Overture

Sequence of events

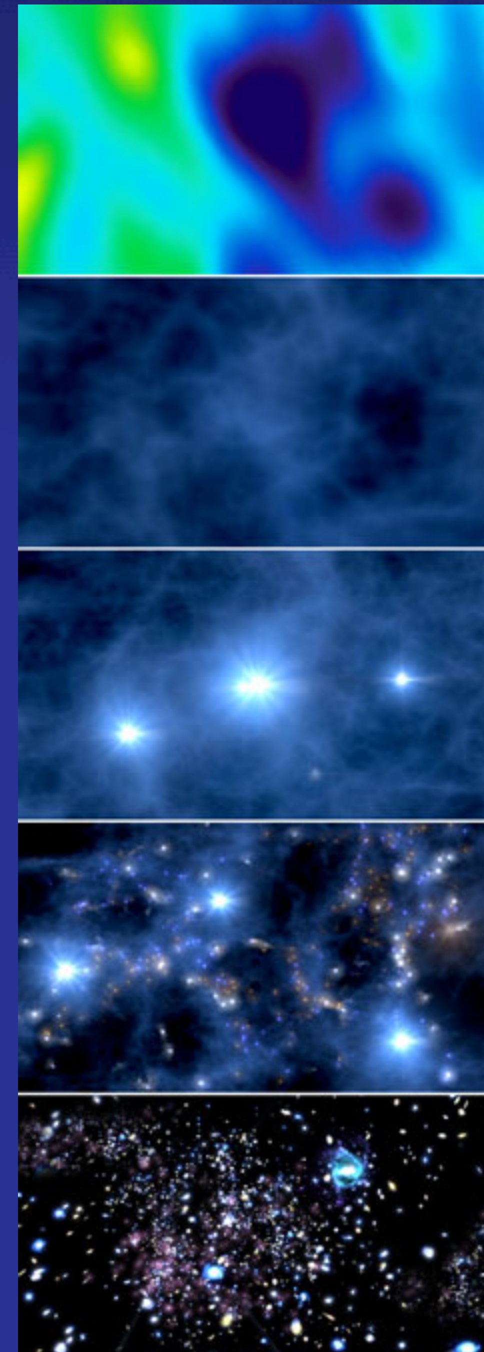
At z=1000 the Universe has cooled down to 3000 K. Hydrogen becomes neutral ("Recombination").

At z < 20 the first "PopIII" star (clusters)/small galaxies form.

At z ~ 6-15 these gradually photo-ionize the hydrogen in the IGM ("Reionization").

At z<6 galaxies form most of their stars and grow by merging.

At z<1 massive galaxy clusters are assembled.



Time

"Historical" material

- Barkana, R. & Loeb, A. 2001, *Phys. Rep.*, **349**, 125
- Bromm, V. & Larson, R. 2004, *ARA&A,* **42**, 79
- Ciardi, B. & Ferrara, A. 2006, *SSRv,* **116**, 625 (updated: Apr 2008)

Recent material

Ferrara, A. 2008, Saas-Fee School, available at web site
http://www.sns.it/it/scienze/menunews/docentiscienze/ferraraandrea/lectures/

Lecture #1

Star Formation in Primordial Gas

Present-day gas

Heavy element mass fraction < 2%

C^+, O, CO, dust grains excellent radiators

Thermal eq. timescale « dynamical timescale

Typical cloud temperature ≈ 10 K

Primordial gas

No heavy elements

H, He poor radiators for $T < 10^4$ K

Cloud evolves almost adiabatically..

..unless H_2 molecules can form

MAIN COOLING PROCESSES IN PRIMORDIAL GAS

1. Radiative recombination

   Thermal energy loss of recombining proton and electron due to photon emitted in the process
   Recombinations to the lowest state lead to ionizing photons, hence net loss = 0
   Total rate obtained by summing over all rates for levels with n >1 (Case B recombination).

2. Collisional ionization

   Thermal energy of electrons converted in ionization energy
   $\sigma_{ion} = a\,(EB)^{-1} \ln(E/B)\,\{1 - b \exp[-c\,(E/B-1)]\} \quad E \geq B = 13.6\ eV$
   Total rate by integrating cross section over Maxwellian distribution

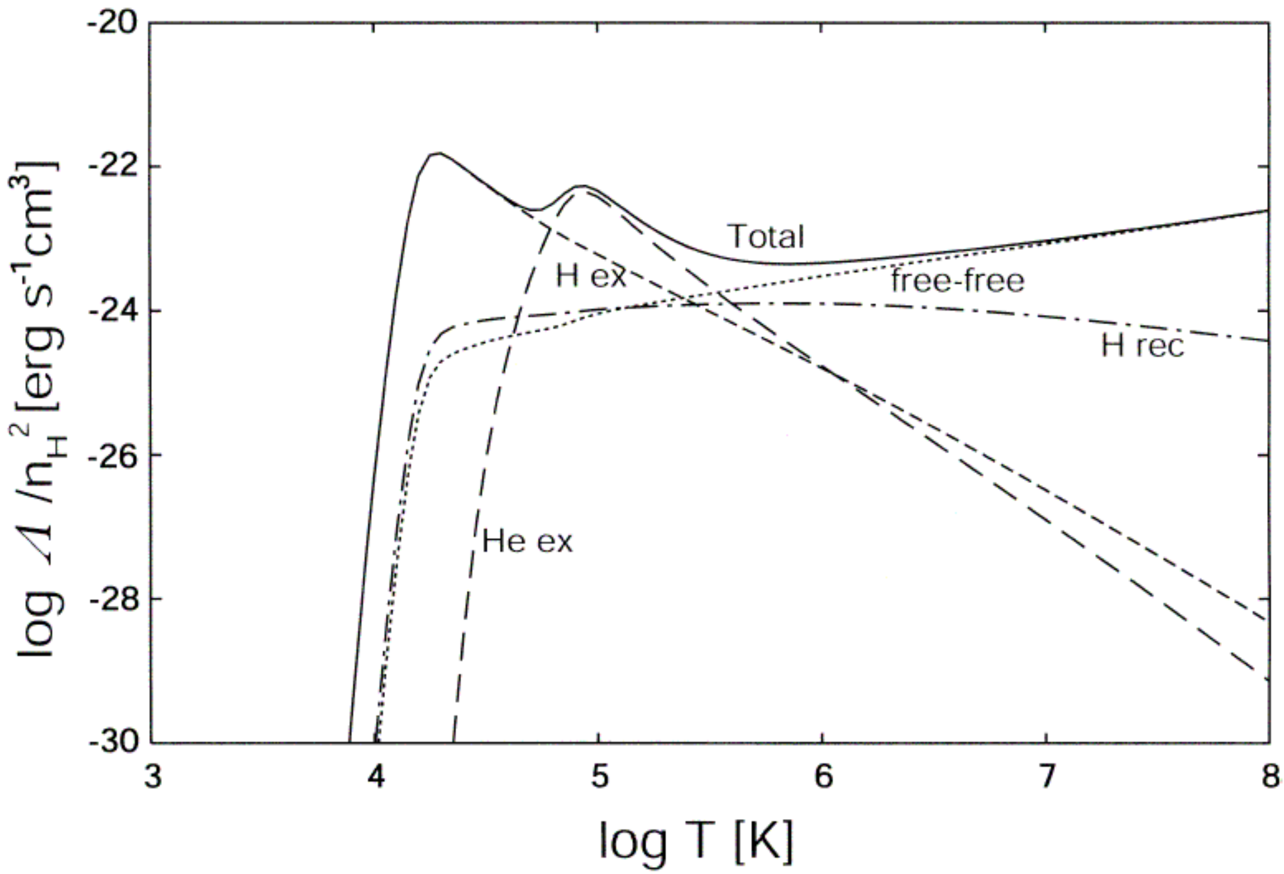
3. Bound-bound transition of hydrogen atom

   Most important cooling process around 10,000 K; collisionally excited.
   Emitted radiation energy equal to energy difference between two levels
   Level population determined by excitation/de-excitation rates for each level

4. Thermal bremsstrahlung emission

   Radiation due to acceleration of a charge in a Coulomb field
   $dE/d\nu\, dV\, dt = (16\,\pi\, e^6 / 3\sqrt{3}\, c^3\, m_e^2\, v)\, n_e\, n_p\, g_{ff}$
   Total rate by integrating cross section over Maxwellian distribution

PRIMORDIAL COOLING FUNCTION



Martin Harwit, 2006: Astrophysical Concepts, Springer-Verlag

FUNDAMENTAL STAR FORMATION TIMESCALES

- Cooling time

$$t_{cool} = 3kT / 2n\Lambda(T)$$

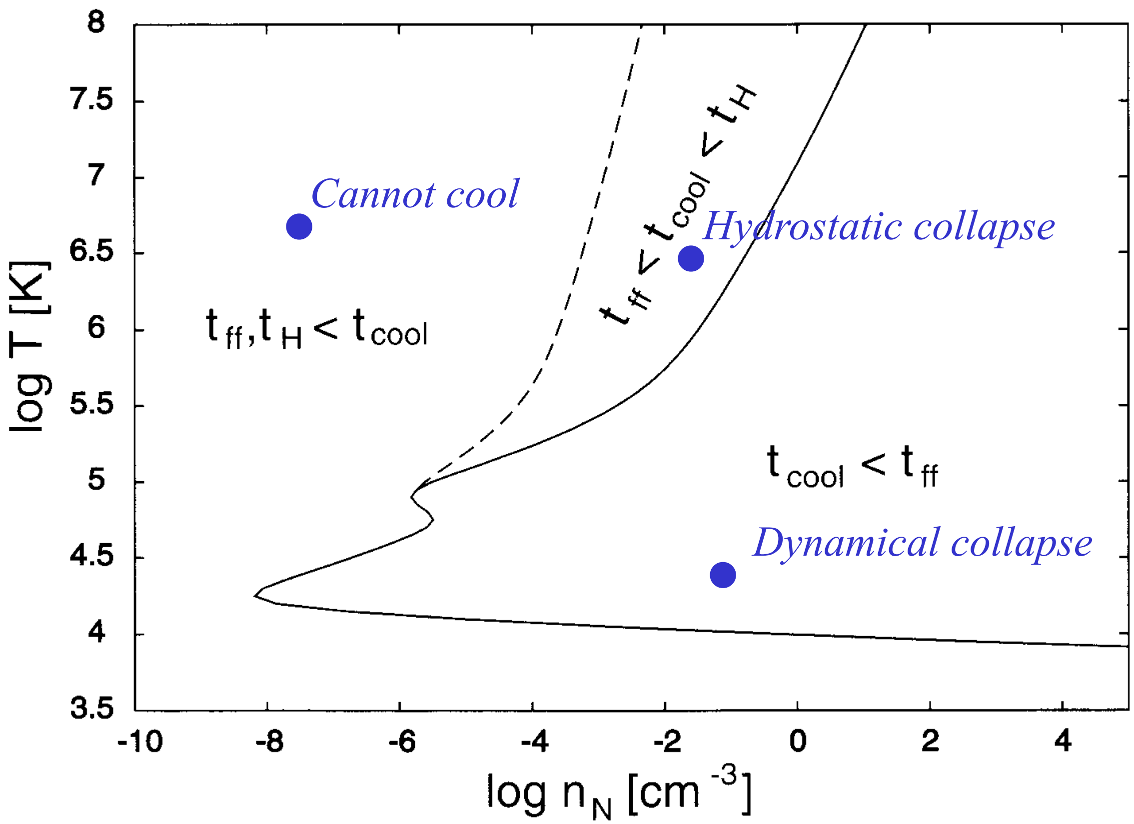
- Free-fall time

$$t_{ff} = (3\pi / 32\, G\rho)^{1/2}$$

- Hubble time

$$t_H = H^{-1}(z)$$

COOLING DIAGRAM



Rees, M. J. & Ostriker, J. P., 1977, MNRAS, 179, 541

COOLING BY HYDROGEN MOLECULES

1. Radiative cooling

Hydrogen molecules have energy levels corresponding to vibrational (10^3 K <T<10^4 K)
and rotational (T<10^3 K) transitions
Einstein's A-coefficient much smaller (no dipole moment) → Absorption coefficient very small

$$\Lambda_{H2} = n_{H2} [n_H L_{vr}^H(n,T) + n_{H2} L_{vr}^{H2}(n,T)]$$

H-H$_2$ | *H$_2$-H$_2$* *collisional excitations*

LEVEL POPULATION

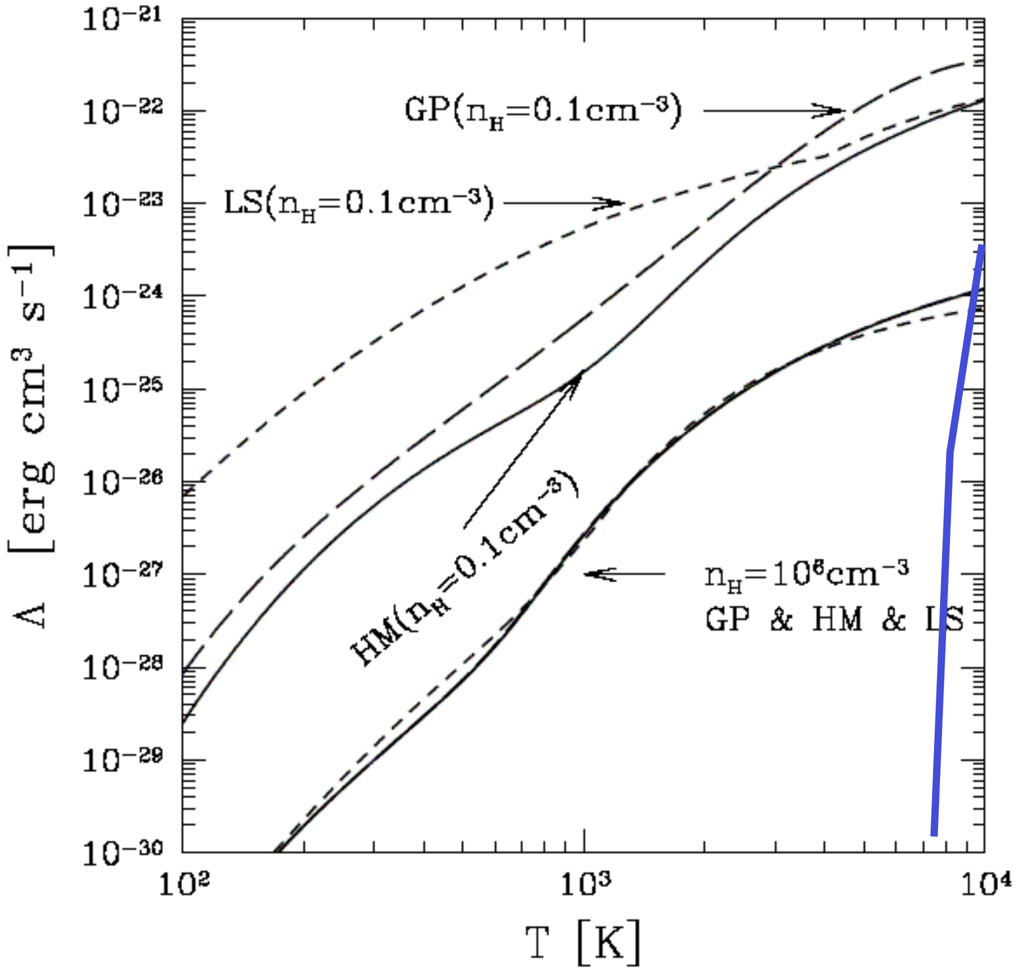
De-excitation rate = Excitation rate

collisional $\propto n^2$ — *collisional*
radiative decay $\propto n$

Critical density n_{crit} :: collisional exc. rate = radiative decay rate

$$\Lambda_{H2} \propto n^2 \quad \text{for } n < n_{crit}$$
$$\Lambda_{H2} \propto n \quad \text{for } n > n_{crit}$$

H_2 COOLING FUNCTION



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COOLING BY HYDROGEN MOLECULES

2. Dissociation cooling/heating

Hydrogen molecules have lower potential energy than the state of two separated neutral H-atoms
H_2 molecules absorb the thermal energy of the colliding particle causing the dissociation

$$\Lambda_{diss} = 7.16 \times 10^{-12} (dn_{H2}/dt)_{-} \quad erg\ s^{-1} cm^{-3}$$

Dissociation of H_2 molecules can occur via three main channels:

- Collisions with H^+ ions — *high ionization level*
- Collisions with H atoms — *low ionization level*
- Collisions with H_2 molecules — *low ionization level*

Heating (reverse process) occurs when H_2 molecules form in an excited state
If collisional de-excitation dominates over radiative decay (high n), energy transported into gas thermal energy

$$\Gamma_{form} = 7.16 \times 10^{-12} (dn_{H2}/dt)_{+} \ (1+n_{cr}/n_H)^{-1} \quad erg\ s^{-1} cm^{-3} \longrightarrow 0 \ \text{for}\ n \ll n_{cr}$$

FORMATION CHANNELS

1. H^- Channel

$$H + e \rightarrow H^- + \gamma$$
$$H^- + H \rightarrow H_2 + e$$

2. H_2^+ Channel

$$H + H^+ \rightarrow H_2^+ + \gamma$$
$$H_2^+ + H \rightarrow H_2 + H^+$$

- *Dipole moment necessary to form H_2 in two-body reactions*
- *Require electrons or protons: ionization degree important*

3. Three body reactions

$$3H \rightarrow H_2 + H$$
$$2H + H_2 \rightarrow 2H_2$$

- *Important at high $n > 10^8$ cm^{-3}, i.e. during prestellar collapse*

4. Direct collision between excited H atoms

$$H(n=1) + H(n=2) \rightarrow H_2 + \gamma$$

- *Important at $z > 10^3$ as CMB photons destroy H_2^+ and H^-*

DISSOCIATION CHANNELS

1. Impact with H / H_2

$$3H \leftarrow H_2 + H \quad or$$
$$2H + H_2 \leftarrow 2H_2$$

• *T > 2000 K, lower T collisions not sufficiently energetic*

2. Impact with H^+

$$H_2^+ + H \leftarrow H_2 + H^+$$
$$2H \leftarrow H_2^+ + e$$

• *Important in hot (T>8000 K) and ionized gas*

3. Impact with electrons

$$2H + e \leftarrow H_2 + e$$

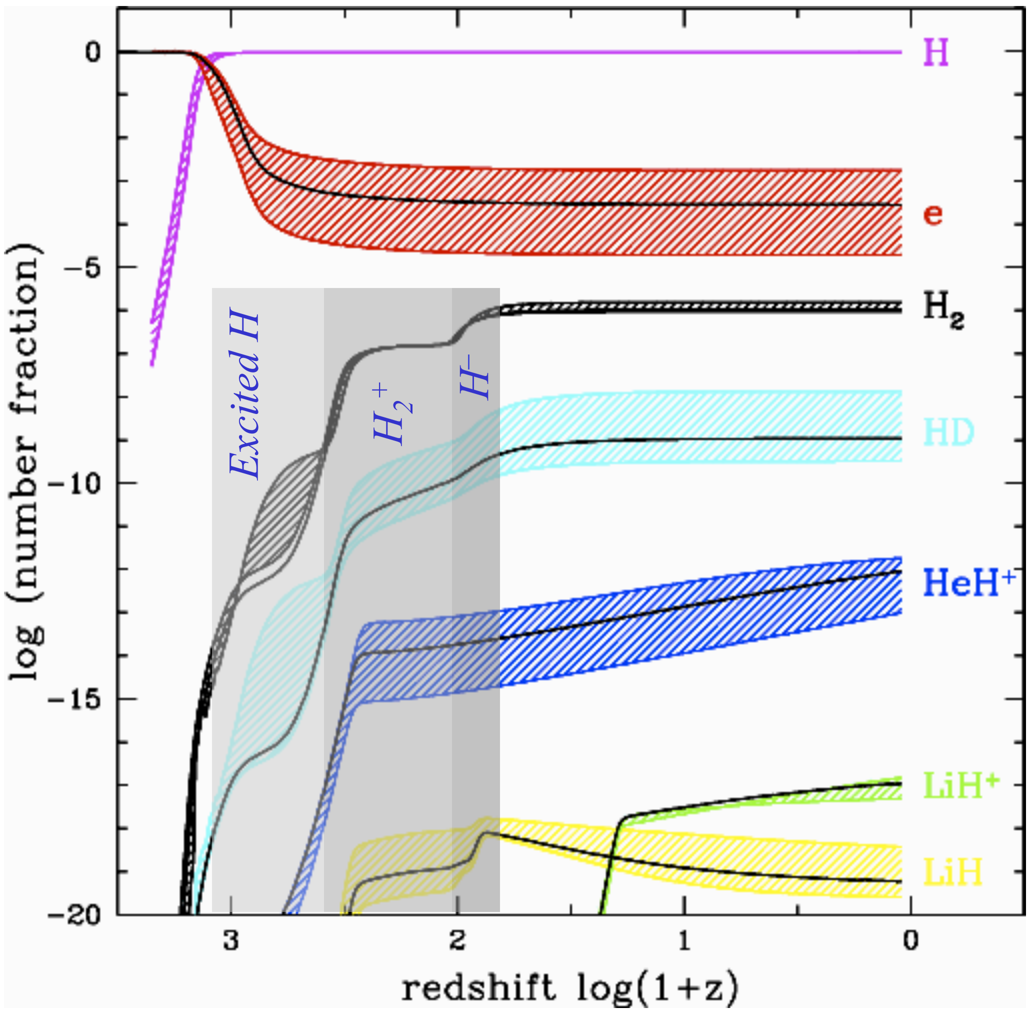
• *Always sub-dominant with respect to 2.*

4. Photodissociation

$$H_2^* \leftarrow H_2 + \gamma$$
$$2H + \gamma \leftarrow H_2^*$$

• *Two step Solomon process; very important.*

COSMIC FREEZE-OUT



Physical hint:

$t_{2body} \propto n^{-1} \propto (1+z)^{-3}$

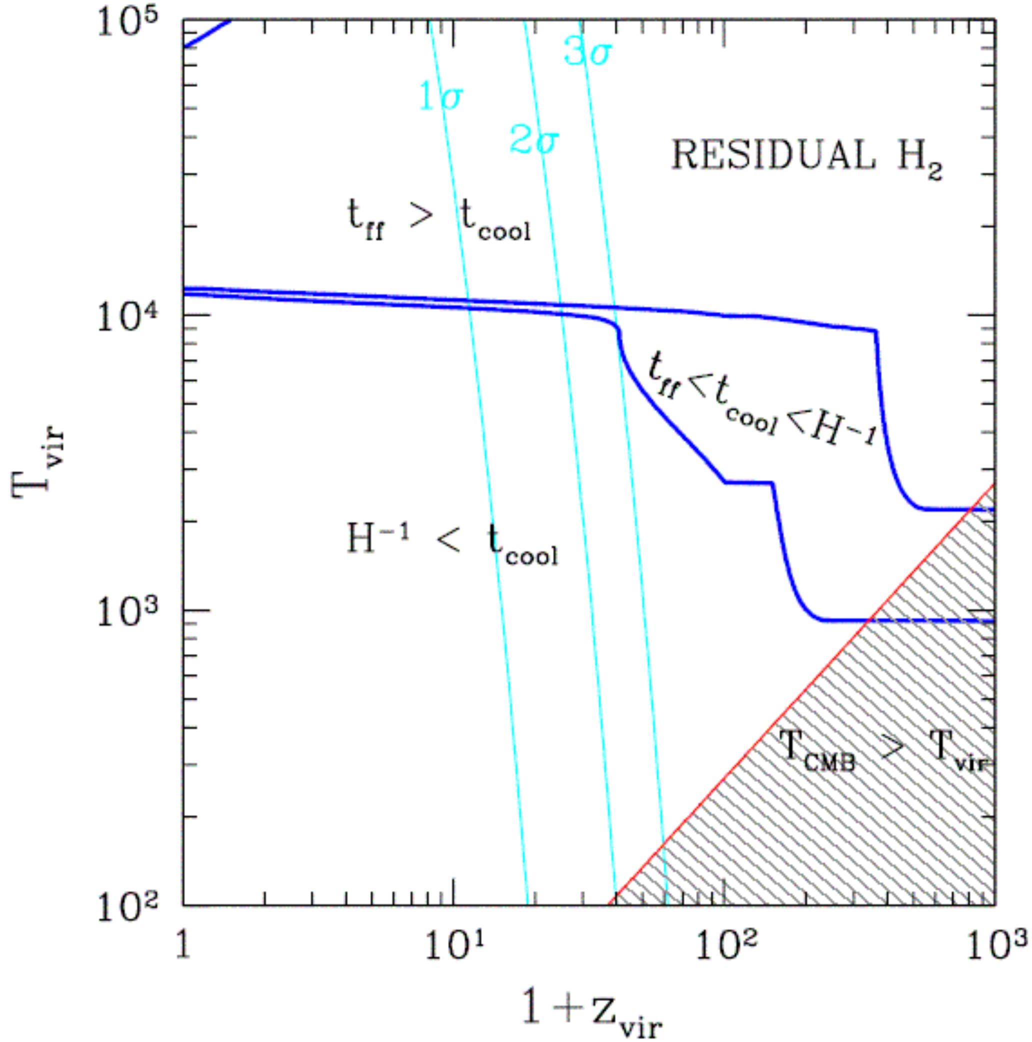
$t_H \propto (1+z)^{-3/2}$

RELIC ELECTRONS

$x_e^{rel} \approx 3 \times 10^{-4}$

RELIC MOL. HYDROGEN

$$y_{H2}^{rel} \approx 1.1 \times 10^{-6}, \quad z < 100$$
$$\approx 1.0 \times 10^{-7}, \quad 100 < z < 250$$
$$\approx 10^{-7} [(1+z)/250]^{-14}, \quad 250 < z$$



10^5
10^4
10^3
10^2
T_{vir}
1
10^1
10^2
10^3
$1+z_{vir}$
1σ
2σ
3σ
RESIDUAL H_2
$t_{ff} > t_{cool}$
$t_{ff} < t_{cool} < H^{-1}$
$H^{-1} < t_{cool}$
$T_{CMB} > T_{vir}$

STRUGGLING FOR MORE H_2: SPHERICAL COLLAPSE

Dynamics

$$\frac{\rho}{\langle\rho\rangle} = \frac{9\,(\alpha - \sin\alpha)^2}{2\,(1 - \cos\alpha)^3} \qquad \textit{where} \qquad \frac{1 + z_{vir}}{1 + z} = [(\alpha - \sin\alpha)/2\pi]^{\,2/3}$$

$$\textit{If} \quad \rho > \rho_{vir} = 18\pi^2\,\langle\rho\rangle \qquad \textit{then} \qquad \rho = \rho_{vir}$$

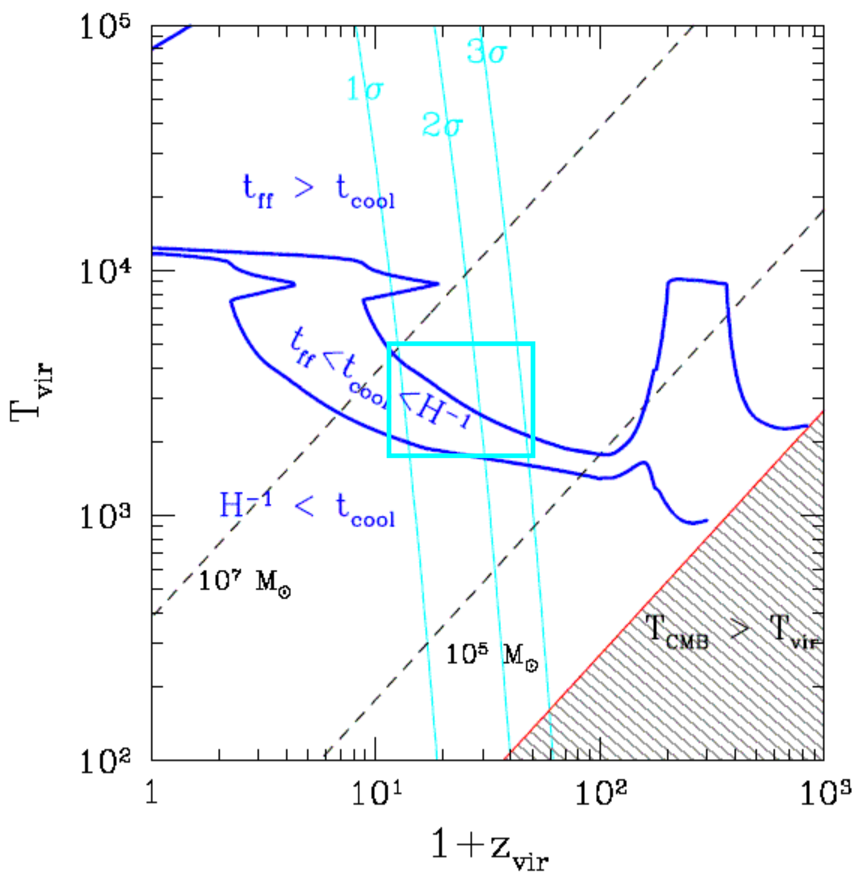
Thermo/chemical evolution

$$\frac{d}{dt}\,\frac{3\,k\,T}{2\mu\,m_p} = (p/\rho^2)\,\frac{d\rho}{dt} - \Lambda\,(T, y_i)$$

$$\frac{dy_i}{dt} = \sum k_j y_j + n_H \sum k_{kl} y_k\, y_l + n_H^2 \sum k_{mns}\, y_m y_n\, y_s$$

ENOUGH FOR COLLAPSE ?

$$t_{cool} = 3kT_{vir} / 2\mu\, n_{vir}\, \Lambda(y_{H2}, T_{vir}) = (3\pi / 32\, G\rho_{vir})^{1/2} = t_{ff}$$



Ferrara, A. 2008, Saas-Fee School: http://www.sns.it/it/scienze/menunews/docentiscienze/ferraraandrea/lectures/

ADDITIONAL PHYSICS: THE ROLE OF HD

- *HD is the second most abundant primordial molecule*

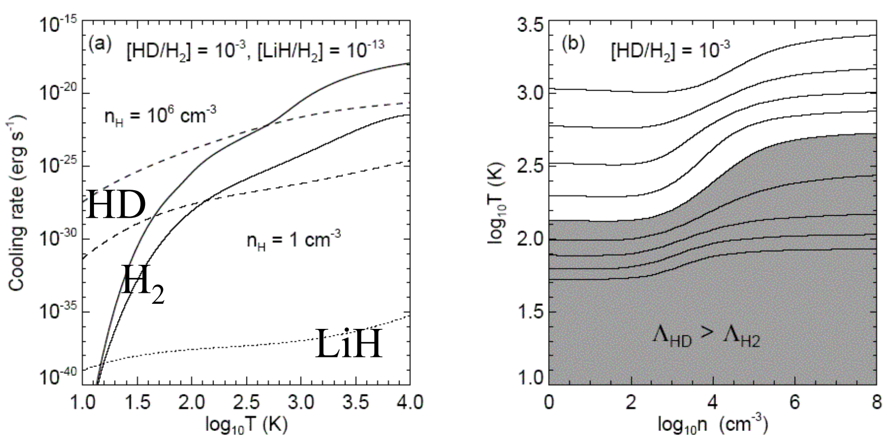
$$n_{HD} \approx 10^{-2} - 10^{-4}\, n_{H2}$$

- *It has a finite dipole moment of* $\mu_{HD} \approx 0.83$ *debye, hence higher transitional probabilities*

- *The energy difference for the lowest rotational transition is*

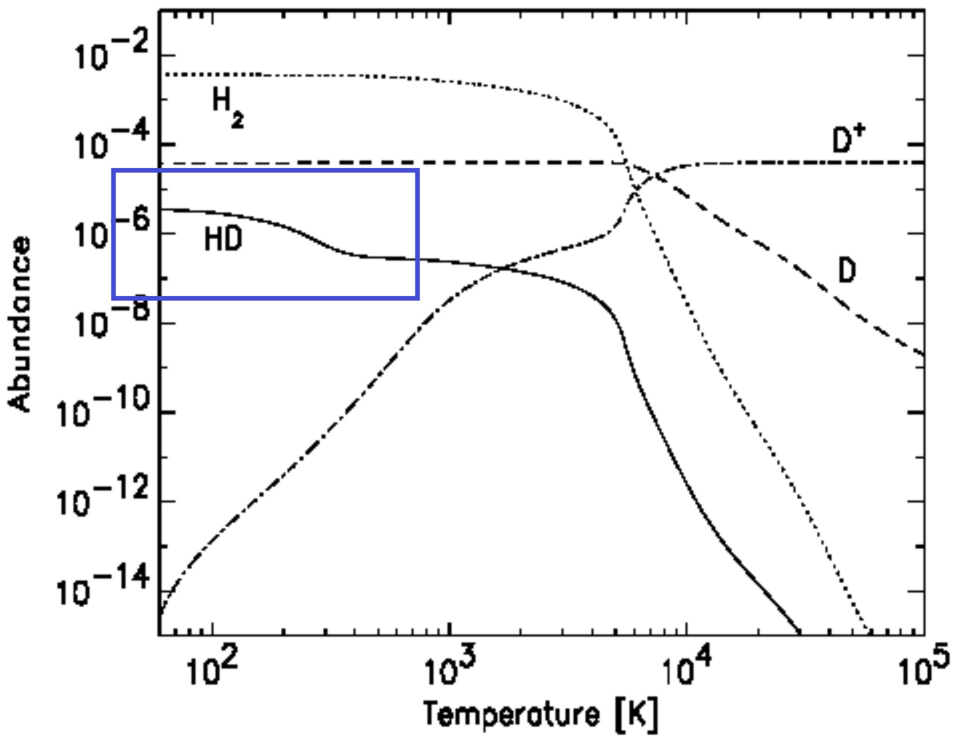
H_2	HD
$\Delta E_{20} / k = 510\ K$	$\Delta E_{20} / k = 128\ K$

Hence HD can reduce the gas temperature to $T < 100K$

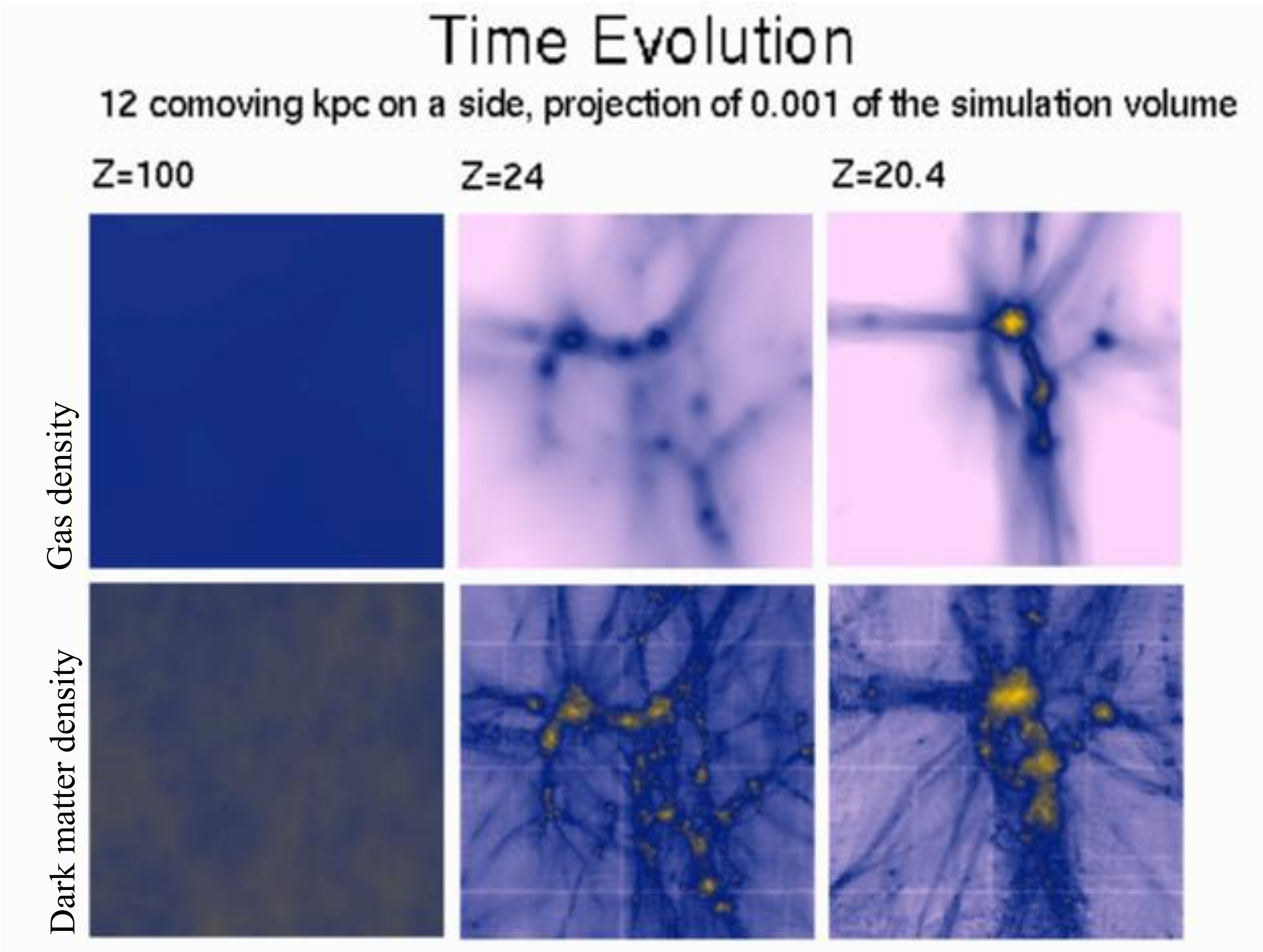


HD FORMATION BEHIND SHOCKS

Shock velocity $v=100\ km\ s^{-1}$, $z=20$



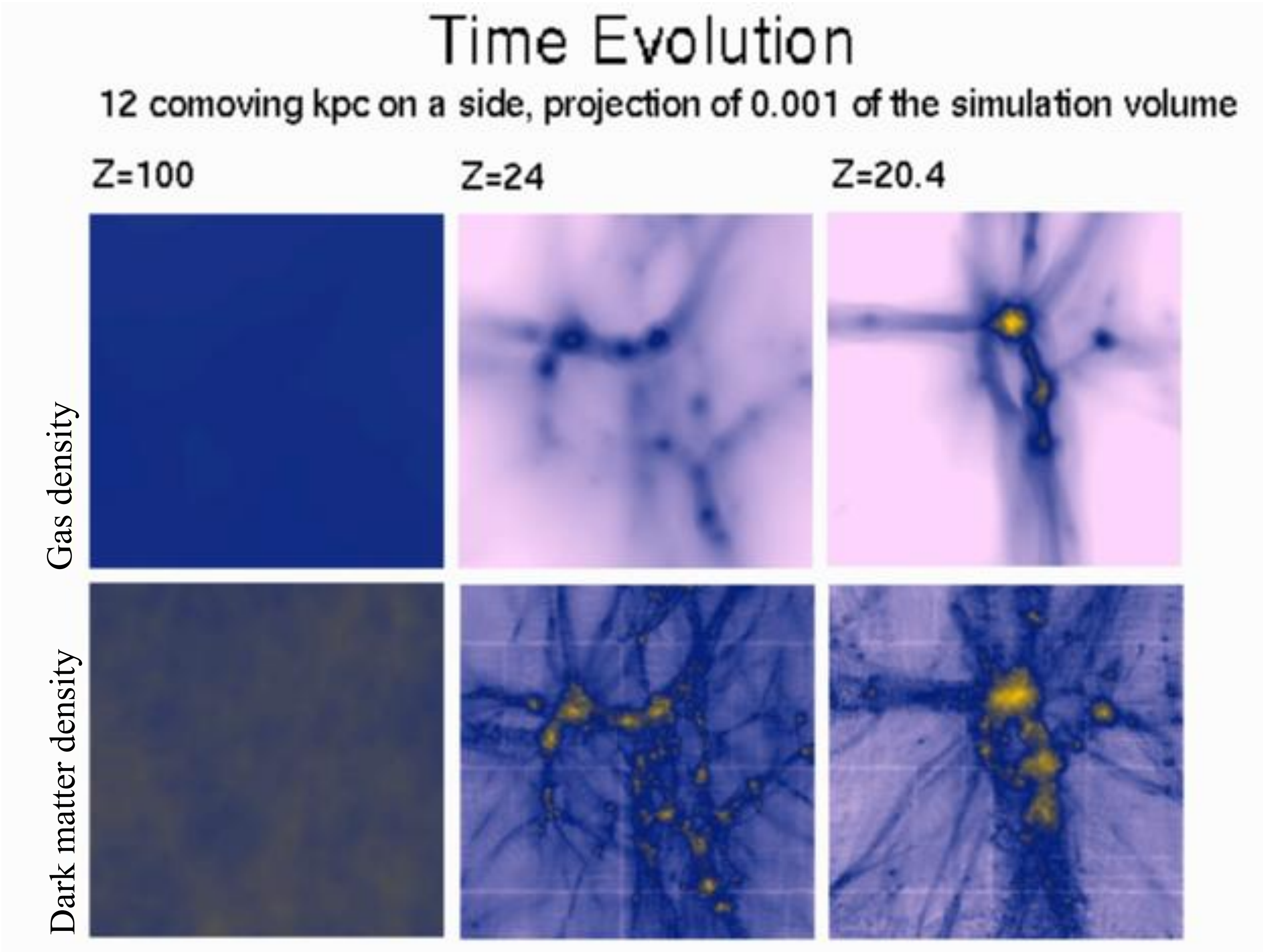
$$D^+ + H_2 \rightarrow HD + H^+$$



Abel, T., Bryan, G. L., & Norman, M. L. 2002

Lecture #2

Formation of the First Stars



Abel, T., Bryan, G. L., & Norman, M. L. 2002

SIMILARITY L-P SOLUTION

- *Isothermal collapse (can be generalized to $p = k\rho^{\gamma}$)*
- *Eulerian formulation of fluid equations*

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{Gm}{r^2} + \mathscr{R}T\frac{d\ln\rho}{dr} = 0,$$

$$\frac{\partial m}{\partial t} + 4\pi r^2 \rho u = 0,$$

$$\frac{\partial m}{\partial r} - 4\pi r^2 \rho = 0,$$

- *Search for solutions of the type*

$$u(r,t) = b(t)u_1(s),$$

$$\rho(r,t) = c(t)\rho_1(s),$$

$$m(r,t) = d(t)m_1(s),$$

$$s = r/a(t).$$

- *This requires the following conditions*

$$b(t) = 1, \qquad c(t) = a(t)^{-2}, \qquad d(t) = a(t).$$

- *Substitute these solutions into fluid equations and eliminate equation for mass definition*

$$\left(\frac{s}{\tau}+u_1\right)\frac{du_1}{ds}+4\pi G\rho_1(s+u_1\tau)+\mathscr{R}T\,\frac{d\ln\rho_1}{ds} = 0,$$

$$\frac{du_1}{ds}+\left(\frac{s}{\tau}+u_1\right)\left(\frac{d\ln\rho_1}{ds}+\frac{2}{s}\right) = 0,$$

- *Here* $\tau = -(da/dt)^{-1}$ *is a fixed constant. Finally reduce these eqs. to nondimensional form by defining:*

$$x = \frac{s}{\tau\sqrt{(\mathscr{R}T)}}, \qquad \xi = \frac{-u_1}{\sqrt{(\mathscr{R}T)}}, \qquad \eta = 4\pi G\rho_1\tau^2.$$

$$\frac{d\xi}{dx} = \frac{x-\xi}{x}\,\frac{\eta x(x-\xi)-2}{(x-\xi)^2-1},$$

$$\frac{d\ln\eta}{dx} = \frac{x-\xi}{x}\,\frac{\eta x-2(x-\xi)}{(x-\xi)^2-1}.$$

BOUNDARY CONDITIONS

- $\xi = 0$ at $x = 0$
- $\eta x = 2$ at $x - \xi = 1$

$$\frac{d\xi}{dx} = \frac{x-\xi}{x}\,\frac{\eta x(x-\xi)-2}{(x-\xi)^2-1},$$

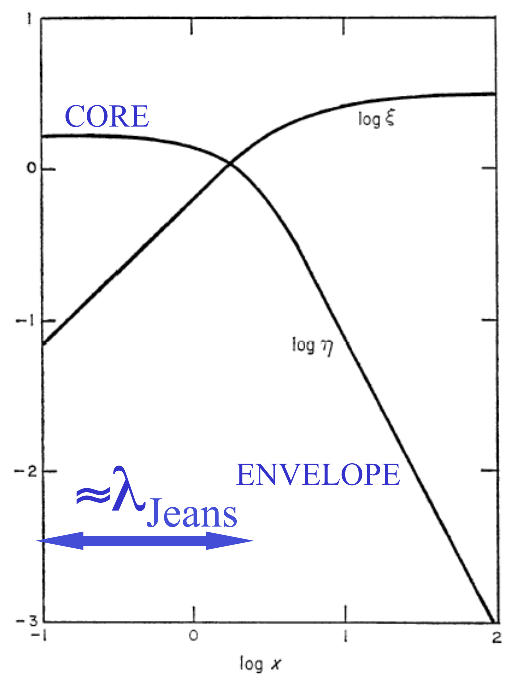
$$\frac{d\ln\eta}{dx} = \frac{x-\xi}{x}\,\frac{\eta x-2(x-\xi)}{(x-\xi)^2-1}.$$

Limiting behavior $x \gg 1$

$$\left.\begin{aligned}\frac{d\ln\xi}{d\ln x} &\to 0\\[1em] \frac{d\ln\eta}{d\ln x} &\to -2\end{aligned}\right\}\ \text{as } x\to\infty.$$

- *u(r) approaches a constant*
- *$\rho(r)$ approaches the form $\rho \propto r^{-2/(2-\gamma)}$*

Ferrara, A. 2008, Saas-Fee School: http://www.sns.it/it/scienze/menunews/docentiscienze/ferraraandrea/lectures/

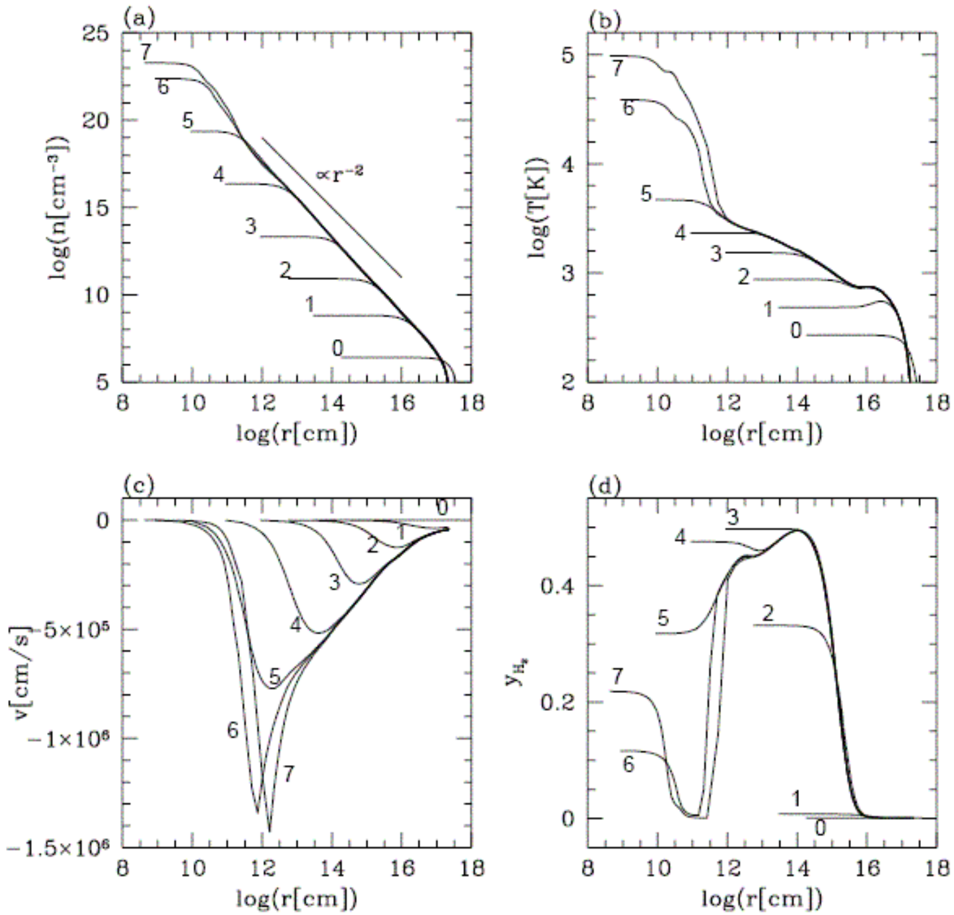


Numerical solution

Collapse proceeds qualitatively as free-fall, but pressure forces important to determine the form of the solution: for $x \ll 1$ |pressure/gravity| = 0.6

CORE EVOLUTION

- *1D, spherically symmetric, no rotation, B-field,, external radiation*
- *Initial conditions: 100 M_o , average $n = 10^6$ cm^{-3}, $x_e = 5\times10^{-4}$, $y_{H2} = 10^{-10}$*



Bromm, Volker; Larson, Richard B., 2004, ARA&A,42,79

Add energy eq. to L-P solution

$$\frac{de}{dt} = -p\frac{d}{dt}\left(\frac{1}{\rho}\right) - \frac{\Lambda_{\rm net}}{\rho},$$

$$e = \frac{1}{\gamma_{\rm ad} - 1}\frac{kT}{\mu m_{\rm H}}$$

$$\Lambda_{\rm net} = \Lambda_{\rm H} + \Lambda_{\rm H_2} + \Lambda_{\rm cont} + \Lambda_{\rm Compt} + \Lambda_{\rm chem}.$$

$$\tau_\nu = \kappa_\nu R_{\rm c} = \kappa_\nu \left(\frac{\lambda_{\rm J}}{2}\right)$$

PHYSICAL ARGUMENTS

Fragmentation condition

$$t_{cool} = 3kT / 2\mu\, n\, \Lambda(y_{H2}, T) \quad \ll (3\pi / 32\, G\rho)^{1/2} = t_{ff}$$

Energy deposited by gravitational contraction cannot balance radiative losses
Temperature decreases with increasing density

♣

$$R_F \approx \lambda_J \propto c_s t_{ff} \propto n^{\gamma/2-1}$$

Fragments form on a scale ensuring pressure equilibrium (Jeans length)

$$M_F \propto n R_F^{\eta} \propto n^{\eta\gamma/2+(1-\eta)}$$

- η = 3 for spheres
- η = 2 for filaments

PHYSICAL ARGUMENTS

Fragmentation stops because:

1. *Critical density for LTE is reached*
2. *Gas becomes optically thick to cooling radiation*

$t_{cool} \geq t_{ff}$

Necessary condition:

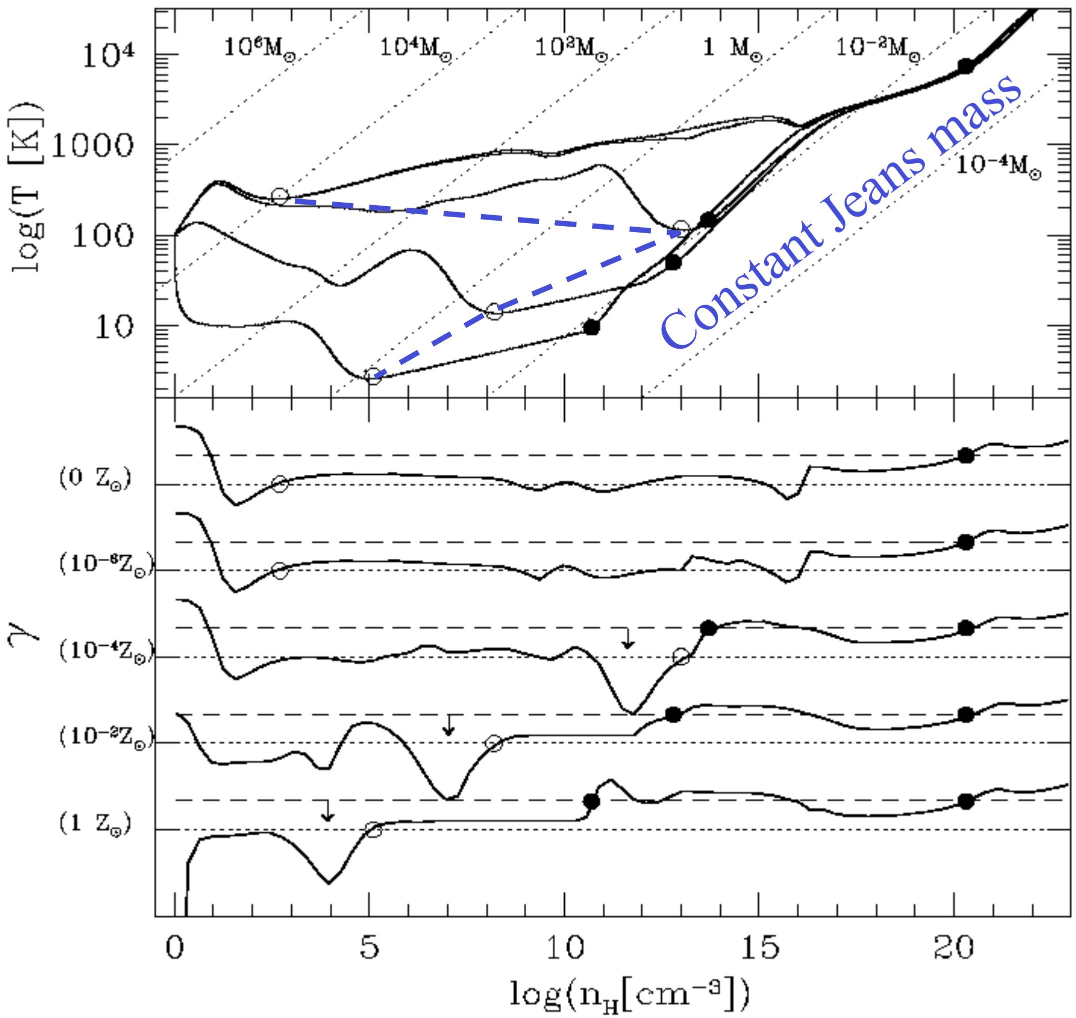
Jeans mass does not decrease any further with increasing density

$$M_F \propto n R_F^{\eta} \propto n^{\eta\gamma/2+(1-\eta)}$$

STABLE IF $\gamma \gtrsim 2\frac{\eta - 1}{\eta}$

- $\gamma \geq 4/3$ …… spherical
- $\gamma \geq 1$ …… cylindrical

CORE THERMAL HISTORY



Chemical Network includes

H,D,He,C,O + Dust
55 species, 496 reactions

○ Fragmentation stops
● Hydrostatic core forms

Ferrara, A. 2008, Saas-Fee School: http://www.sns.it/it/scienze/menunews/docentiscienze/ferraraandrea/lectures/

Critical metallicity for IMF transition

$$Z = 10^{-5 \pm 1} Z_{\odot}$$

determined by fragmentation physics

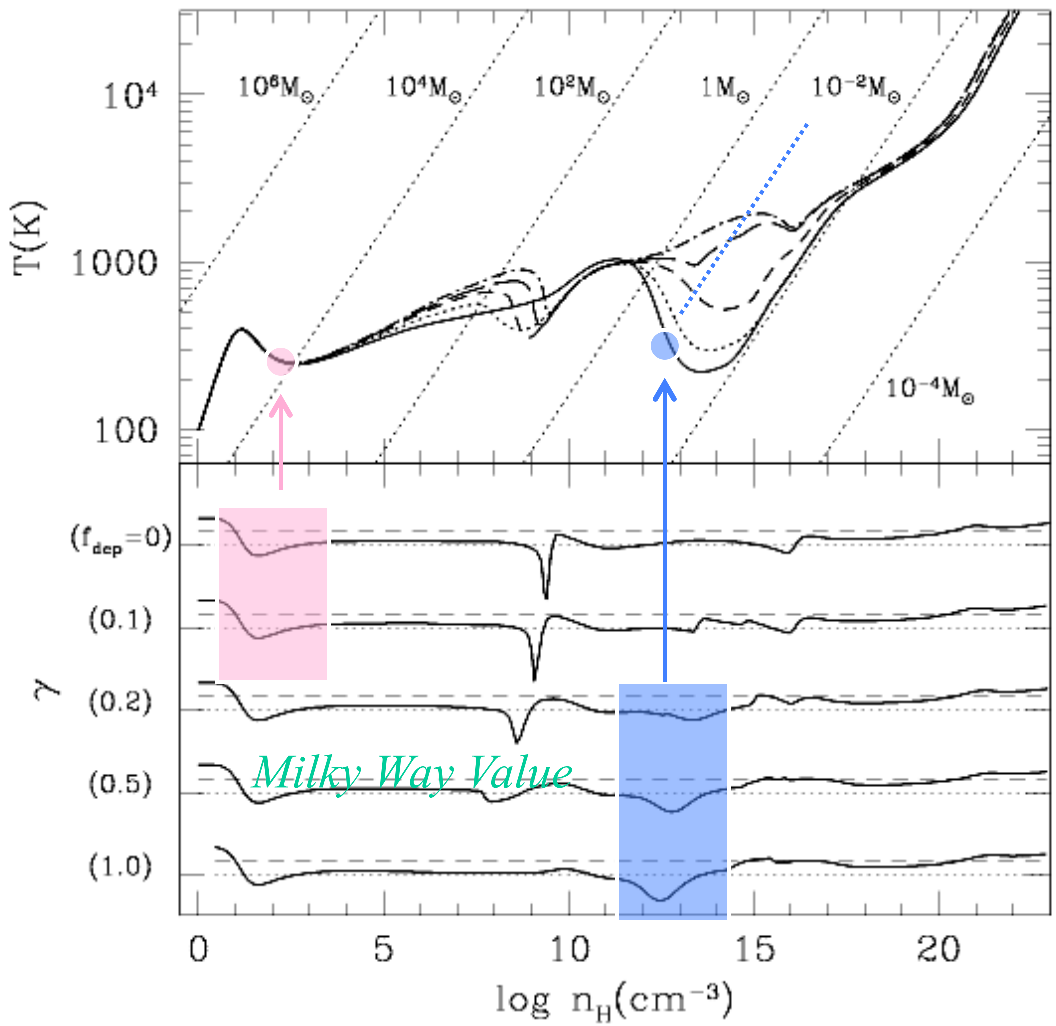
DUST EFFECTS

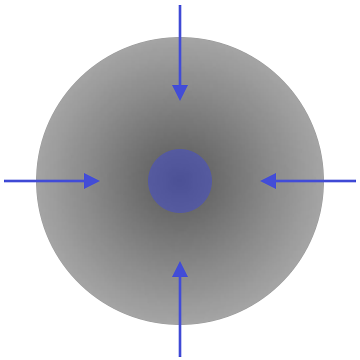
$$Z = 10^{-5.1}\ Z_\odot$$

- *Pair-instability SN abundances*

Metal Depletion onto Dust

$$f_{dep} = \frac{M_Z^{dust}}{M_Z}$$





- *Run-away collapses produces a core + accreting envelope structure*
- *Initial conditions:* $M_c \approx 10^{-3}\, M_\odot$, $M_{env} \approx 10^{3}\, M_\odot$

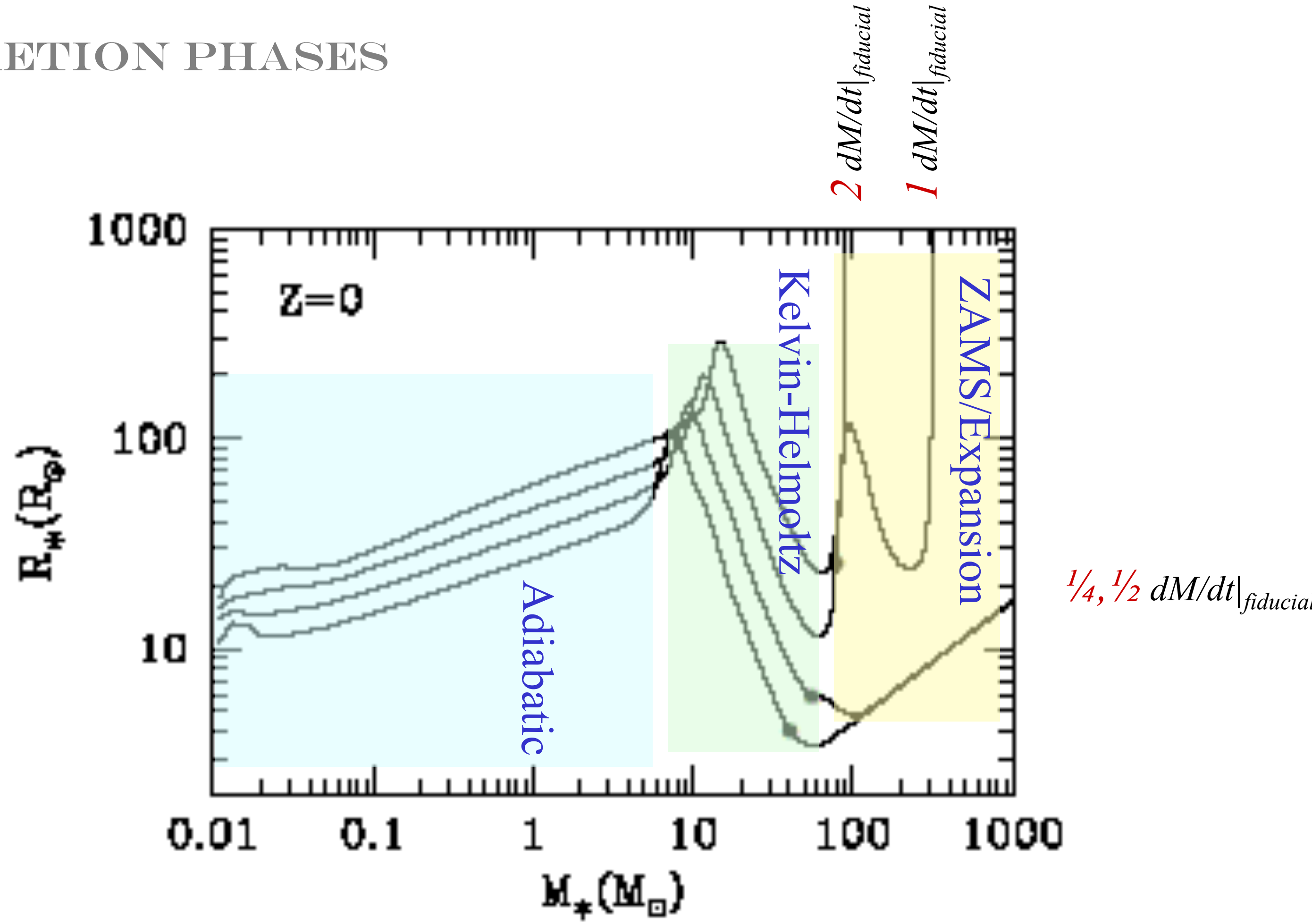
Accretion rate

$$dM/dt \approx M_J / t_{ff} \propto \lambda_J^3 \rho / t_{ff} \propto c_s^3 \rho \; t_{ff}^2 \propto c_s^3 G^{-1}$$

Numerical estimate

$$dM/dt \approx 4.4 \times 10^{-3}\ M_\odot/\mathrm{yr} \qquad (T = 1600\ \mathrm{K})$$

ACCRETION PHASES



$dM/dt|_{fiducial} = 4.4 \times 10^{-3}\ M_\odot/\mathrm{yr}$

Bromm, Volker; Larson, Richard B., 2004, ARA&A,42,79

PHYSICAL ARGUMENTS

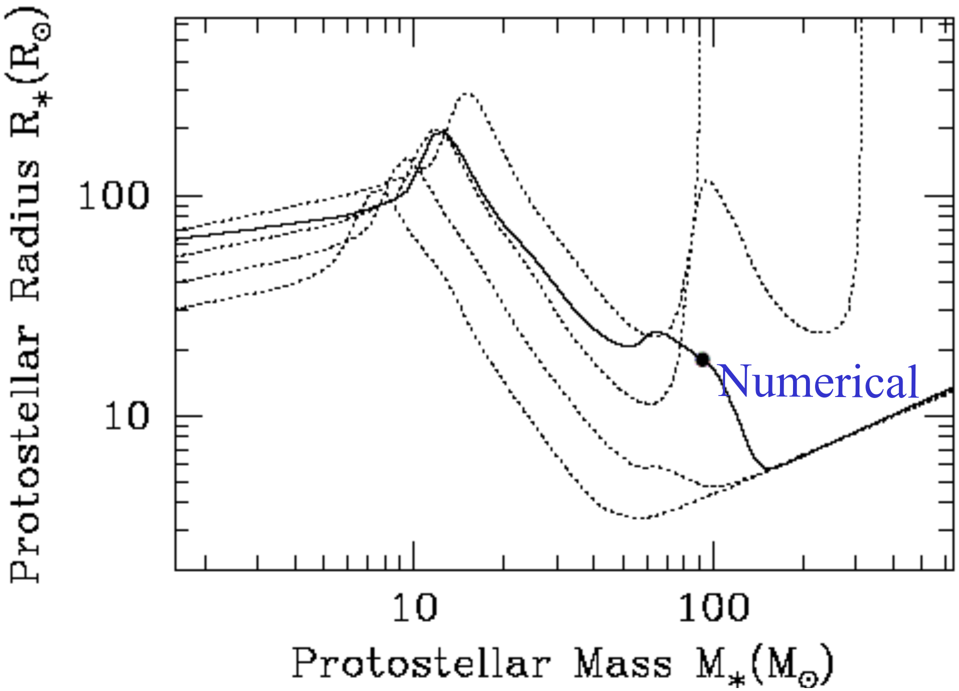
- Adiabatic accretion
  - *High opacity, low interior luminosity*

$$t_{KH} = (GM_\star^2/R_\star)\, L_\star^{-1} \gg M_\star/(dM/dt) = t_{acc}$$

  - *Accreted material piles up without cooling. Temperature increase makes opacity fall rapidly.*
  - *Luminosity wave reaches surface, causing a sudden swelling of star*
- Kelvin-Helmholtz contraction
  - *KH time becomes similar to accretion time*
  - *Opacity dominated by electron scattering, leading to* $L_\star \propto M_\star^3$ *hence* $t_{KH} \propto 1/M_\star R_\star$
  - *Stability condition leads to the relation* $R_\star \propto (dM/dt) / M_\star^2$
- ZAMS settling
  - *Low accretion rate: central T high enough to synthesize C (catalyst of CN cycle) H burning begins. Star relaxes to ZAMS, unique mass-luminosity-radius relation*
  - *High accretion rate: Violent swelling due to radiation pressure exceeding the Eddington limit before H ignition. Blow away of the accreting envelope.*

REALISTIC ACCRETION RATES

Omukai, Kazuyuki; Palla, Francesco, 2003, ApJ,589,677



Stellar growth limited only by capability to stop accretion

STOPPING THE INFALL

- Radiation Pressure (Opacity ?)
- Bipolar Outflows (Magnetic fields ?)
- Angular Momentum Barrier (Disk formation?)
- Competitive Accretion by Companions

Top-Heavy Primordial IMF ?

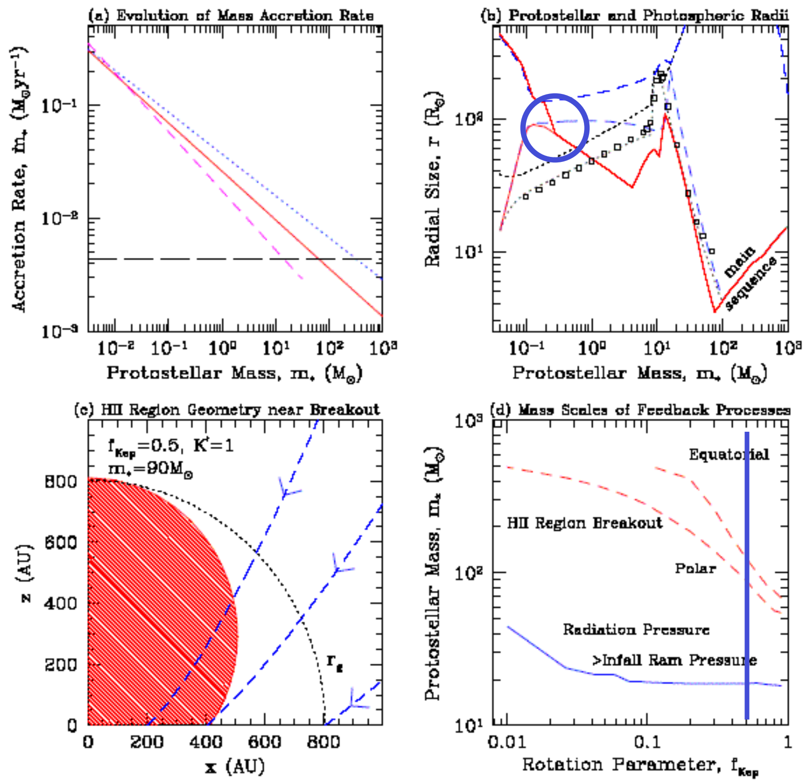
EFFECTS OF ROTATION

- *Protostellar cores are seen in rotation in numerical simulations with* $v_{rot} \approx \frac{1}{2}\, v_{Keplerian}$
- *Most mass reaches the star via an accretion disk → optical depth decreases*
- *Photosphere shrinks to stellar surface*
- *Higher fluxes of ionizing and FUV radiation*

Isentropic

1D Omukai

1D Ripamonti



Tan, Jonathan C.; McKee, Christopher F., 2004, ApJ,603,383

“Historical” material

- Barkana, R. & Loeb, A. 2001, *Phys. Rep.*, **349**, 125
- Bromm, V. & Larson, R. 2004, *ARA&A,* **42**, 79
- Ciardi, B. & Ferrara, A. 2006, *SSRv,* **116**, 625 (updated: Apr 2008)

Recent material

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- Tan, Jonathan C.; McKee, Christopher F., 2004: The Formation of the First Stars. I. Mass Infall Rates, Accretion Disk Structure, and Protostellar Evolution, ApJ,603,383
- Omukai, Kazuyuki; Palla, Francesco, 2003: Formation of the First Stars by Accretion, ApJ,589,677