Stellar structure and evolution

Fundamental and Unified Understandings for Stellar Structure and Evolution

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Difference in approach

before computer age

~ ca. 1970

- looked for generalized understandings
- processes idealized to know what are essential (idealized model)
- more easily foresee cases with different values of parameters
- can be extended to global physics as a system and structure formation

after computer age

ca. 1970 ~

- Ask computer what will be the result
- every detail included (fine model) for local physics but their effects hard to know
- One can play more with computers (computer games?)
- global physics hard to know , but the results compared with observation in more detail

Solving for Stellar Structure

Multi-timescale problem

- mechanical (hydrodyn) eql: τ_{sound} ~ days
- heat transport to steady state: τ_{heat} ~ M yrs
- secular change (nuclear): τ_{nuclear} ~ G yrs

Strongly non-linear problem

Hydrostatic equilibrium – multiplications only

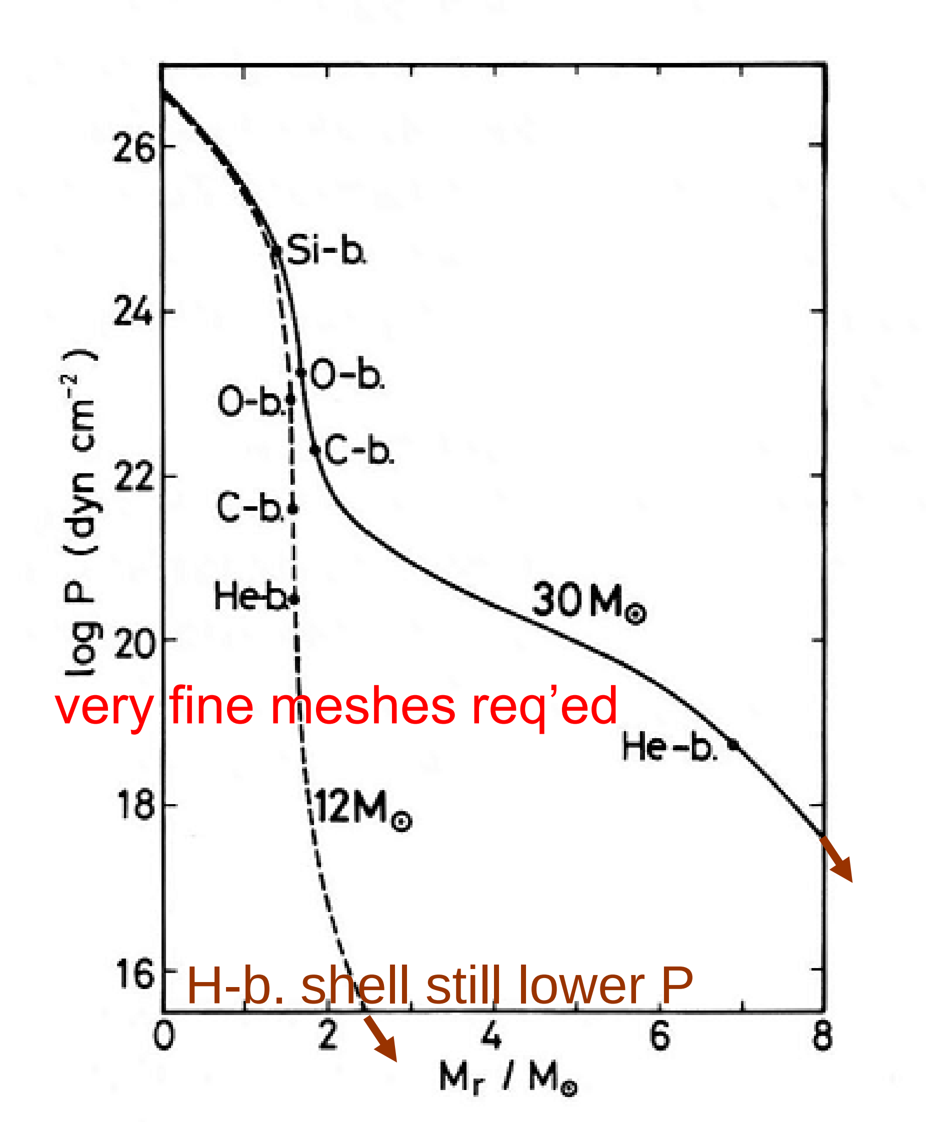
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2}$$

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2\rho$$

$$1 + \frac{1}{N} = \frac{\mathrm{d}\log P/\mathrm{d}\log r}{\mathrm{d}\log \rho/\mathrm{d}\log r}$$

(not dy/dt = A + B
according to main terms
approx 1: dy/dt = A
2: dy/dt = B
3 (steady state): A+B=0)

or eqs of heat transport
& energy conservation



very fine meshes req'ed

H-b. shell still lower P

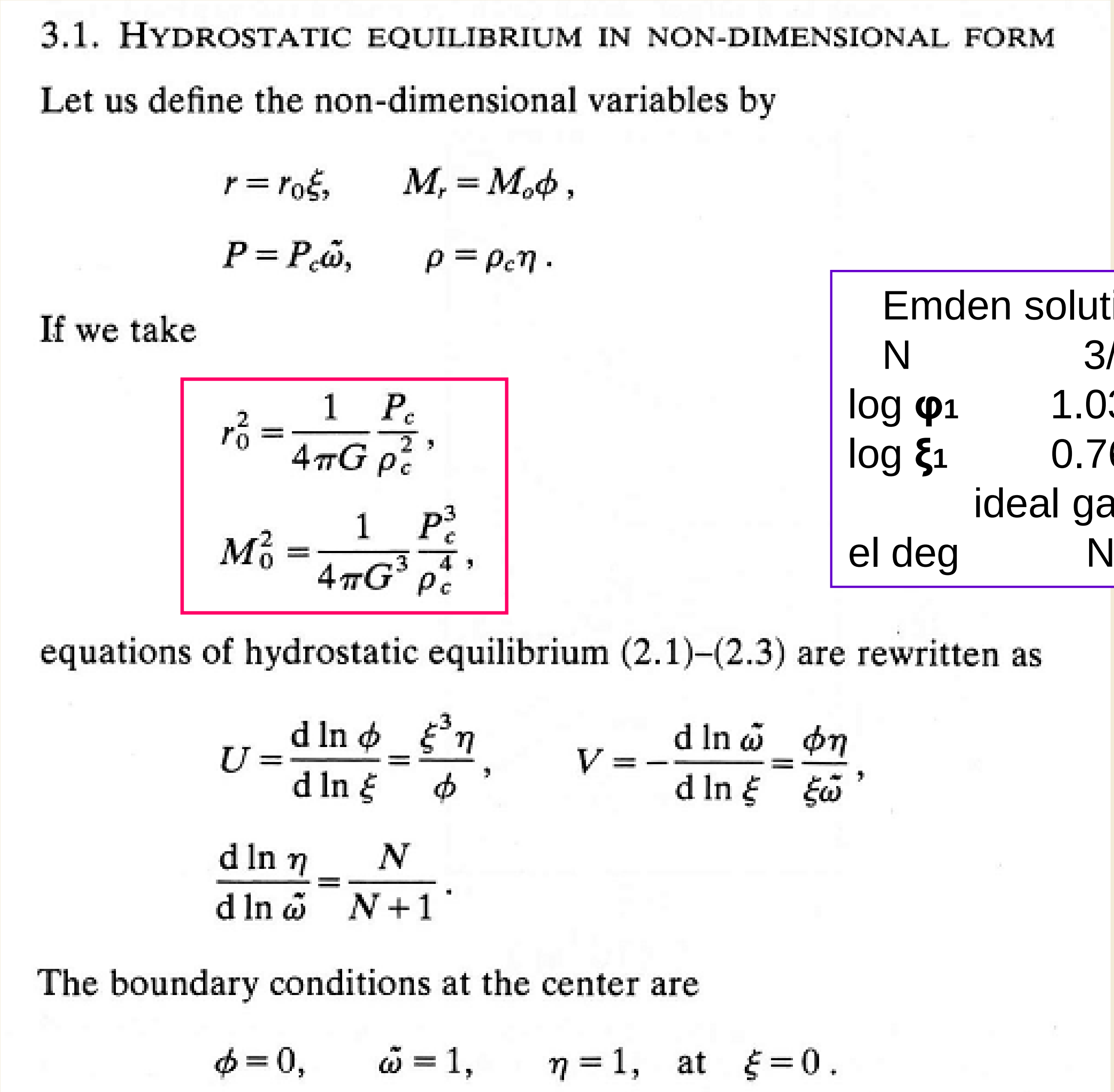
Sugimoto D.,NomotoK.,1979,SSRv,25,155

Extremely wide
dynamic range
due also to
infinite range
of grav force

Pressure distribution
in the core
of 12 & 30 $M_\odot$
Nomoto et al (1979)

Results
in many concepts
out of
common sense

Homology (transf)



3.1. HYDROSTATIC EQUILIBRIUM IN NON-DIMENSIONAL FORM

Let us define the non-dimensional variables by

$$r = r_0 \xi, \qquad M_r = M_o \phi ,$$

$$P = P_c \tilde{\omega}, \qquad \rho = \rho_c \eta .$$

If we take

$$\boxed{\begin{aligned} r_0^2 &= \frac{1}{4\pi G} \frac{P_c}{\rho_c^2} , \\ M_0^2 &= \frac{1}{4\pi G^3} \frac{P_c^3}{\rho_c^4} , \end{aligned}}$$

equations of hydrostatic equilibrium (2.1)–(2.3) are rewritten as

$$U = \frac{d \ln \phi}{d \ln \xi} = \frac{\xi^3 \eta}{\phi} , \qquad V = -\frac{d \ln \tilde{\omega}}{d \ln \xi} = \frac{\phi \eta}{\xi \tilde{\omega}} ,$$

$$\frac{d \ln \eta}{d \ln \tilde{\omega}} = \frac{N}{N+1} .$$

The boundary conditions at the center are

$$\phi = 0, \qquad \tilde{\omega} = 1, \qquad \eta = 1, \quad \text{at} \quad \xi = 0 .$$

Emden solution of polytrope

N	3/2	3
log $\boldsymbol{\varphi}_1$	1.031	1.208
log $\boldsymbol{\xi}_1$	0.762	1.140
	ideal gas conv	rad press
el deg	NRL	REL

Important !

Hydrostatic equilibrium described only with P & ρ

Complications arises in T

thru Eq of States, Heat Transport etc

Find solutions only with P & ρ

and then

translate them to T

eq of state $$P = \boxed{\frac{\Lambda}{\mu\beta}} \left(\frac{k}{m_{\rm amu}}\right) \rho \boxed{T}$$

can absorb complications arising from electron degeneracy, radiation pressure, and changing chemical compositions

Gravitational Contraction

When interior of the star is cooled (entropy s decreased) the temperature rises (gravothermal)

Important Transformations

$$M_1^2 = \frac{1}{4\pi G^3}\boxed{\frac{P_c^3}{\rho_c^4}}\varphi_1^2$$

$$r_1^2 = \frac{1}{4\pi G}\frac{P_c}{\rho_c^2}\xi_1^2$$

(core) mass & radius

φ_1 and ξ_1

depend weakly on *N*

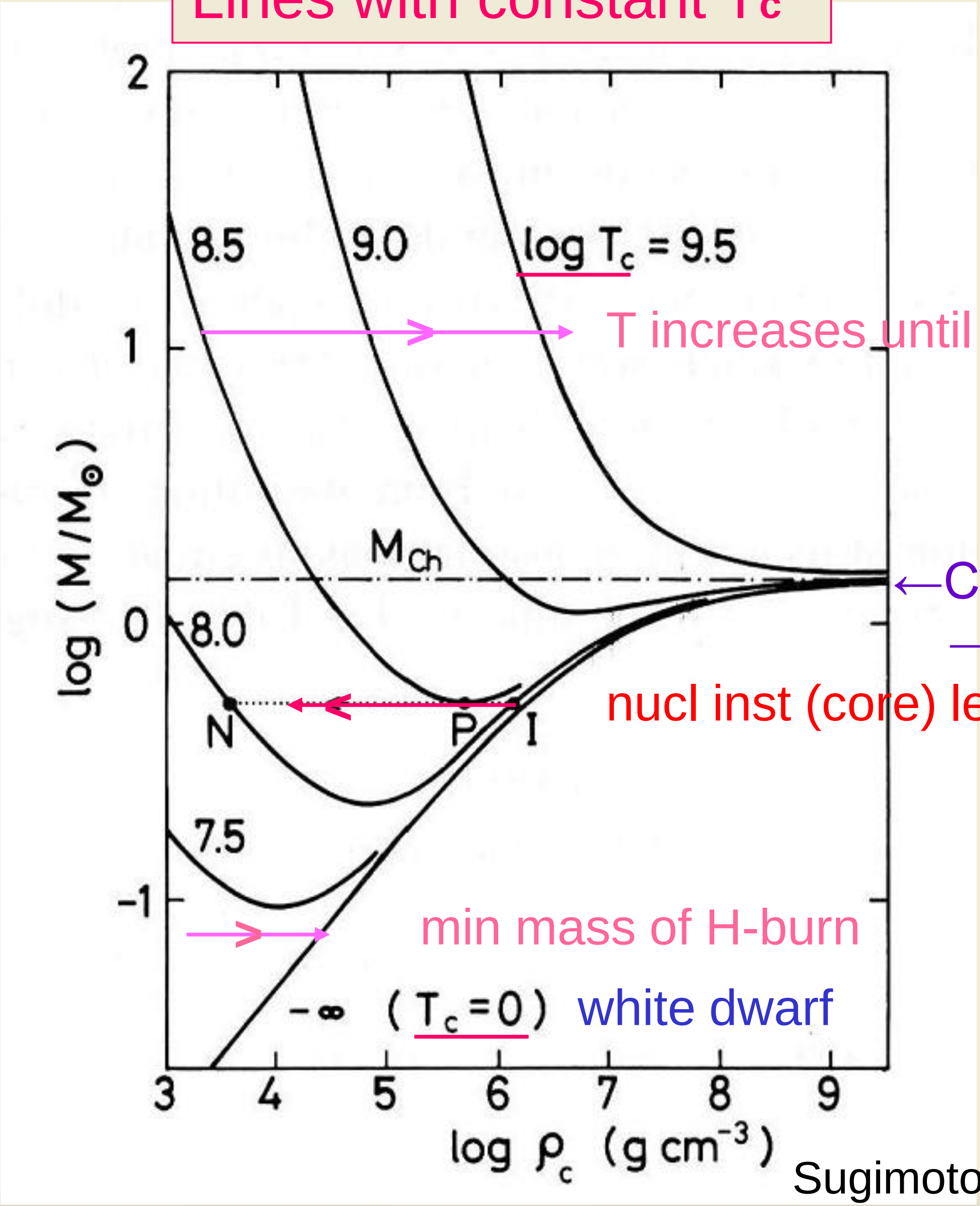
& M1 determine everything

φ_1 almost determined by Nc / by

Lines with constant T_c

$$M_1^2 = \frac{1}{4\pi G^3}\frac{P_c^3}{\rho_c^4}\varphi_1^2$$

and Eq of state determine



T increases until iron photodiss SN

←Chandrasekhar limit; REL,
→ electron capture SN

nucl inst (core) leading to He-flash, C-def SN

min mass of H-burn

white dwarf

brown dwarf
planet

Sugimoto D.,NomotoK.,1979,,SSRv,25,155

Stable nuclear burning

(main sequence stars) and Luminosity

$$\frac{M}{R} \sim \frac{P}{\rho} \sim T \quad (\mathrm{gas}), \qquad \sim \frac{T^4}{\rho} \quad (\mathrm{rad})$$

$$\tau \sim \kappa\rho R$$

$$L \sim \frac{aT^4V}{t_{\mathrm{diff}}} \sim \frac{T^4R^3}{\tau R/c} \sim \frac{T^4R}{\kappa\rho} \sim M^3 \quad (\mathrm{gas}),$$

$$\sim \frac{M}{\kappa}(\mathrm{rad:Eddington\ limit})$$

Wrong reasoning

Mass larger

→ gravity stronger

→ Temperature higher

→ Nuclear burning stronger

→ Luminosity higher

Right reasoning

Mass larger,

but Temperature (almost) the same (nuclear E generation)

→ grav potential (almost) the same

→ Density lower

→ Optical depth smaller

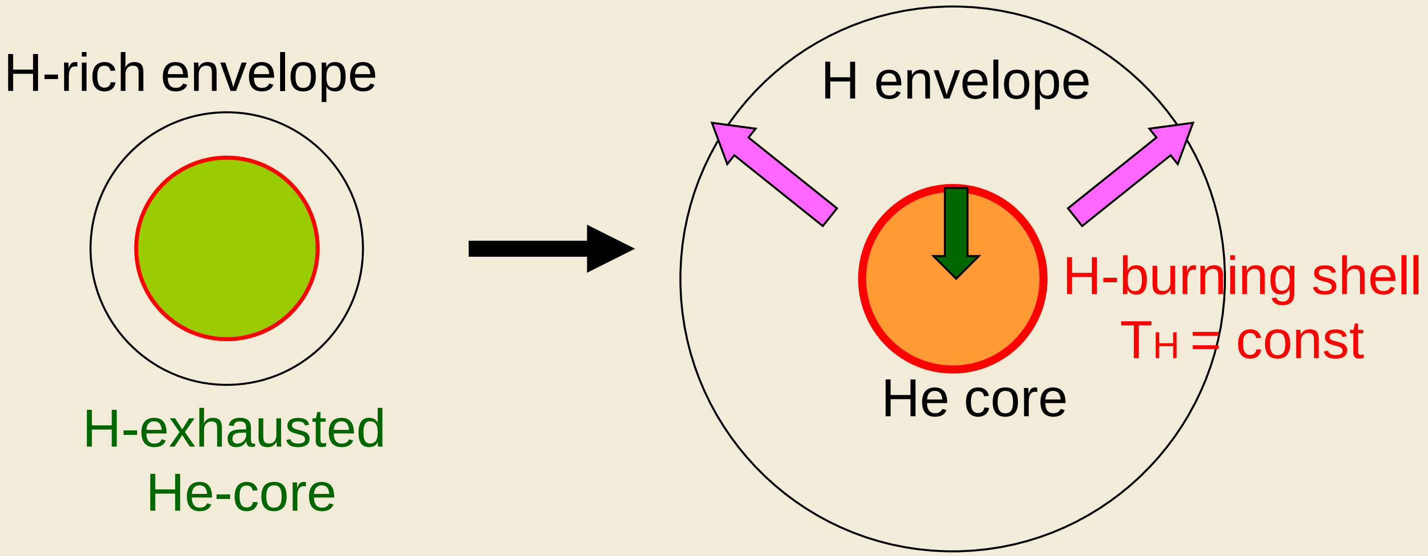
→ Photons more easily diffuse out

→ Luminosity higher

→ slightly higher Temperature required

(evolution towards) Red Giant Stars
consisting of core and envelope of different compositions

Core contracts – Envelope expands (to core-halo str)



Denoting with subscript c: the center of the core
1: core edge just inside of the H-burning shell

core contracts but T_1 stays const

→ r_1 and the volume of the core stays const

→ $\rho_c\uparrow$ but $\rho_1\downarrow$ → P_1 lowered

nevertheless

the same mass of the env must be sustained against gravity

→ for equilibrium solution (boundary value problem):
Place the bulk of the env-mass where the gravity weak,
i.e., where the radius larger

NB: To realize it (initial value problem):
raise T_1 slightly for a little while to supply add'l nucl energy

hydrostatic eql
described
only with *P* & *ρ*

homology
invariants

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2}$$

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2\rho$$

$$U = \frac{\mathrm{d}\log M_r}{\mathrm{d}\log r} = \frac{4\pi r^3\rho}{M_r}$$

$$V = -\frac{\mathrm{d}\log P}{\mathrm{d}\log r} = \frac{GM_r\rho}{rP}$$

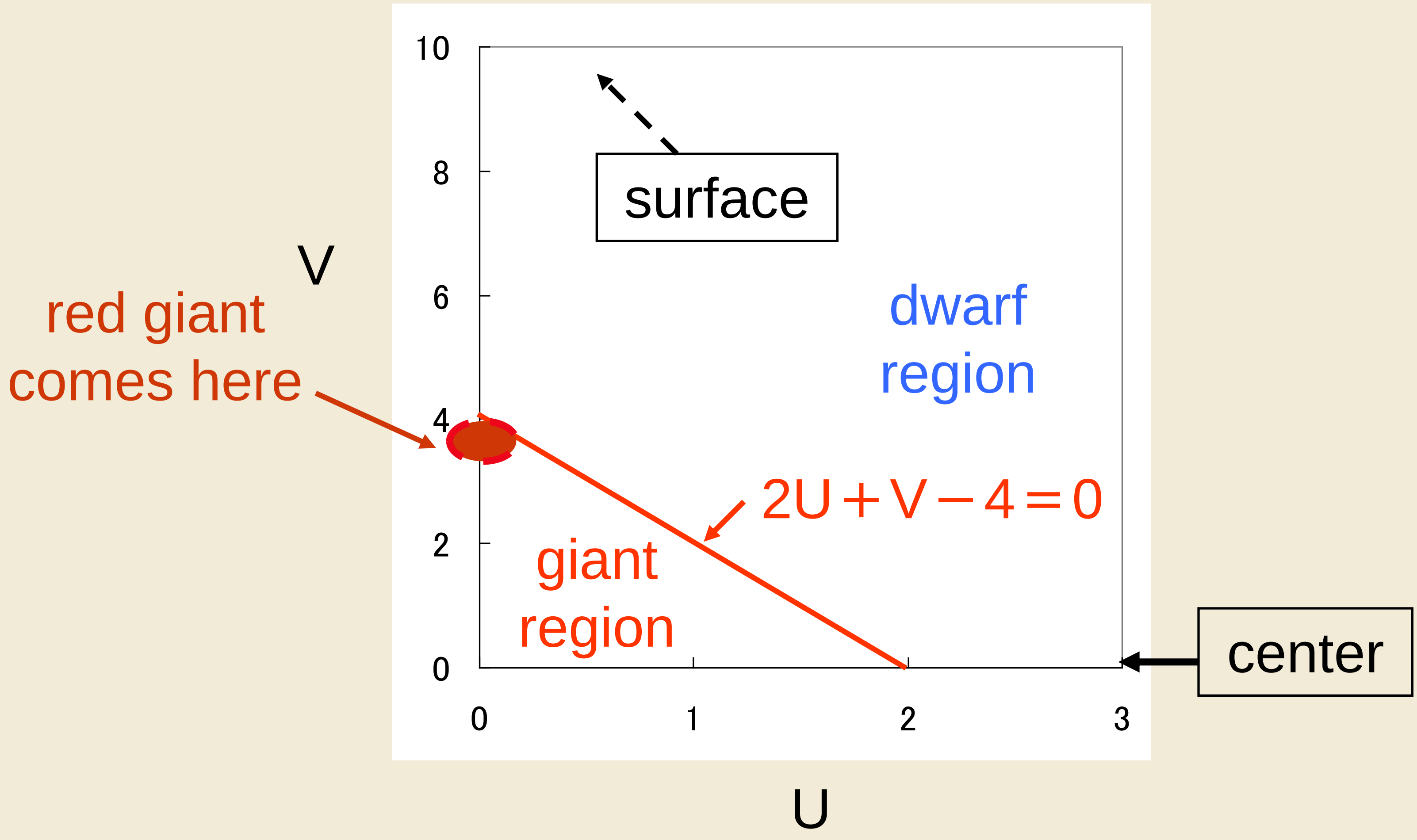
(local)
polytropic index

$$1 + \frac{1}{N} = \frac{\mathrm{d}\log P/\mathrm{d}\log r}{\mathrm{d}\log \rho/\mathrm{d}\log r}$$

singularity
on 2U + V − 4=0

$$\boxed{d\ln M_r = -U\frac{d\ln U - d\ln V}{2U + V - 4}}$$

U-V plane


10
8
6
4
2
0
0
1
2
3
V
U
surface
center
dwarf region
giant region
2U＋V－4＝0
red giant comes here

polytropes (N=const)

N=1.5

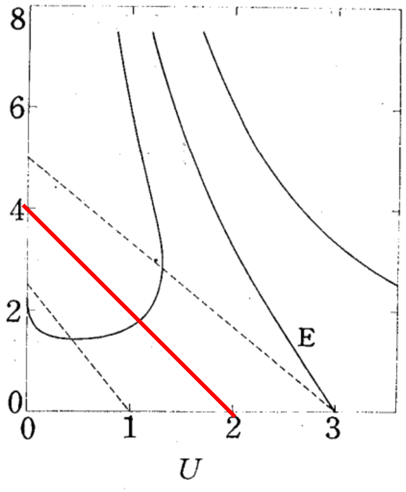


Fig. 3–2. The U-V curves for $N=1.5$.

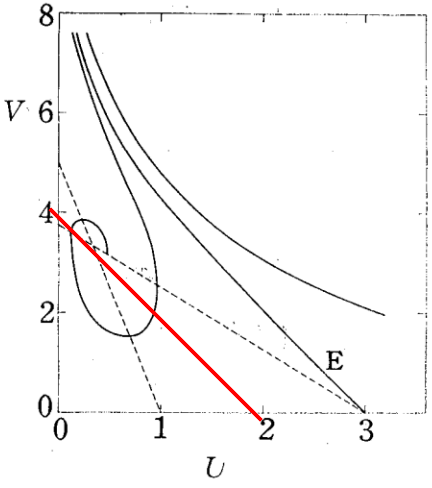


Fig. 3–3. The U-V curves for $N=4$.

N=4

N=∞

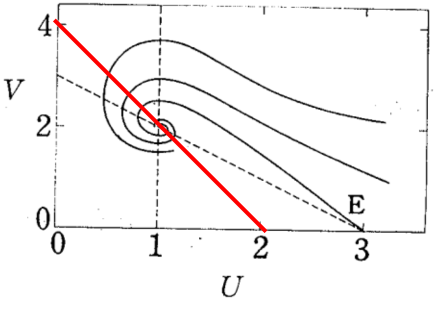
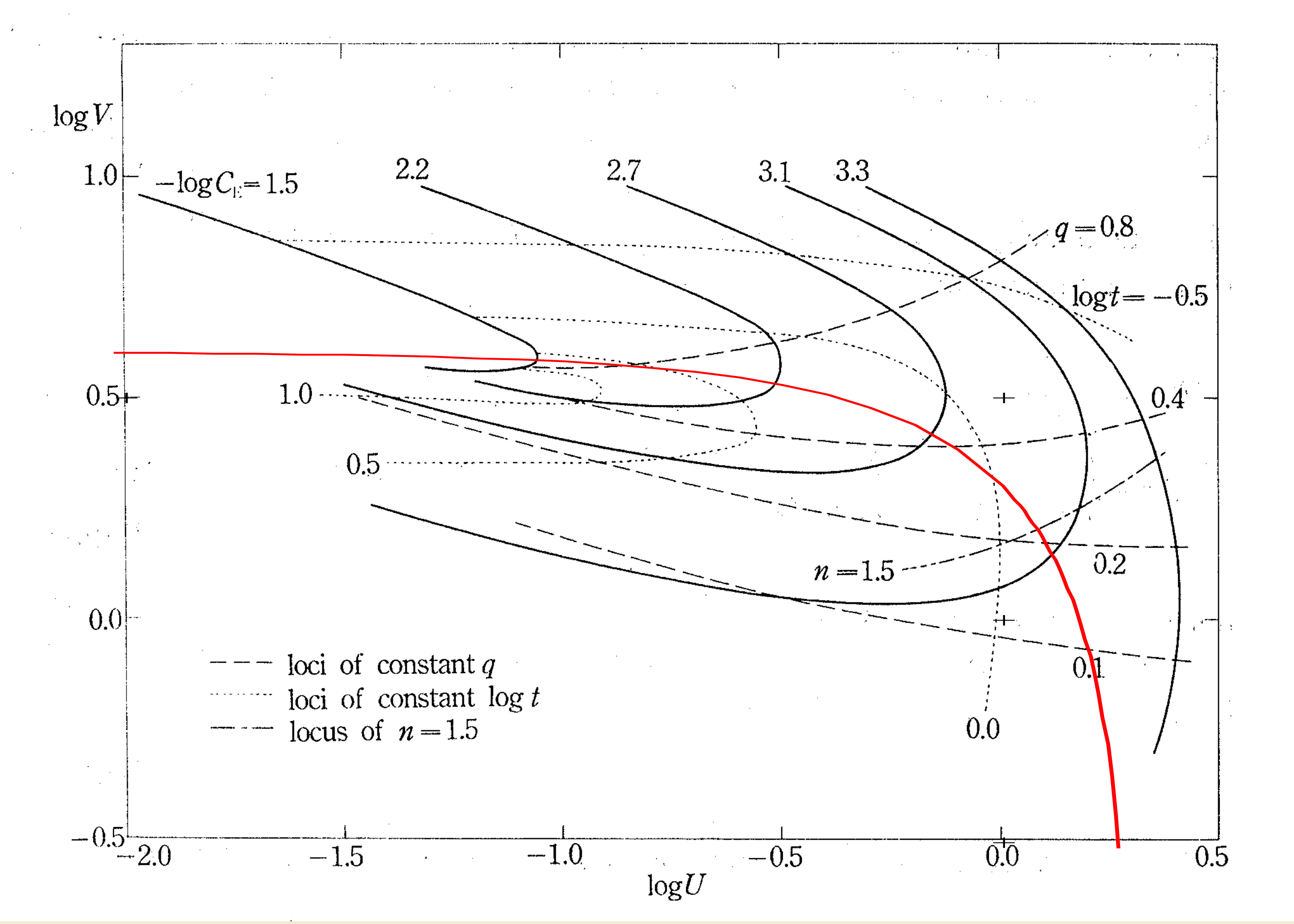


Fig. 3–4. The U-V curves for $N=\infty$.

Hayashi C.,Hoshi R.,Sugimoto D.,1962,PThP,22,1

envelope solutions
for el scattering opacity



Hayashi C.,Hoshi R.,Sugimoto D.,1962,PThP,22,1

core solution

contracting core with T gradient

$$P = \frac{\Lambda}{\mu\beta}\left(\frac{k}{m_{\mathrm{amu}}}\right)\rho T$$

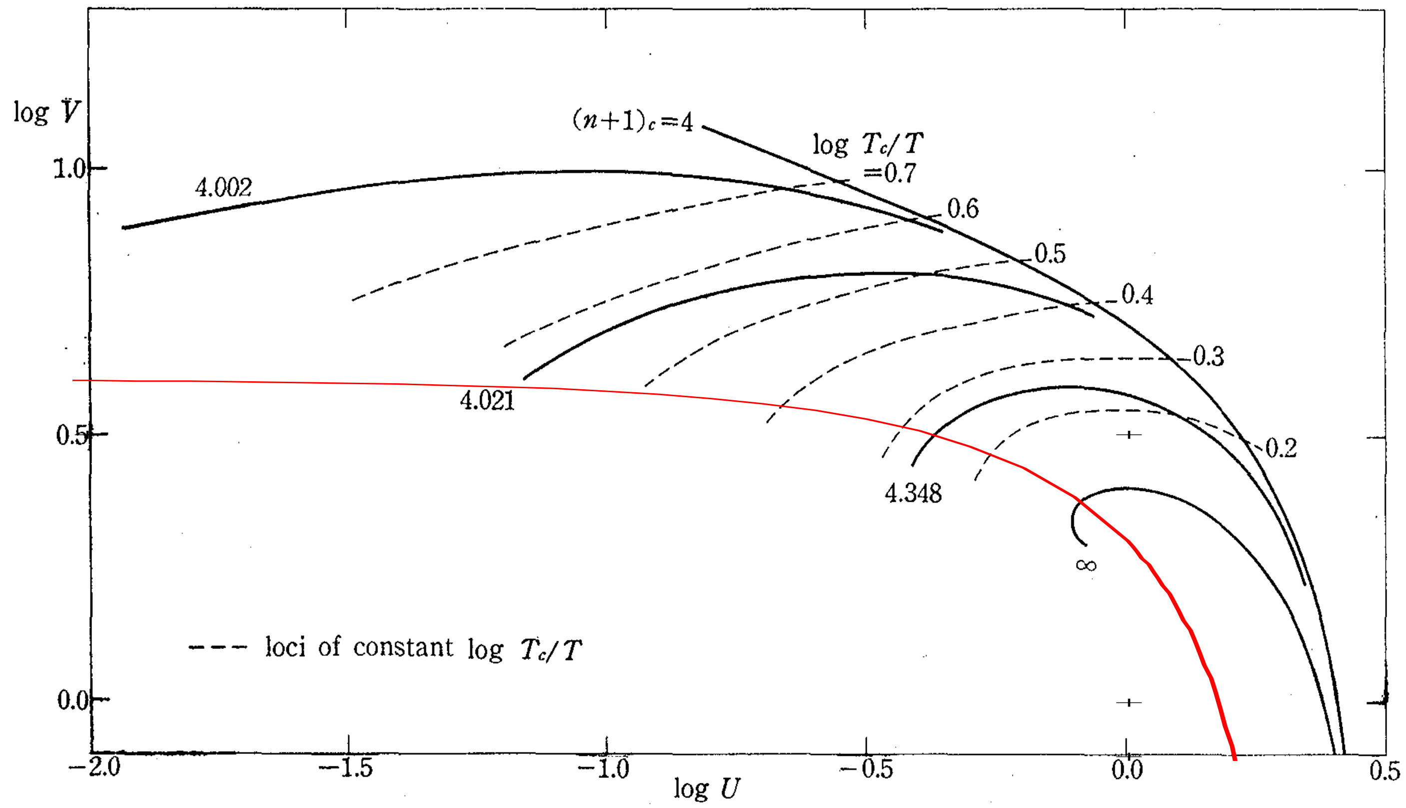


Fig. 4-9. The U-V curves of contracting cores ($\kappa L_r/M_r$ = constant, β = 1).

Hayashi C.,Hoshi R.,Sugimoto D.,1962,PThP,22,1

electron degenerate core with Λ (and μ) gradient

$$P = \frac{\boxed{\Lambda}}{\mu\beta}\left(\frac{k}{m_{\rm amu}}\right)\rho T$$

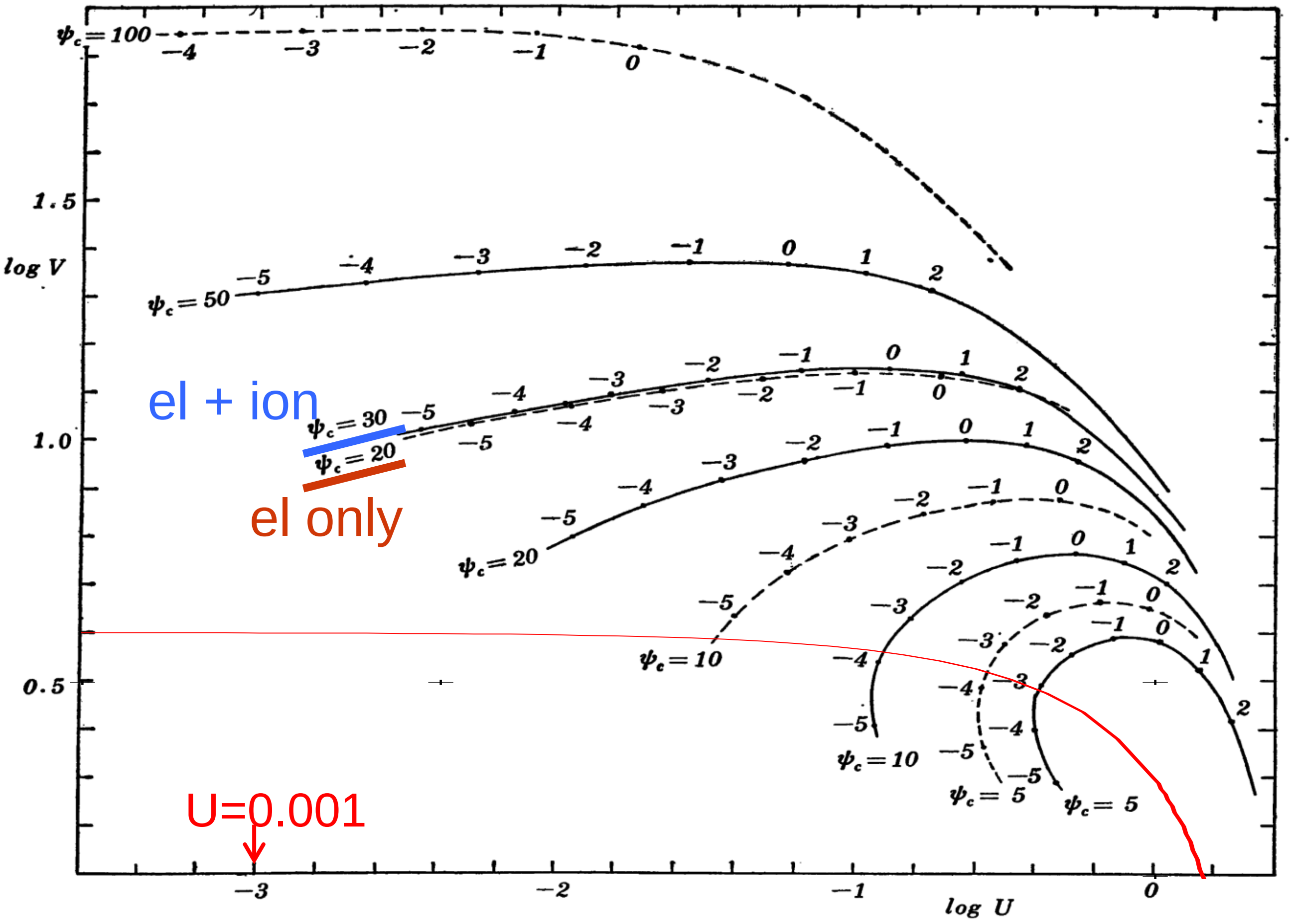


Fig. 1. U–V curves of the solutions of the partially degenerate isothermal cores. Full and dotted curves show the cases $\mu_n/\mu_e=2$ and $\mu_n/\mu_e=\infty$, respectively. Values of ψ are shown on the points on these curves.

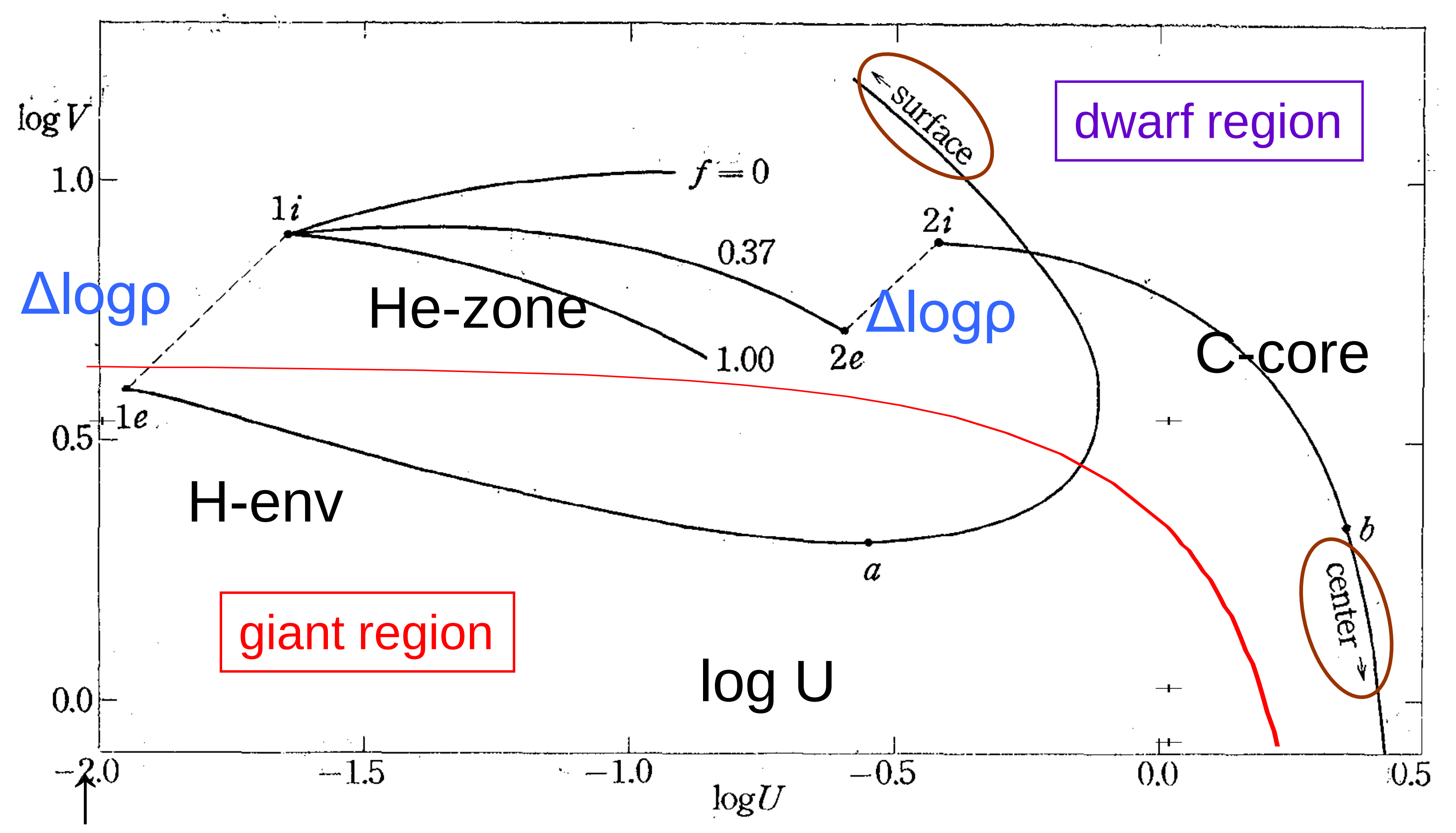
Hayachi C.,1957:PThPh,17,727

log U
vs
log V
plane

admired
by Martin
as
Hayashi's
invention

Fitting of envelope to core

giant sol has a topology with loop



U=0.01

Hayashi C.,Hoshi R.,Sugimoto D.,1962,PThP,22,1

Required for giant-type solution
- giant-type topology with loop in U-V plane

- Large concentration in the core,
  i.e., large ratio of $(P/\rho)_c$ ／ $(P/\rho)_1$
  and Large mass fraction of the envelope

Not the case of giant-type solution

- Small envelope mass fraction:
  - much mass has been lost in the preceding phase
  - accreting WD (nova), X-ray burst of n-star
- Non-degenerate isothermal core smaller than S-C limit
- Chemically homogeneous star
  - the point $(P/\rho)_1$ comes
    not close to the singularity but to the surface

Processes in competition - 1

$$P = \boxed{\frac{\Lambda}{\mu\beta}}\left(\frac{k}{m_{\rm amu}}\right)\rho\boxed{T}$$

Comparison: Effect of ion pressure in electron degenerate core

center: Λ_c large for el,
$1/\mu \simeq 1/\mu_{el}$; $P_{ion} \ll P_{el}$

core edge
ion neglected: $\Lambda=1$, $1/\mu= 1/\mu_{el}$
ion taken into account:
$\Lambda=1$, $1/\mu=1/\mu_{el} + 1/\mu_{ion}$ larger
→ higher Λ_c required for
the same difference in P/ρ

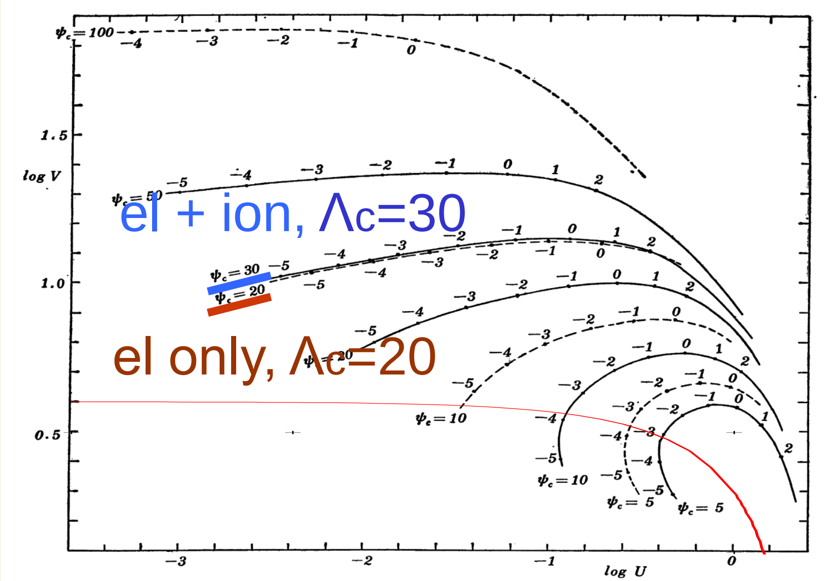
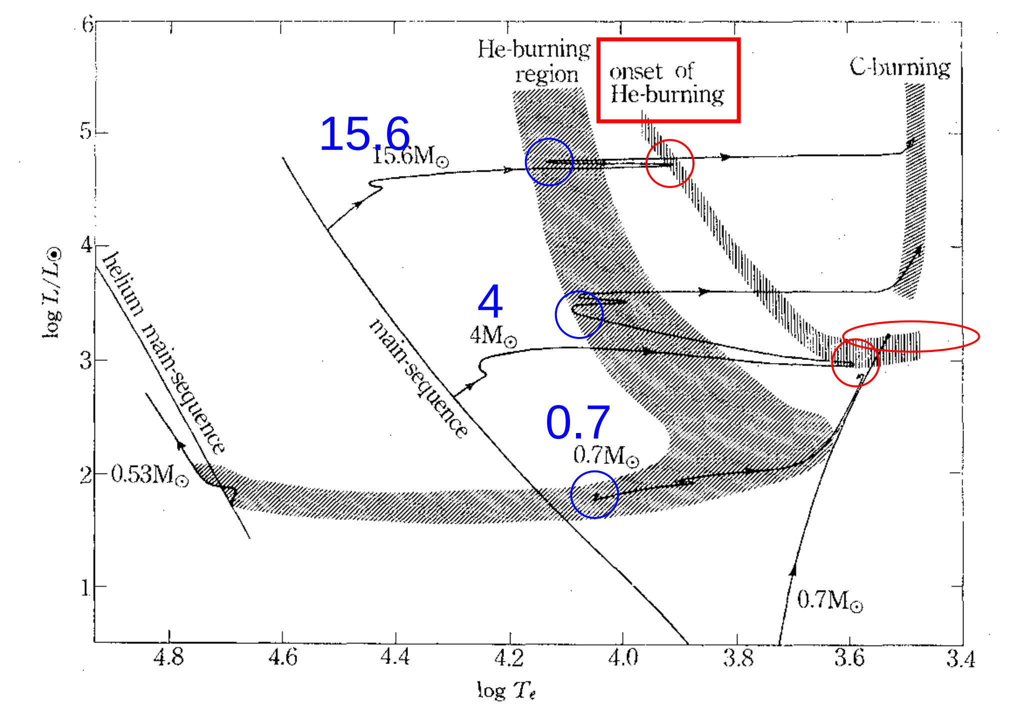


Fig. 1. U–V curves of the solutions of the partially degenerate isothermal cores. Full and dotted curves show the cases $\mu_n/\mu_e=2$ and $\mu_n/\mu_e=\infty$, respectively. Values of ψ are shown on the points on these curves.

Processes in competition - 2

$$P = \boxed{\frac{\Lambda}{\mu\beta}}\left(\frac{k}{m_{\mathrm{amu}}}\right)\rho\boxed{T}$$

Helium-burning phase:
competition between T and μ(center)



Hayashi C.,Hoshi R.,Sugimoto D.,1962,PThP,22,1

○ onset, Y=1.0

⬭ He flash, Y=1.0

↓

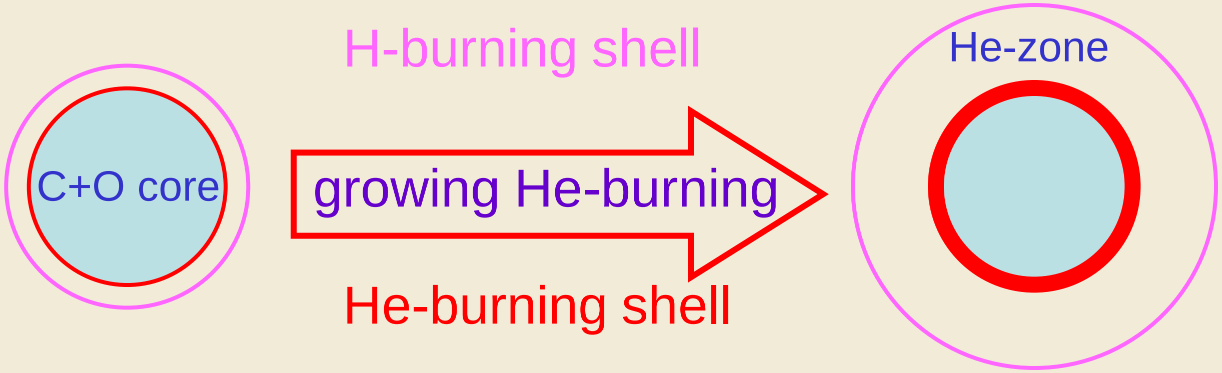
○ min R, Y~0.3

as He consumed,
both T and μ
increases.
compete each other
at about Y=0.3
(for He-flash Λ_c decr)

Processes in competition - 3

$$P = \boxed{\frac{\Lambda}{\mu\beta}}\left(\frac{k}{m_{\mathrm{amu}}}\right)\rho\boxed{T}$$

Thermal pulses of helium shell-burning (AGB stars)



H-burning shell

C+O core

growing He-burning

He-zone

He-burning shell

unstable
(planer config
= non-gravothermal,
i.e., dwarf type)

Deposit of entropy
→He-zone expands
T_{He}/T_H increases

instability ceases
(spherical config
= gravothermal,
i.e., giant-type
2U+V-4~0)

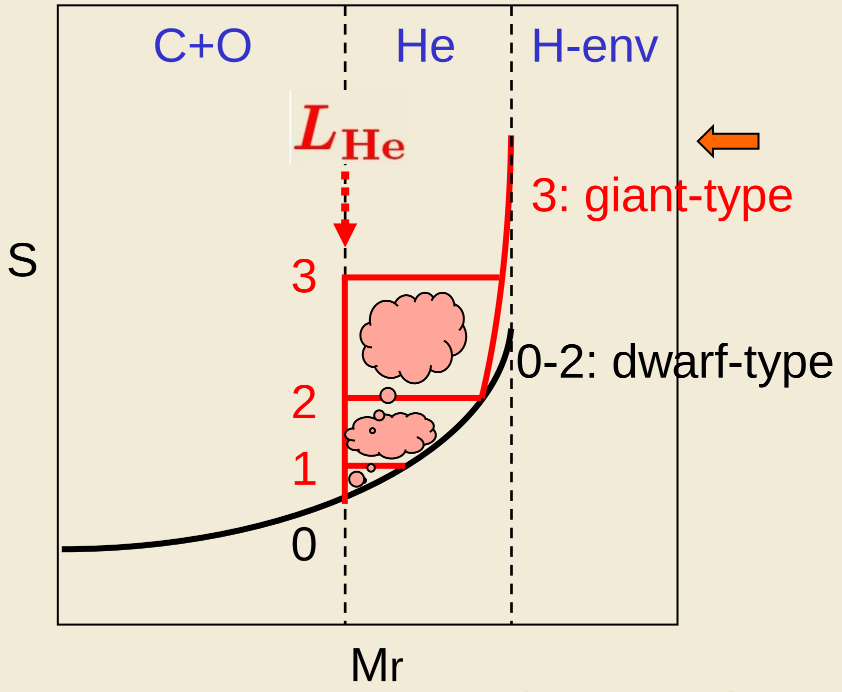
← cooling down

relaxation osc
↔

Processes in competition – 4

Entropy aspects

$$P = \boxed{\frac{\Lambda}{\mu\beta}}\left(\frac{k}{m_{\rm amu}}\right)\rho\boxed{T}$$



0-3: outer edge of C+O core

T~P/ρ stays const

1-3: bottom of He-zone

T~P/ρ increases

1-3: S increases in He-zone

He-zone expands

2-3: heat diffusion

3: top of He-zone

S becomes very large

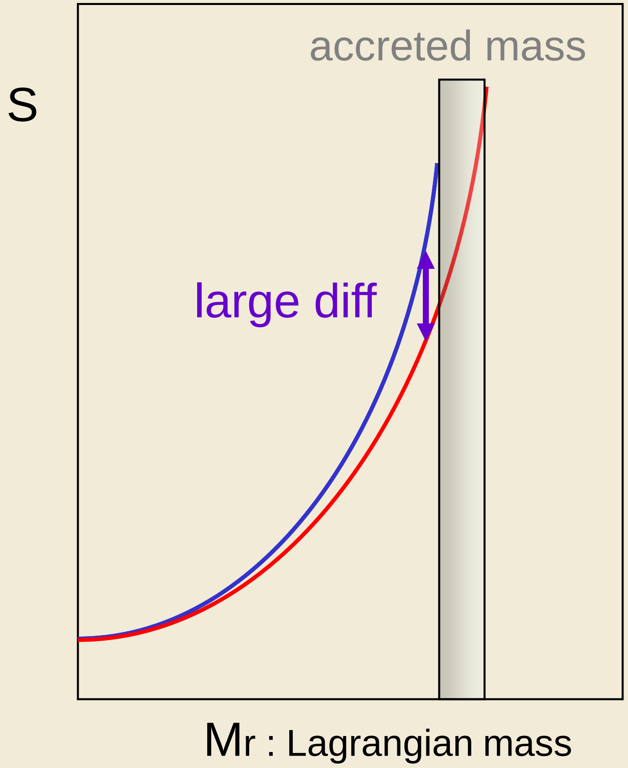
When S at the core edge becomes as high as
S in the surface convection zone (⇦)
it invades into the helium zone

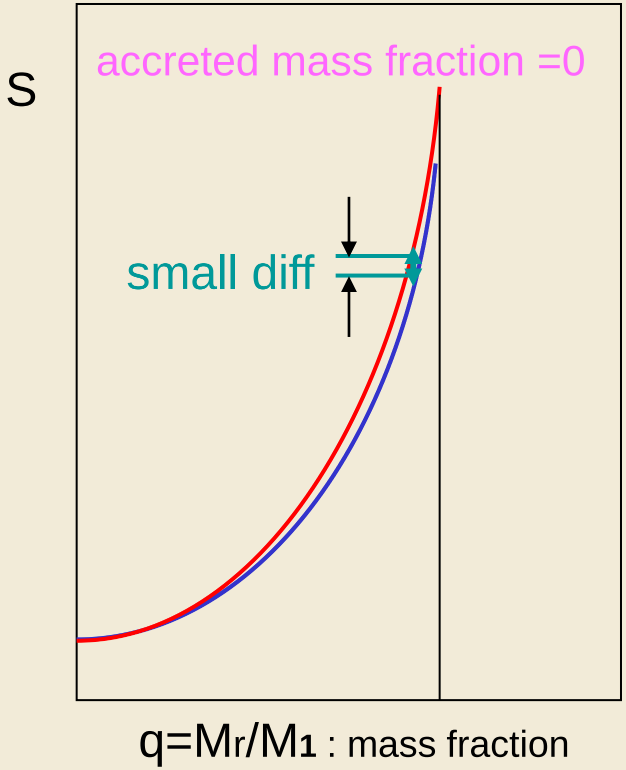
more details later

Mass loss / accretion / flow in close binary / repeating thermal pulses/ etc

$$s = s(M_r, t) = s(q, t); \quad M_r = M_1 q$$

$$\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)_{M_r} = \underbrace{\left(\frac{\partial s}{\partial t}\right)_q}_{\text{small}} - \frac{\mathrm{d}\ln M_1}{\mathrm{d}t}\underline{\left(\frac{\partial s}{\partial \ln q}\right)}_t$$



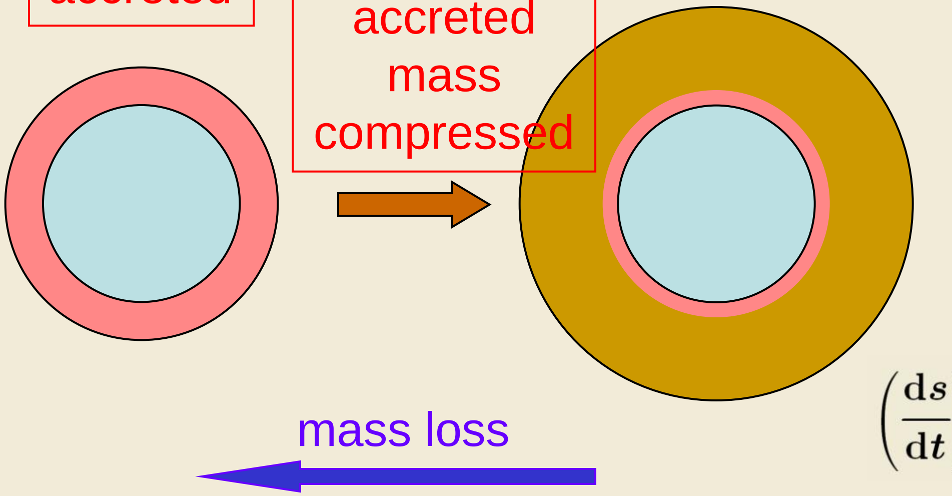


eg, ~ 1000 but computed easily (it has nothing to do with numerical stability, since *t* not contained)

Accretion and Mass exchange in binary stars

Wrong Statement
The star puffs up (since its radius increases)

Right Statement
Accreted matter just piling-up and compressed; radius for a Lagrange mass shell is shrinking



accreted

accreted mass compressed

accreted more

mass loss

Energy
released / required
for mass flow

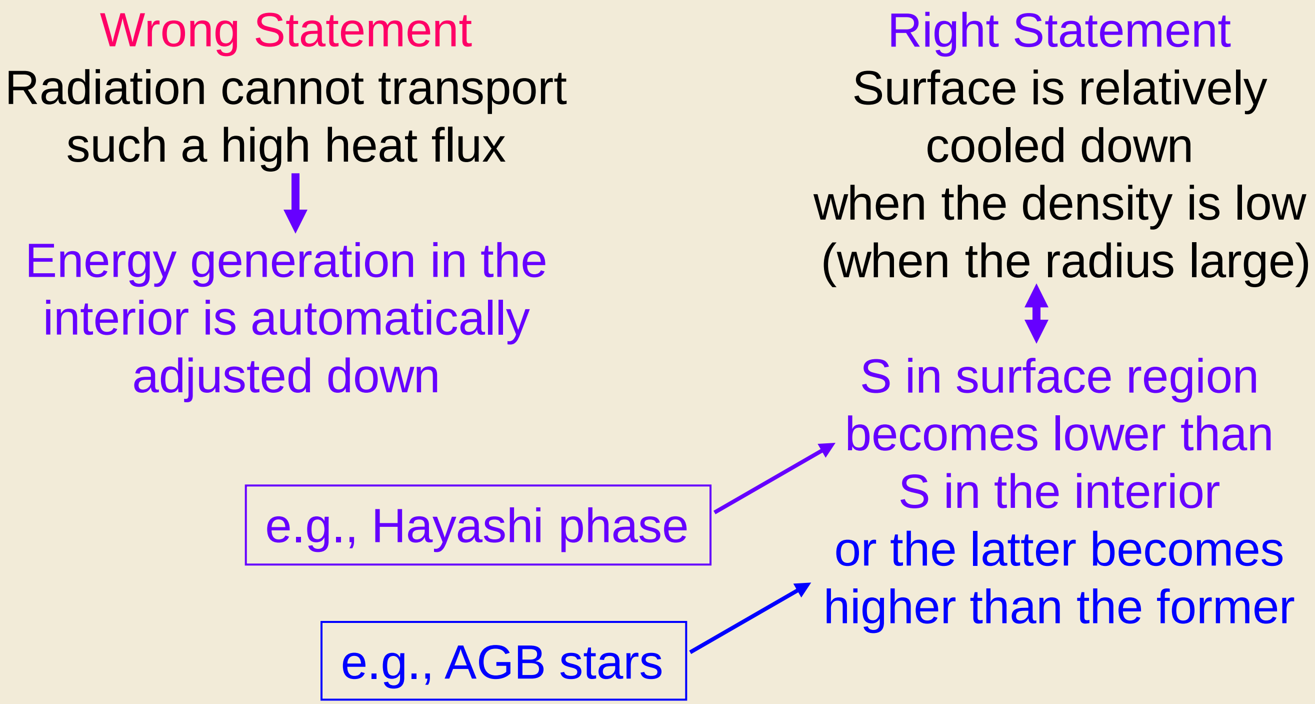
$$L_r = \int_0^r T\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right) \mathrm{d}M_r$$

$$\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)_{M_r} = \underbrace{\left(\frac{\partial s}{\partial t}\right)_q}_{\text{small}} - \frac{\mathrm{d}\ln M_1}{\mathrm{d}t}\boxed{\left(\frac{\partial s}{\partial \ln q}\right)_t =0}$$

If *S* is const (convective),
no energy required to
pushing-out the env mass

: rapid mass exchange

Development and invasion of Surface Convection Zone



Wrong Statement

Radiation cannot transport such a high heat flux

↓

Energy generation in the interior is automatically adjusted down

Right Statement

Surface is relatively cooled down when the density is low (when the radius large)

↕

S in surface region becomes lower than S in the interior

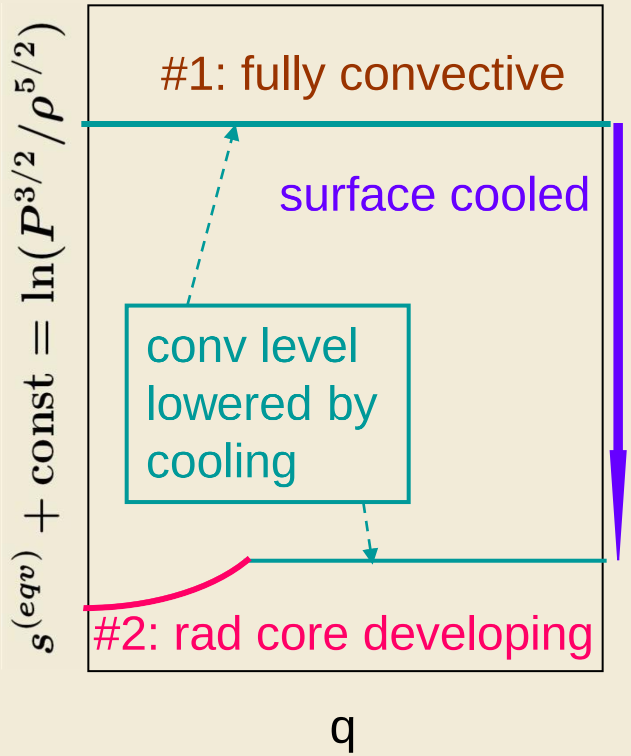
or the latter becomes higher than the former

e.g., Hayashi phase

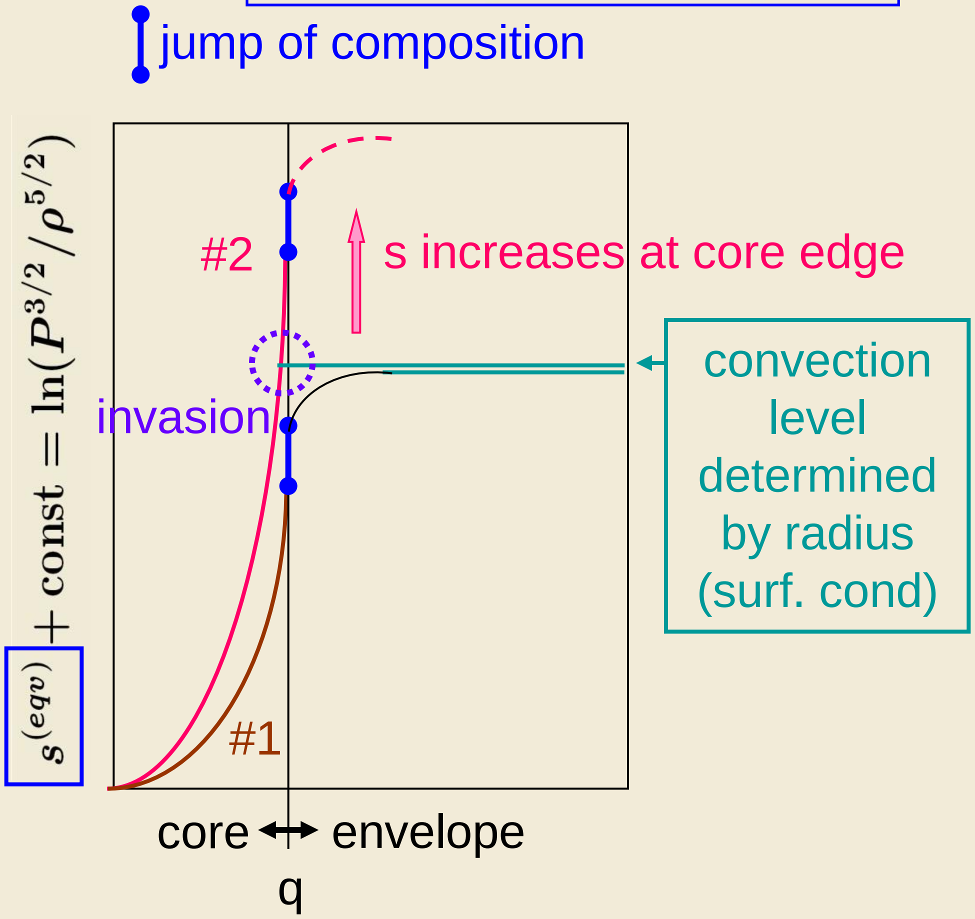
e.g., AGB stars

Invasion of surface conv into the core

to include mixing of diff compositions: $\boxed{\mathrm{d}s^{(eqv)} = \frac{\mathrm{d}q}{T} = \mathrm{d}s + \Sigma_k \frac{\mu_k}{T}\mathrm{d}N_k}$



homogeneous;
pre-main-seq contr

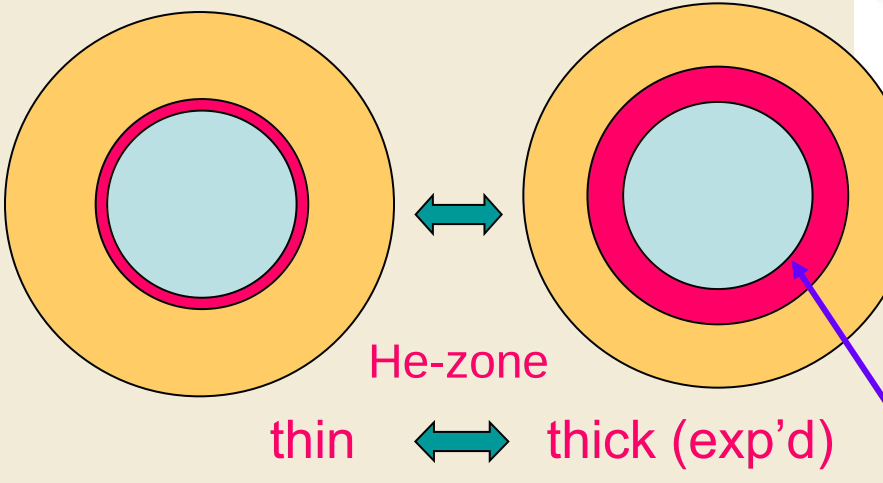


Thermal Pulses of He-shell burning in Asymptotic Giant Branch (AGB)

shell flash – non-linear oscillation

H-rich env

C+O core



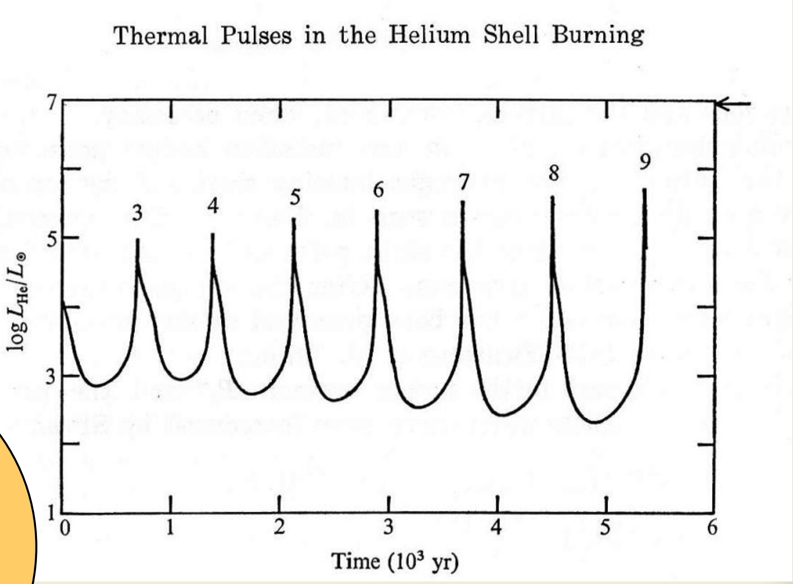
He-zone

thin ↔ thick (exp'd)

dwarf-type giant-type

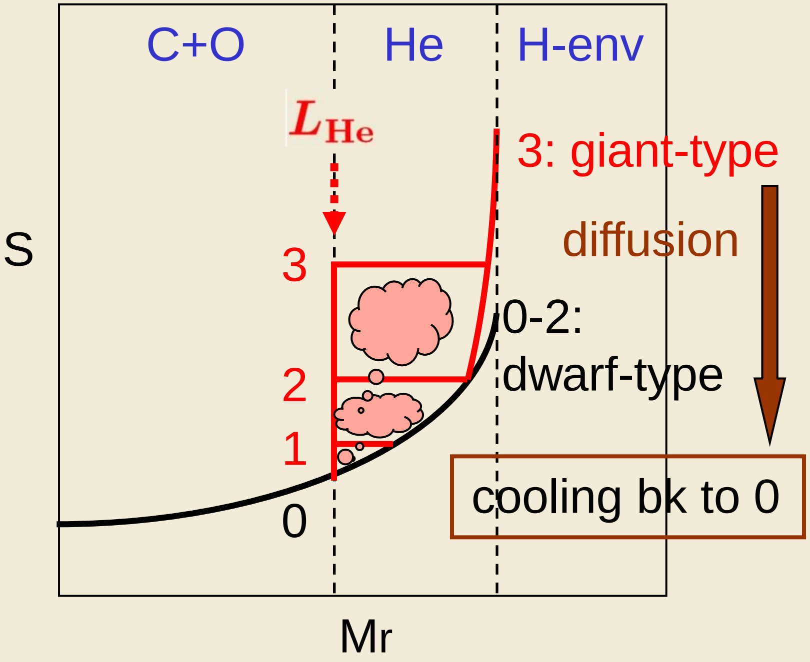
unstable stable

approaches $2U + V - 4 = 0$



Fujimoto & Sugimoto (1979)

progress of the flash
and
non-linear oscillation



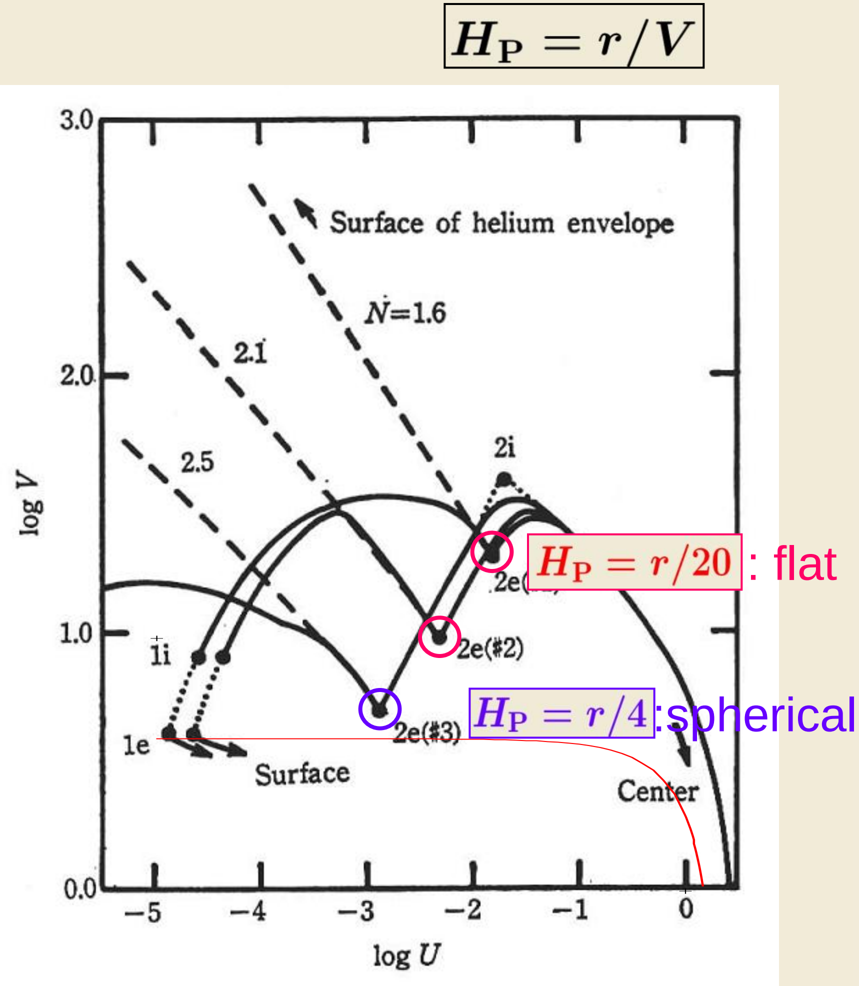
3: giant-type

diffusion

0-2:
dwarf-type

cooling bk to 0

conv invades afterwards
→ mixing and dredging-up
s-process elements



$H_P = r/V$

$H_P = r/20$: flat

$H_P = r/4$:spherical

Sugimoto D.,Fujimoto M. Y.,PASJ,20,467

Unified & Similarity understanding
for repeating nuclear shell-burning instabilities

Thermal pulse

- e.g., C+O core
- He-burning
- added by H-shell burning
- thin-shell instability
- non-degenerate
- ceased by transit to spher geometry
- period controlled by L_H and mixing
- formation of C-star and dredging-up of s-process elements

Nova Explosion

- White dwarf
- H-burning
- added by accretion
- ← same
- med. degeneracy
- ← same or by explosion
- period controlled by accretion rate
- slower accr results stronger explosion due to higher deg
- fate; SN Ia

X-ray Burst

- Neutron star
- He-burning
- Accreted H burned quietly into He
- ← same
- strong degeneracy
- ceased by fuel He-exhaustion
- ← same
- If time should allow, its fate would be collapse into BH

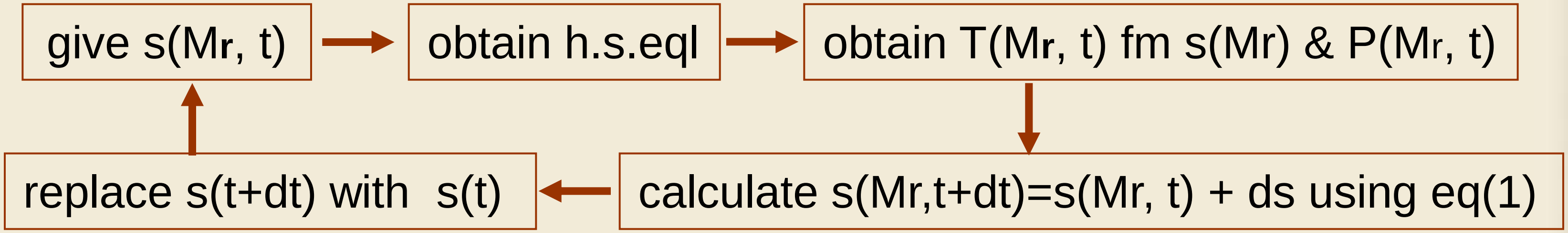
Effect of rapid neutrino loss beyond C-burning

- Lph << Lv~= Ln; Radiative heat transport negligible
- Time-scale for heat diffusion too long $\tau_{\mathrm{ph}} (\gg \tau_\nu \simeq \tau_{\mathrm{nucl}})$
- Entropy distribution $s(M_r)$ is determined by

eq(1) : $$T\frac{\mathrm{d}s}{\mathrm{d}t} = -\varepsilon_\nu + \varepsilon_n + \boxed{\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)} \leftarrow \text{negligible}$$

- Thermal process separable from hydrostatic equilibrium

  (decoupled; gravothermal but not in thermal equilibrium)
- Computation becomes easier because of the decoupling



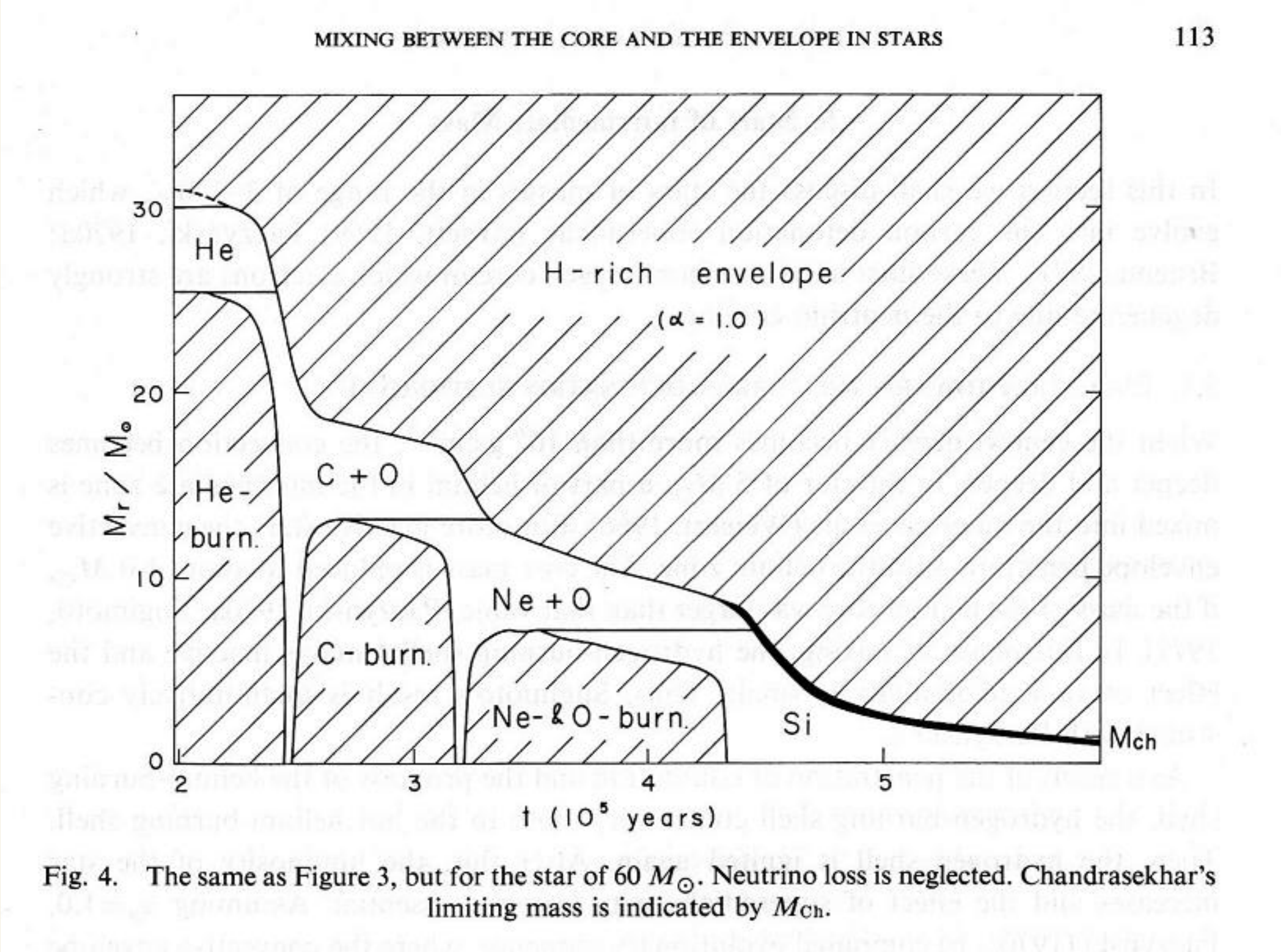
give s(Mr, t) → obtain h.s.eql → obtain T(Mr, t) fm s(Mr) & P(Mr, t)

↓

calculate s(Mr,t+dt)=s(Mr, t) + ds using eq(1) → replace s(t+dt) with s(t)

↑ give s(Mr, t)

If neutrino loss is not included in computation, strong invasion of the surface conv 60 $M_\odot$ (1974)



MIXING BETWEEN THE CORE AND THE ENVELOPE IN STARS 113

Fig. 4. The same as Figure 3, but for the star of 60 $M_\odot$. Neutrino loss is neglected. Chandrasekhar's limiting mass is indicated by M_{Ch}.

Sugimoto D.,Nomoto K.,IAUS,66,105

With neutrino loss included; 30 $M_\odot$
surf conv stops invading (No time for redistributing s)

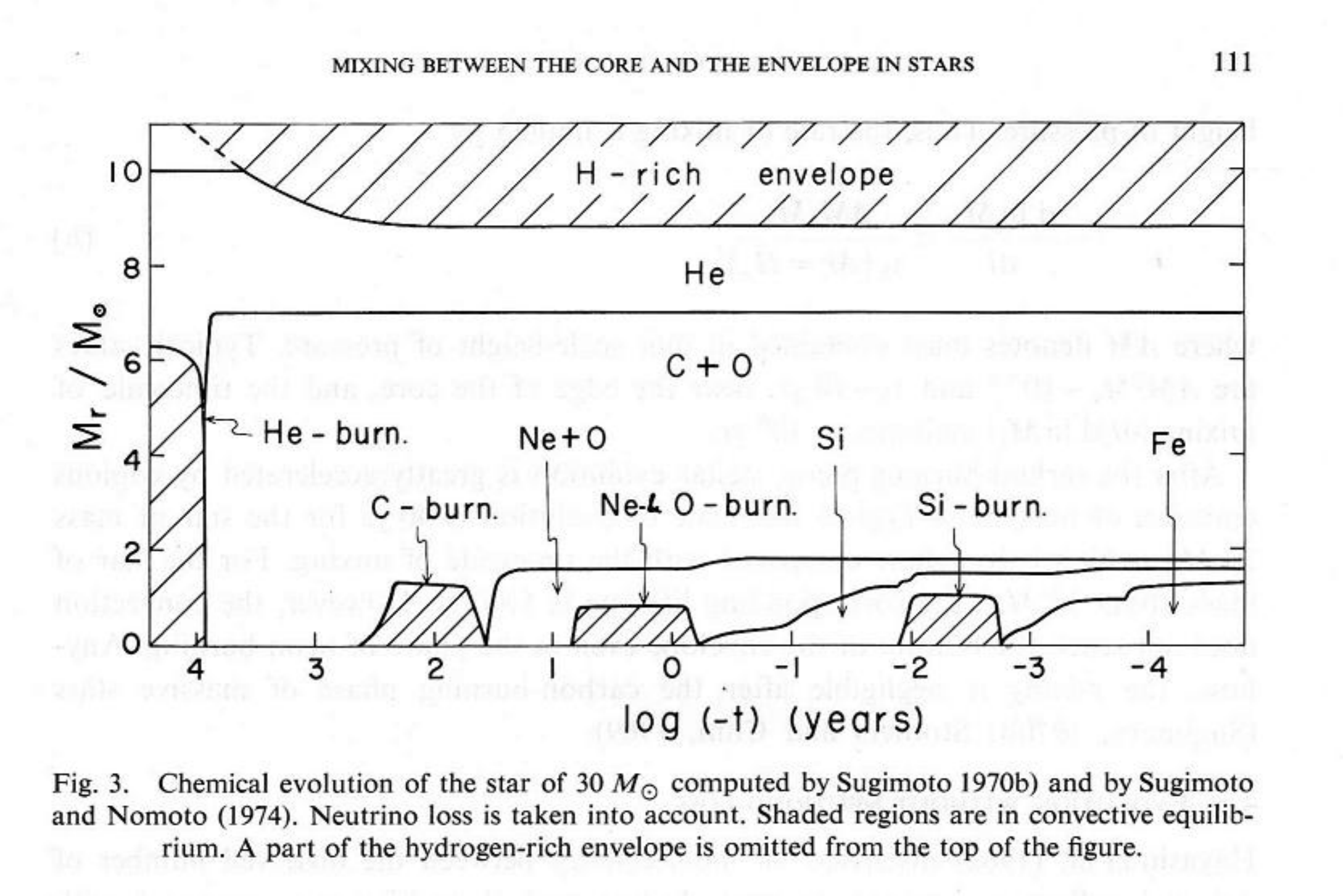


Fig. 3. Chemical evolution of the star of 30 $M_\odot$ computed by Sugimoto 1970b) and by Sugimoto and Nomoto (1974). Neutrino loss is taken into account. Shaded regions are in convective equilibrium. A part of the hydrogen-rich envelope is omitted from the top of the figure.

Sugimoto (1971) & Nomoto (1974)

Conclusion

- Referring to

  global understanding

  for the structure of non-linear system and

  for resulting gravothermal nature,

- it is easy to explain stellar structure and evolution in various phases, systematically

Vote, please: agree / disagree / abstain

Another Vote, please:

Do you think a text book necessary
which describes such a theory in a unified form?

yes / no / uncertain

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