SATELLITE FORMATION

SUPPLY OF SOLID MATERIAL ONTO CIRCUM-PLANETARY DISKS

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Satellite systems

- Regular and irregular satellites
  - Regular satellites:
    - Large fraction of total mass
    - Co-planner and circular orbits
    - Formed in circum-planetary disks
Structure of circum-planetary disks
Structure of circum-planetary disks

Tanigawa and Watanabe 2002

Machida 2009
Previous studies

- **Traditional model**
  - **Closed disk** model with the “Minimum Mass Sub-Nebula”
  - Several severe problems
    - Temperature, accretion time, type I migration ...

- **Canup and Ward model (2002, 2006)**
  - **Open disk** model based on the knowledge of gas accretion flow onto gas giant planets
    - Solid material is steadily supplied to circum-planetary disks
    - $M_{\text{satellites}} / M_{\text{planet}}$ is consistent with the real systems
  - Sasaki et al. is trying to explain the difference between Jovian and Saturnian systems.
Canup and Ward model

Steady mass supply

Planet

Gas
Canup and Ward model

Steady mass supply
Growth from outside

Planet

Gas
Canup and Ward model

Steady mass supply

Growth from outside

Larger planets move inward

Inner objects are swept
Canup and Ward model

- Steady mass supply
- Growth from outside
- Larger planets move inward
- Inner objects are swept

Continue until the mass supply terminates.

Current satellites are the last generation of this cycle.
Canup and Ward model

They reproduces total mass of satellite systems, but hard to explain the difference between Jovian and Saturnian systems
Sasaki, Stewart, and Ida model:
Did inner edge determine the difference between Jovian and Saturnian systems?

- Analogy of star formation
  - CTTS stage $\rightarrow$ strong magnetic field
    - Jupiter?
    - Inner edge exists
  - WTTS stage $\rightarrow$ magnetic field weakens
    - Saturn?
    - No disk edge?

- How about gas giant planets?
  - Jupiter can terminate its growth by forming a gap
    - Mass supply suddenly stop
    - Frozen in the stage corresponds to CTTS?
    - Satellites are stacked?
  - Saturn mass is insufficient to form a gap
    - Mass supply gradually decreases with dissipation of proto-planetary disks
    - Evolved through the stage corresponds to WTTS?
    - Satellites fall to the planet easily.
    - Large satellites are likely to be at outer region
Previous studies

- **Traditional model**
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- **Canup and Ward model (2002, 2006)**
  - *Open disk* model based on the knowledge of gas accretion flow onto gas giant planets
    - Solid material is steadily supplied to circum-planetary disk
    - $M_{\text{satellites}} / M_{\text{planet}}$ is consistent with the real systems.
    - Sasaki et al is trying to explain the difference between Jovian and Saturnian systems.

- **Assumptions**
  - Solid material is supplied uniformly on the disks.
Objective

To determine distribution of supplying rate of solid material onto circum-planetary disks from proto-planetary disks.

Two manners of supplying solid material

- Smaller size ( < m-size )
  - Strongly entrained by gas accretion flow
- Larger size ( > m-size )
  - Weakly affected by gas drag

An analytical estimation for larger size is shown.
Analytical model
Setting

Assumptions

- Axisymmetry of circum-planetary gas disk with power-law surface density distribution
- Pericenter of orbit just after captured in the Hill sphere does not change in the course of circularization.

Captured by gas drag with the disks
Capturing process

Before
Gravitational focusing

\[ r_{\text{min}} \propto r^2 \] (centered near the planet)
Capturing process

Before

Gravitational focusing

$r_{\text{min}} \propto r^2$ (centered near the planet)

Critical radius to be captured

$$R_c \propto m^{-1/3(p+1)} \quad \Sigma_{\text{gas}} = \Sigma_{\text{gas,0}} \left(\frac{r}{r_0}\right)^{-p}$$

Dissipation energy due to gas drag

Energy necessary to be captured by the gravitational potential
Capturing process

Before

Gravitational focusing

$\rightarrow r_{\text{min}} \propto r^2$ (centered near the planet)

Critical radius to be captured

$R_c \propto m^{-1/3(p+1)} \quad \Sigma_{\text{gas}} = \Sigma_{\text{gas,0}} \left( \frac{r}{r_0} \right)^{-p}$

Dissipation energy due to gas drag

$\parallel$

Energy necessary to be captured by the gravitational potential

After

Eccentricity and inclination decrease with keeping the pericenter
Supplying rate of solid material

For a single size swarm

\[ \text{Surface density / time for } m \propto r^{-1} \]

\( R_c = \text{Critical radius to be captured} \)
Supplying rate of solid material

For a single size swarm

\[
\text{Surface density / time for } m \propto r^{-1}
\]

(Larger size)

(Smaller size)

\(R_c = \text{Critical radius to be captured}\)
Supplying rate of solid material

For a single size swarm:

\[ \frac{\text{Surface density}}{\text{time for m}} \propto r^{-1} \]

Larger size \quad Smaller size

For a power-law size distribution:

\[ \dot{\sigma}_{\text{solid}} \propto r^{-1-3(p+1)(2-s)} \]

\[ \sigma_{\text{gas}} \propto r^{-p} \]

\[ n(m) \propto m^{-s} \]

\( R_c \) = Critical radius to be captured

A typical case (\( p=1, s=11/6 \))

\[ \dot{\sigma}_{\text{solid}} \propto r^{-2} \]
Supplying rate of solid

\[ \dot{\sigma}(\tilde{r}) = \frac{1}{2\pi} \left( \frac{9\pi}{128} C_D^3 (A(i))^3 \right)^{2-\alpha} \left( \frac{\sigma_{g,0}^3}{\rho_s^2 m_{\text{max}}} \right)^{2-\alpha} \left( \frac{\Sigma_d}{\Omega_K^{-1}} \right)^{\tilde{r}^{-1}-3(p+1)(2-\alpha)} \frac{d}{d\tilde{r}} P_{\text{col}}(\tilde{r}), \]

\[ A(i) = \frac{2(3 - 2\sqrt{2} \cos i)^{1/2}}{3 \sin i} (\sqrt{2} - \cos i) \]

Typically \( (\alpha = 11/6, \ a = 5\text{AU}) \)

\[ \dot{\sigma}(r) \sim 10^3 \text{g cm}^{-2} \text{ yr}^{-1} \left( \frac{\sigma_{g,0}}{10^3 \text{g cm}^2} \right)^{1/2} \left( \frac{m_{\text{max}}}{10^{18} \text{g}} \right)^{-1/6} \left( \frac{\Sigma_d}{10 \text{g cm}^{-2}} \right) \left( \frac{\rho_s}{1 \text{g cm}^{-3}} \right)^{-1/3} \left( \frac{r}{10 R_J} \right)^{-2} \]

Tanigawa and Ikoma 2007

Mass supplying rate \( \propto (\text{gas surface density})^{1/2} \)

Dust/gas ratio increases with decreasing disk gas?

Satellite formation promotes late stage of formation of gas giant?
Migration due to gas drag?

After circularization with short timescale, objects slowly spiral toward the planets by gas drag

Migration velocity due to the gas drag with disk gas:

\[ \nu_{r,s} = -2\eta\Gamma v_K \]

\[ \Gamma = \frac{\tau_K}{\tau_{\text{stop}}} \]

\[ \eta \sim \left(\frac{c}{v_K}\right)^2 \]

How about the steady state distribution?
Steady state distribution considering radial migration due to gas drag

For a single size swarm

\[ \dot{\sigma}_s \propto r^{-1} \]

\[ v_{r,s} = -2\eta \Gamma \nu_K \]

Surface density / time for \( m \)

Supplying rate

\( R_c \)
Steady state distribution
considering radial migration due to gas drag

For a single size swarm

Supplying rate

Surface density / time for $m$

\[ \dot{\sigma}_s \propto r^{-1} \]

\[ v_{r,s} = -2\eta \Gamma v_K \]

Steady state distribution

Surface density for $m$

\[ \sigma_s = \frac{\dot{M}_s(r, m)}{2\pi rv_{r,s}} \]

\[ \sigma_s \propto r^{q/2-p-1} \]

\[ \dot{M}_s(r, m) = \int_r^{R_c} 2\pi r' \dot{\sigma}_s(r', m)dr' \]

$R_c$
Steady state distribution considering radial migration due to gas drag

For a single size swarm

Supplying rate

\[ \hat{\sigma}_s \propto r^{-1} \]

\[ \nu_{r,s} = -2\eta \Gamma v_K \]

Surface density / time for \( m \)

\[ \sigma_s = \frac{M_s(r, m)}{2\pi r \nu_{r,s}} \]

Steady state distribution

Surface density for \( m \)

\[ \sigma_s \propto r^{q/2 - p - 1} \]

Surface density

\[ \dot{M}_s(r, m) = \int_{r'}^{R_c} 2\pi r' \hat{\sigma}_s(r', m) dr' \]

For a power-law size distribution

\[ \Sigma_s(r) = \int_{m_1}^{m_c(r)} \sigma_s(r, m) dm \]

\[ \Sigma_s \propto r^\theta \]

\[ \theta = p - q/2 \]

\[ -\max((p+1)(7-3s), 0) \]

\[ T_{\text{gas}} \propto r^{-q} \]

\[ \sigma_{\text{gas}} \propto r^{-p} \]

\[ n(m) \propto m^{-s} \]

\[ p = 1, \quad q = 1/2, \quad s = 11/6: \]

\[ \Sigma_s \propto r^{-9/4} \]
Test orbital calculations for captured satellitesimals
Basic equations

Equation of motion
\[ \frac{d\vec{v}}{dt} = -\nabla \tilde{\Phi}_{\text{hill}} - 2e_z \times \vec{v} + \tilde{a}_{\text{drag}} \]

Hill’s potential
\[ \tilde{\Phi}_{\text{hill}} = -\frac{3}{r} - \frac{3}{2} \tilde{x}^2 + \frac{1}{2} \tilde{z}^2 + \frac{9}{2} \]

Gas drag term
\[ \tilde{a}_{\text{drag}} = -\frac{3}{8} C_D \left( \frac{\rho_g}{\rho_S} \right) \tilde{r}_S^{-1} \Delta \tilde{u} \Delta \tilde{u} \quad (\text{Only inside the Hill’s sphere}) \]

Hydrostatic equilibrium in z-direction and axisymmetric

\[ \rho_g(r, z) = \rho_0 r_{\text{AU}}^p \exp\left(-\frac{z^2}{2h_g^2}\right) \]

\[ \Omega_g(r, z) = \Omega_{K,\text{mid}} \left[ 1 + \frac{1}{2} \left( \frac{h_g}{r} \right)^2 \left( p + q + \frac{q z^2}{2h_g^2} \right) \right] \]

\[ c^2(r) = c_0^2 r_{\text{AU}}^q \]
Example orbits

Hill’s coordinate
(A local coordinate that rotates with the planet)
Example orbits \( (e=i=0, b=2.35, 2.41) \)

- Prograde
- Retrograde

\[ j_z = r \times v_z \]
Summary

- Solid supply onto circum-planetary disks
  - Capture of planetesimals by gas drag with circum-planetary disks
  - Analytical estimation
    - Distribution of solid supplying rate
      - Gradients of solid and gas surface density is generally different.
        - Dust/gas ratio is a function of radius
        - Dependence of solid supplying rate on gas surface density
          - Proportional to (gas surface density)$^{1/2}$
          - → Dust/gas ratio increases in the late stage

\[
\dot{\sigma}_{\text{solid}} \propto r^{-1-3(p+1)(2-s)}
\]

Typical case

\[
\dot{\sigma}_{\text{solid}} \propto r^{-2}
\]

\[
\dot{\sigma}_{\text{solid}} \propto r^{-1}
\]

cf. \(\sigma_{\text{gas}} \propto r^{-p}\)

for m – km size (s=11/6)

for larger than 1km size (s=8/3)

generally different.

Typical case