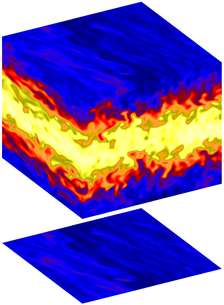
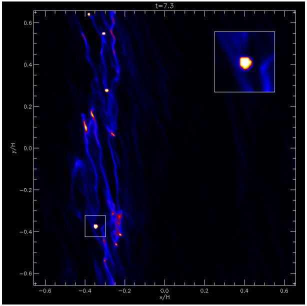
Planetesimal formation in turbulent protoplanetary discs





Anders Johansen
Leiden Observatory, Leiden University

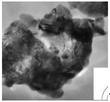
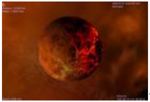
"Workshop on the Magnetorotational Instability in Protoplanetary Disks" (Kobe University, June 2009)

Collaborators: Andrew Youdin, Hubert Klahr, Wladimir Lyra, Mordecai-Mark Mac Low, Thomas Henning

Planet formation

Planets form in protoplanetary discs from dust grains that collide and stick together (planetesimal hypothesis of Safronov, 1969).

- From dust to planetesimals
  μm $\rightarrow$ m: Contact forces in collisions cause sticking
  m $\rightarrow$ km: ???
- From planetesimals to protoplanets
  km $\rightarrow$ 1,000 km: Gravity
- From protoplanets to planets
  Terrestrial planets: Protoplanets collide
  Gas planets: Solid core attracts gaseous envelope

 $\longrightarrow$  $\longrightarrow$  $\longrightarrow$  $\longrightarrow$

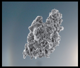
Planetesimals

- Kilometer-sized objects massive enough to attract each other by gravity (two-body encounters)
- Building blocks of planets
- Formation:
  - μm → cm: Dust grains collide and stick
    (Blum & Wurm 2000)
  - cm → km: Sticking or gravitational instability
    (Safronov 1969, Goldreich & Ward 1973, Weidenschilling & Cuzzi 1993)
- Dynamics of turbulent gas important for modelling dust grains and boulders

William K. Hartmann

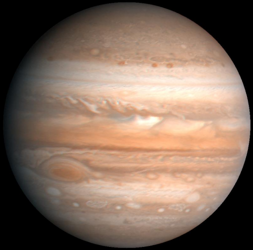
Overview of planets

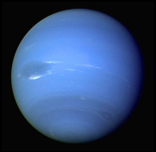
Protoplanetary discs



Dust grains

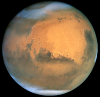
Pebbles





Gas giants and ice giants

Terrestrial planets



Dwarf planets

+ Countless asteroids and Kuiper belt objects
+ Moons of giant planets
+ More than 300 exoplanets

Particle dynamics

Gas accelerates solid particles through drag force:

$$\frac{\partial \mathbf{w}}{\partial t} = \ldots - \frac{1}{\tau_{\mathrm{f}}}(\mathbf{w} - \mathbf{u})$$

Particle velocity   Gas velocity

In the Epstein drag force regime, when the particle is much smaller than the mean free path of the gas molecules, the friction time is (Weidenschilling 1977)

$$\tau_{\mathrm{f}} = \frac{a_\bullet \rho_\bullet}{c_{\mathrm{s}} \rho_{\mathrm{g}}}$$

$a_\bullet$: Particle radius

$\rho_\bullet$: Material density

c_{s}: Sound speed

ρ_{g}: Gas density

Important nondimensional parameter in protoplanetary discs:

$\Omega_{\mathrm{K}} \tau_{\mathrm{f}}$ (*Stokes number*)

At $r = 5$ AU we can approximately write $a_\bullet/\mathrm{m} \sim 0.3\Omega_{\mathrm{K}}\tau_{\mathrm{f}}$.

Diffusion-sedimentation equilibrium

Diffusion-sedimentation equilibrium:

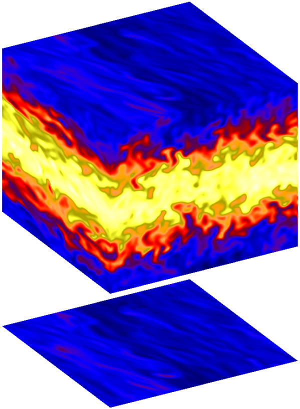
$$\frac{H_{\rm dust}}{H_{\rm gas}} = \sqrt{\frac{\delta_{\rm t}}{\Omega_{\rm K}\tau_{\rm f}}}$$

$H_{\rm dust}$ = scale height of dust-to-gas ratio

$H_{\rm gas}$ = scale height of gas

$\delta_{\rm t}$ = turbulent diffusion coefficient, like α-value

$\Omega_{\rm K}\tau_{\rm f}$ = Stokes number, proportional to radius of solid particles



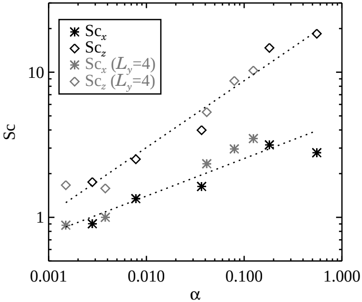
(Johansen & Klahr 2005)

Diffusion coefficient

Definition of Schmidt number:

$\mathrm{Sc} = \nu_{\mathrm{t}}/D_{\mathrm{t}} = \alpha_{\mathrm{t}}/\delta_{\mathrm{t}}$

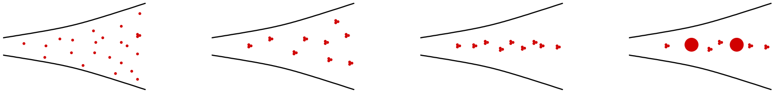
- From the scale-height of the dust one can calculate the diffusion coefficient: $\delta_{\mathrm{t}} = \delta_{\mathrm{t}}(H_{\mathrm{dust}})$



- Johansen & Klahr (2005): $\mathrm{Sc}_z \simeq 1.5$, $\mathrm{Sc}_x \simeq 1$

  (Turner et al. 2006: $\mathrm{Sc}_z \simeq 1$; Fromang & Papaloizou 2006: $\mathrm{Sc}_z \simeq 3$)
- Carballido, Stone, & Pringle (2005): $\mathrm{Sc}_x \simeq 10$
- Johansen, Klahr, & Mee (2006):
  The ratio between diffusion and viscosity depends on the strength of an imposed magnetic field

The role of the Schmidt number



Safronov (1969):

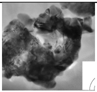
- Dust grains coagulate and gradually decouple from the gas
- Sediment to form a thin mid-plane layer in the disc
- Planetesimals form by continued coagulation or self-gravity (or combination) in dense mid-plane layer

HOWEVER:
MRI-driven turbulence very efficient at diffusing dust

Need to look at how larger particles react to turbulence

Dust nomenclature

- My suggestion for naming solid particles (not official):

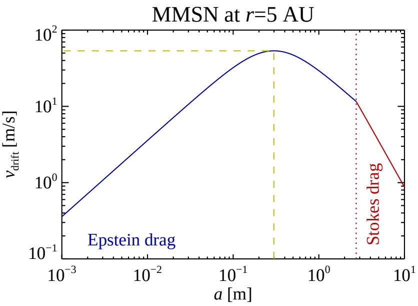
Diameter	Name	
<1 mm	Dust	
1 mm	Sand	
1 cm	Pebble, gravel	
10 cm	Cobble, rock	
>1 m	Boulder	

Radial drift

Balance between drag force and head wind gives radial drift speed (Weidenschilling 1977)

$$v_{\rm drift} = -\frac{2}{\Omega_{\rm K}\tau_{\rm f} + (\Omega_{\rm K}\tau_{\rm f})^{-1}}\eta v_{\rm K}$$

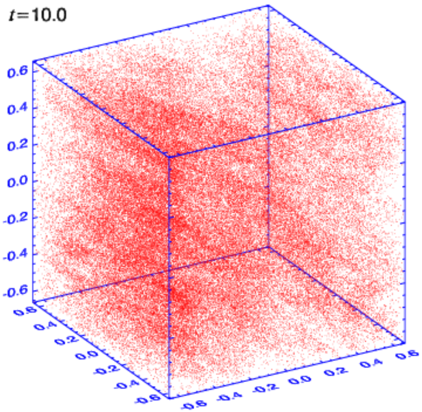
for Epstein drag law (solids smaller than gas mean free path).



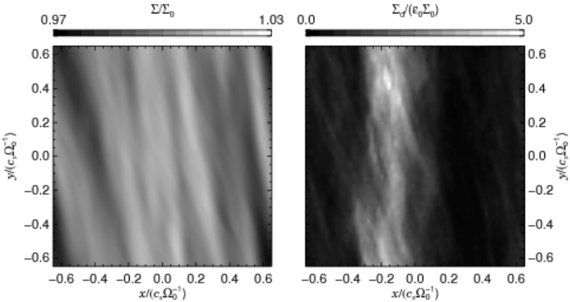
- MMSN η from Cuzzi et al. 1993
- Maximum drift speed of 50 m/s
- Drift time-scale of 50-100 orbits for solids of 30 cm in radius at 5 AU, but 1 cm at 100 AU

Boulders in turbulence

Johansen, Klahr, & Henning (2006):
2,000,000 boulders moving in magnetorotational turbulence



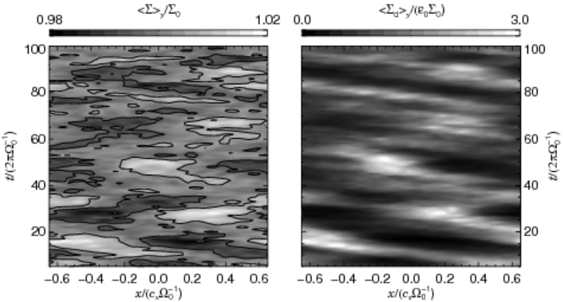
Gas density bumps



- Strong correlation between high gas density and high particle density (Johansen, Klahr, & Henning 2006)
- Solid particles are caught in gas overdensities

  (Whipple 1972, Klahr & Lin 2001, Haghighipour & Boss 2003)
- Gravoturbulent formation of planetesimals

Gas density bumps



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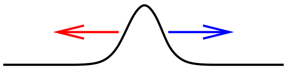
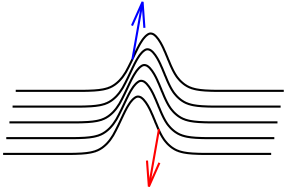
Pressure gradient trapping



- Outer edge:
  Gas sub-Keplerian. Particles forced by gas drag to move inwards.
- Inner edge:
  Gas super-Keplerian. Particles forced by gas drag to move outwards.

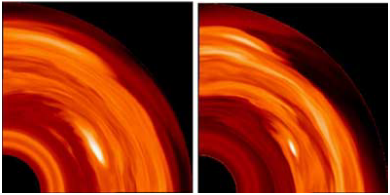


Global models

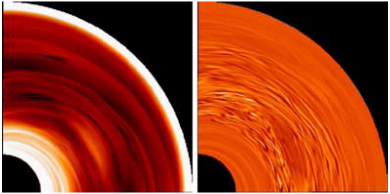
Fromang & Nelson (2005):

Dust concentrates in long-lived vortex

Dust density (5 cm and 25 cm):

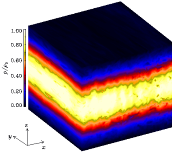


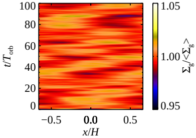
Gas density and vorticity (ω_z):

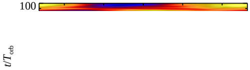


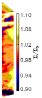
Increasing box size

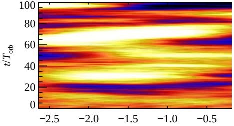
- Stratified shearing box simulations with increasing box size

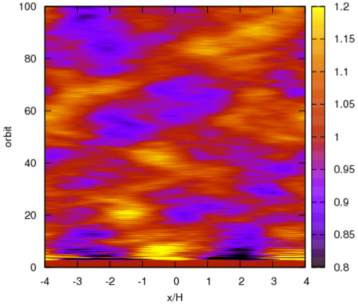












Orbital advection algorithm wi

(Fourier interpolate the Kepler

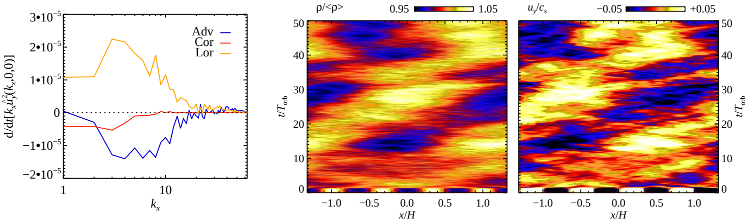
- No spurious density depre

  (Johnson et al. 2008)

- Pressure bumps of few pe reappear at time-scales of

Plot by T. Sano

Zonal flow

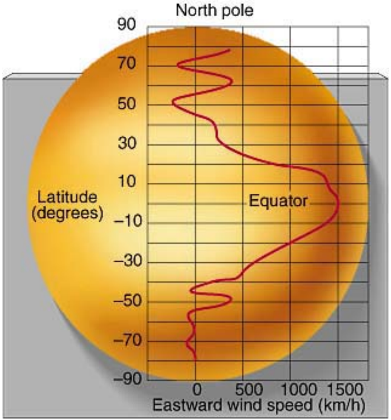


- Large scale variation in Maxwell stress launches zonal flows
- Pressure bumps form as zonal flows are slightly compressive
- Balance between turbulent diffusion and compression gives $|\hat{\rho}| \propto k_x^{-2}$
- Johansen, Youdin, & Klahr (2009): *Zonal flows in accretion discs*

Examples of zonal flow – planets

Definition of zonal flow:

Axisymmetric large scale variation in rotation velocity



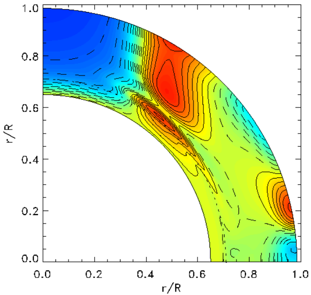
- Saturn and Jupiter show steady zonal flows
- Driven by convection (inverse hydrodynamical cascade)

Examples of zonal flow – the Sun

- On top of the Sun's differential rotation there is a zonal flow of amplitude approximately 3 m/s
- Discovered in 1980 from very precise measurements of the solar rotation

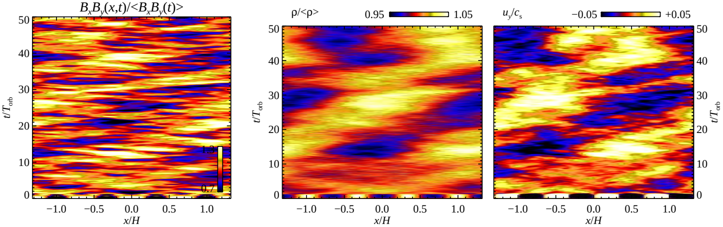
  (Howard & Bonte 1980)
- Migrates with the solar cycle
- Zonal flows (or torsional oscillations) are launched by the magnetic tension associated with large scale magnetic fields

  (Schüßler 1981, Yoshimura 1981)



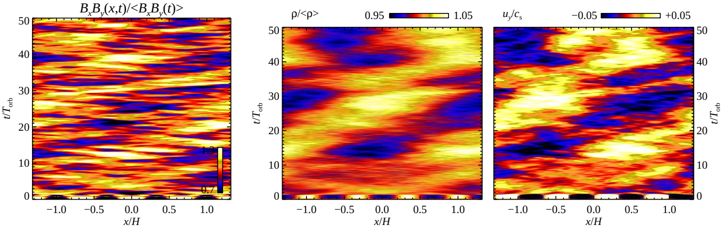
Stress variation

Resistive $2.56H \times 2.56H \times 1.28H$ simulation at $256 \times 256 \times 128$ grid points ($\mathrm{Re_M} = 12500$, $\mathrm{Pm} = 3.75$):



- Turbulent viscosity $\alpha \approx 0.005$
- Stress variation of 10%–20%
- Stress correlation time of a few orbits
- Density bumps and zonal flows correlated on tens of orbits

Analytical model



Analytical model of zonal flow excitation and saturation

Need to connect a known (measured) stress and stress variation to amplitude of density bumps and zonal flows

- Forcing of the zonal flow by stress variation
- Geostrophic balance between pressure bump and zonal flow envelope
- Damped random walk model

Variation in stress

Linearised, axisymmetric evolution equation for u_y:

$$\frac{\partial u_y'}{\partial t} = -\frac{1}{2}\Omega u_x' + T'$$

The tension term T' describes momentum transport by Maxwell stress:

$$T' = \frac{1}{\rho_0}\frac{1}{\mu_0}\frac{\partial \langle B_x B_y \rangle}{\partial x}$$

$$M = -\mu_0^{-1}\langle B_x B_y \rangle$$

In shearing sheet the tension is simply the derivative of the Maxwell stress variation:

$$T' = -\frac{1}{\rho_0}\frac{\partial M'}{\partial x}$$

Zonal flow dynamical equations

Linearised equation system for zonal flow excitation

(hats denote wave amplitudes):

$$
\begin{aligned}
0 &= 2\Omega \hat{u}_y - \frac{c_s^2}{\rho_0} \mathrm{i} k_0 \hat{\rho} \\
\frac{\mathrm{d}\hat{u}_y}{\mathrm{d}t} &= -\frac{1}{2}\Omega \hat{u}_x + \hat{T} \\
\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} &= -\rho_0 \mathrm{i} k_0 \hat{u}_x - \frac{1}{\tau_{\mathrm{mix}}}\hat{\rho}
\end{aligned}
$$

- Assumed geostrophic balance between zonal flow and pressure bump
- Density evolution includes turbulent diffusion term acting on time-scale τ_{mix}

Solutions

Combine the three equations to get

Master equation

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = \frac{1}{1 + k_0^2 H^2}\left(\hat{F} - \frac{\hat{\rho}(t)}{\tau_{\rm mix}}\right)$$

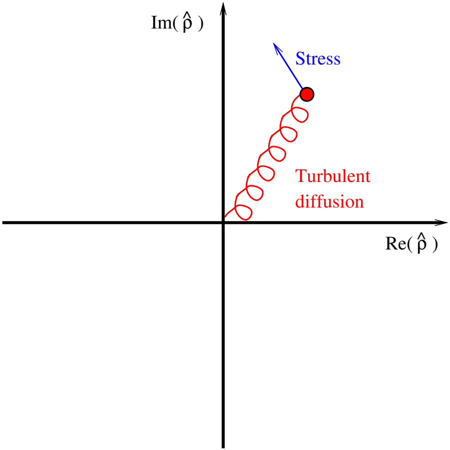
$$\hat{F} = -2\mathrm{i}k_0\rho_0\Omega^{-1}\hat{T}$$

Straight forward solution:

$$\hat{\rho}_{\rm eq} = \tau_{\rm mix}\hat{F}$$

Only valid if correlation time of stress variation larger than mixing time-scale. Need to model as damped random walk. Exciting at time-scale $\tau_{\rm for}$ and damping on time-scale $\tau_{\rm mix}$.

Damped random walk



- Correlation time equal to turbulent diffusion time-scale
- What is the amplitude?

Random walk solution

Solution involves product of forcing and mixing time-scales:

Random walk solution

$$\frac{\hat{\rho}_{\rm eq}}{\rho_0} = 2\sqrt{c_k \tau_{\rm for} \tau_{\rm mix}} H k_0 \frac{\hat{T}}{c_{\rm s}}$$

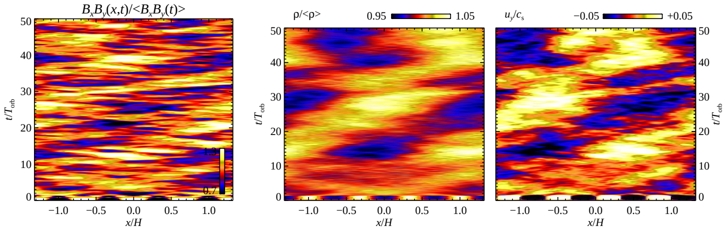
$$c_k = \frac{1}{1 + k_0^2 H^2}$$

$$\hat{\rho}_{\rm eq} \propto k_0^{-1} \quad \text{for } k_0 H \gg 1$$
$$\hat{\rho}_{\rm eq} \propto \text{const} \quad \text{for } k_0 H \ll 1$$

How to find amplitude of zonal flow:

- Take ρ_0, H, Ω from disc model
- Read off $\hat{T}$, $\tau_{\rm mix}$ and $\tau_{\rm for}$ from simulation
- Solution gives $\hat{\rho}_{\rm eq}$ at a given scale k_0
- Geostrophic balance gives $\hat{u}_y$ from $\hat{\rho}$

Comparison to simulation

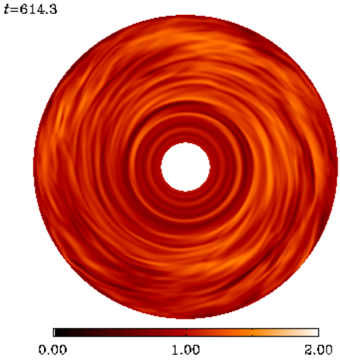


- Turbulent mixing time-scale $\tau_{\rm mix} \approx 1/(k_0^2 D) \approx 6\ T_{\rm orb}$
- Stress variation of $\widehat{B_x B_y} \sim 10^{-3}$
- Stress correlation time of a few orbits
- Formula predicts pressure bump amplitude of $\hat{\rho}_{\rm eq} \approx 0.08$
- In fairly good agreement with the measured $\hat{\rho}_{\rm eq} \approx 0.05$

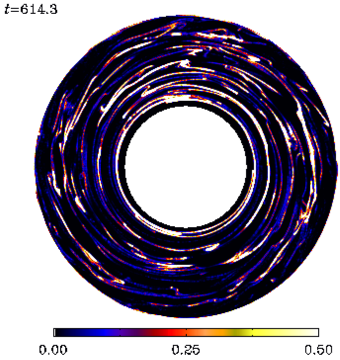
Global models

Lyra, Johansen, Klahr, & Piskunov (2008):

- Global disc with boulders on Cartesian grid (disk-in-a-box)



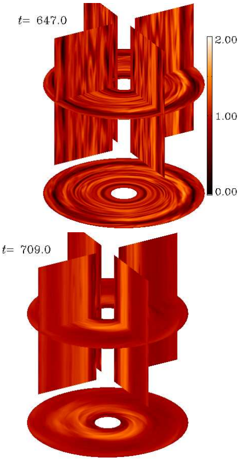
Gas density (320 $\times$ 320 $\times$ 32)

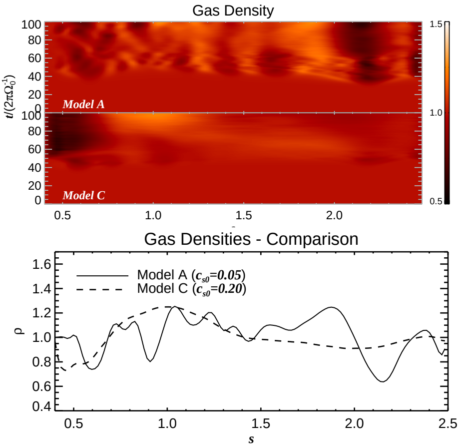


Particle density (10^6 particles)

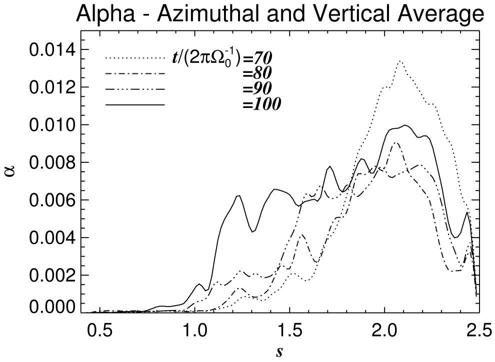
Space-time plots

Gas density structure from Lyra et al. (2008):





Stress variation

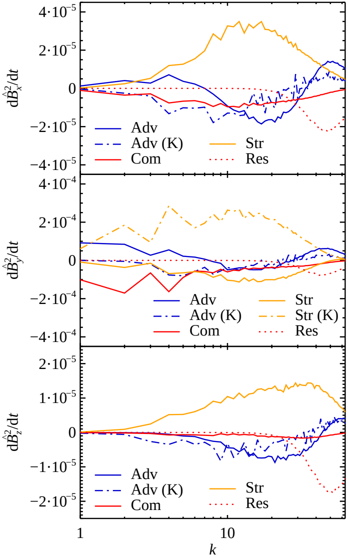


- At any given time there are approximately 10% variations in the α-value
- This is enough to launch zonal flows
- Similar variations reported in Fromang & Nelson (2006)

Inverse cascade

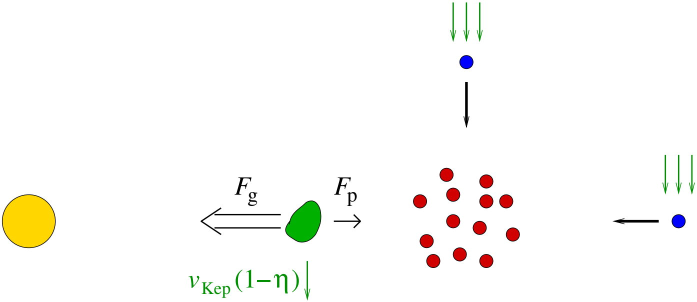
Plots show power contribution of different terms in the induction equation:

- Magnetic energy cascades to largest scales in the box
- Happens through the advection term
- Excites large scale variation in Maxwell stress
- Very little large scale activity in the vertical field component



Streaming instability

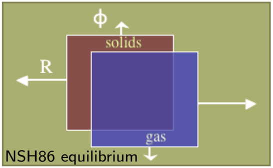
- Gas rotates slightly slower than Keplerian
- Particles lose angular momentum due to headwind
- Particle clumps locally reduce headwind and are fed by isolated particles

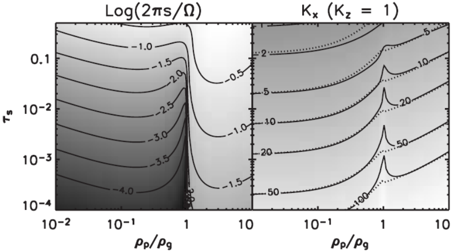


- Nakagawa, Sekiya, & Hayashi (1986): Equilibrium flow solution
- Youdin & Goodman (2005): "Streaming instability" (also Goodman & Pindor 2000)
- Johansen, Henning, & Klahr (2006); Youdin & Johansen (2007); Johansen & Youdin (2007); Ishitsu, Inutsuka, & Sekiya (2009)

Streaming instability

Youdin & Goodman (2005) :
"Streaming Instabilities in Protoplanetary Disks"

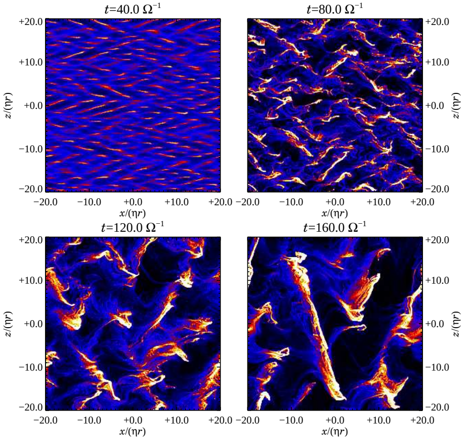




- Gas rotates slower than Keplerian because of radial pressure gradient
- Gas and solid components "stream" relative to each other
- Radial drift flow of solids is *linearly unstable*
- Growth on dynamical time-scale for marginally coupled solids (rocks/boulders)

Clumping

Linear and non-linear evolution of radial drift flow of meter-sized boulders ($\Omega_{\rm K}\tau_{\rm f} = 1$):

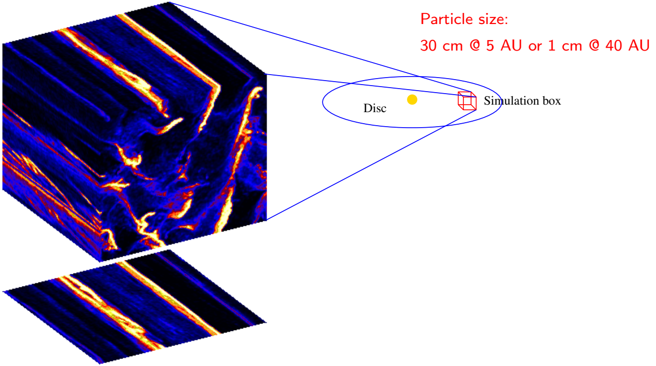


Strong clumping in non-linear state of the streaming instability

(Youdin & Johansen 2007, Johansen & Youdin 2007)

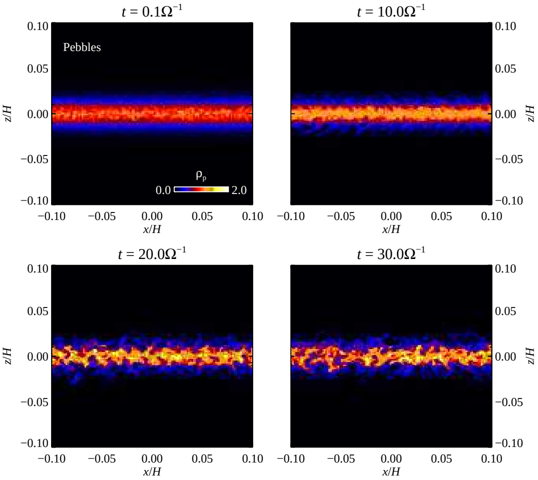
Clumping in 3-D

3-D evolution of the streaming instability:



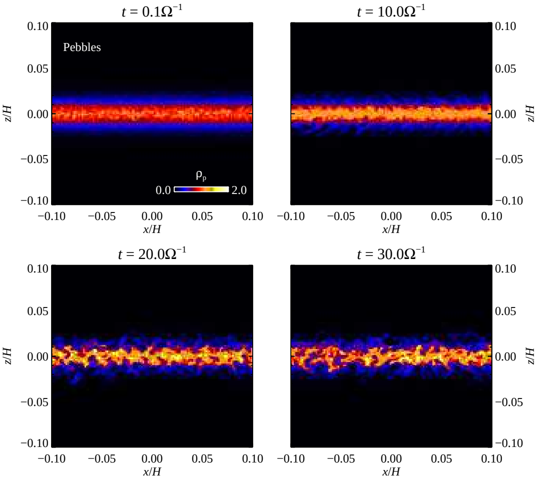
- Particle clumps have up to 100 times the gas density
- Clumps dense enough to be gravitationally unstable
- But still too simplified: no vertical gravity

Pebbles



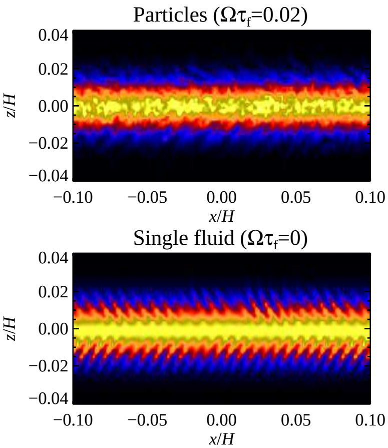
- Some overdense regions occur, but weak, and coupling with gas too strong for self-gravity to be important

Pebbles



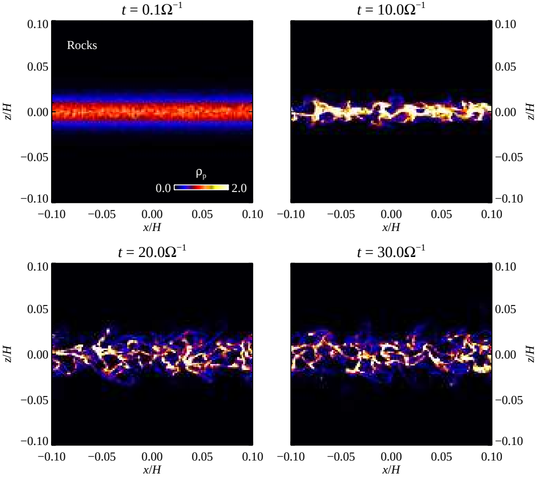
- Baroclinic instability of $u_y(z)$ shear? (Ishitsu & Sekiya 2002; Ishitsu et al. 2009)

Baroclinic instability?



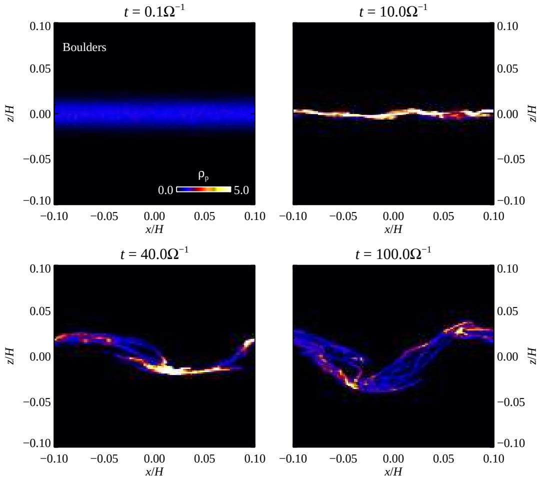
- Ishitsu & Sekiya (2002), Ishitsu et al. (2009)

Rocks



- Higher overdensities, due to the streaming instability, but still with short correlation times

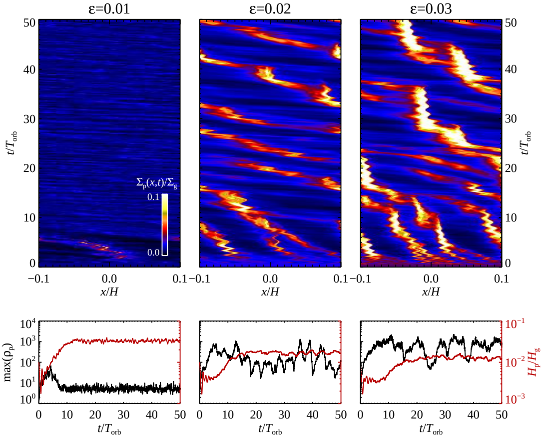
Boulders



- Almost no overdensities. Violent turbulent motion puffs up and dilutes mid-plane layer.

Clumping depends strongly on metallicity

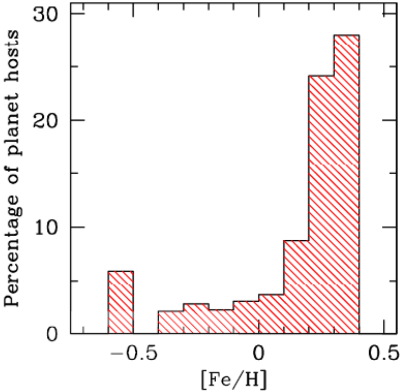
- Increase $\Sigma_{\rm par}/\Sigma_{\rm gas}$ from 0.01 to 0.03
- All particles between 1.5 and 15 centimetres



Johansen, Youdin, & Mac Low (in preparation)

The exoplanet zoo

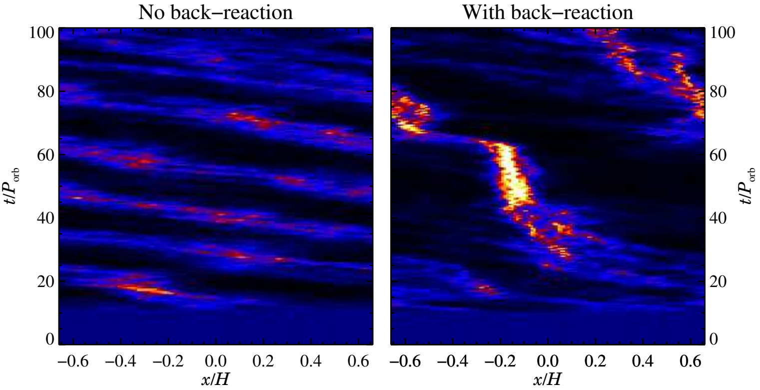
- First planet around solar-type star discovered in 1995 (Mayor & Queloz)
- Since then 340 planets discovered
- Exoplanet probability rises steeply with heavy element abundance of host star:



(Santos et al. 2004)

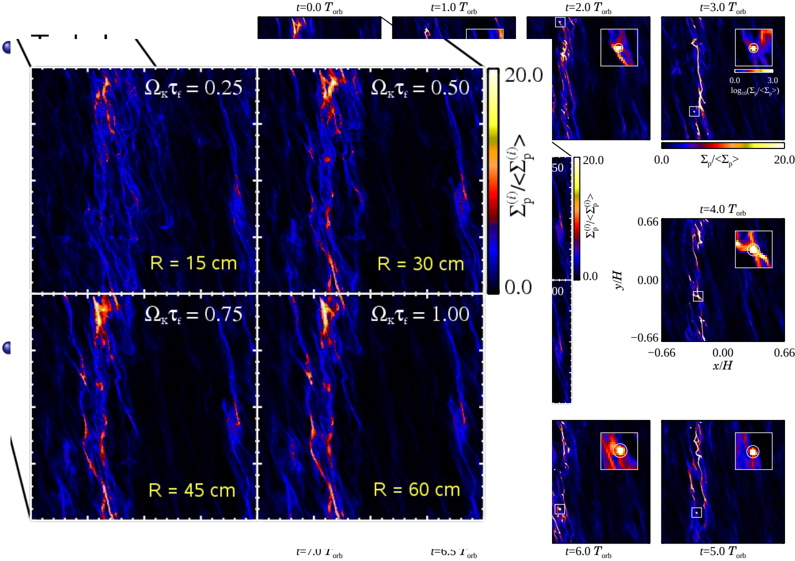
Overdense seeds

Dust column density as a function of radial coordinate x and time t measured in orbits:



Turbulent overdensities combined with streaming instability create transient, overdense "seeds" where self-gravity is important.

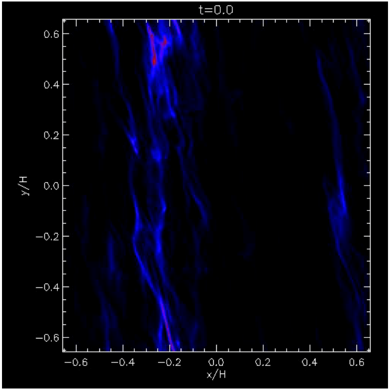
Formation of Ceres-mass object from rocks and boulders



Forming planet embryos

Time is in Keplerian orbits (1 orbit $\approx$ 10 years)

Keplerian flow



Keplerian flow

Johansen et al. 2007 (Nature, 448, 1022)

Forming planet embryos

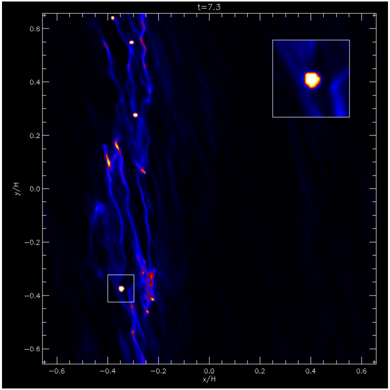
Planetesimal formation in turbulent protoplanetary discs

Anders Johansen

- Planet formation
- Dust in MHD turbulence
- Planetesimal formation
- Zonal flows
- Analytical model
- Global models
- **Streaming and self-gravity**
- Dead zones
- Conclusions

Time is in Keplerian orbits (1 orbit $\approx$ 10 years)

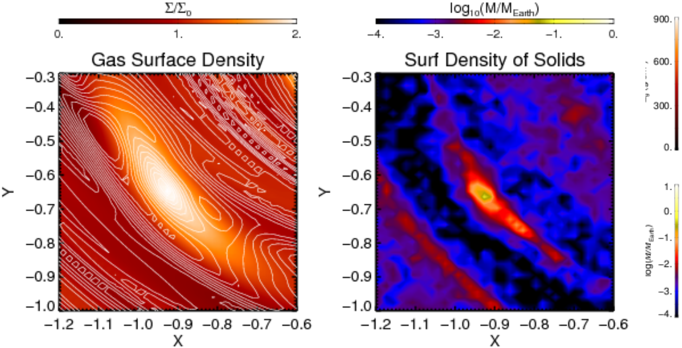
Keplerian flow



Keplerian flow

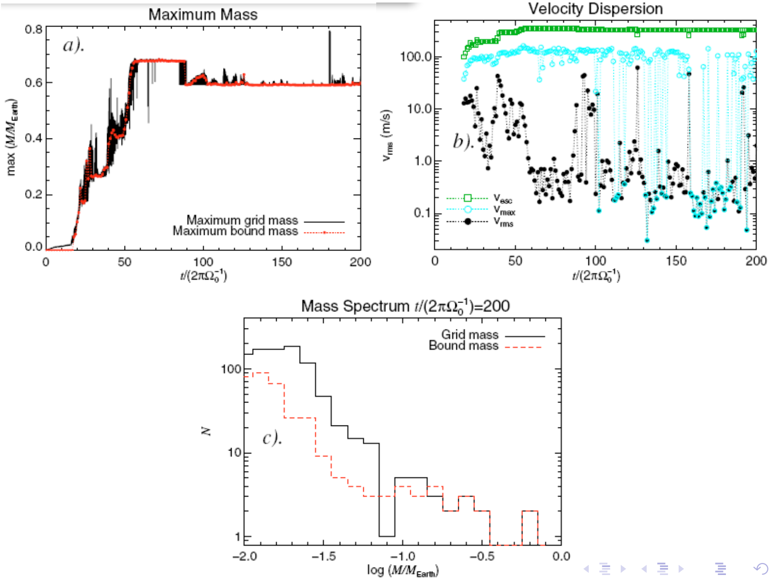
Johansen et al. 2007 (Nature, 448, 1022)

Dead zones

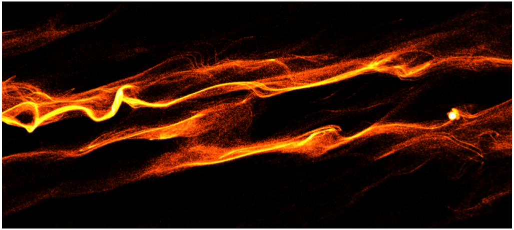


- Transition from active accretion to dead zones triggers Rossby wave instability in pile up of gas (Varnière & Tagger 2006; Inaba & Barge 2006)
- Rossby vortices trap particles
- Formation of Mars or Earth size planets by self-gravity
- Lyra et al. (2008, 2009)

Mass spectrum



Conclusions



- MRI can play a crucial role in the formation of planets
- Zonal flows are excited by ≈10% radial variation in the Maxwell stress of magnetorotational turbulence
- MRI and streaming instability can interact constructively
- Convergence zones concentrate solids and allow the formation of 1000 km sized planet embryos by gravity
- MRI good for planet formation even in its absence – Rossby vortices excited at transition from dead to active regions

Open questions

- What sets the scale of zonal flows?
- Do collision speeds of MRI turbulence lead to growth or to destruction of dust agglomorates?
- Can we even assume MRI to be operative in planet forming regions?
- Would turbulent simulations of dead zones lead to Rossby wave instability and vortices?
- How do you grow enough pebbles to launch the streaming instability?
- How does coagulation and fragmentation proceed in a gravitationally contracting clump?
- What is the relative importance of streaming, Kelvin-Helmholtz and baroclinic instabilities in the mid-plane layer?
- ...