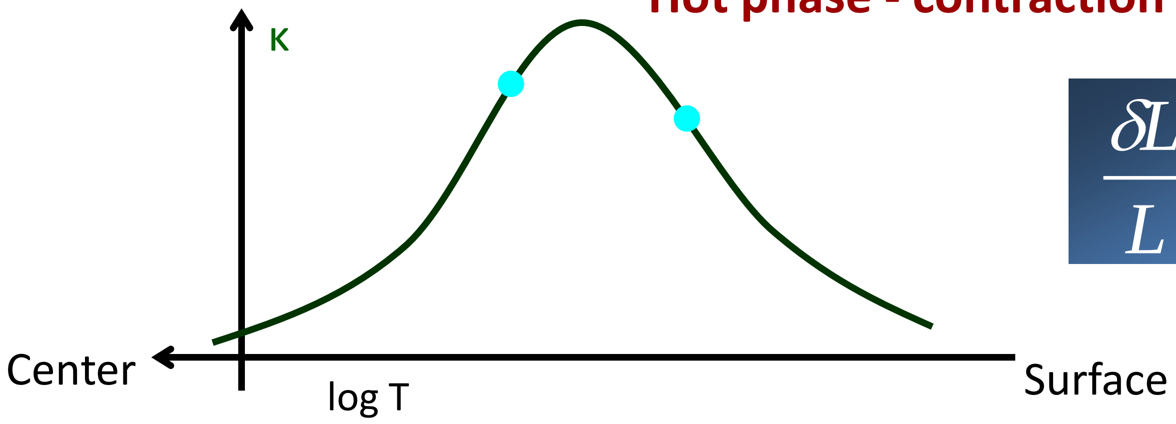
Functioning condition of a thermodynamical engine

Excitation mechanism - κ mechanism

$$\frac{dW}{dr} = -\frac{1}{2}\frac{\delta T}{T}\frac{\partial \delta L}{\partial r} > 0 \Rightarrow driving\,, < 0 \Rightarrow damping$$

ionization zone ➔ opacity bump

Hot phase - contraction

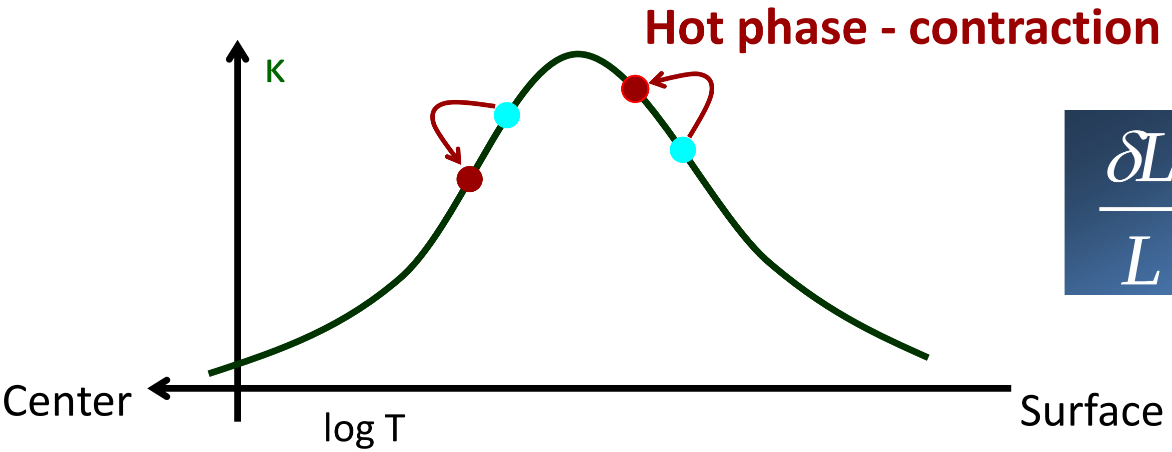


$$\frac{\delta L}{L} \approx -\frac{\delta \kappa}{\kappa}$$

Excitation mechanism - κ mechanism

$$\frac{dW}{dr} = -\frac{1}{2}\frac{\delta T}{T}\frac{\partial \delta L}{\partial r} > 0 \Rightarrow driving\ , < 0 \Rightarrow damping$$

At the hot phase, heat can enter but cannot go out



$$\frac{\delta L}{L} \approx -\frac{\delta \kappa}{\kappa}$$

κ↘ ➔ δκ/κ < 0 ➔ δL > 0

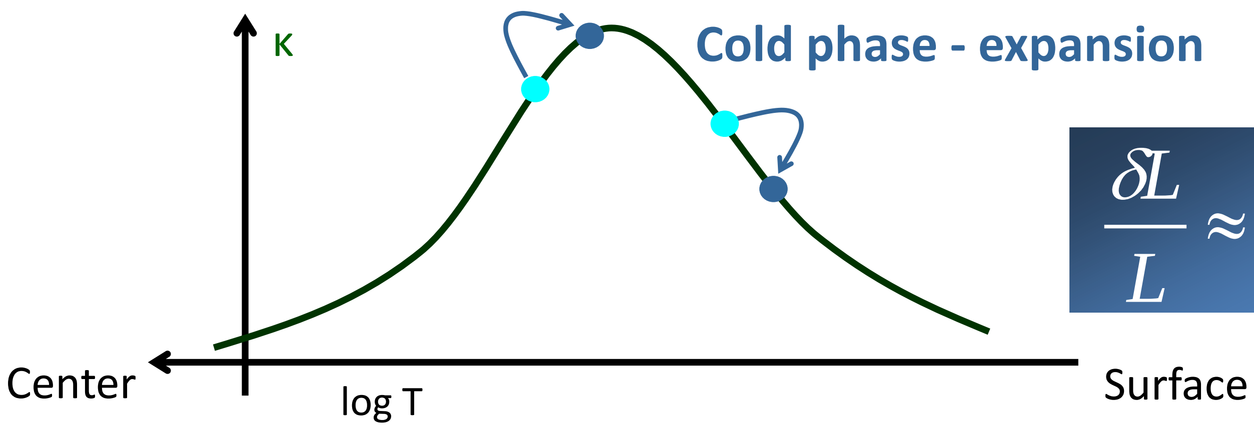
κ↗ ➔ δκ/κ > 0 ➔ δL < 0

dδL/dr < 0 , δT/T > 0 ➔ dW/dr > 0

Excitation mechanism - κ mechanism

$$\frac{dW}{dr} = -\frac{1}{2}\frac{\delta T}{T}\frac{\partial \delta L}{\partial r} > 0 \Rightarrow driving\ , < 0 \Rightarrow damping$$

At the cold phase, heat cannot enter but can go out

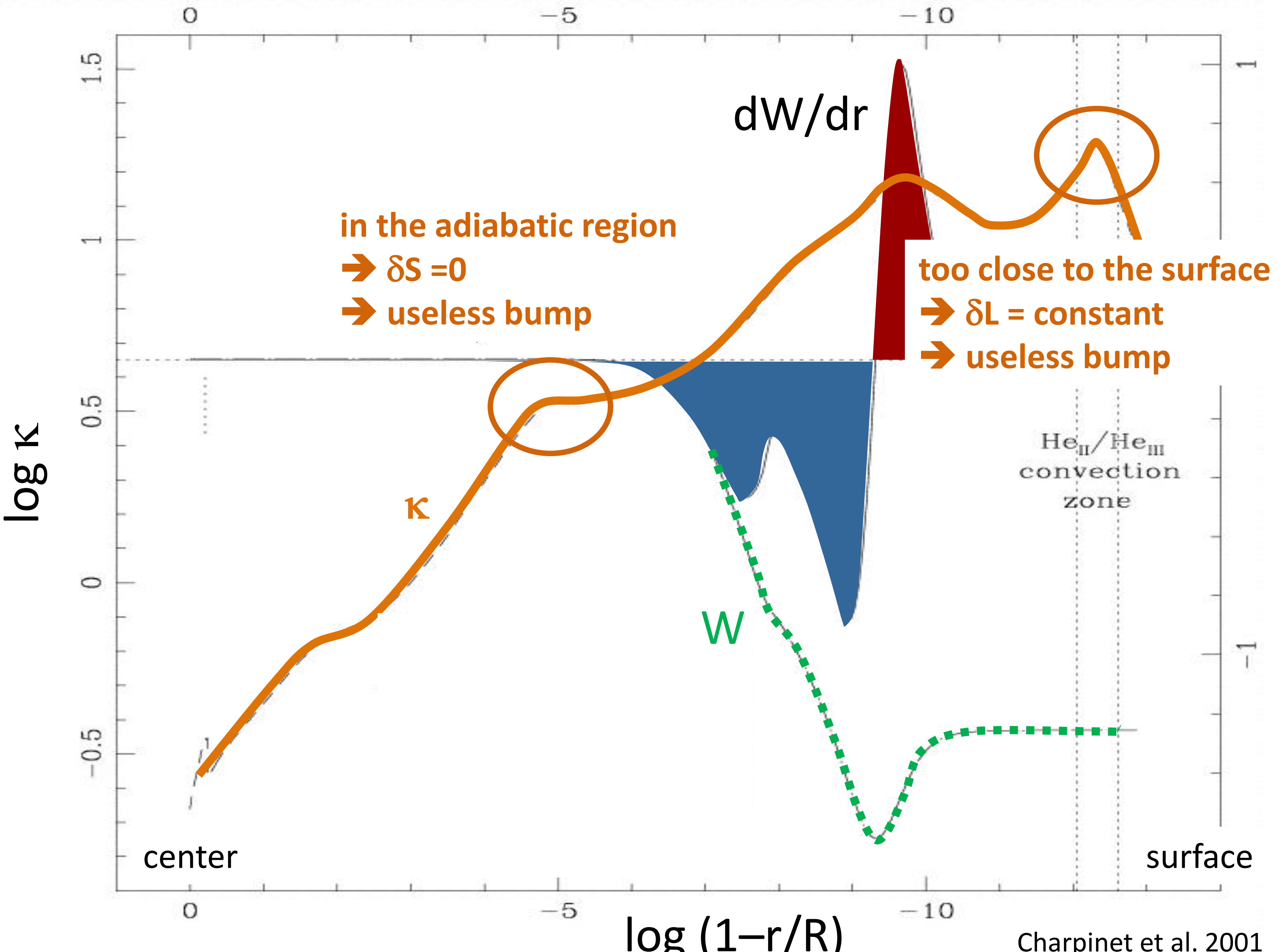


$$\frac{\delta L}{L} \approx -\frac{\delta \kappa}{\kappa}$$

κ↗ ➔ δκ/κ > 0 ➔ δL < 0

κ↘ ➔ δκ/κ < 0 ➔ δL > 0

dδL/dr > 0 , δT/T < 0 ➔ dW/dr > 0

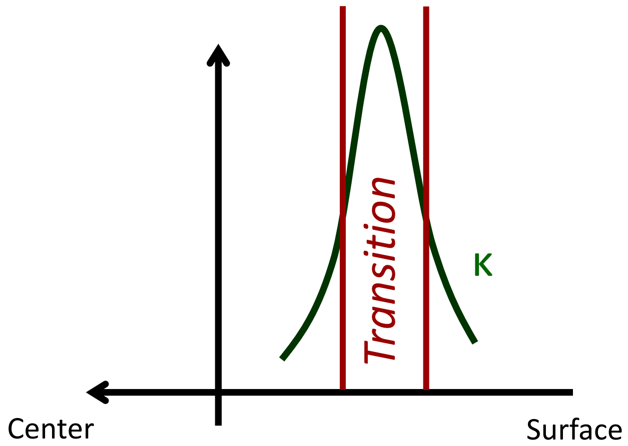


Charpinet et al. 2001

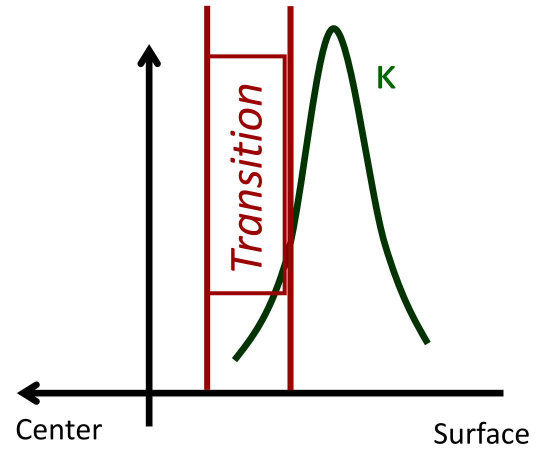
Excitation mechanism - κ mechanism

The opacity bump must be in the transition region

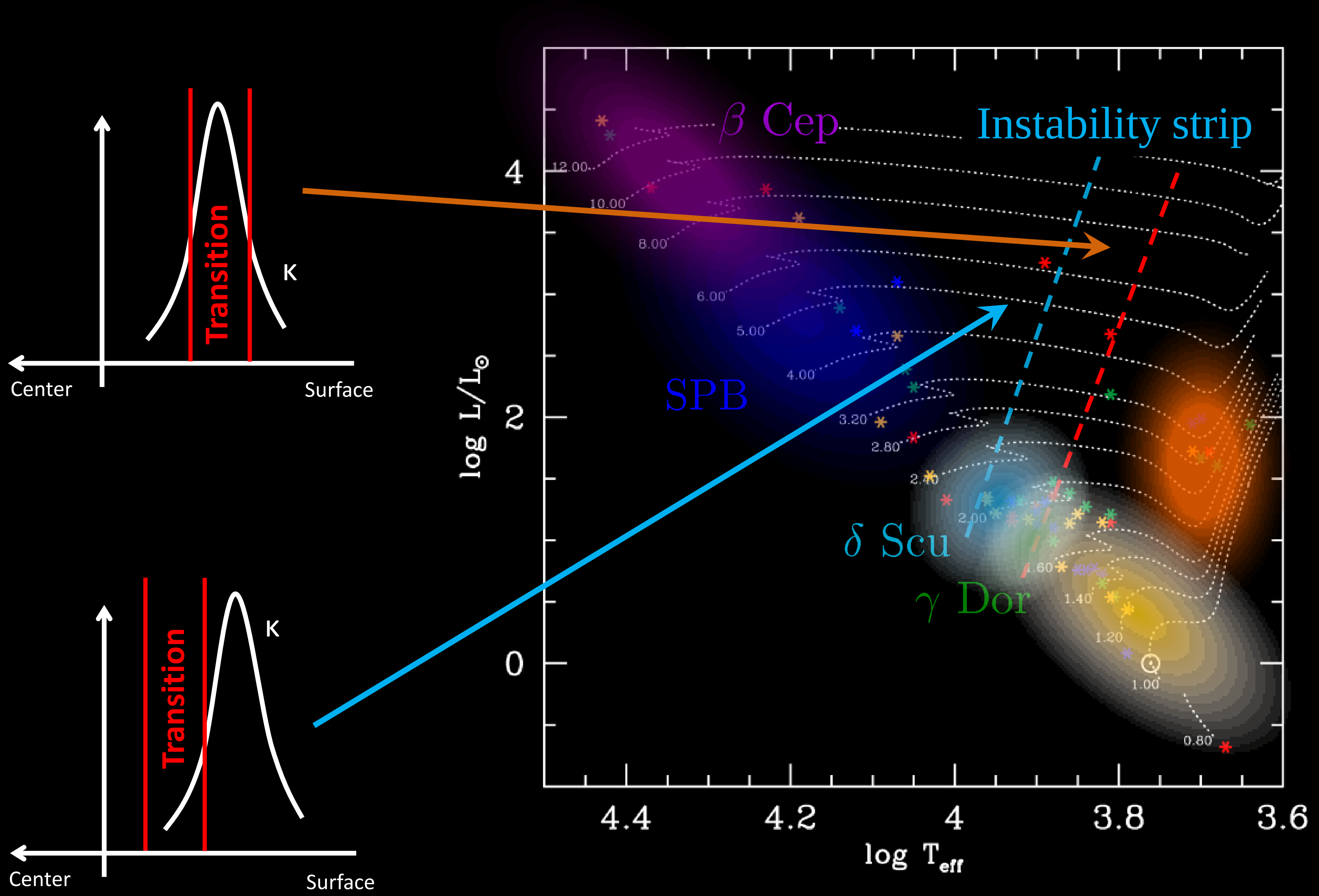
Driving



No driving, model too hot



→ ***Instability strip***



Transition
K
Center
Surface
Transition
K
Center
Surface
β Cep
Instability strip
SPB
δ Scu
γ Dor
log L/L⊙
log Teff
4.4
4.2
4
3.8
3.6
4
2
0
12.00
10.00
8.00
6.00
5.00
4.00
3.20
2.80
2.40
2.00
1.60
1.40
1.20
1.00
0.80

Stochastic excitation – solar-like oscillations

Solar-like oscillations result from **damped** acoustic modes

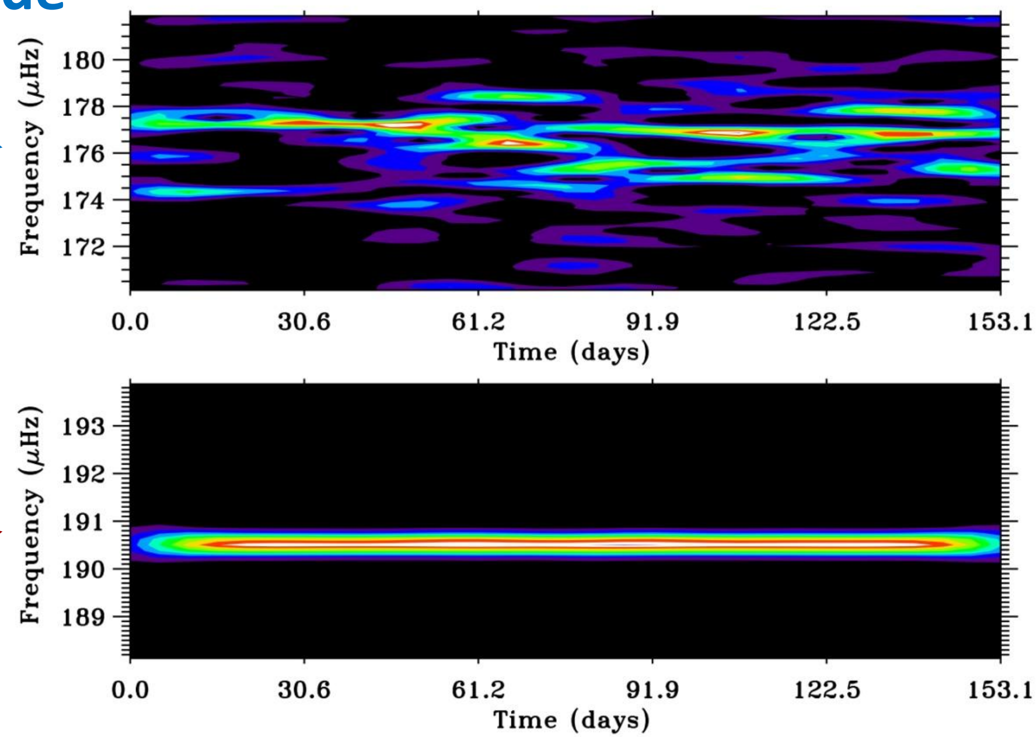
Turbulent motions at the top of the convective envelope stochastically hit the photosphere

Stochastically excited mode

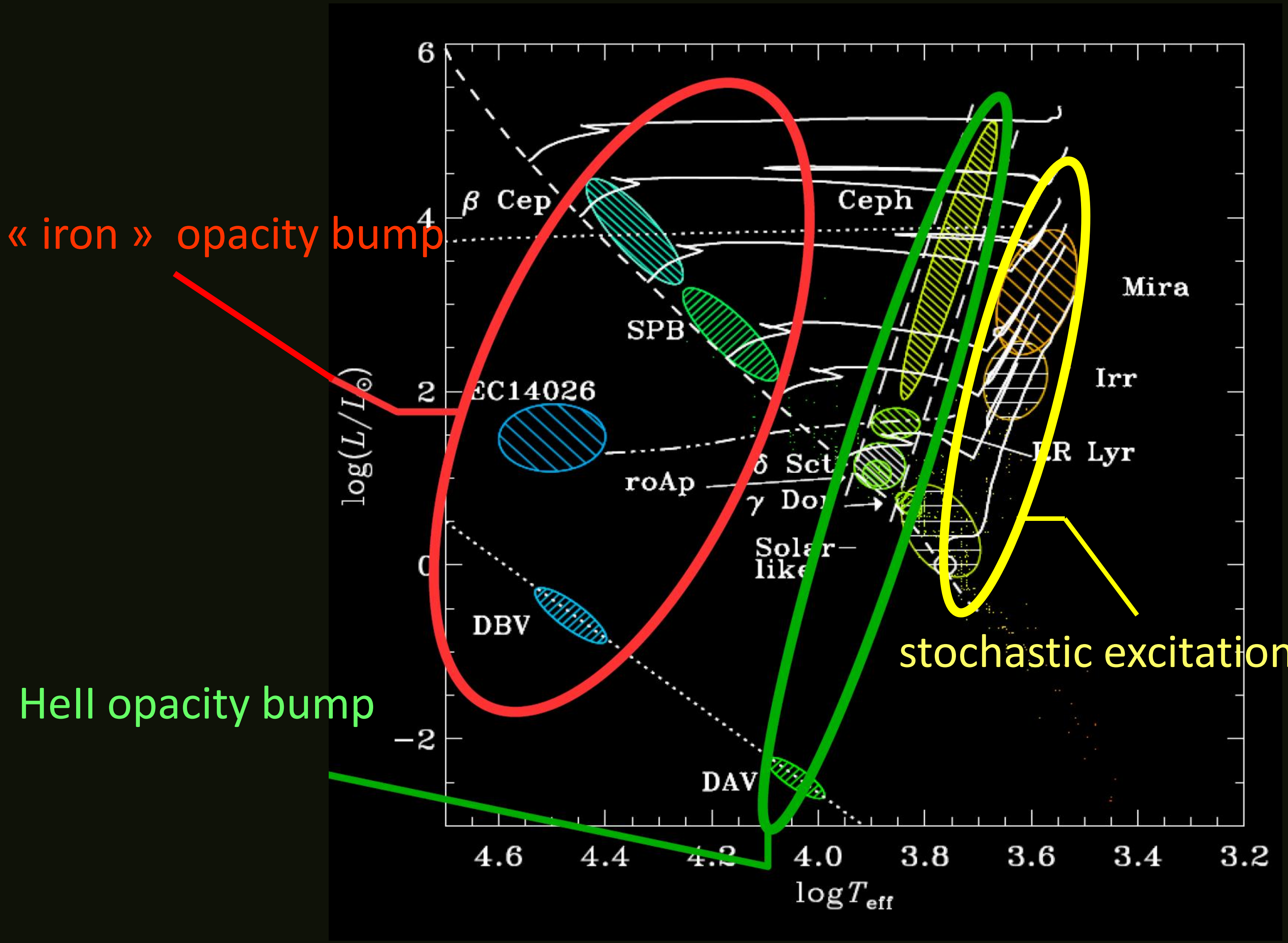
η: damping rate

1/η : mode life-time

Excited mode



Excitation mechanism



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