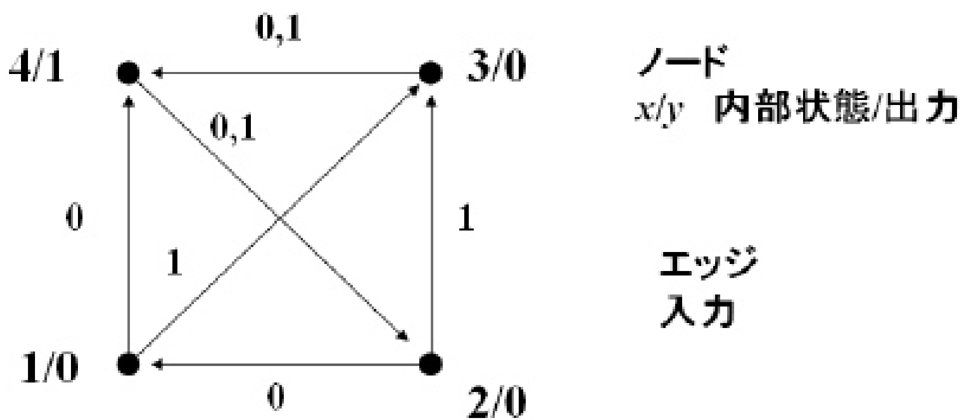
Computational Complementarity

Moore (1956), Finkelstein (1983), Svozil(1993), Calude et al. (1997), Calude et al. (2002)

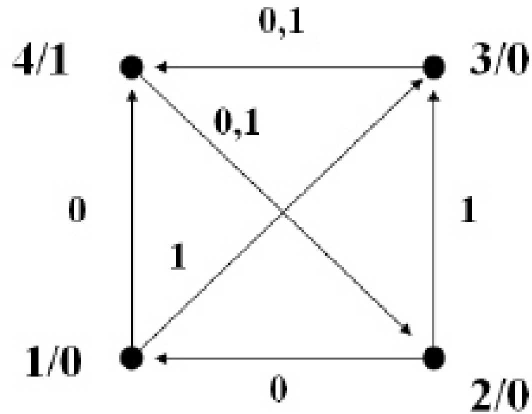
There exists a Moore automaton M such that any pair of its distinct states are distinguishable, but there is no experiment which can determine what state the machine was in at the beginning of the Experiment.



オートマトン　（計算機）

内部状態の集合　Σ
入力記号の集合　P
出力記号の集合　Q

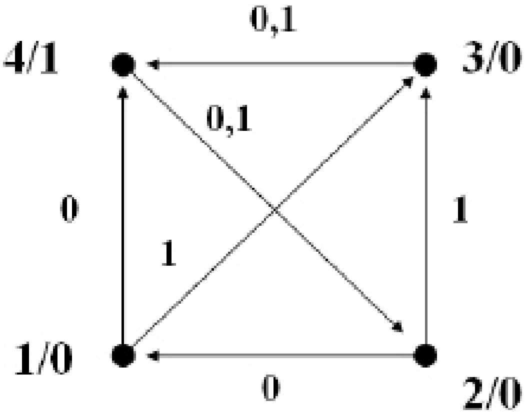
遷移規則　$\sigma: P \times \Sigma \to \Sigma$
出力関数　$f: \Sigma \to Q$

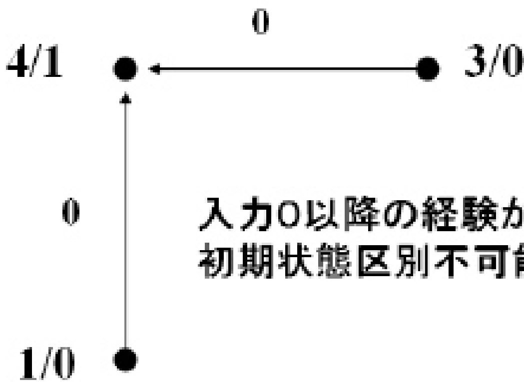


$\sigma 0$	1	2	3	4
	4	1	4	2

$\sigma 1$	1	2	3	4
	3	3	4	2

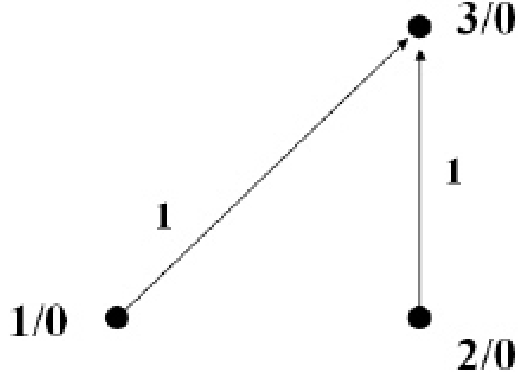
f	1	2	3	4
	0	0	0	1

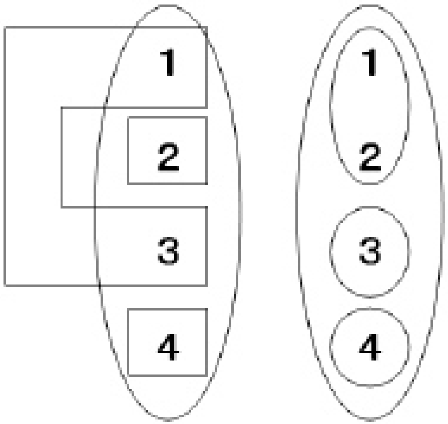


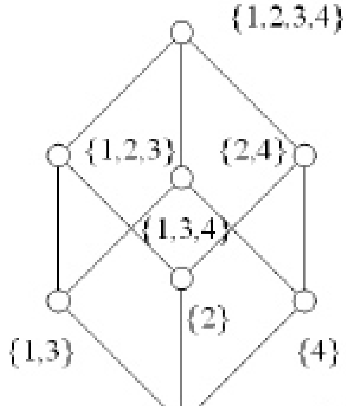


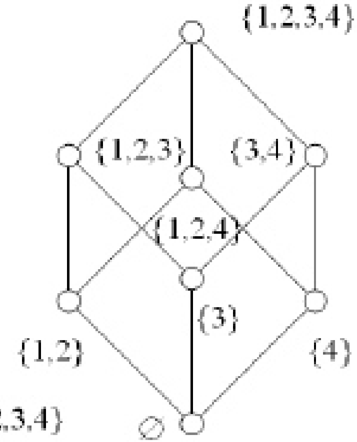
入力0以降の経験から
初期状態区別不可能

入力1以降の経験から
初期状態区別不可能



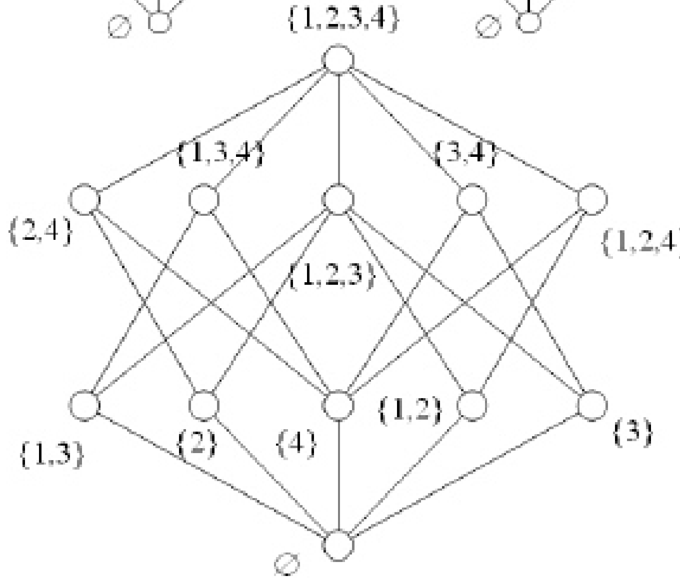




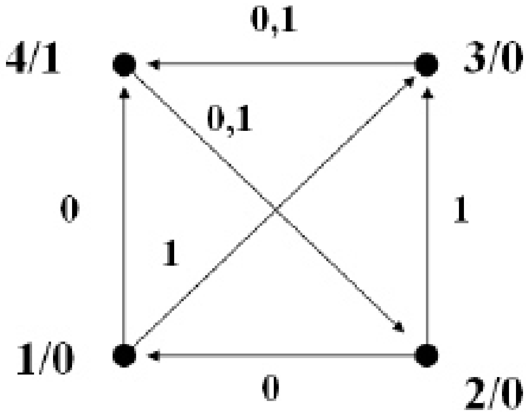


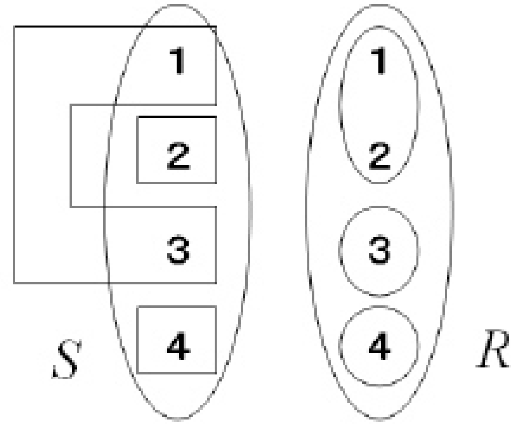
相補性が全体の構造に寄与しない

相補性に普遍性がない



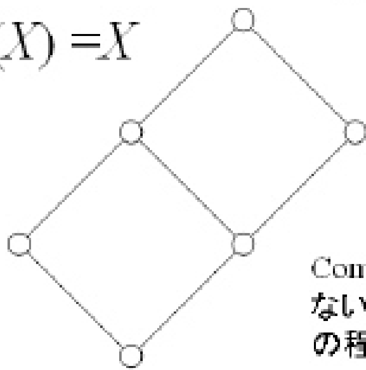
Svozil (1993)



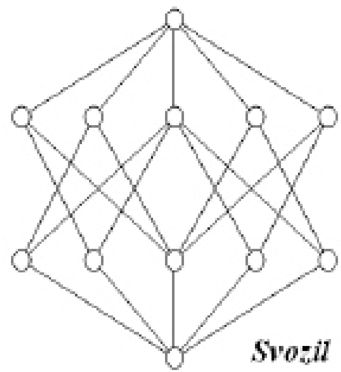


入力0の遷移規則　入力1の遷移規則

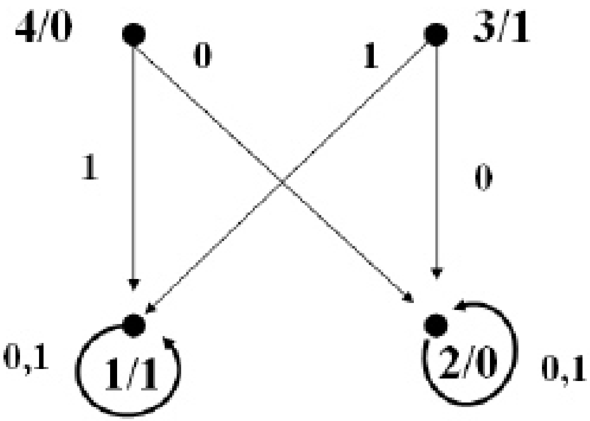
$S_*R^*(X) = X$

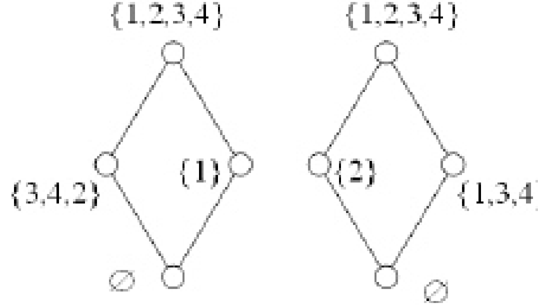


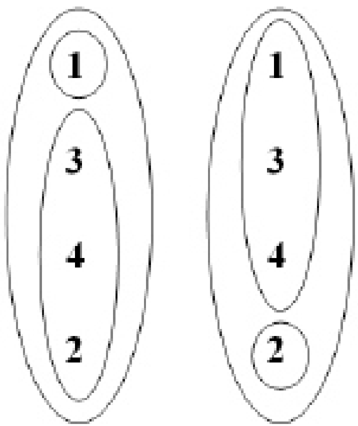
Complement
ない場合あり
の程度？

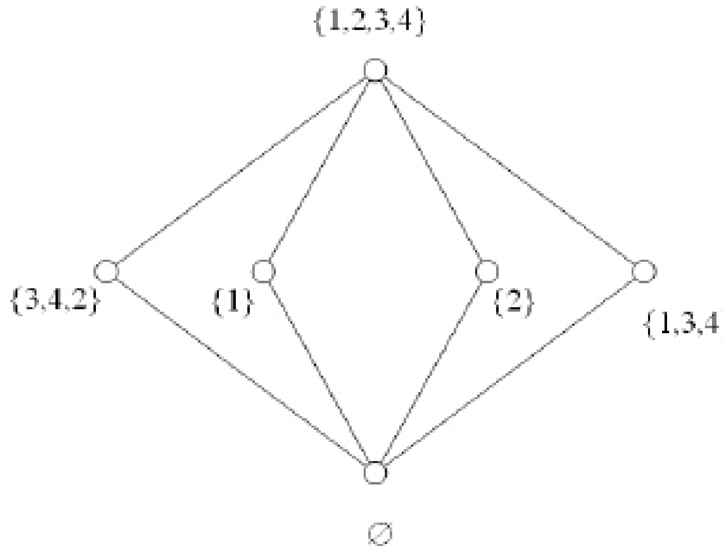


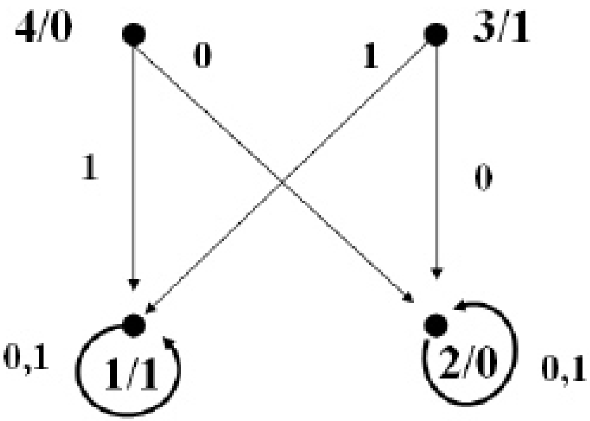
Svozil (1993)

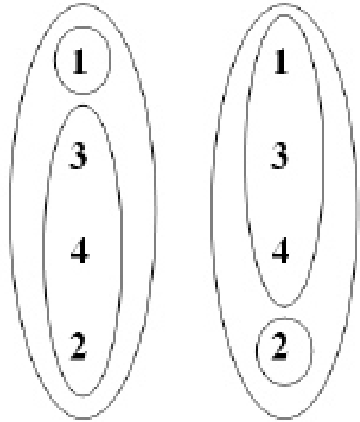

4/0
3/1
0
1
1
0
0,1
1/1
2/0
0,1


{1,2,3,4}
{3,4,2}
{1}
∅
{1,2,3,4}
{2}
{1,3,4}
∅

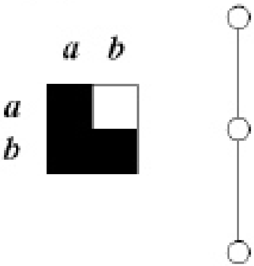

1
3
4
2
1
3
4
2

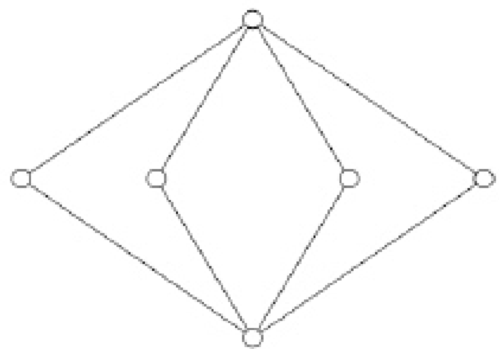

{1,2,3,4}
{3,4,2}
{1}
{2}
{1,3,4}
∅

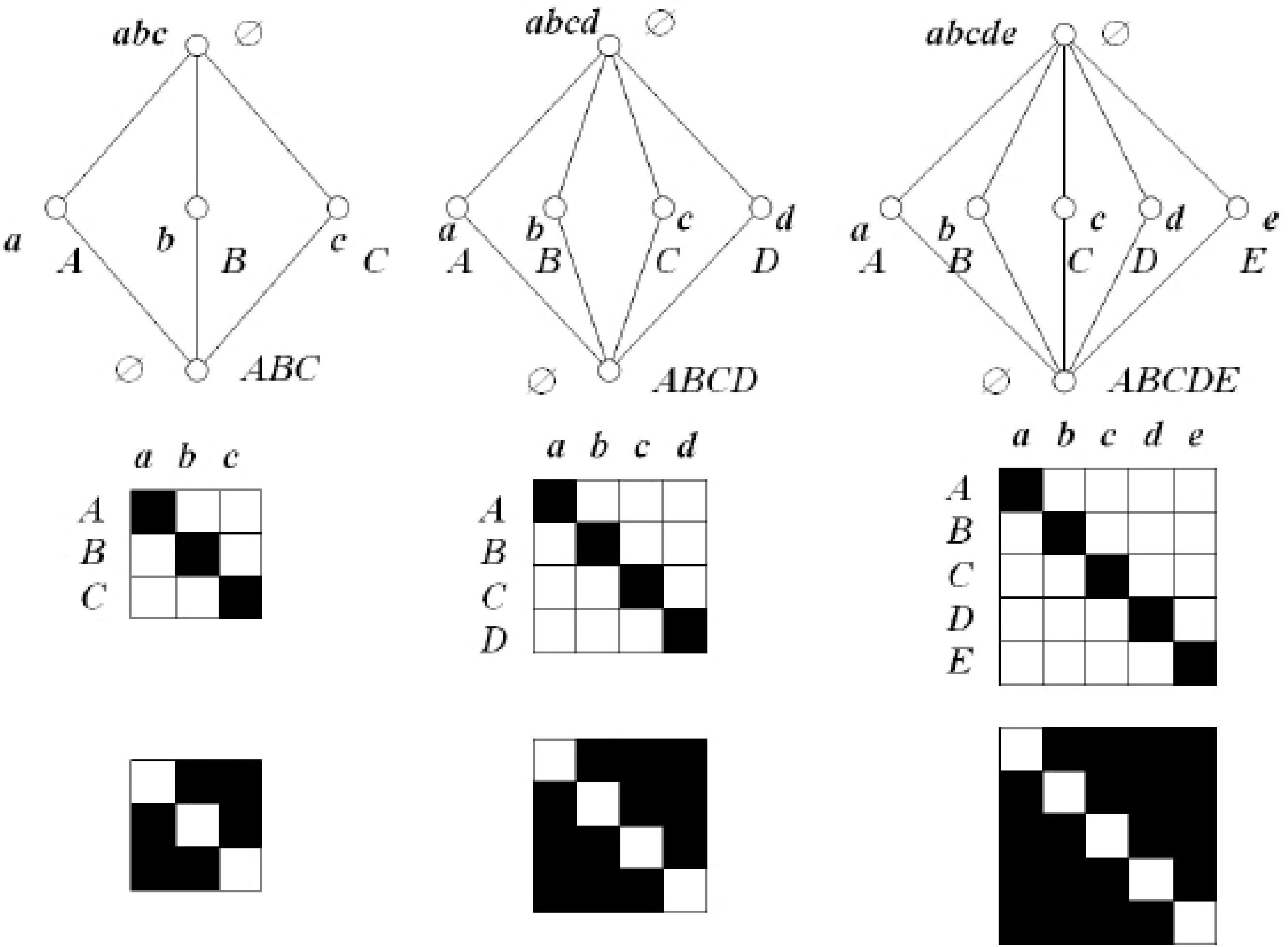




$$S_*R^*(X) = X$$



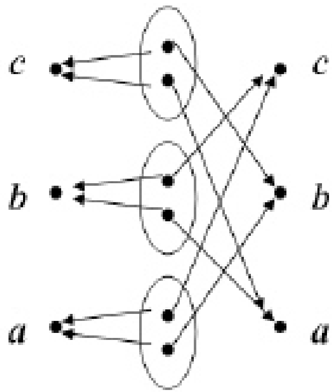


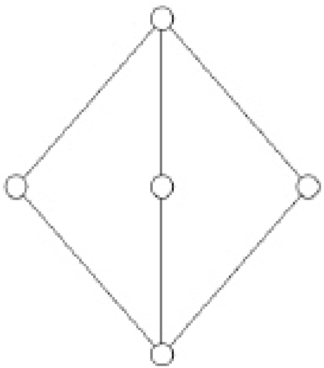

abc
∅
a
A
b
B
c
C
∅
ABC
abcd
∅
a
A
b
B
c
C
d
D
∅
ABCD
abcde
∅
a
A
b
B
c
C
d
D
e
E
∅
ABCDE
a b c
A
B
C
a b c d
A
B
C
D
a b c d e
A
B
C
D
E

$f, g : U \to S$

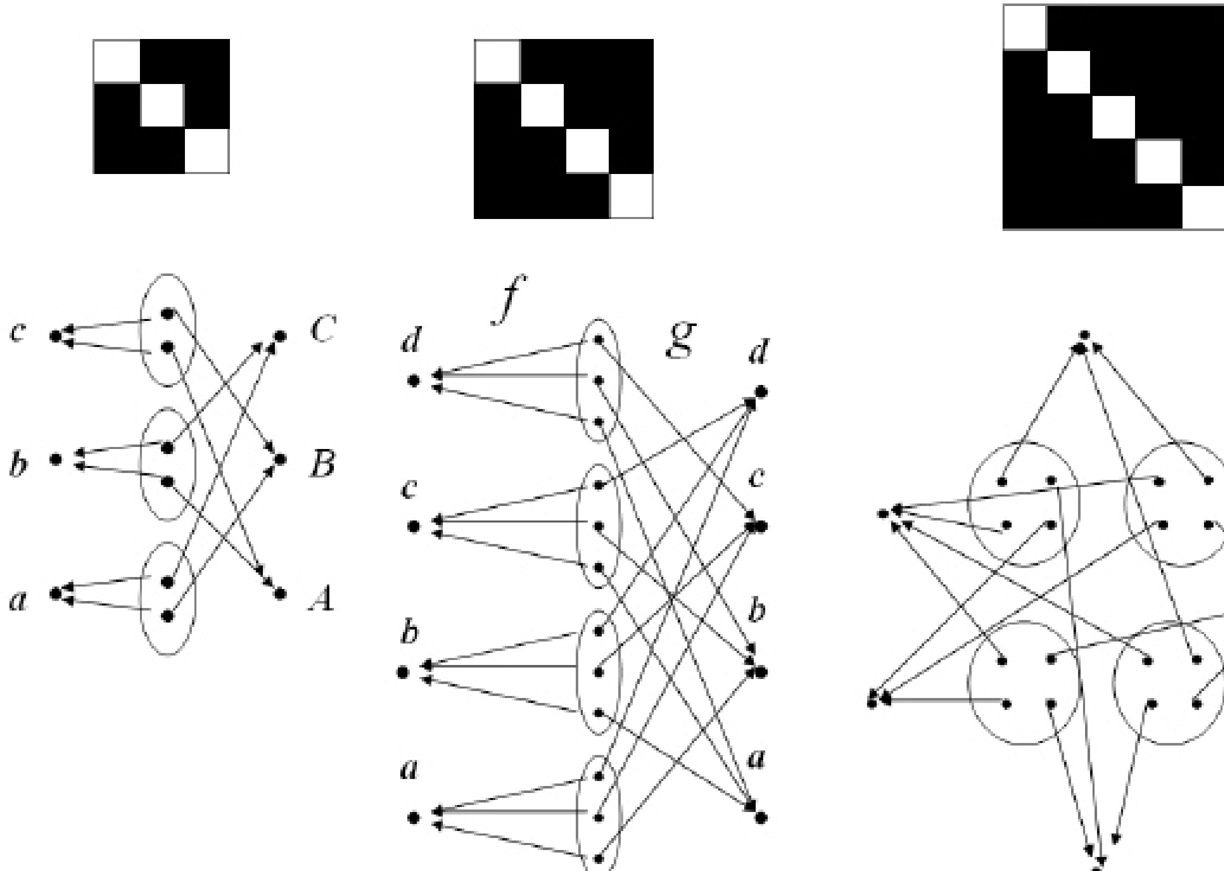
(1) $f(x)=a$なるxにおいて$g(x)=b$ならば
$f(y)=b$なるyにおいて$g(y)=a$

(2) $f(x)=g(x)$なるxが存在しない



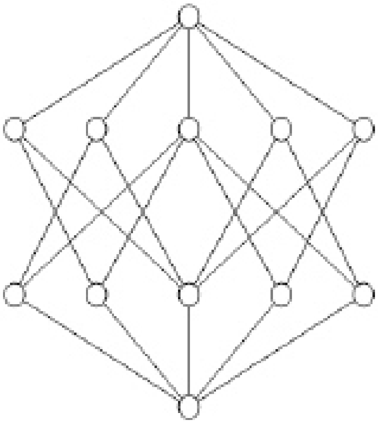


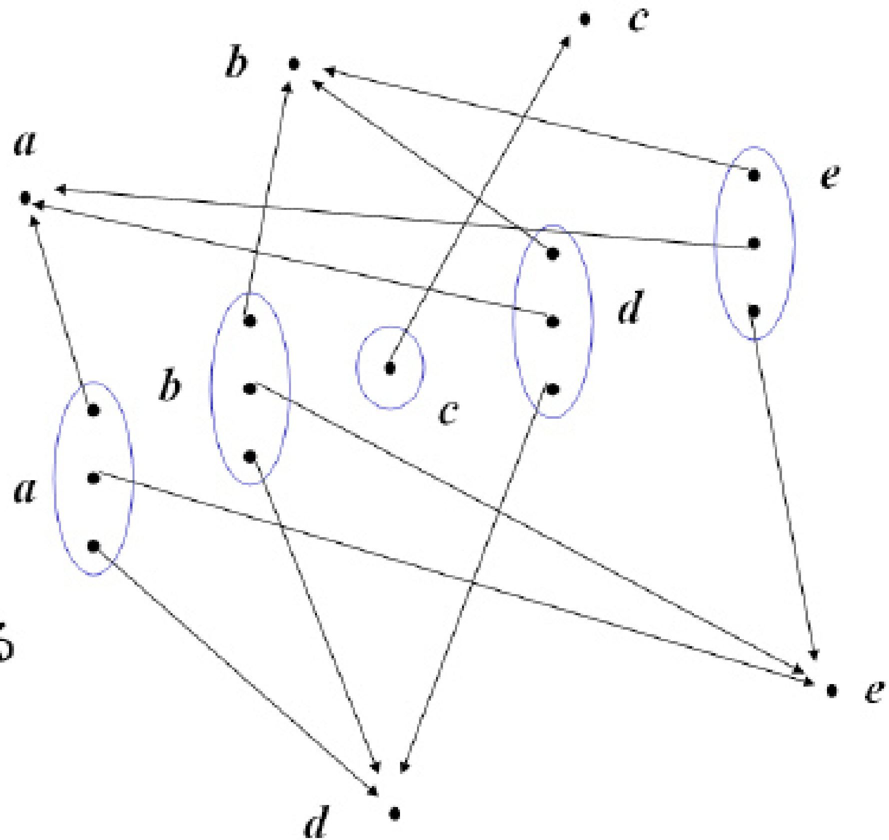
(1)は
満たすが
(2)と
同時に
満たすのは
不可能

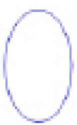


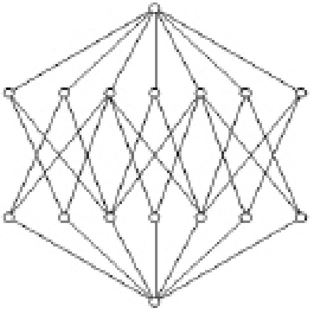
或る入力環境で、全ての内部状態が区別でき、別の入力環境で全て一致するオートマトン

$f(x)=a$なるxにおいて$g(x)=b$ならば
$f(y)=b$なるyにおいて$g(y)=a$



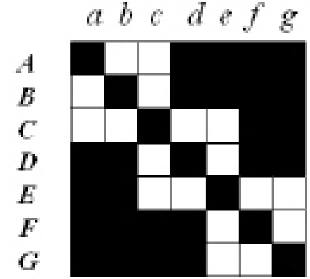


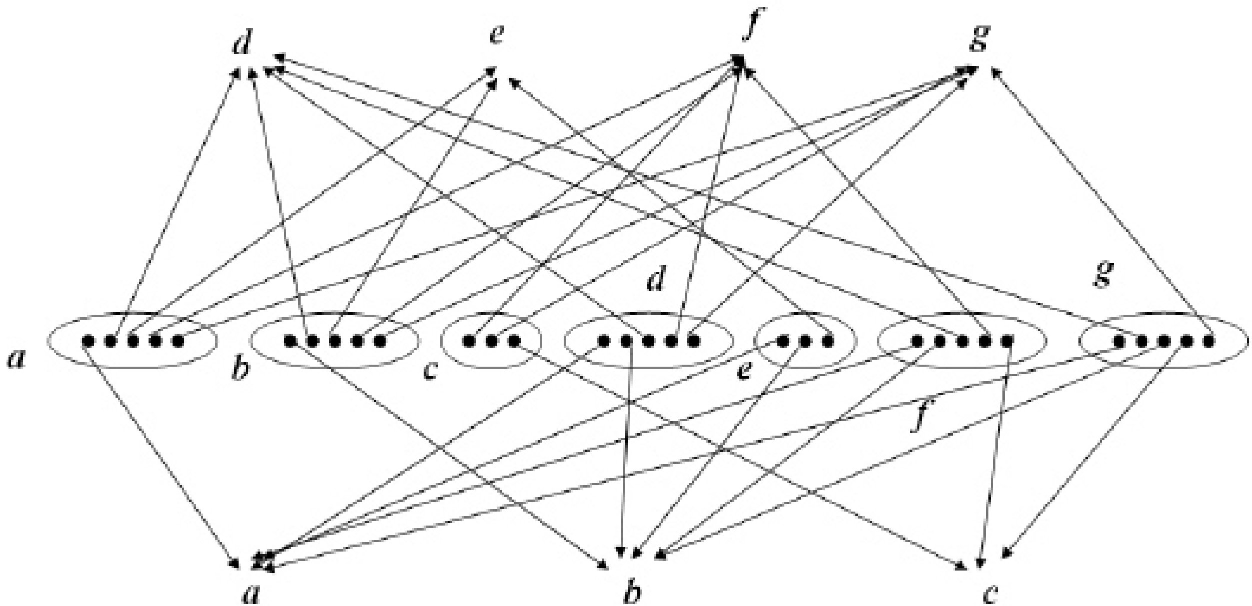

はfで誘導される
同値類



(1) $f(x)=a$なるxにおいて$g(x)=b$ならば
$f(y)=b$なるyにおいて$g(y)=a$

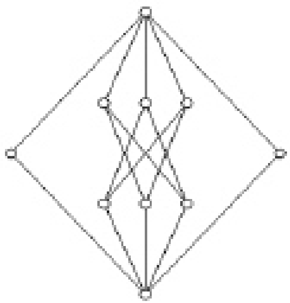
(2) $f(x)=g(x)$なるxが存在する

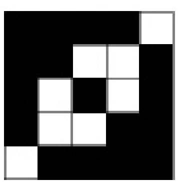


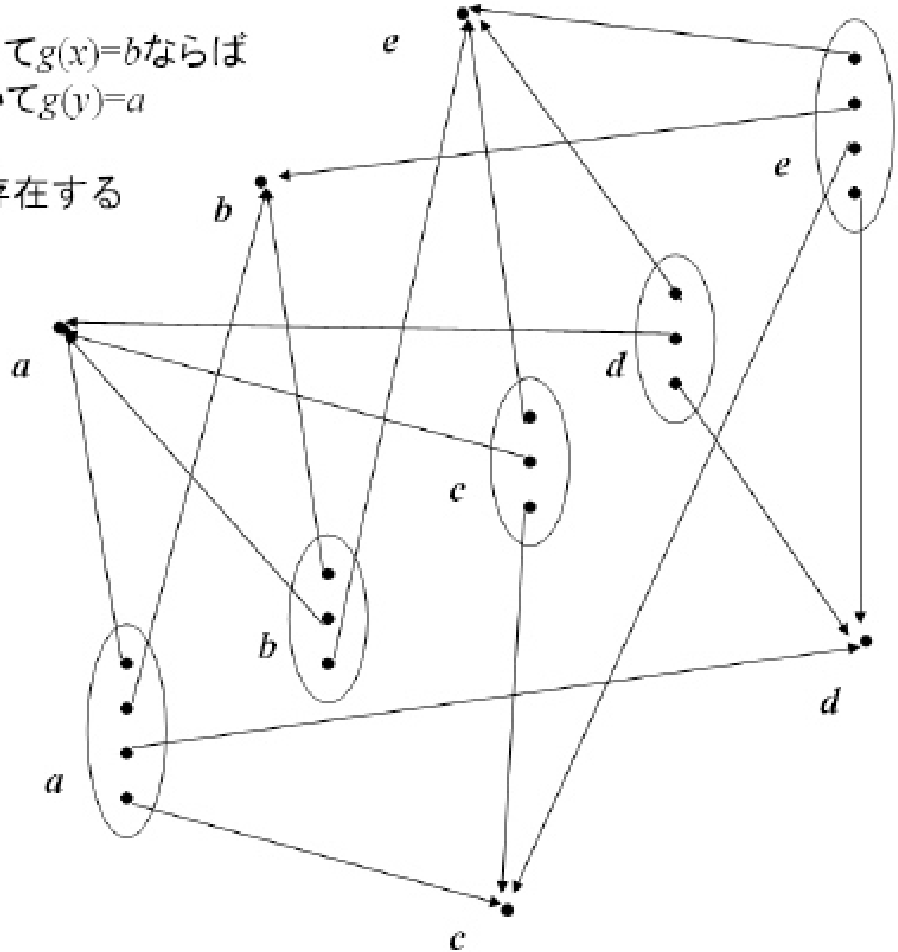


(1) $f(x)=a$なるxにおいて$g(x)=b$ならば
$f(y)=b$なるyにおいて$g(y)=a$

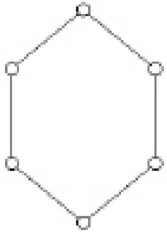
(2) $f(x)=g(x)$なるxが存在する

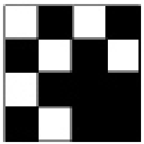


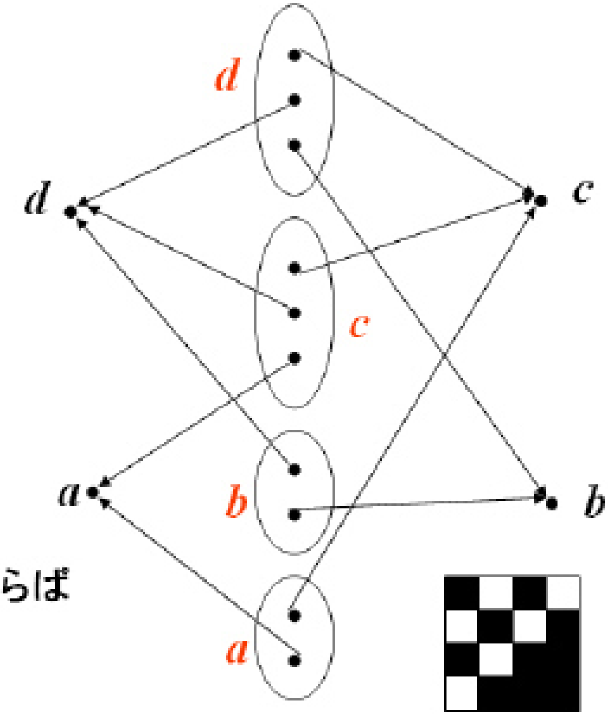




赤字:f の同値類
黒字:gの同値類

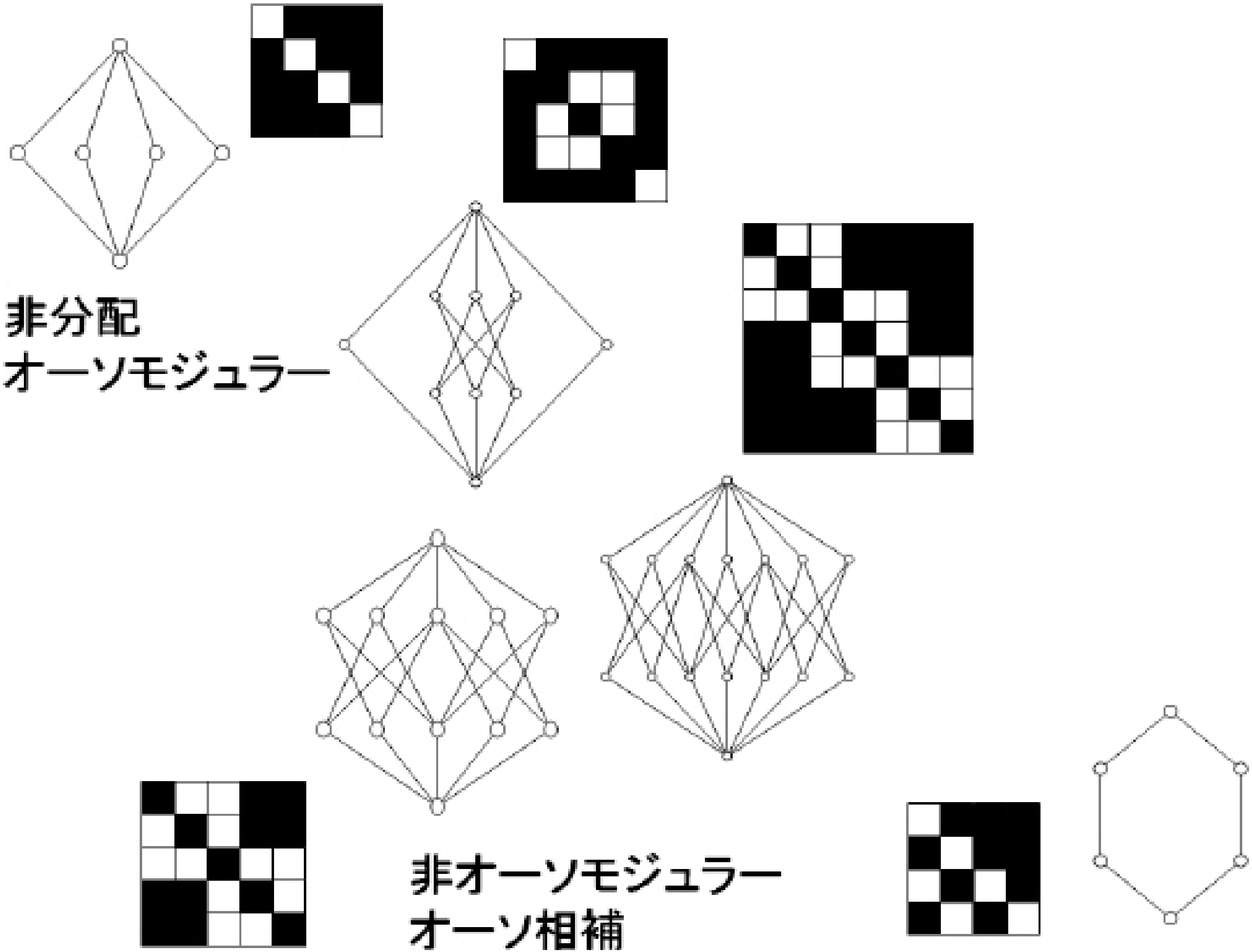






(1) $f(x)=a$なるxにおいて$g(x)=b$ならば
$f(y)=b$なるyにおいて$g(y)=a$

(2) $f(x)=g(x)$なるxが存在する


非分配
オーソモジュラー
非オーソモジュラー
オーソ相補

Elementary Cellular Automata

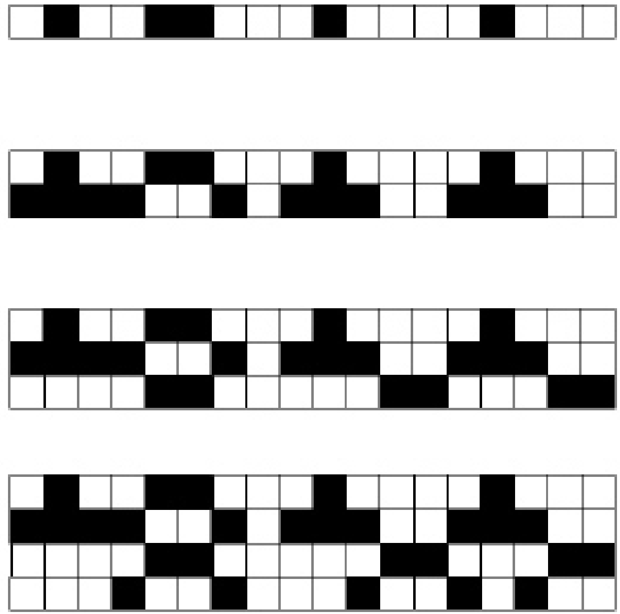
000	**001**	**010**	**011**
0	**1**	**1**	**0**

100	**101**	**110**	**111**
1	**0**	**0**	**0**

RN = 2+4+16=22

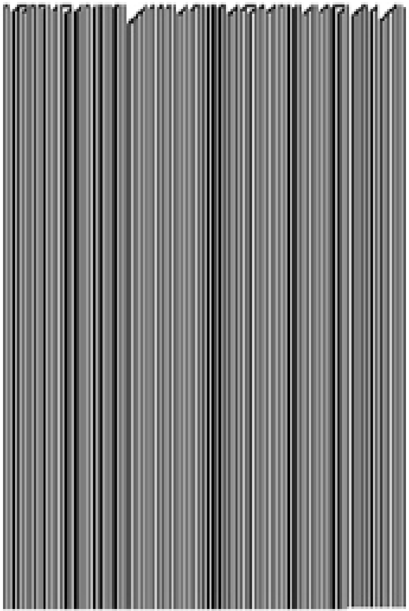
d_0 d_1 d_2 d_3

d_4 d_5 d_6 d_7

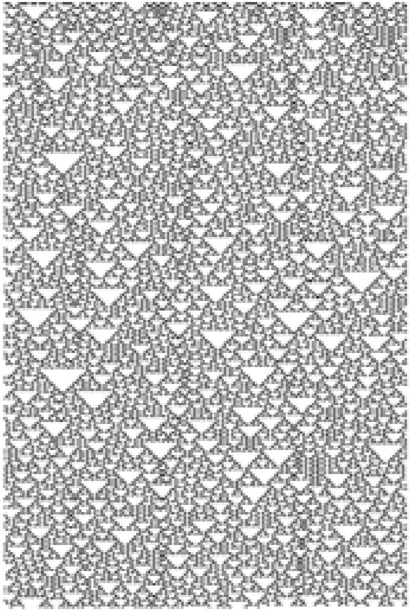


Rule Number = $\Sigma\, \boldsymbol{d_k 2^k}$

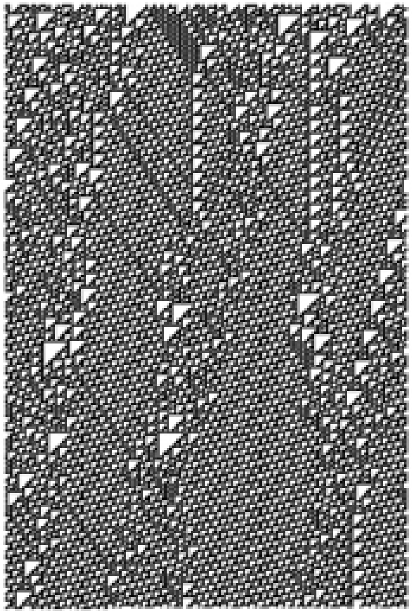
クラス2



クラス3



クラス4



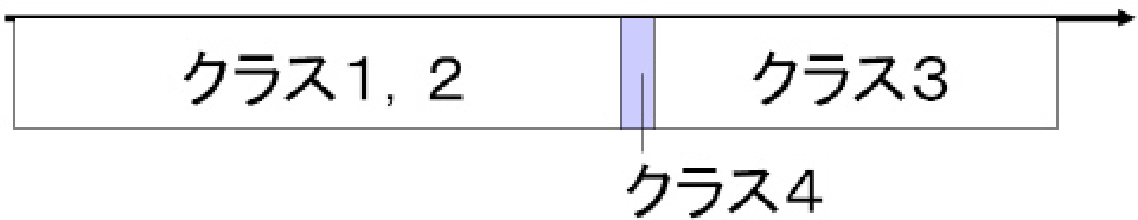
セル・オートマトンによる世界観

- システムの挙動は基本的に局所的規則で尽くせる
- 挙動には様々なものがあり、全体が均質なクラス１、局所安定なクラス２、カオスに対応するクラス３、２と３の中間４で尽くされる
- 生命のような複雑な挙動はクラス４だろう

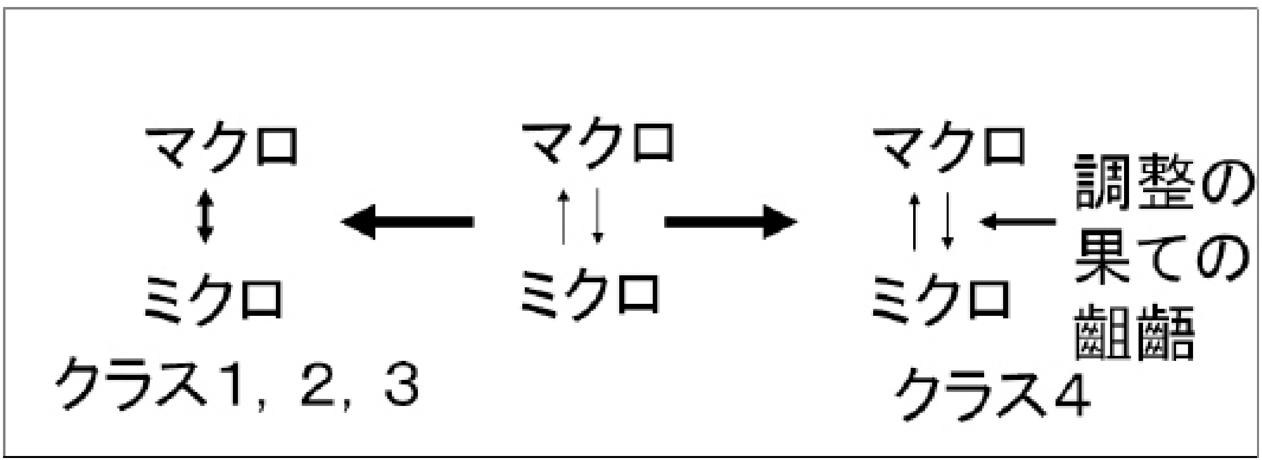
- 挙動の違いはルールの違いである。ルールをうまく並べた空間を想定すれば、２と３の臨界点として４を、生命を理解できる

（階層性も何もない！）

ルール空間



以下のようなモデルは可能か



マクロな制約（相補性による弱い分配性）

↓

齟齬がつくる関係（文脈）の多重性

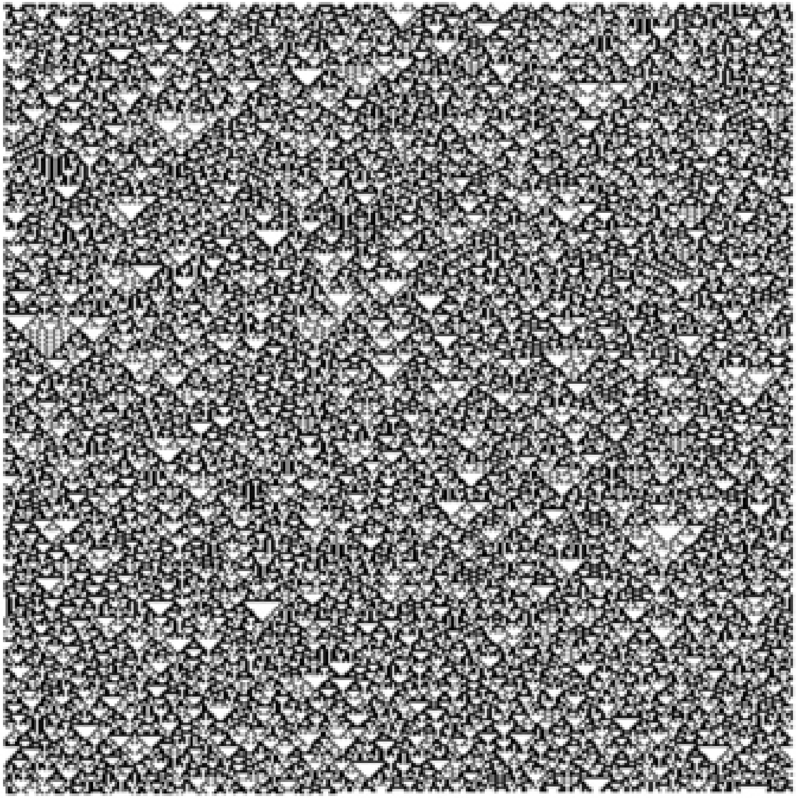
↑

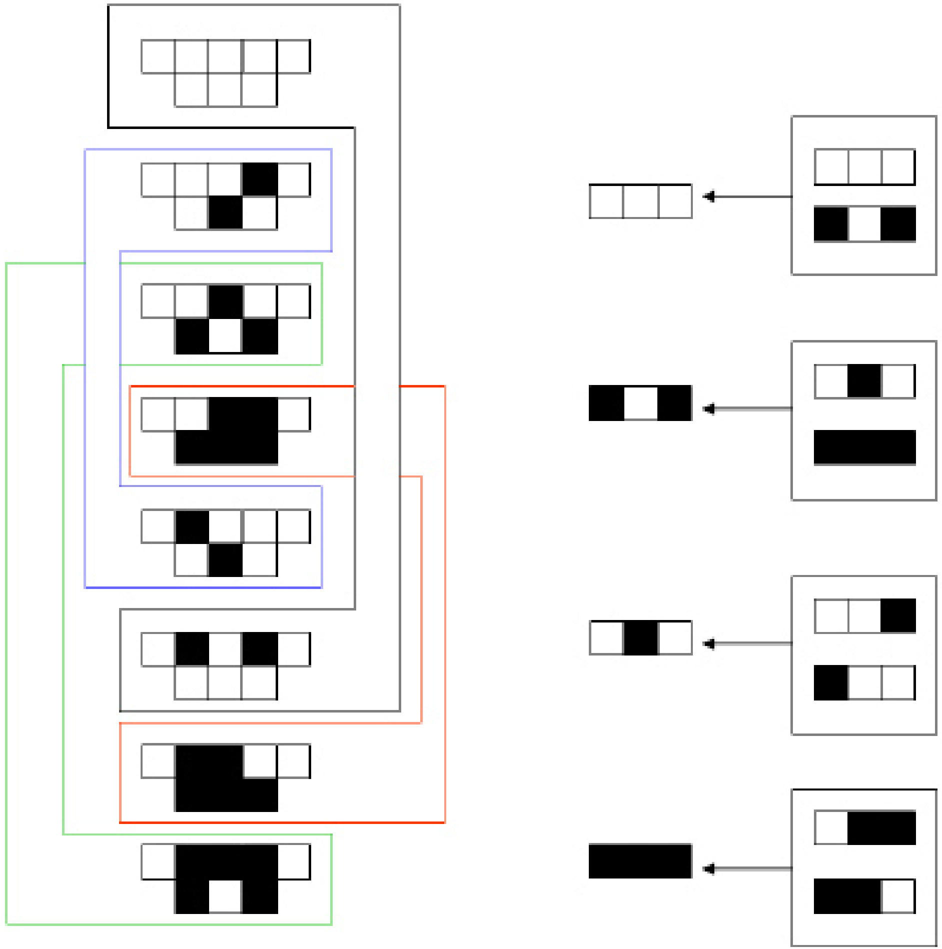
ミクロな規則（非分配性）

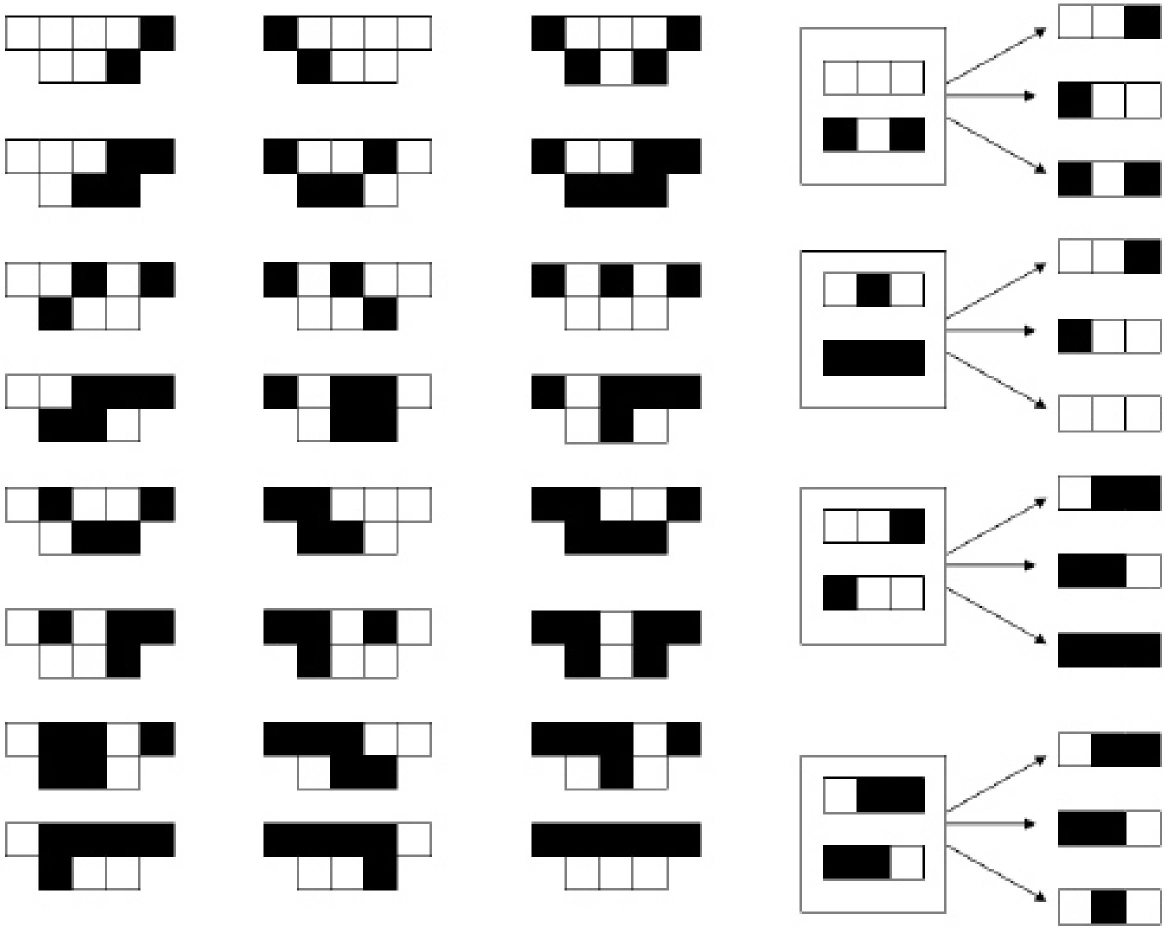
R90

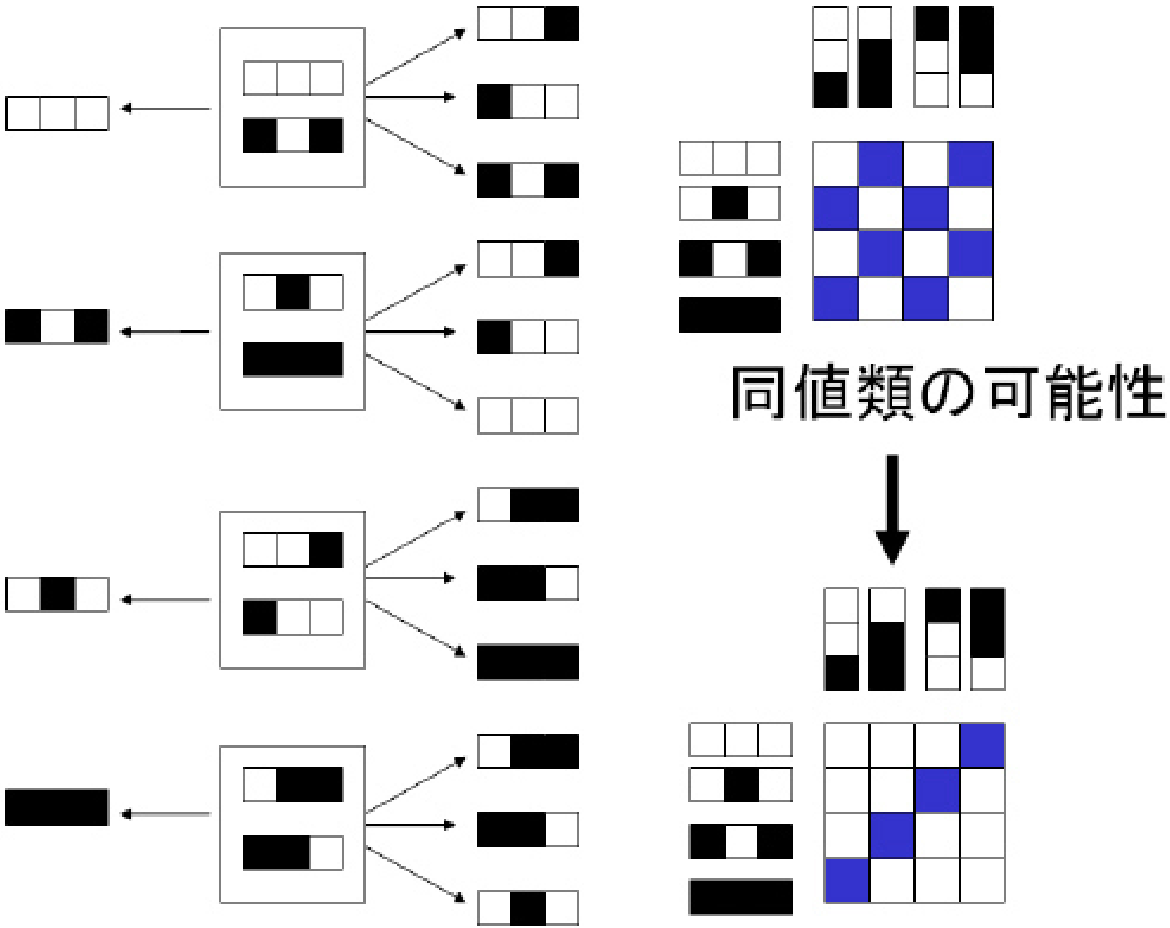
000	001	010	011
0	1	0	1

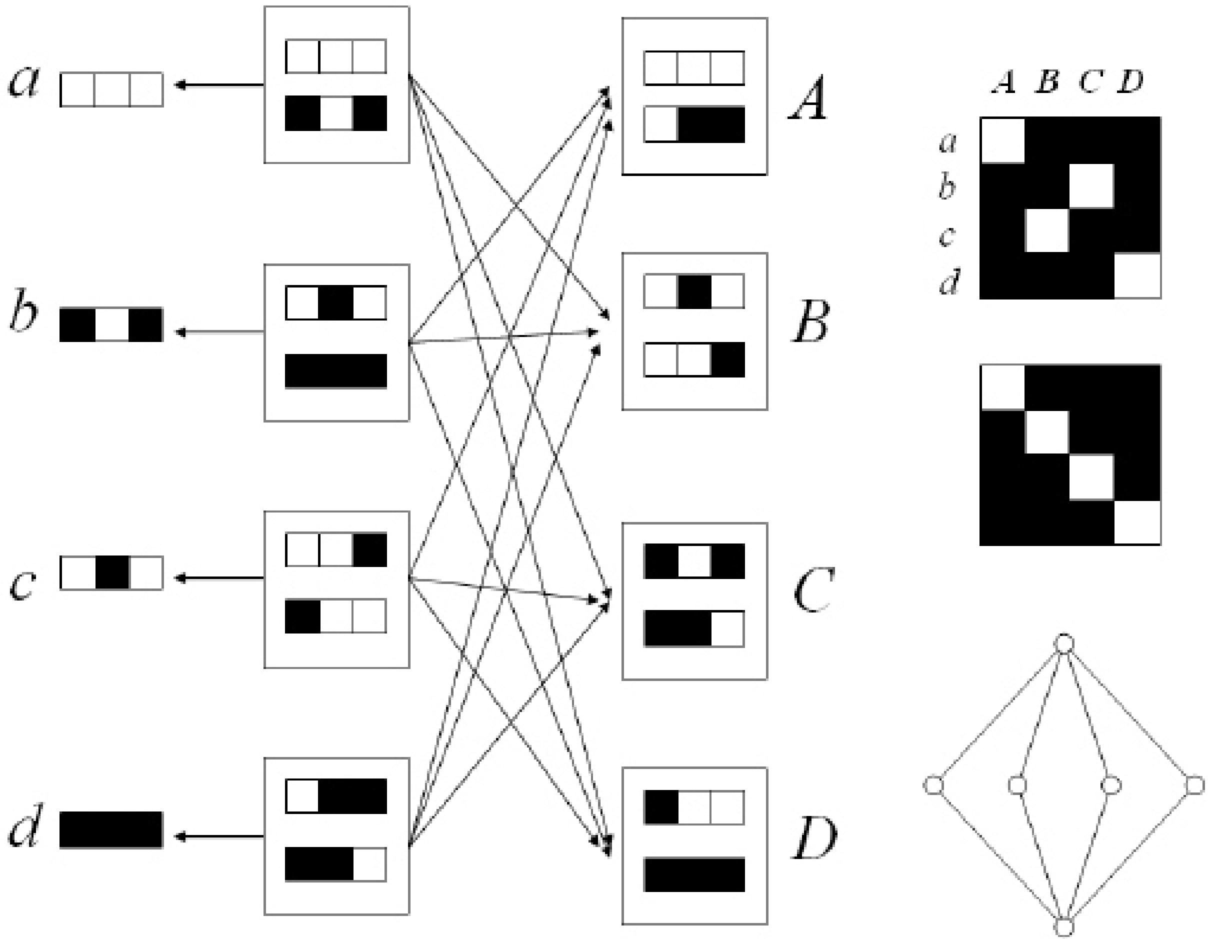
100	101	110	111
1	0	1	0





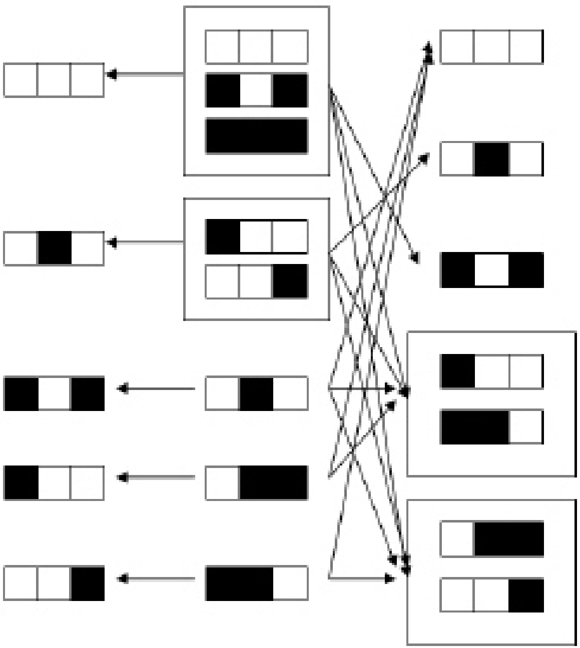


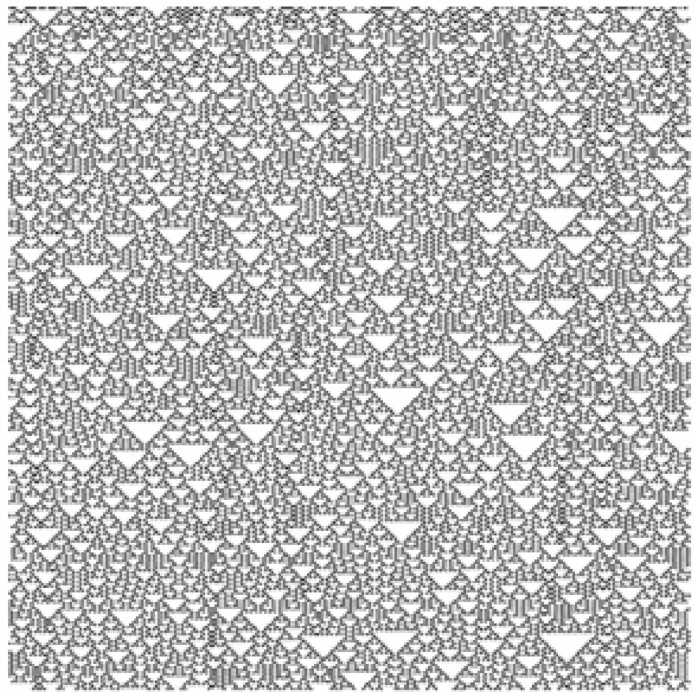

同値類の可能性


a
b
c
d
A
B
C
D
A B C D
a
b
c
d

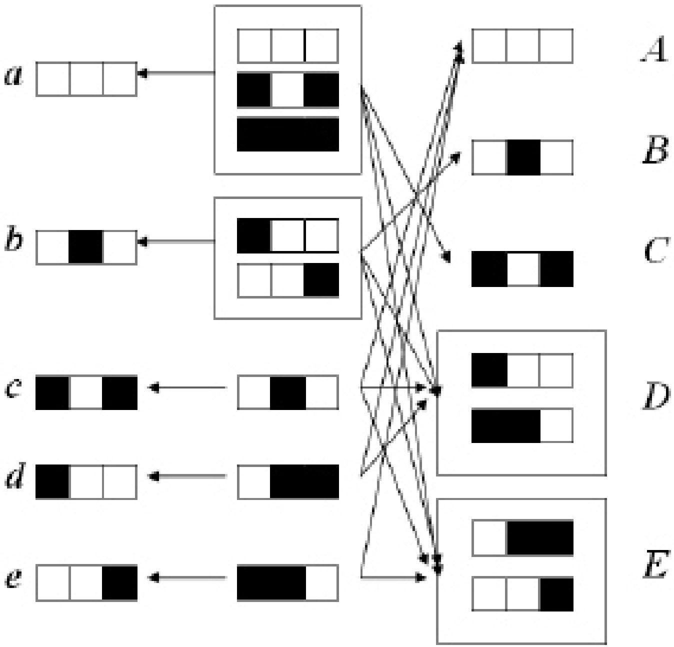
R18

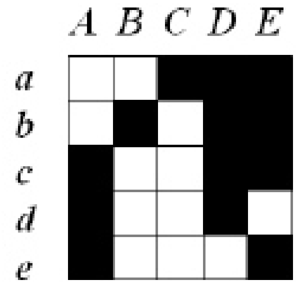
000	**001**	**010**	**011**
0	**1**	**0**	**0**
100	**101**	**110**	**111**
1	**0**	**0**	**0**

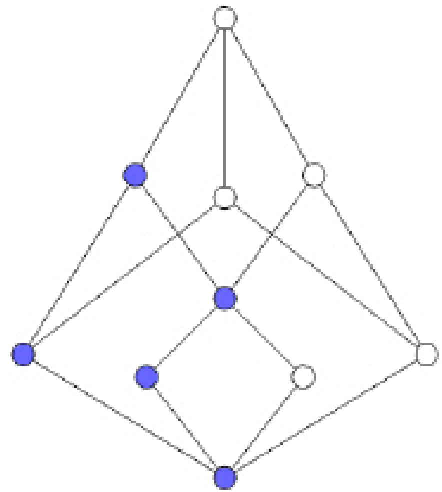




R18



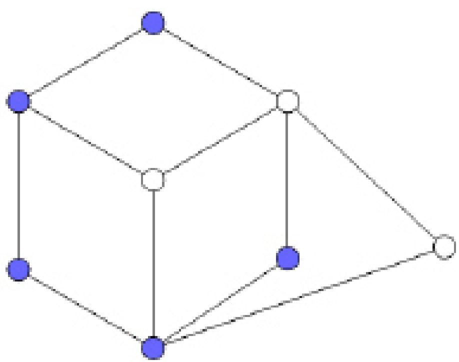




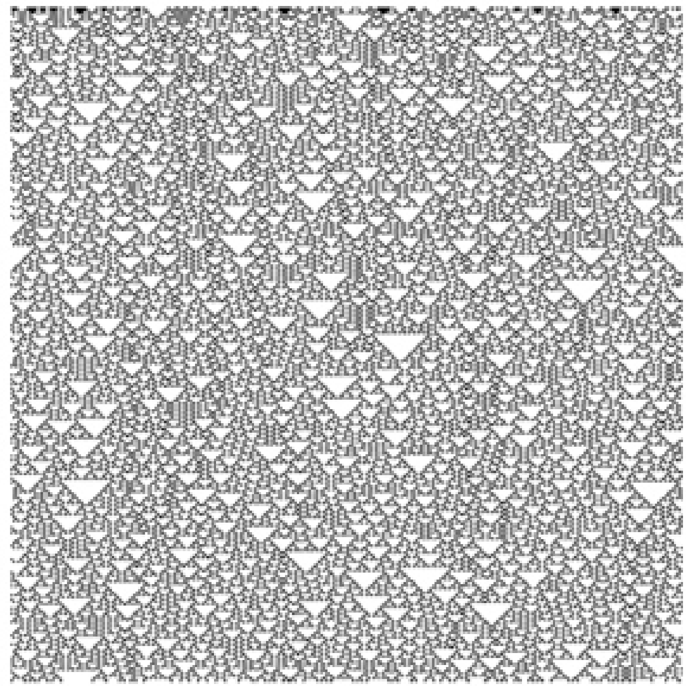
非分配束

R146

000	001	010	011
0	1	0	0
100	101	110	111
1	0	0	1



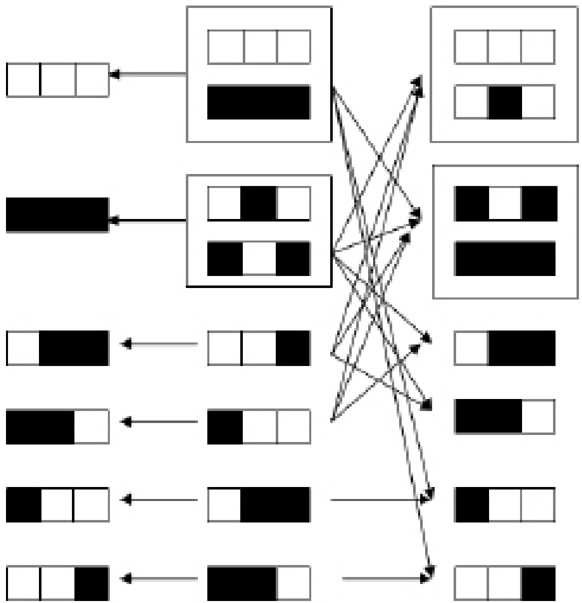
非分配束

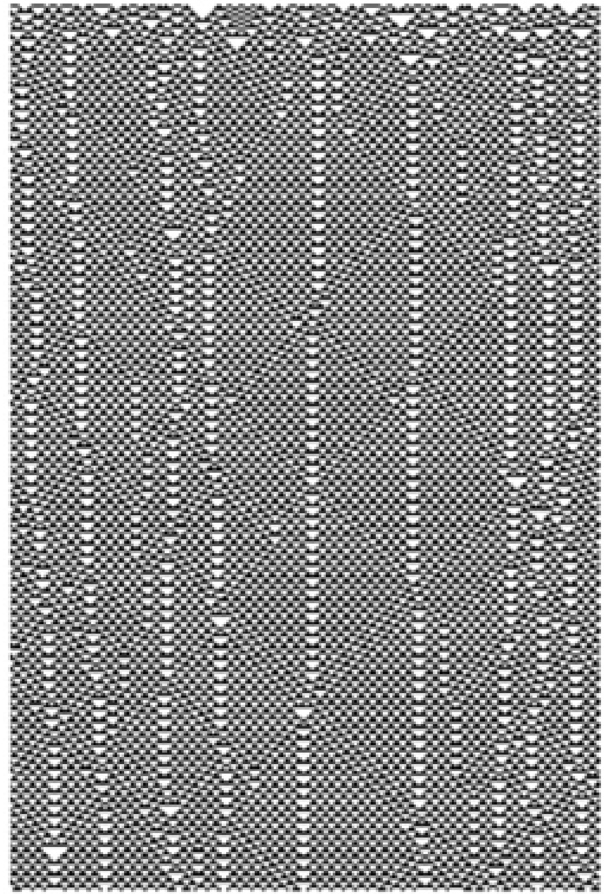


R54

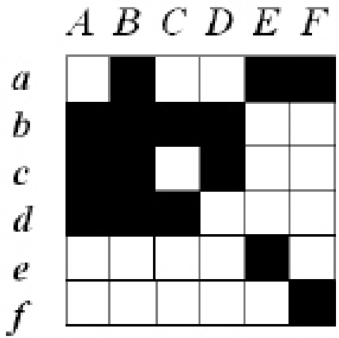
000	001	010	011
0	1	0	0

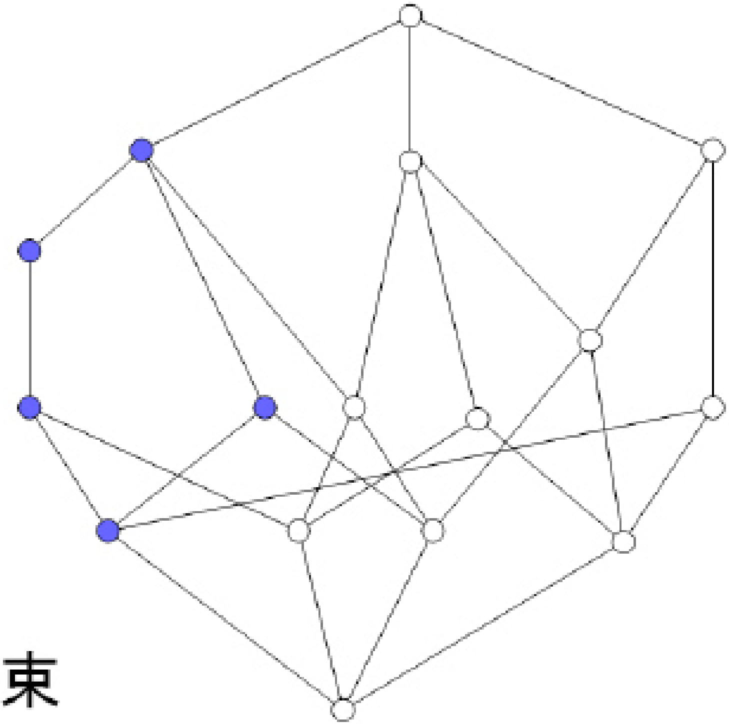
100	101	110	111
1	0	0	1





R54



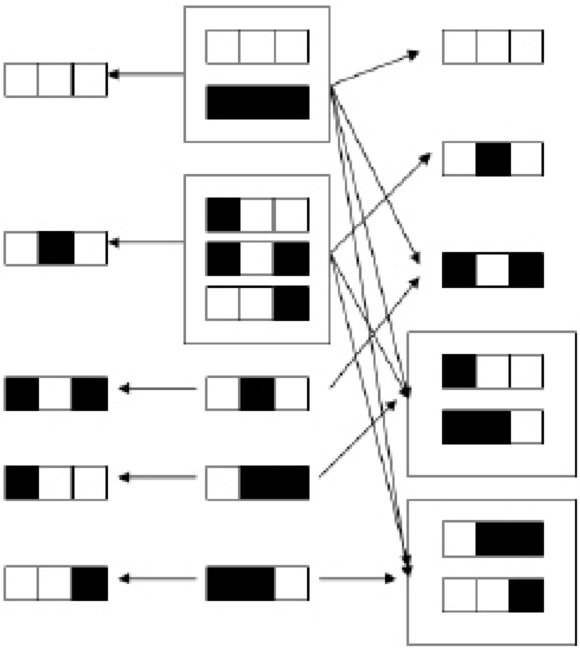


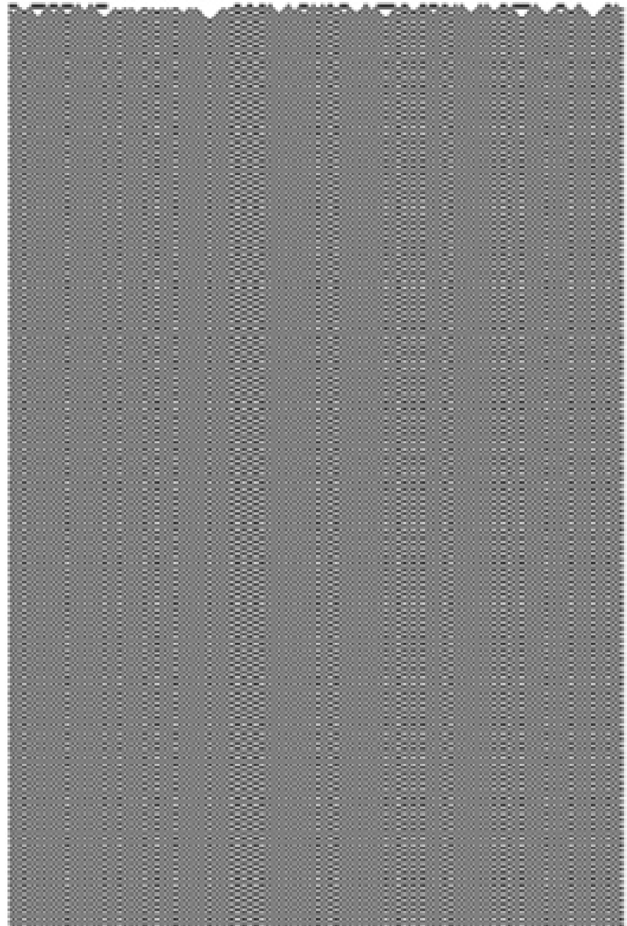
非分配束

R50

000	001	010	011
0	1	0	0

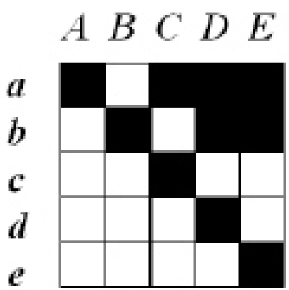
100	101	110	111
1	1	0	0

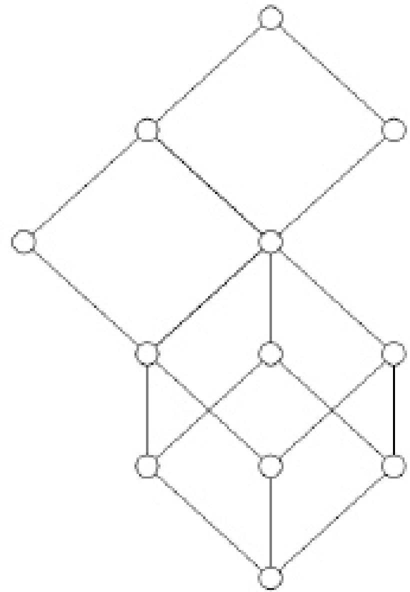




R50

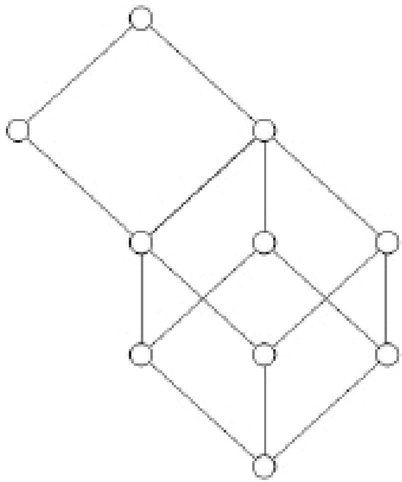
000	**001**	**010**	**011**
0	**1**	**0**	**0**
100	**101**	**110**	**111**
1	**1**	**0**	**0**





分配束

R32



分配束

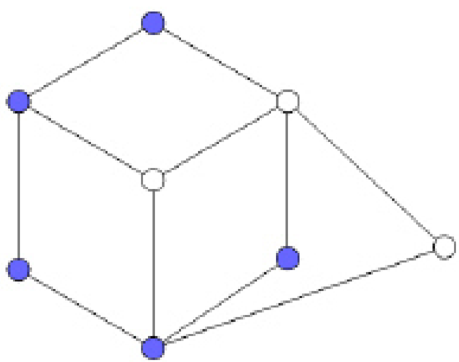
クラス１

クラス１，２　　分配束

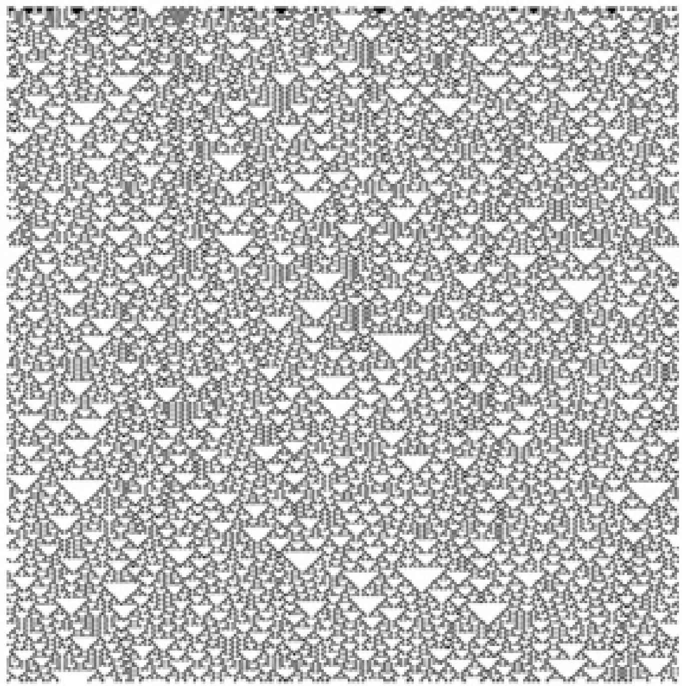
クラス３，４　非分配束

R146

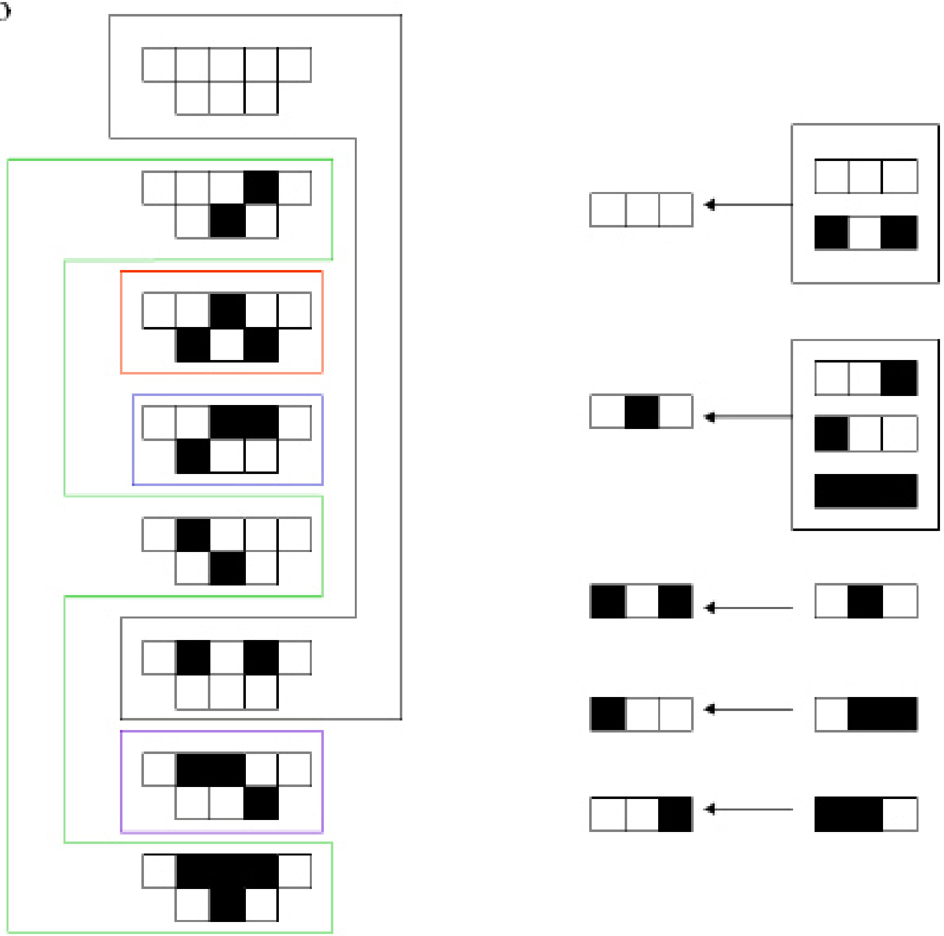
000	001	010	011
0	1	0	0
100	**101**	**110**	**111**
1	0	0	1



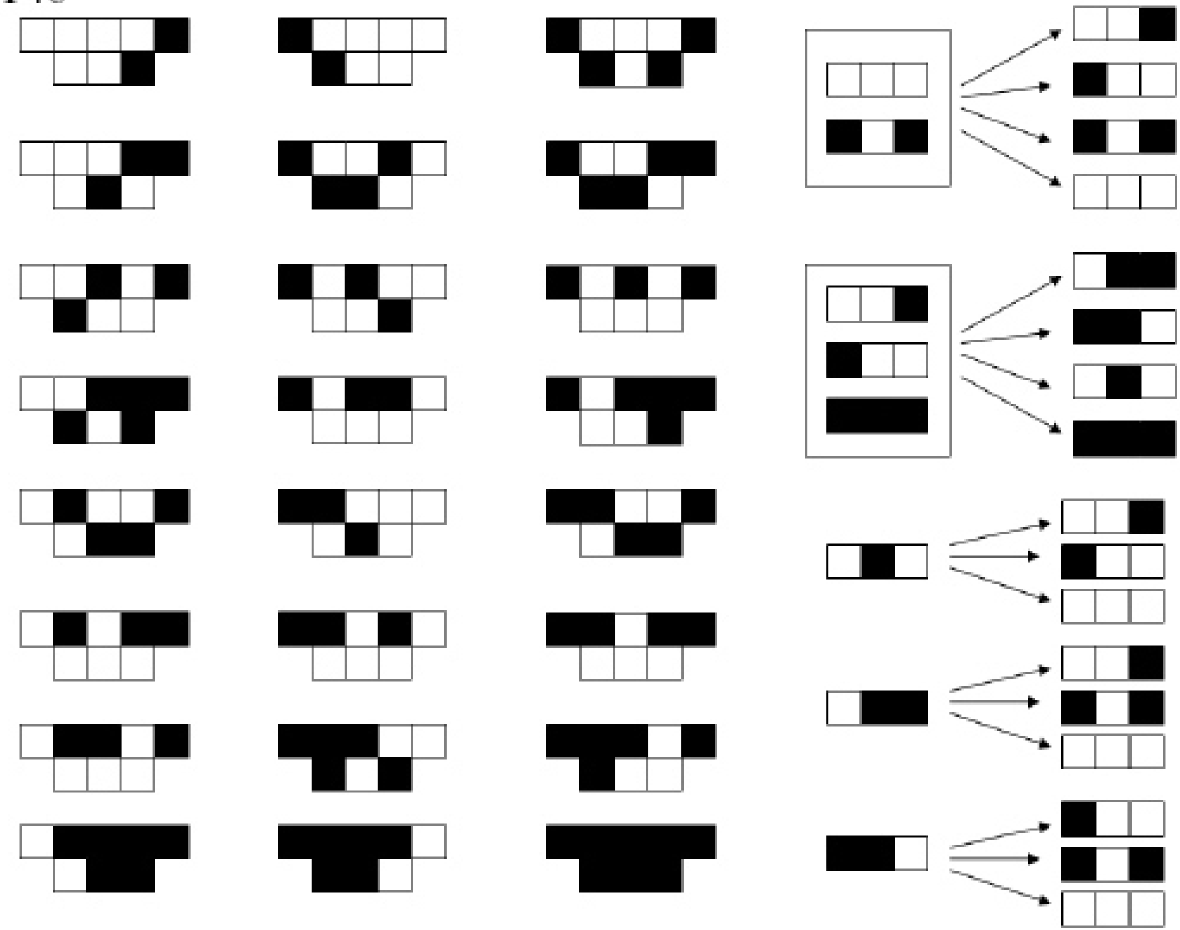
非分配束



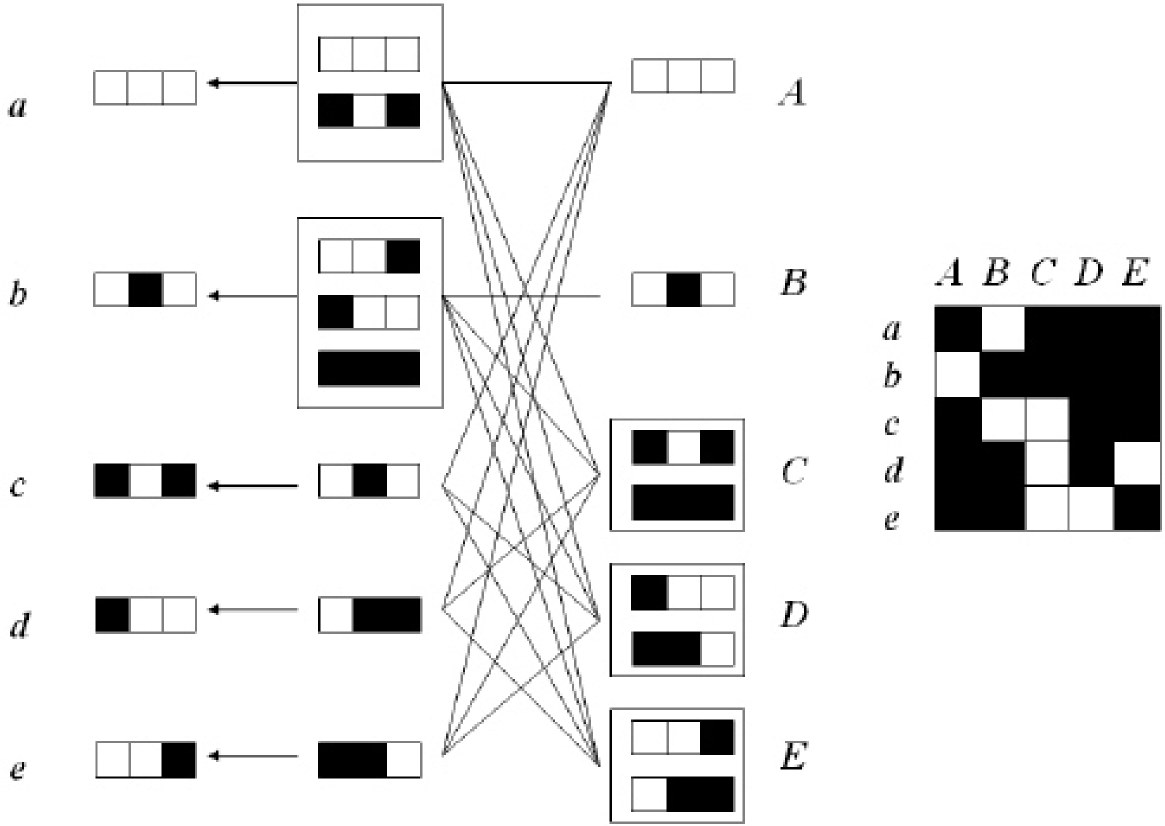
R146



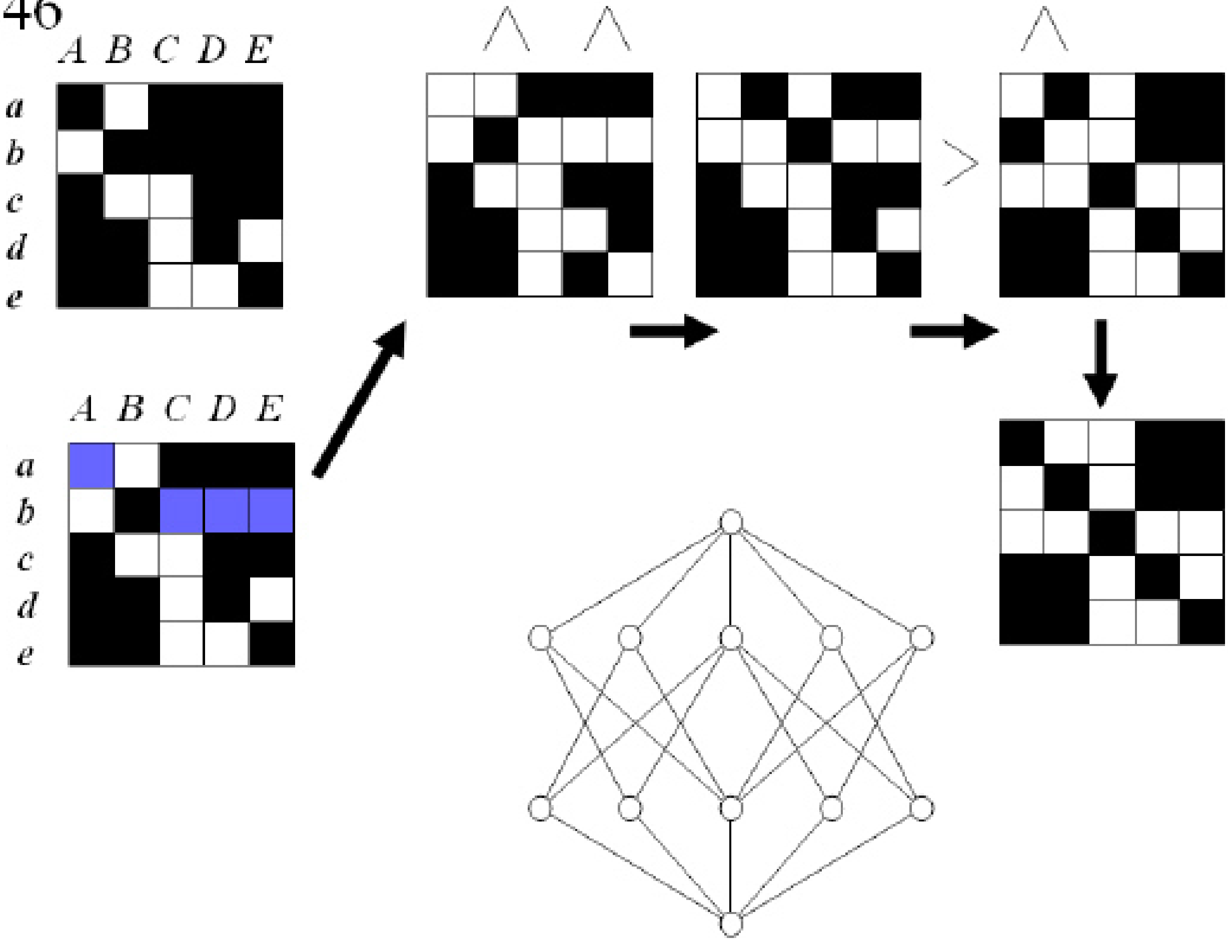
R146

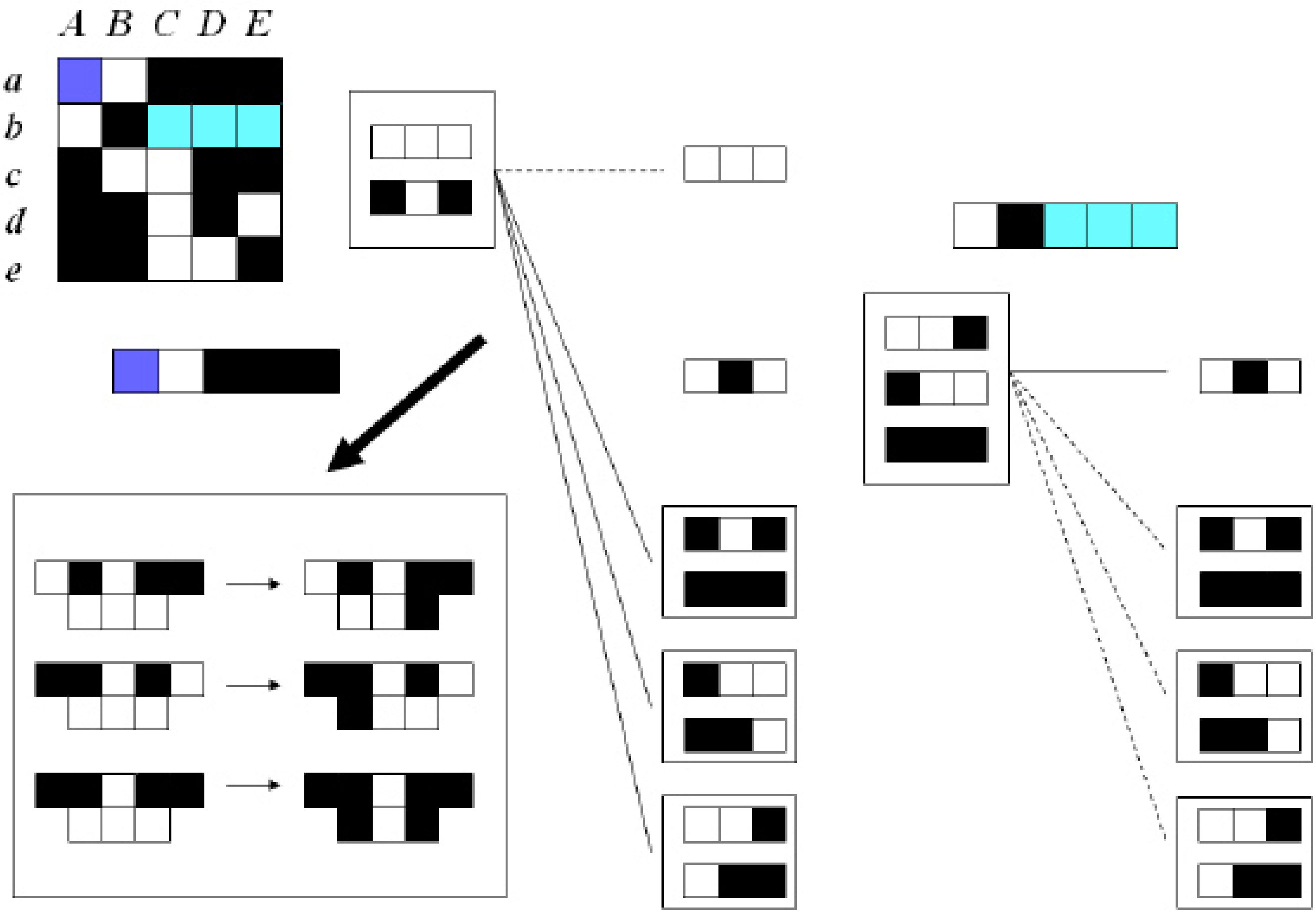


R146

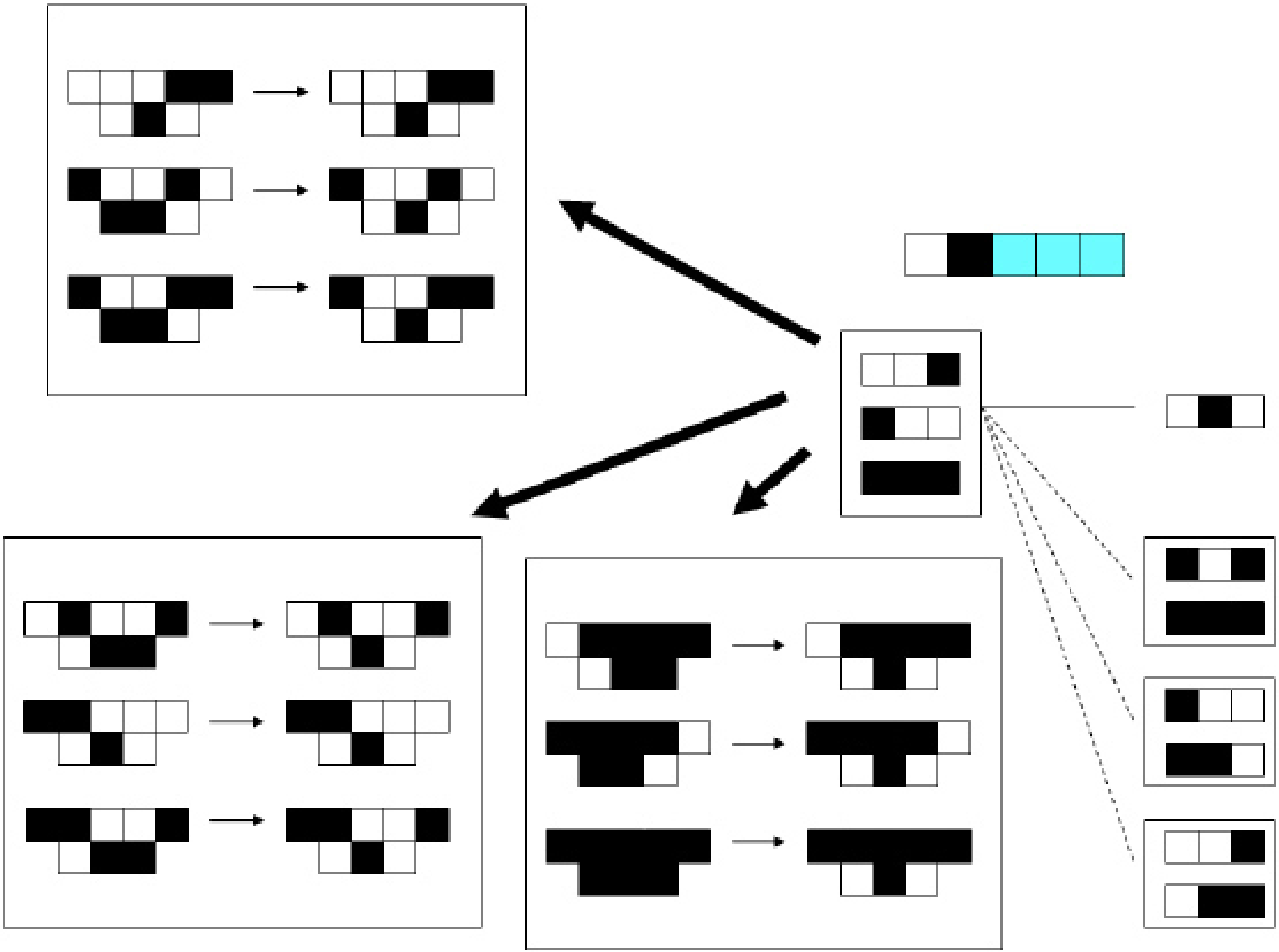


R146

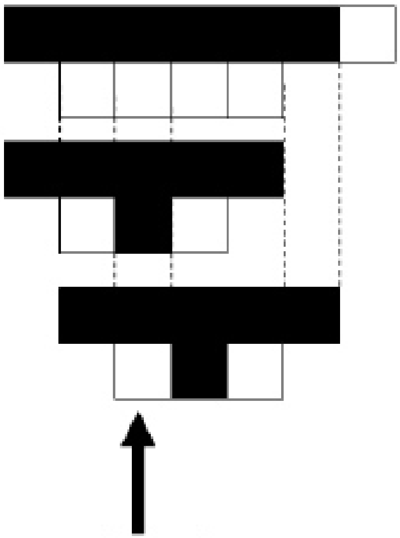




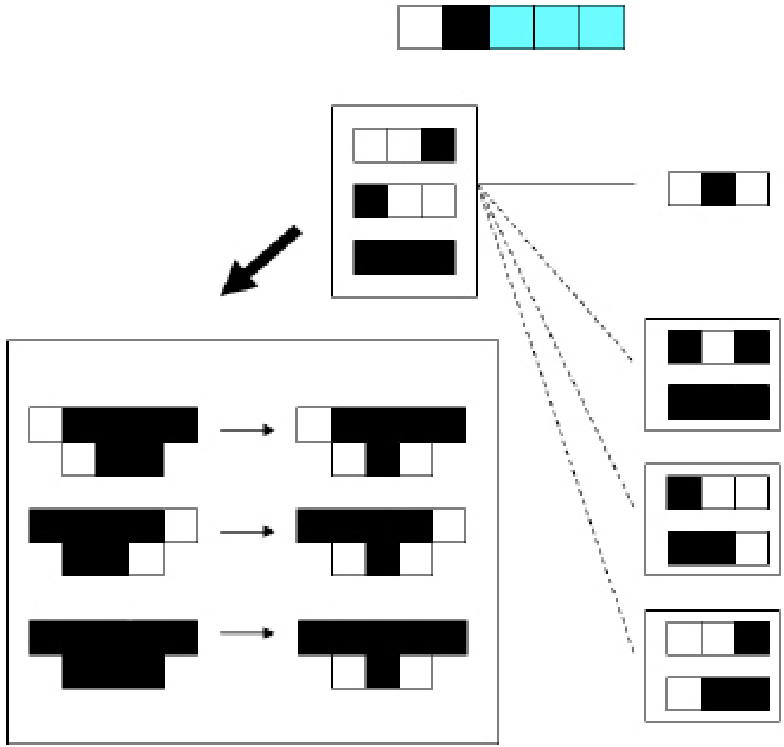
A B C D E
a
b
c
d
e



ミクロとマクロは整合的であり得ない



0か1か
決定論的では
なくなる



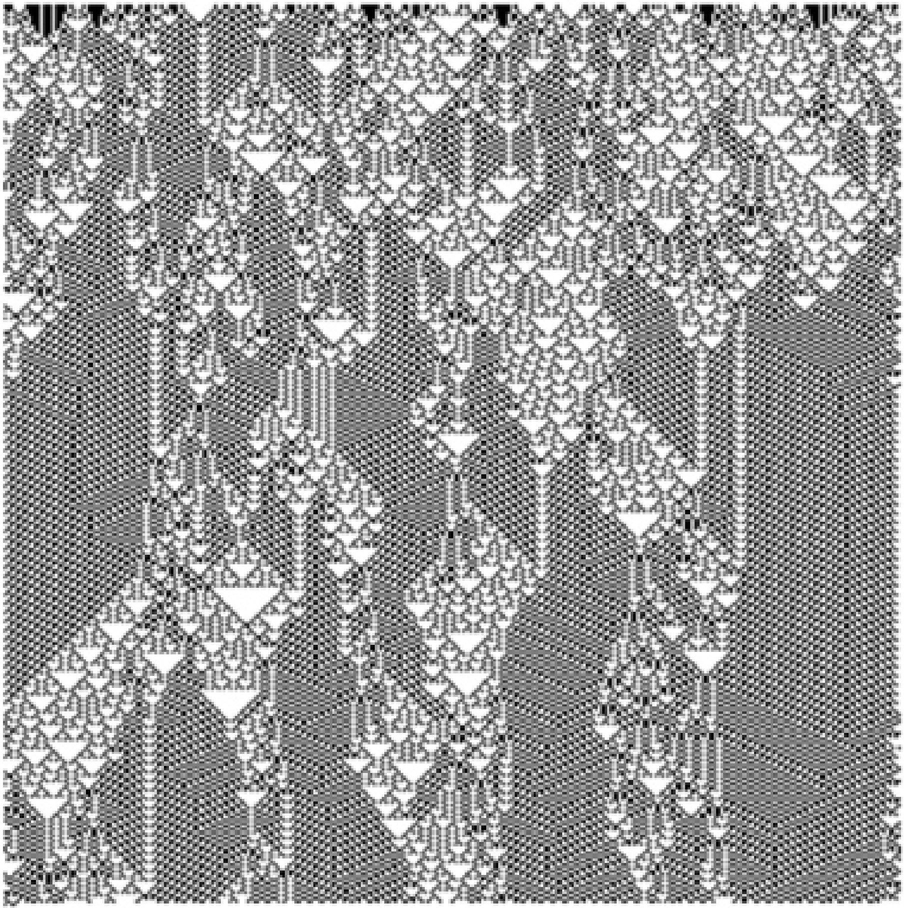
R146

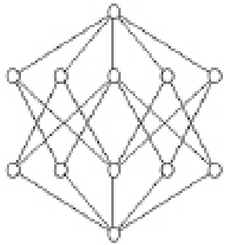
000	001	010	011
0	1	0	0

100	101	110	111
1	0	0	1

＋

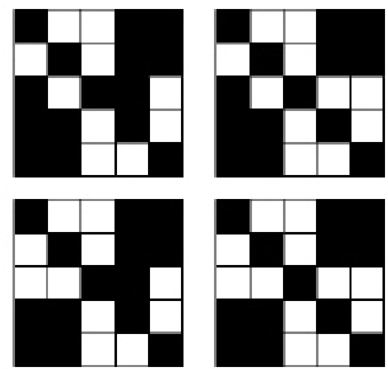
オーソ相補束の構造の制約（マクロ）



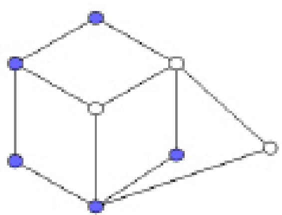


マクロな制約（相補性による弱い分配性）

齟齬＋調整 → 多重関係

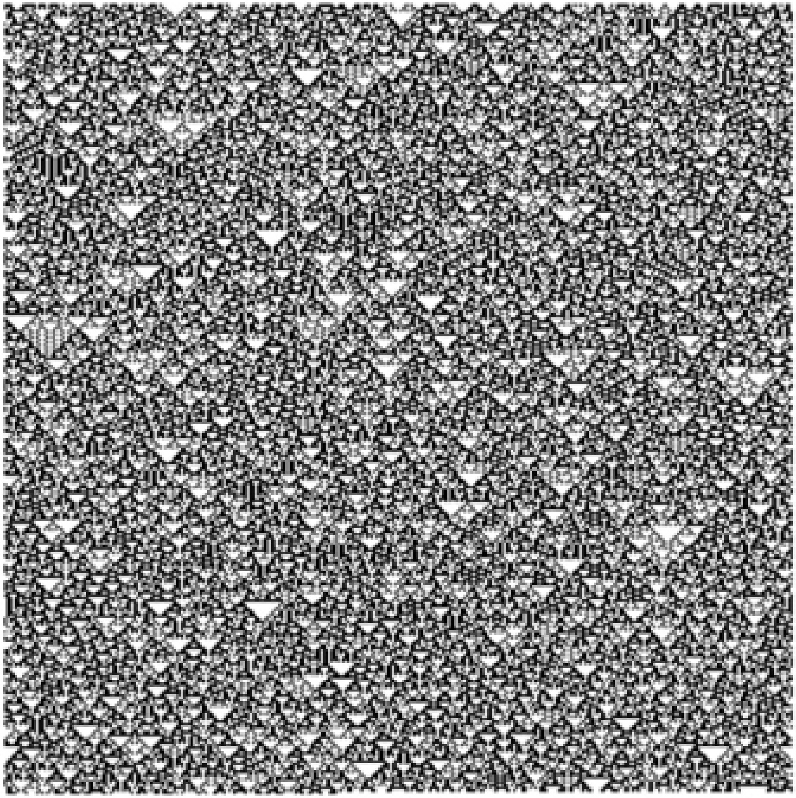


ミクロな規則（非分配性）



R90

000	**001**	**010**	**011**
0	**1**	**0**	**1**
100	**101**	**110**	**111**
1	**0**	**1**	**0**

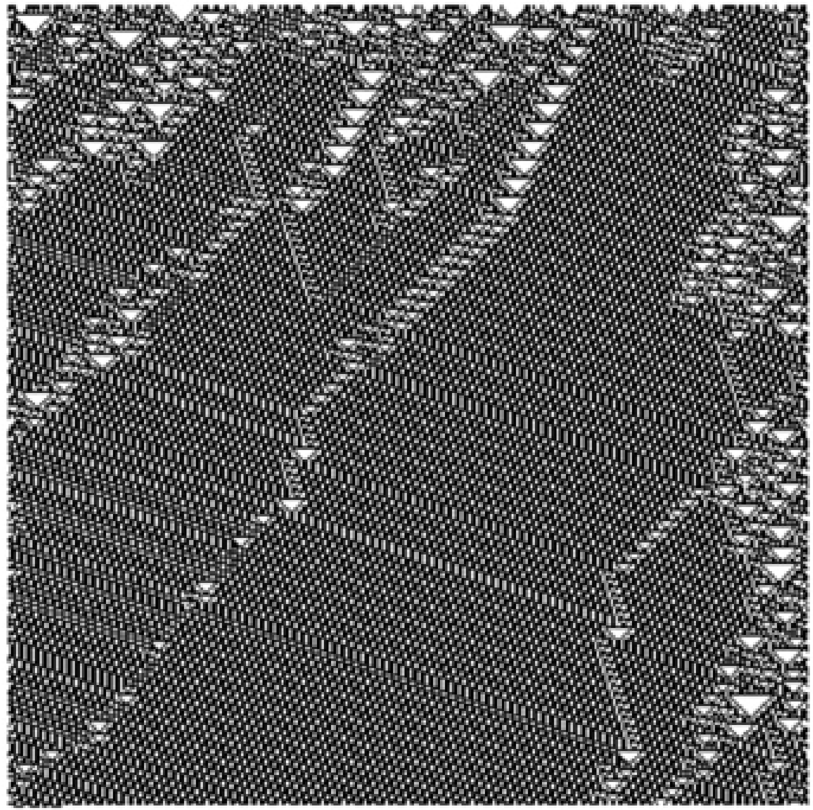


R90 + social constraint

000	001	010	011
0	1	0	1
100	**101**	**110**	**111**
1	0	1	0

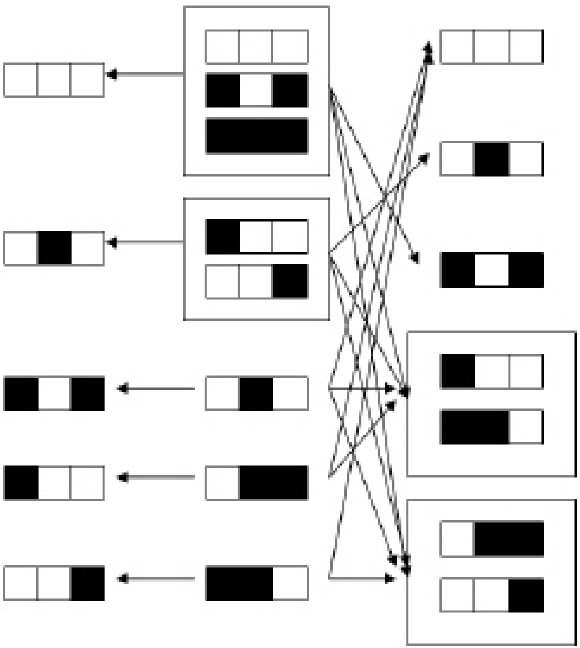
+

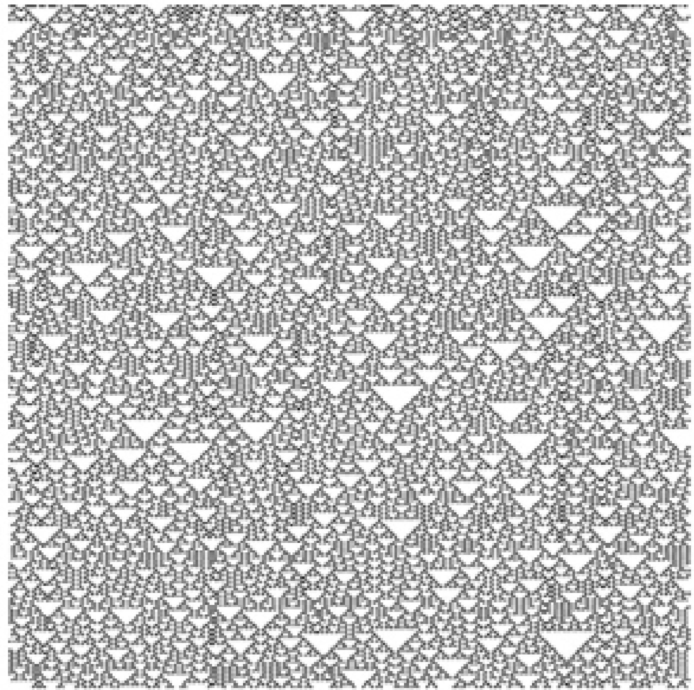
10101	10111
111	011
11111	11101
010	000



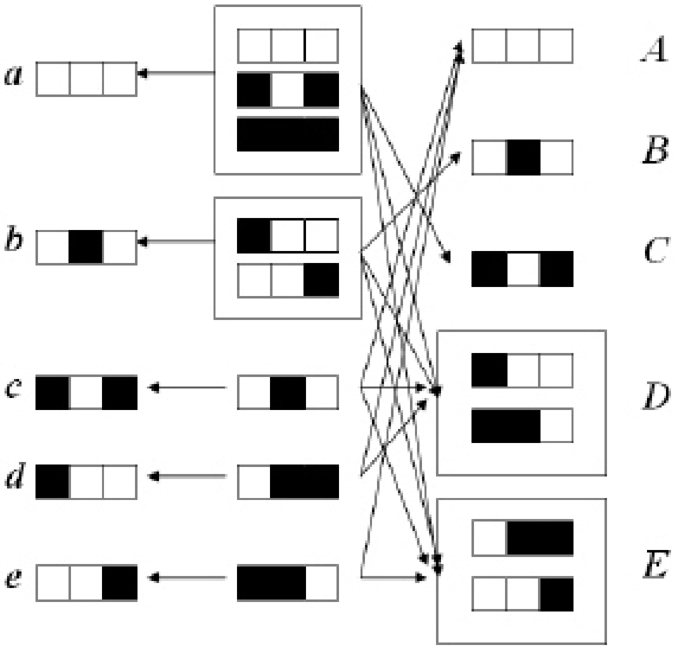
R18

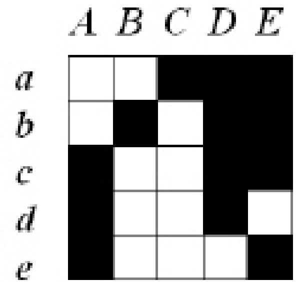
000	001	010	011
0	1	0	0
100	**101**	**110**	**111**
1	0	0	0

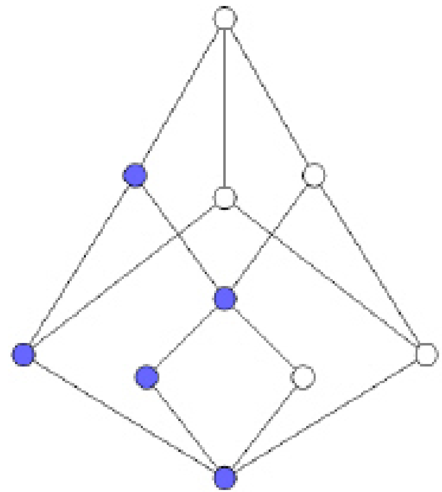




R18





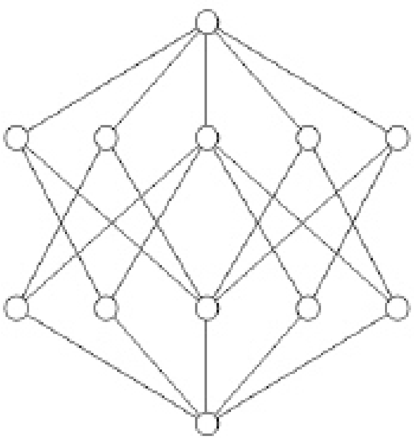


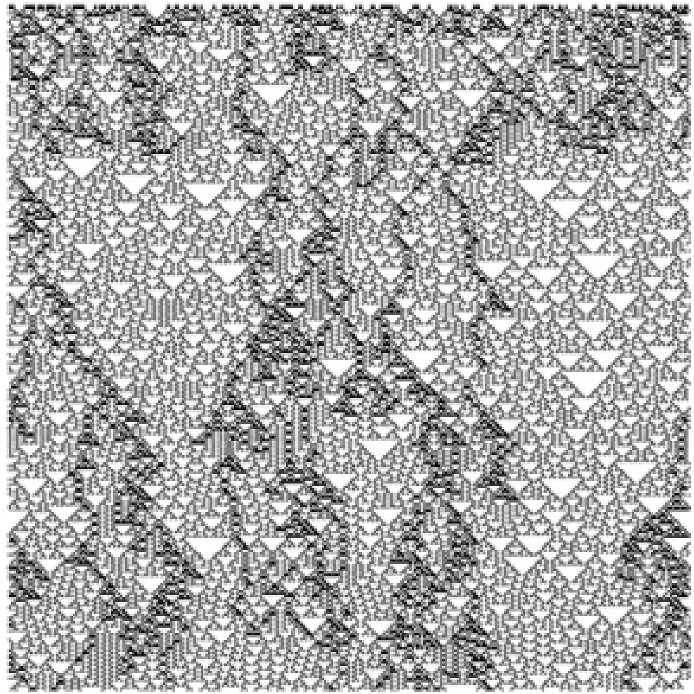
非分配束

R18

000	001	010	011
0	1	0	0
100	101	110	111
1	0	0	0

+

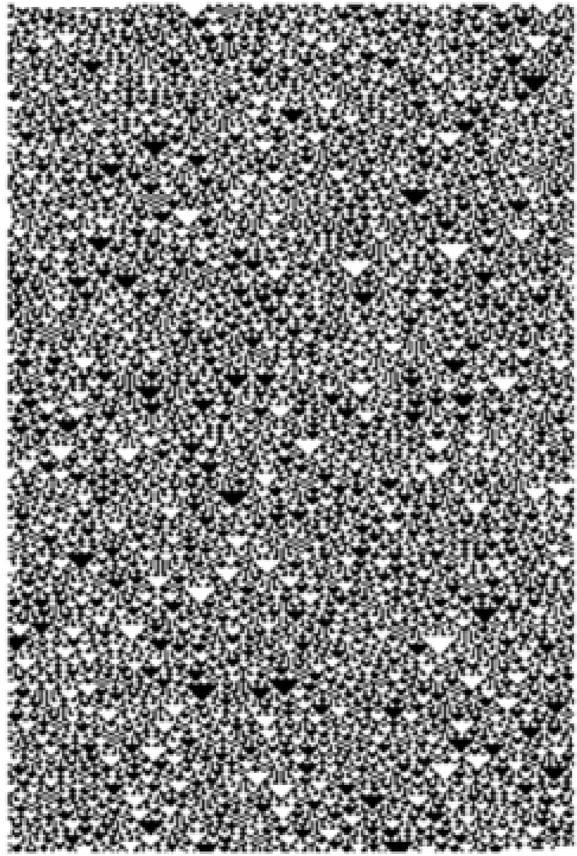


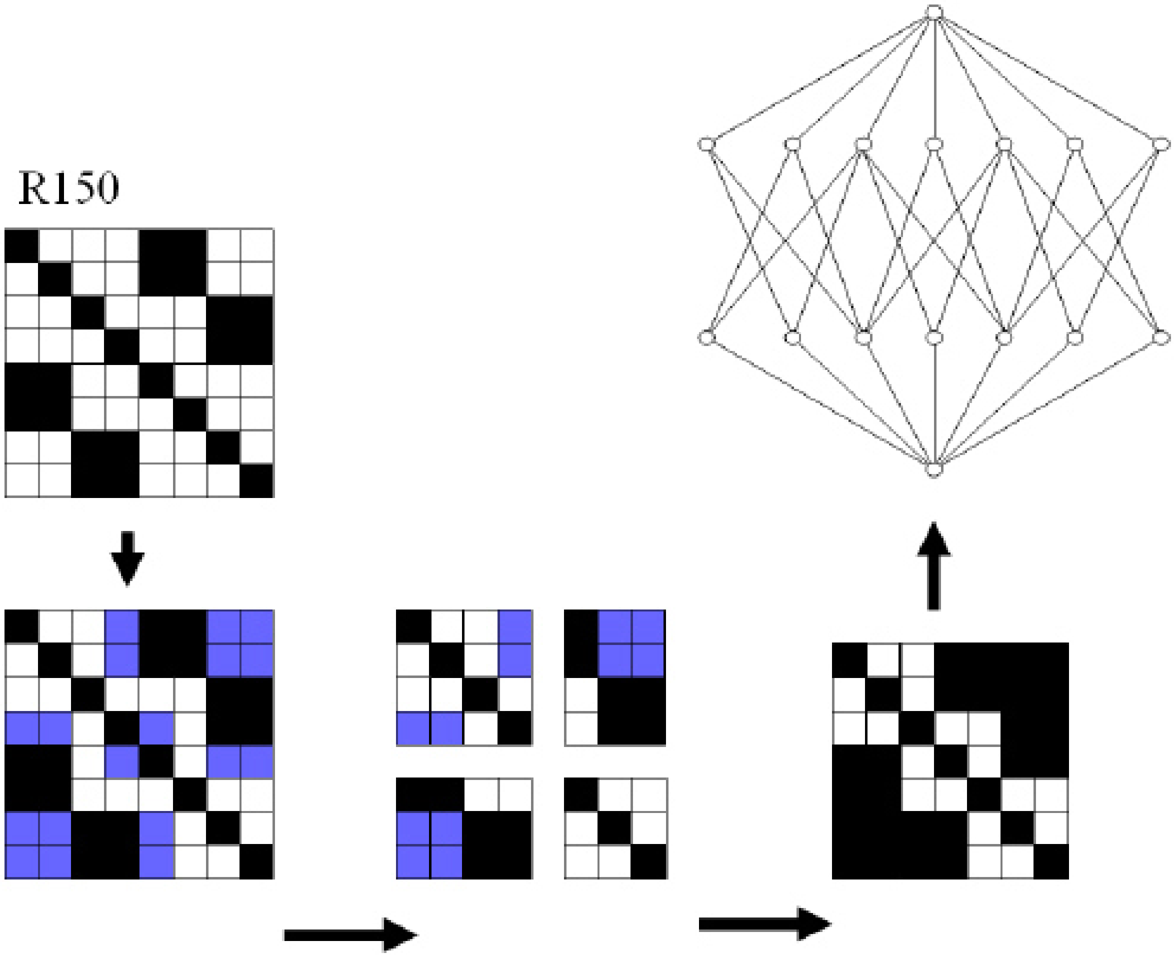


R150

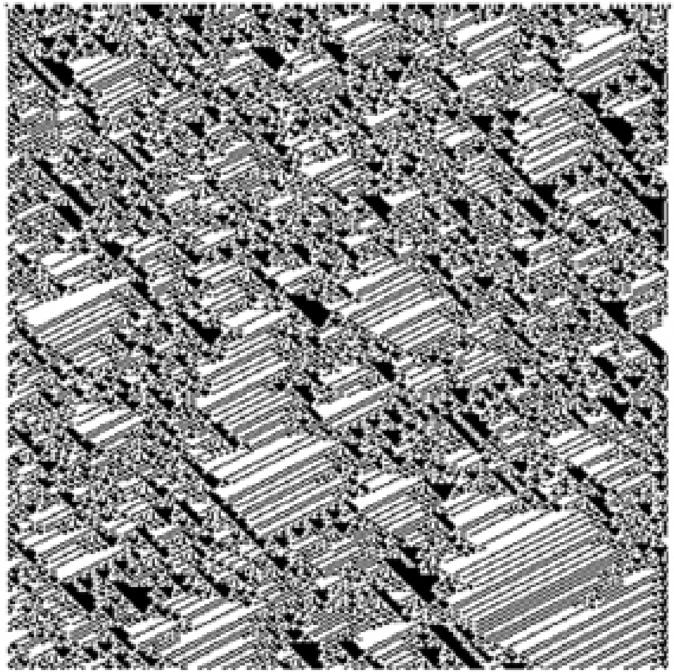
000	001	010	011
0	1	1	0

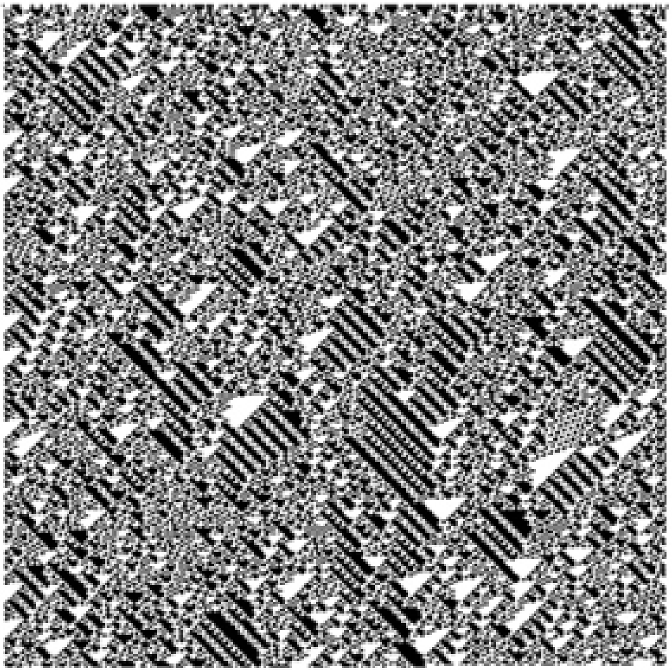
100	101	110	111
1	0	0	1




R150

R150





結論

マクロ（分配束）
↕
ミクロ（クラス１・２ルール）

マクロ（分配束）
↕
ミクロ（クラス３）