Electric Charging of Dust Aggregates and its Effect on Dust Coagulation in Protoplanetary Disks

Satoshi Okuzumi (Kyoto U.)
Hidekazu Tanaka (Hokkaido U.)
Masa-aki Sakagami (Kyoto U.)

Introduction: Dust charging in protoplanetary disks
✓ Dust is an “MRI killer”

- Evolution of dust charge & gas ionization states
  ➡ Analytical solutions

- “Electric barrier” against dust growth
  ✓ Without turbulence, uniform (orderly) dust growth is likely to stall at its very early stage.

- Coagulation simulation with porosity evolution
  ➡ Bimodal growth induced by the electric barrier
Dust Coagulation: The First Step toward Planet Formation

One way to go further:
More “realistic” modeling of dust aggregates
Porosity Evolution of Aggregates

“Classical” dust model: compact sphere ($\rho=\text{const}$)

In reality, aggregates can be very porous!

Low-velocity collision
- Merger w/o restructuring
- Fractal Structure: $D \approx 2$
  \[ a \approx a_0 N^{1/2} \]

✓ Fractal growth lasts until porous size reaches $0.1\text{-}10\text{cm}$ (Blum 04, Suyama+ 08)

Experimental: Wurm & Blum 98; Blum+ 98
Statistical: Ormel+ 07; Zsom & Dullemond 08
N-body: Kempf+ 99; Wada+ 07,08; Suyama+08

High porosity $\Rightarrow$ Large surface area
- Strong coupling to gas, Efficient ion & electron capturing
MRI provides disk turbulence which may cause:
- Fragmentation of large aggregates
- Stirring-up of small aggregates/fragments

Small grains/aggregates greatly reduce $x_e$ by negative charging

Wide “dead zone” (Sano+ 00)

Dust = “MRI Killer”

Dust Growth & Settling ↔ MRI-driven Turbulence

Interact via Dust Charging
Question:

What’s the role of the electrostatic interaction of aggregates in their collisional growth?

1: How the dust charge state (& gas ionization state) evolve with dust growth?

2: How the dust charging feed back to the growth?
Step 0: Dust Charging in Neutral Plasmas

Thermal velocity: Electron >> Ion

Negatively Charged

Charge Equilibrium Condition:

\[ Z \sim -\frac{Ze^2}{a} \sim k_B T \]

\[ Z \sim -\frac{ak_B T}{e^2} \sim \left( \frac{a}{0.1\mu m} \right) \left( \frac{T}{100K} \right) \]

☐ An exact solution exists for spherical bodies (Spitzer 1941)
☐ Roughly holds for fluffy aggregates (Matthews & Hyde 2008)
Step 1: Charge State in Weakly Ionized Gas

★ Dust charge state : $n_d(Z)$
★ Gas ionization state : $n_i$ & $n_e$
Must be computed **consistently**

Previous Work (Sano+ 00, Ilgner & Nelson 06)
- Solved charge transfer **numerically**
- Heavy to compute

This Work: solves charge transfer as analytically as possible
✓ Single eq. for a “master” parameter
✓ Arbitrary size & porosity distribution
✓ Agrees with numerical solutions very well (see next slide)
✓ Enables coupled simulation of dust growth/settling & MRI turbulence
Example: \( r = 5 \text{AU}, \ z = \text{H}, \) fractal growth (\( D=2 \)) : \( a \approx a_0 N^{1/2} \)

**Evolution of Dust Charge State**

Small Dust \( \rightarrow \) **Ion-Dust Plasma (IDP)** : \( n_i \approx -\langle Z \rangle n_d \),

Large Dust \( \rightarrow \) **Ion-Electron Plasma (IEP)** : \( n_i \approx n_e \),

\[
\frac{\langle Z \rangle e^2}{a} \ll k_B T
\]

\[
\frac{\langle Z \rangle e^2}{a} \sim k_B T
\]
Fractal Dust Maintains a Wide Dead Zone

\( \text{D} \leq 2 \rightarrow \text{Total surface area } \sigma n_d \text{ conserved } \rightarrow n_e \text{ kept low} \)

**Compact \( (D=3) \) Dust Model**

- MRI active for size \( a > 10 \mu \text{m} \)

**Fractal \( (D=2) \) Dust Model**

- MRI inactive for all sizes!!

\( \rightarrow \text{Dead zone is maintained until aggregates are compressed } (a \sim \text{cm}) \)
Step 2: A Simple Estimation of the Effect of Electrostatic Repulsion on Dust Growth

Collisional Cross Section:

\[ E_{\text{kin}} = \frac{1}{2} \mu v_{\text{rel}}^2 \]

Relative Kinetic Energy

\[ E_{\text{el}} = \frac{Z Z' e^2}{a + a'} \]

“Electric Energy” before Contact

\[ \sigma_{\text{eff}} = \pi (a + a')^2 \left( 1 - \frac{E_{\text{el}}}{E_{\text{kin}}} \right) \]

Growth Possible if

\[ E_{\text{kin}} > E_{\text{el}} \]

✓ Dust Motion: Brownian Motion + Sedimentation + Turbulence

✓ Growth Mode: Monodisperse (=uniform) & Fractal (D=2)

\( \Leftarrow \) valid for early evolutionary stages \((a < 1\text{cm})\)

✓ Disk Model: MMSN + Ionization sources

(cosmic ray + X-ray + radionuclides)
Kinetic Energy vs Electrostatic Energy

Example: \( r = 5\text{AU}, z = H, \) no turbulence

\[
\begin{align*}
E_{el} & : Z_1 = Z_2 = \langle Z \rangle \\
E_{el,3\sigma} & : Z_1 = \langle Z \rangle, \ Z_2 = \langle Z \rangle + 3\langle \Delta Z^2 \rangle^{1/2}
\end{align*}
\]

Both \( E_{el} \) & \( E_{el,3\sigma} \) quickly converge to the neutral plasma limit, exceeding \( E_{\text{kin}} \) much before collisional compaction (★) begins!

Uniform growth stalls at size as small as 10\( \mu \text{m} \)!
If MRI turbulence is present, dust is likely to overcome the electric barrier.

Disk turbulence enhances the collisional velocity $\Delta v$. (see, e.g., Ormel & Cuzzi 07)

The electric barrier ($E_{el}>E_{kin}$) is removed for $\alpha \geq 10^{-2}$.

But...
Disk Turbulence: a Friend or Foe?

At later stages, such strong turbulence is very likely to cause **collisional fragmentation** of aggregates!!

- $\Delta v \sim \alpha^{1/2} c_s \sim 100\text{m/s}$ for $t_{\text{fric}} \sim T_k$ & $\alpha \sim 10^{-2}$
  
  (Ormel & Cuzzi 07)

- Collision of aggregates results in fragmentation for $\Delta v > 30$-60 m/s
  
  (Wada+ 08,09(submitted))

Laminar $\rightarrow$ Electric barrier at early growth stages

Turbulent $\rightarrow$ Fragmentation barrier at late stages
Evolution of dust charge & gas ionization states with dust growth is investigated analytically.

Fractal growth (valid for size < cm) tends to maintain a large dead zone.

“Electric barrier” against dust growth:
- Without turbulence, uniform (orderly) dust growth is likely to stall at its very early stage.
- With turbulence, dust can overcome the barrier, but then suffers from collisional fragmentation.
A Possible Path to Evolve: Bimodal Growth

Small “field” aggregates (mass \(m\))
Large “test” aggregates (mass \(M\))

Test aggregates can collide with “frozen” field aggregates if

- Brownian-motion dom. \(\Rightarrow (\Delta v)_{Mm} > (\Delta v)_{mm}\)
- Sedimentation-dom. \(\Rightarrow (\Delta v)_{Mm} \geq (\Delta v)_{mm}\)

A small fraction of massive aggregates are allowed to continue growing even if the growth of the others has “frozen out.”
Step 3: Simulation of Porous Dust Coagulation

- Extended Smoluchowski Equations:
  Evolve mass DF $n(M)$ & mass-volume rel. $V(M)$ consistently
- Porosity change $\xleftarrow{}$ Empirical formula from N-body simulations

(Okuzumi, Tanaka, & Sakagami, coming soon!)

\[ \partial_t n_M = \int_0^{M/2} K_{M-M',V_{M-M'}} n_{M'} n_{M-M'} dM' \]
\[ - n_M \int_0^\infty K_{M',V_M} n_{M'} dM' \]
\[ \partial_t V_M = \int_0^{M/2} \left[ V_{1+2}(V_{M'}, V_{M-M'}) - V_M \right] \]
\[ \times K_{M-M',V_{M-M'}} n_{M'} n_{M-M'} dM' \]

$V_{1+2}(V_m, V_M)$: volume of collisional outcome

Decreasing $V_m/V_M$

Small Size Ratio $\downarrow$

Small Porosity Increase
Result

\[ r = 5\text{AU}, \ z = H, \ \alpha = 0, \ \text{Ionization Rate} = 0.1 \times (X\text{-ray Ionization}) \]

Mass Spectrum

Brownian dom. Sedimentation dom.

Mass-Radius Relation

Brownian dom. Sedimentation dom.
Summary

- Evolution of dust charge & gas ionization states with dust growth is investigated analytically.
  ✓ Fractal growth (valid for size < cm) tends to maintain a large dead zone.

- “Electric barrier” against dust growth
  ✓ Fractal growth is likely to stall in the absence of strong turbulence

- Bimodal growth induced by the electric barrier is confirmed by a coagulation simulation including porosity evolution.
VAM reproduces full-2D results in surprisingly good accuracy.

Full-2D Monte Carlo Method (Ormel+ 07)

Volume-Averaging Method

CPU Time: 2 Hours

CPU Time: 1 Min
Is Bimodal Growth Favorable for Planetesimal Formation?

Recent Lab Experiment (Teiser & Wurm 08)

Collision of large (1-10cm) target and small(<mm) projectile

→ Target can achieve net growth even if $v_{imp} > 50$cm!!

Teiser & Wurm (2008)

Bimodal growth might overcome the fragmentation barrier!
Porous Dust Can Overcome the Radial Drift Barrier

\[ r = 5\text{AU}, \, z = 0 \]

Compact Dust Model (Porosity = 0%)
- Epstein
- Stokes

Porous Dust Model (Porosity = 99%)
- Epstein
- Stokes

\[ t_{\text{stop}} \propto \frac{ma}{\sigma} \propto \bar{\rho}a^2 \]

\[ t_{\text{stop}} \sim \Omega_K^{-1} \Rightarrow a_c \propto \bar{\rho}^{-1/2} \]

\[ \sigma \propto \bar{\rho}^{-1} \]

\[ m \propto \bar{\rho} a_c^3 \propto \bar{\rho}^{-1/2} \]

\[ \frac{\dot{m}}{m} \propto \frac{\sigma}{m} \propto \bar{\rho}^{-1/2} \]