Waves in Partially Ionized Solar Atmosphere

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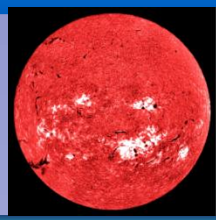

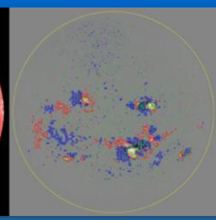

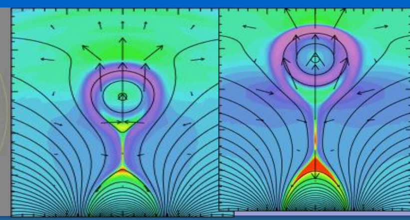

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Abstract

The partially ionized part of the solar atmosphere is investigated, in order to study the wave propagation, within the framework of a single- fluid MHD description including the non-ideal effects such as the Hall and the ambipolar diffusion in the generalized Ohm's law.

MHD Equations

Momentum eq. for electron, ion &neutral

$$m_e n_e\left[\frac{\partial \mathbf{V}_e}{\partial t}+(\mathbf{V}_e\cdot\nabla)\mathbf{V}_e\right]=-e n_e\left[\mathbf{E}+\frac{\mathbf{V}_e\times\mathbf{B}}{c}\right]-\gamma_{en}\rho_e(\mathbf{V}_e-\mathbf{V}_n)-\gamma_{ei}\rho_e(\mathbf{V}_e-\mathbf{V}_i)$$

$$m_i n_i\left[\frac{\partial \mathbf{V}_i}{\partial t}+(\mathbf{V}_i\cdot\nabla)\mathbf{V}_i\right]=e n_i\left[\mathbf{E}+\frac{\mathbf{V}_i\times\mathbf{B}}{c}\right]-\gamma_{in}\rho_i(\mathbf{V}_i-\mathbf{V}_n)-\gamma_{ie}\rho_i(\mathbf{V}_i-\mathbf{V}_e)$$

$$m_n n_n\left[\frac{\partial \mathbf{V}_n}{\partial t}+(\mathbf{V}_n\cdot\nabla)\mathbf{V}_n\right]=-\gamma_{ni}\rho_n(\mathbf{V}_n-\mathbf{V}_i)-\gamma_{ne}\rho_n(\mathbf{V}_n-\mathbf{V}_e)$$

The electrons are inertialess (i.e. m_e =0). For δ << 1, the ion dynamics can be ignored. This gives Ohm's Law in the electron's case as

$$\mathbf{E}=-\frac{\mathbf{V}_e\times\mathbf{B}}{c}-\frac{\gamma_{en}\rho_e}{e n_e}(\mathbf{V}_e-\mathbf{V}_n)-\frac{\gamma_{ei}\rho_e}{e n_e}(\mathbf{V}_e-\mathbf{V}_i)$$

The ion force balance equation now becomes

$$0=e n_i\left[\mathbf{E}+\frac{\mathbf{V}_i\times\mathbf{B}}{c}\right]-\gamma_{in}\rho_i(\mathbf{V}_i-\mathbf{V}_n)-\gamma_{ie}\rho_i(\mathbf{V}_i-\mathbf{V}_e)$$

These equations ultimately lead to

$$\rho_n\left[\frac{\partial \mathbf{V}_n}{\partial t}+(\mathbf{V}_n\cdot\nabla)\mathbf{V}_n\right]=\frac{\mathbf{J}\times\mathbf{B}}{c}$$

and an induction equation

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times\left[\mathbf{V}_n\times\mathbf{B}-\frac{\mathbf{J}\times\mathbf{B}}{e n_e}+\frac{(\mathbf{J}\times\mathbf{B})\times\mathbf{B}}{c\,\gamma_{in}\rho_i}-\eta(\nabla\times\mathbf{B})\right]$$

The Dispersion Relation

$$\omega^2-\omega\left[\pm\eta_H+i(\eta+\eta_A)\right]k_z^2-k_z^2 V_{Ai}^2\delta=0$$

where

$$\delta=\frac{\rho_{oi}}{\rho_{0n}},\ V_{Ai}=\frac{B_0}{(4\pi\rho_{oi})^{0.5}}$$

$$\eta=\frac{c^2}{\omega_{pe}^2}(\gamma_{ei}+\gamma_{en}),\ \eta_H=\frac{c B_0}{4\pi e n_e},\ \eta_A=\frac{V_{Ai}^2}{\gamma_{in}}$$

and the collision frequencies are given by (Khodachenko et al. 2004)

$$\gamma_{ei}=5.89\times10^{-24}\frac{n_i \Lambda Z^2}{k_B^{1.5}T^{1.5}},$$

$$\gamma_{en}=n_n\sqrt{\frac{8k_BT}{\pi m_{en}}}\Sigma_{en},$$

$$\gamma_{in}=n_n\sqrt{\frac{8k_BT}{\pi m_{in}}}\Sigma_{in},$$

$$\Sigma_{en}\sim10^{-15}cm^2,\ \Sigma_{in}\sim5\times10^{-15}cm^2$$

$$m_{kn}=\frac{m_k m_n}{m_k+m_n},\ k=e,i$$

$$\omega=\bar{\omega}\omega_0,\ \bar{k}=k_z\lambda_i,\ V=\bar{V}\omega_0\lambda_i,\ \eta=\bar{\eta}\omega_0\lambda_i^2$$

Physical parameters in the solar atmosphere

h (in 10^5 cm)	T (K)	ρ_i (g cm^{-3})	ρ_n (g cm^{-3})	B (G)	β_i
0	6520	1.0×10^{-10}	1.90×10^{-7}	1200	9.4×10^{-4}
50	5790	1.2×10^{-11}	1.59×10^{-7}	1125.77	1.2×10^{-4}
125	5270	1.18×10^{-12}	1.00×10^{-7}	980.16	1.3×10^{-5}
175	5060	3.39×10^{-13}	7.04×10^{-8}	880.33	4.6×10^{-6}
250	4880	9.37×10^{-14}	3.89×10^{-8}	737.21	1.7×10^{-6}
400	4560	1.12×10^{-14}	1.09×10^{-8}	503.71	4.2×10^{-7}
490	4410	4.37×10^{-15}	4.84×10^{-9}	394.42	2.6×10^{-7}
560	4430	4.72×10^{-15}	2.47×10^{-9}	322.27	4.2×10^{-7}
650	4750	2.29×10^{-14}	1.00×10^{-9}	246.31	3.7×10^{-6}
755	5280	1.08×10^{-13}	3.79×10^{-10}	183.67	3.5×10^{-5}
855	5650	1.75×10^{-13}	1.66×10^{-10}	143.40	1.0×10^{-4}
980	5900	1.78×10^{-13}	6.57×10^{-11}	108.65	1.8×10^{-4}
1065	6040	1.67×10^{-13}	3.61×10^{-11}	90.88	2.5×10^{-4}

Mixed Modes: a more general case

Including the compression term $-\nabla p_n$ and the oblique propagation, we show that the modes are mixed.

The pressure-perturbation term is $p_{n1} = C_S^{\,2} \rho_{1n}$ and

the sound speed is $C_s = \sqrt{\dfrac{\gamma\, p_{0n}}{\rho_{0n}}}$

From the continuity equation, we have $\dfrac{\rho_{1n}}{\rho_{0n}} = \dfrac{(\boldsymbol{k} \cdot \boldsymbol{V}_{1n})}{\omega}$

Using the continuity equation, momentum equation for the neutrals and the induction equation for a partially ionized plasma, a determinant *D* (*ω*) is given by:

$$D(\omega) = [\omega^2 - (V_{Ai}^{\,2}\delta + i\eta_A\omega)k_z^{\,2} - i\eta k^2\omega][\omega^4 - i(\eta_A+\eta)k^2\omega^3 - (V_{Ai}^{\,2}\delta + C_s^{\,2})k^2\omega^2 + i(\eta_A+\eta)C_s^{\,2}k^4\omega$$
$$k^2 k_z^{\,2} C_s^{\,2} V_{Ai}^{\,2}\delta] - \eta_H^{\,2} k^2 k_z^{\,2}\omega^2\left(\omega^2 - k^2 C_s^{\,2}\right)$$

It can be seen that the magnetoacoustic and Alfvén-like modes are mixed.

Typical plasma parameters in a weakly ionized solar atmosphere

h (in 10^5 cm)	δ	γ_{ei} (s^{-1})	γ_{en} (s^{-1})	γ_{in} (s^{-1})	η ($cm^2\ s^{-1}$)	η_H ($cm^2\ s^{-1}$)	η_A ($cm^2\ s^{-1}$)
0	5.08×10^{-4}	6.22×10^{9}	5.92×10^{9}	9.78×10^{8}	4.46×10^{7}	7.74×10^{7}	1.2×10^{6}
50	7.96×10^{-5}	9.41×10^{8}	4.51×10^{9}	7.45×10^{8}	7.8×10^{7}	2.82×10^{8}	1.1×10^{7}
125	1.18×10^{-5}	1.0×10^{8}	2.71×10^{9}	4.48×10^{8}	1.02×10^{8}	6.26×10^{8}	1.4×10^{8}
175	4.8×10^{-6}	3.07×10^{7}	1.86×10^{9}	3.07×10^{8}	1.08×10^{8}	8.84×10^{8}	5.9×10^{8}
250	2.4×10^{-6}	8.96×10^{6}	1.0×10^{9}	1.66×10^{8}	1.09×10^{8}	1.38×10^{9}	2.7×10^{9}
400	1.02×10^{-6}	1.18×10^{6}	2.74×10^{8}	4.53×10^{7}	1.06×10^{8}	3.4×10^{9}	3.9×10^{10}
490	9.03×10^{-7}	4.87×10^{5}	1.19×10^{8}	1.97×10^{7}	1.02×10^{8}	5.94×10^{9}	1.4×10^{11}
560	1.91×10^{-6}	5.22×10^{5}	6.11×10^{7}	1×10^{7}	9.86×10^{7}	9.06×10^{9}	1.7×10^{11}
650	2.27×10^{-5}	2.28×10^{6}	2.58×10^{7}	4.26×10^{6}	8.83×10^{7}	1.36×10^{10}	4.9×10^{10}
755	2.86×10^{-4}	9.22×10^{6}	1.02×10^{7}	1.68×10^{6}	5.67×10^{7}	9.42×10^{9}	1.5×10^{10}
855	1.05×10^{-3}	1.34×10^{7}	4.63×10^{6}	7.65×10^{5}	4.24×10^{7}	9×10^{9}	1.2×10^{10}
980	2.7×10^{-3}	1.28×10^{7}	1.87×10^{6}	3.09×10^{5}	3.64×10^{7}	4.72×10^{9}	1.7×10^{10}
1065	4.61×10^{-3}	1.16×10^{7}	1.04×10^{6}	1.72×10^{5}	3.41×10^{7}	4.31×10^{9}	2.3×10^{10}

Using Cox et al. (2000), Khodachenko et al. (2004) and Table 1
B_0 = 1200 G

Conclusions

- **The damping of Alfvén- like mode in a weakly ionized part of the solar atmosphere is mainly caused by the electron-neutral collisions and the ion-neutral collisions (through Cowling diffusivity).**

- **Cowling diffusivity is dominant beyond the height 175 km above the solar surface for the solar model given by Cox (2000) and chosen magnetic field.**

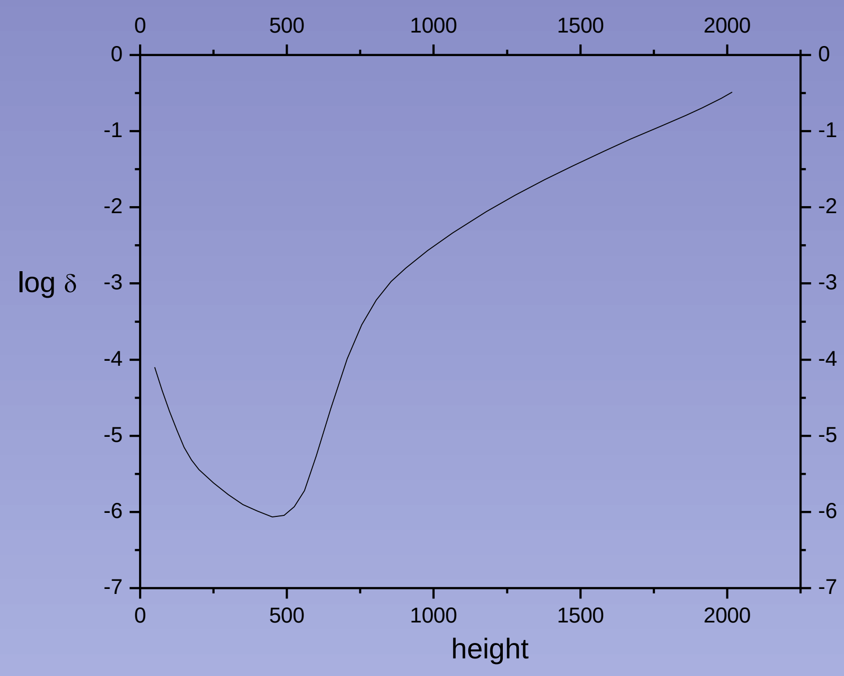
- **The Hall effect introduces a strong dispersion to the Alfvén- like mode in a partially ionized solar atmosphere. It has been shown clearly that the symmetry between co- and counter-propagating wave modes breaks** *at the length scale approaching the Hall length scale*

- **In the presence of Hall effect the Alfvén- like mode is *circularly* polarized whereas in the absence of it, the Alfvén- like mode is linearly polarized. The, Hall effect facilitates propagation of short- wavelength modes required for the heating of the solar plasma.**

Acknowledgements

KAPS gratefully acknowledges Prof. Vinod Krishan for enlightening on the topic. A detailed version of the poster has been accepted for publication in the journal NEW ASTRONOMY.

Figures

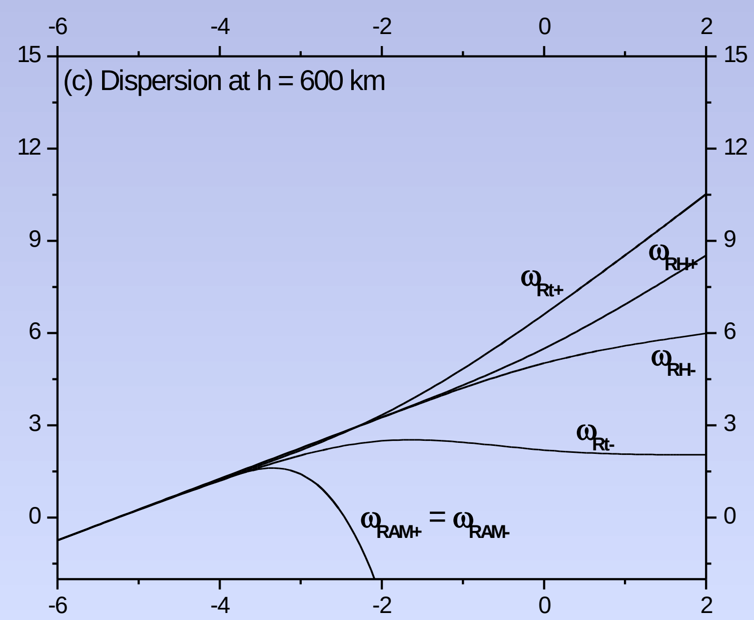


The ratio of ion to neutral density is shown (log axis) as a function of height (in units of 10^5 cm) in the solar atmosphere

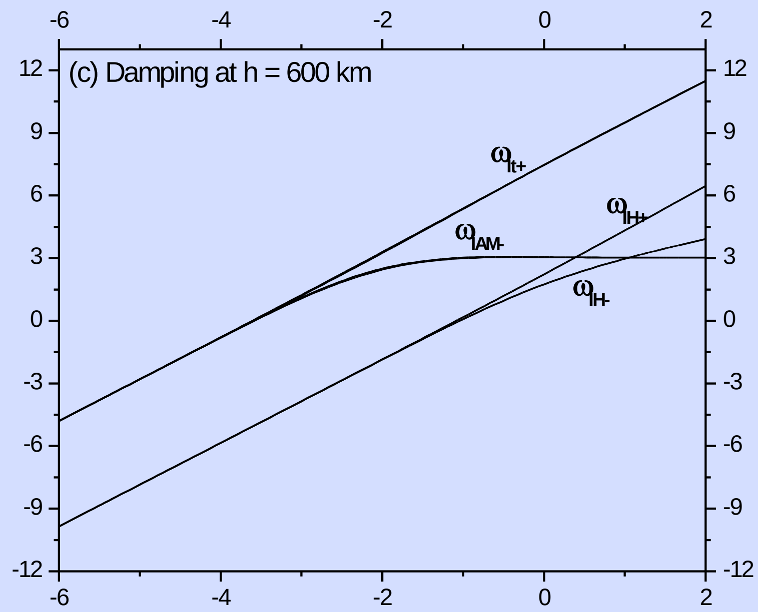
The lines $\omega_{RH\pm}$ are the dispersion curves for co- and counter-propagating Hall Alfvén waves

The lines $\omega_{RAM\pm}$ are the the dispersion curves for Alfvén waves including , ambipolar diffusion

The lines $\omega_{Rt\pm}$ are the the dispersion curves for Alfvén waves including both the Hall and ambipolar diffusion



Dispersion of wave modes at h = 600 km



Damping of wave modes at h = 600 km