The effect of temporally varying charges of Dusty Plasmas on the linear stage of Magneto-Rotational Instability

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[1] Abstract
We report several results of linear analysis of Magneto-Rotational Instability (MRI) under the effect of Dusty Plasmas. Recent study has shown that effects of fluctuating dust charges are strongly affected by the equilibrium state of rotating disc. We also assume that dust particles temporarily vary their charges through collisions with ions and considered the effect of fluctuating dust charges. We found that effect of fluctuating dust charges are strongly affected by the equilibrium state of rotating disc.

[2] Introduction
'Single-fluid' MRI [Balbus & Hawley, 1991]

Dusty Plasmas
- Our space \(\Rightarrow 99\%\) electron-ion plasma
- \(1\%\) of dust particles \((<10^{-10} m_p, \text{size} < 1 \mu m)\)
- [a] collisions with electrons
- [b] Photoelectric effect
- [c] Secondary electron emission
- Collisions with electrons is dominant process
- Can be treated as the third component of plasma
- Modifications on plasma phenomena (e.g. wave propagation, Instabilities)
- 'Multi-fluid' MHD approach should be performed to understand the effect of dusty plasma on MRI

[3] Linear analysis (1) - fixed dust charge -
Solve 3 fluid (electron, ion, dusty plasma) MHD equation

- Linearize 3 fluid MHD equations
- Neglect radial gradient by assuming 'Local' approximation
- Assume axisymmetry

Derived 3 fluid MRI dispersion relation

\[ \frac{\Omega^2}{k^2} \left( \sum_a \frac{1}{\nu_A^2} - 1 \right) > \frac{1}{3} \]

- Shorter wavelength can be unstable compared to ordinary 1 fluid MRI

- Consider radial gradient and numerically calculate fluctuation satisfying boundary conditions
- Solve eigen value-eigen function problem by changing \(\omega\)
- Calculate growth rate from derived eigen value \(\Omega\)

Dusty Plasma can destabilize MRI
- Tidal force is enhanced by dust (massive component).
- Magnetic tension is unchanged by increase of dust number density.

[4] Linear analysis (2) - temporally varying dust charges -
Model of Charge fluctuation

- Consider two component (ion, dust) plasma
- Temporal variation of dust charge is caused by capture/release of ions
- Ion-Rich region \(\Rightarrow\) inflow of ion exceeds
- Ion-Less region \(\Rightarrow\) outflow of ion exceeds
- Tidal force is enhanced by dust (massive component).
- Magnetic tension is unchanged by increase of dust number density.

- Effect of fluctuating of dust charge is strongly affected by the equilibrium state of rotating disk

- Result of linear analysis implied that existence of dust would destabilize MRI
- Effect of charge fluctuation is strongly affected by equilibrium state of rotating disk.
- Linear analysis considering radial, azimuthal wave propagation should be done to consider interaction with new modes arose by dusty plasma
- Modeling of capture/release process, estimation of capture/release rate should be done
- 'Multi fluid' simulation to study nonlinear stage as a future study

[5] Summary and future works
- Result of linear analysis implied that existence of dust would destabilize MRI
- Effect of charge fluctuation is strongly affected by equilibrium state of rotating disk.
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\[ B = \xi(r) \exp(j(\omega t + kz - \omega t)) \]

\[ \delta q_d = \frac{q_i}{n_i} \delta n_i \]

\[ \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}) = S \]

Equilibrium State
Linearized Lorentz force term in dusts’ equation of motion

\[ \frac{\partial q}{\partial t} + m_d (E + \nu_d \times B) - \frac{\delta q_d}{m_d} (E + \nu_d \times B) + \frac{\partial q}{\partial t} (E + \delta q_d \times B + v_d \times \delta B) \]

Effect of fluctuating of dust charge cannot be determined unless the equilibrium state of rotating disk is known.

The equilibrium state (electric field, angular velocity) in rotating disk is not known (cannot be determined without an appropriate assumption).

[Nekrazov 2007, Phys. of Plasmas]

Fig.1 The case \(E_0 = 0\)
Fig.2 The case \(E_0 + \nu_d \times B_0 = 0\)
Effect of dusts’ charge fluctuation is significant when \(E_0 + \nu_{d0} \times B_0 \neq 0\)