粉体の破砕と粒径分布関数

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Heilbrunner & Keulen 2006

- 断層破砕物の大きさ分布はべキ的
- 指数はひずみ量に伴い増大
particle-size distribution = power law

- Exponent \( D = 2.0 - 3.0 \)
- depends on the strain?

(Chester et al. 1993)


NOTE: exponent different from impact fragmentation (e.g. \( D = 1.3 - 1.6 \))
laboratory experiments
experiments on wear and comminution of rock

\[ D = 2.6(3) \]

Biegel, Sammis, Dieterich 1989

\[ D = 1.9 - 2.9 \]

Marone & Scholz 1989


exponent “D” depends on the strain
numerical experiment

exponent “D” depends on the strain
“D” approaches 3?
model: elementary process

an elementary process

a grain is broken up into “daughters”

1. mass distribution of daughters: \( F(m, M) \)

\[
\text{mass conservation: } \int_0^M dmF(m, M)m = M
\]

2. \( k(M) \): frequency of fracture ("reaction" rate)
time evolution of particle-size distribution

\[ \dot{n}_t(m) = \int_{m}^{M_0} dM k(M) n_t(M) F(m, M) - k(m) n_t(m) \]

trivial steady state \[ n_{ss}(m) = \delta(m_{\text{min}}) \]
scale-free assumption

\[ F(m, M) = M^{-1} f \left( \frac{m}{M} \right) \]

\[ \dot{n}_t(m) = \int_0^1 dx \left( x^{-1} k \frac{m}{x} \right) n \left( \frac{m}{x} \right) f(x) - k(m)n(m) \]

simplified equation

\[ x \equiv \frac{m}{M} \]
**nontrivial steady state**

\[ \int_0^1 dxx^{-1}k\left(\frac{m}{x}\right)n\left(\frac{m}{x}\right)f(x) - k(m)n(m) = 0 \]

\[ n_{ss}(m) \propto m^{-2}k^{-1}(m) \]

irrespective of \( f(x) \)!

**k(m):** "fracture frequency"

if uniform probability \( (k(m) = \text{const.}) \)

\[ n_{ss}(m) \propto m^{-2} \quad \text{D}=6 \quad \text{too large!} \]

\[ \text{(mass)} \propto \text{(size)}^3 \]

more plausible estimate of \( k(m) \)?
$k(m)$ depends on the configuration

large differential stress $\rightarrow$ fracture

hydrostatic

large differential stress

**experimental observation**
most grains undergo tensile fracture under uniaxial compression

(e.g. Marone & Scholz 1989)
probability of having similar-sized neighbors

one similar-sized neighbor

\[ k(m) \propto \frac{n(m)}{N_{tot}} \]

(population)

two similar-sized neighbors

\[
\left( \frac{n(m)}{N_{tot}} \right)^2 \ll \frac{n(m)}{N_{tot}}
\]

negligible
exponent of steady-state solution

\[ n_{ss}(m) \propto m^{-2} k^{-1}(m) \]

\[ k(m) \propto \frac{n(m)}{N_{tot}} \]

\[ \rightarrow n_{ss}(m) \propto m^{-1} \]

PSD exponent
(mass) \( \propto \) (size)\(^3\)

obtain exponent “D=3”
transient behavior

\[ \tau \dot{n}_t(m) = \int_m^{M_0} dM k(M) n_t^2(M) M^{-1} f\left(\frac{m}{M}\right) - n_t^2(m) \]

a model: \[ f(x) = \text{const.} \quad n_0(m) = \text{const.} \]

explain experiments and field observations