Dust in Planetary Systems, Jena, 2010

25min

Electric Processes in Protoplanetary Disks

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Part 0 The Science is in Crisis

The Hakubi Center, Kyoto University



- Unique, long-sighted, young researchers wanted from all the world, to save the science
- max. 20 people / year, 5 years position
- Natural Science, Social Science, Engineering... all OK
- Salary of Assistant Prof. / Associate Prof. + research funding
- No mid-career assessment & lay-off
- No education duty
- No PhD required to apply
- No tenure track

Dust–dust collisional charging and lightning in protoplanetary discs

Takayuki Muranushi Mon. Not. R. Astron. Soc. 401, 2641–2664 (2010)

Part 1 Introduction To Lightning

Charging Mechanism Unique to Ice Surface, Crucial to Lightning



 Ice surface exchanges charge when collided together, or when collided with other materials

Charging Mechanisms for Solid Surface

Triboelectric Charge

- exchange of electron between matters due to different electron affinity
- Disk lightning mechanism candidate (Desch & Cuzzi, 2000)



n

Surface Charge

- free electron wave function spill-out
- widely seen in conductors and semiconductors (Somorjai 1994)

 10^{-7} cm

2 X

10⁻⁴cm

Ice Surface Charge

- charge transport by molecular ions H₃O⁺
- Unique to H₂O (e.g. NH₃
 lacks this mechanism:
 Goncalves+ 1999)
- deep, large separation

 $6.2 \times 10^9 \,\mathrm{e} \,\mathrm{cm}^{-2}$



\star Lightning on Earth \star

Rapid generation of small ice dust : Dusts with Rough surface, Large Charge Separation, coupled to fluid

2.

Slow mantle accretion of large dust : Dusts with Smooth surface, Little Charge Separation, decoupled from fluid

3.

Collision and Charging of Ice dust

Charge Separation of ~ 50[C] , in height ~ km

(electric force \sim 1% of gravity)



Krehbiel et al. (1983), Preceedings in Stomospheric Electricity (From Okuzumi's Talk)

Radial drift may be a FRIEND of dust growth!



If so, collision of very different-sized aggregates will be important.

Dust Charging Processes in Protoplanetary Disks



Charge by plasma absorption (Okuzumi, 2009)

All dusts are weakly negative Blocks the dust growth at μ m stage



Charge by dust-dust collision (Gaskell et.al. 1978 ...)

Dusts are charged in both sign, and very large compared to plasma absorption.

Part 2 Lightning in Protoplanetary Disks

Lightning in Protoplanetary discs?

(Horejsi 1997)

Chondrules

are silicate spherical grains of mm size, found today in wide area of our solar system. They are probably formed in protoplanetary nebula, when fluffy silicate dust aggregates are subject to ~1200K heat, melted, become spheric by surface tension, then frozen.

has long been studied

- as a candidate mechanism of chondrule heating
- Literatures generally says that lightning do not take place, or do not contribute to chondrule formation.
- There are problems ...

Three Problems against Lightning and Chondrule Formation by Lightning

①Neutralization Problem (Gibbard ,Levy & Morfill 1997)

- Lightning do not occur. Since PPD is weakly ionized and conductive, and dust is few, generated E field is too weak. $E \propto n^2$ so you need unnaturally high dust number density.

②Energetics Problem (e.g. Weidenschilling 1997)

 If lightning do occur, it isn't energetic enough to melt all the chondrules observed today, because the budget is turbulent energy --- a little portion of total kinetic energy

③Destruction Problem (Güttler+ 2008)

 If lightning do occur, experiments show that it destroys the dust aggregates rather than melts them. 13

New pictures in protoplanetary disc



Suyama, Wada et.al. ,2007



Cuzzi & Zahnle, 2003

- Dust aggregates are fluffy (low fractal dimension).
- Regions with higher dust number density than MMSN are possible. Disks with more dust are found

- e.g. $3 \times 10^{-3} M_{SUN}$ dust for a $0.5 M_{SUN}$ star, Sauter+ 2009

- Let us take these into consideration and re-estimate if lightning do take place (Muranushi, 2010)
- Default location is 2.7AU@MMSN equatorial

The 'Circuit' within protoplanetary disk



- Large dust(L); Small dust(S); ions(i); electrons(e); are involved
- In this talk e.g. Small dust(S) is Cationic (receives positive charge from collision); Large dust(L) is Anionic. The other polarity is also possible.

Estimation of Currents[esu cm⁻³ s⁻¹]

Gas-phase Ionization

Plasma Absorption

$$J_{i,e} = -e\zeta n_g. \qquad J_{L,i} = -Q_i n_L \sigma_{Cou} \left(\frac{Q_L}{n_L}, e, r_L, k_B T\right) v_i$$

$$J_{\rm L,e} = -Q_{\rm e} n_{\rm L} \sigma_{\rm Cou} \left(\frac{Q_{\rm L}}{n_{\rm L}}, -e, r_{\rm L}, k_{\rm B} T\right) v_{\rm e}$$

where

 $\zeta=10^{-18}$ (Umebayashi & Nakano, 2009)

$$J_{\mathrm{S,i}} = -Q_{\mathrm{i}}n_{\mathrm{S}}\sigma_{\mathrm{Cou}}\left(\frac{Q_{\mathrm{S}}}{n_{\mathrm{S}}}, e, r_{\mathrm{S}}, k_{\mathrm{B}}T\right)v_{\mathrm{i}}$$

$$\sigma_{\text{Cou}}(q) = \pi a^2 \exp\left(-\frac{qq'}{ak_{\text{B}}T}\right) \qquad (qq' > 0),$$

$$J_{\rm S,e} = -Q_{\rm e} n_{\rm S} \sigma_{\rm Cou} \left(\frac{Q_{\rm S}}{n_{\rm S}}, -e, r_{\rm S}, k_{\rm B} T\right) v_{\rm e}$$

$$\sigma_{\text{Cou}}(q) = \pi a^2 \left(1 - \frac{qq'}{ak_{\text{B}}T} \right) \qquad (qq' < 0)$$

Dust-Dust Collisional Charging and Collisional Neutralization

Three particles involved: fractal dimension and radius

Large dust: $D_{\rm L}$, $r_{\rm L}$

Small dust: $D_{\rm S}$, $r_{\rm S}$

 \bigcirc Monomer : $r_{\rm m}$

Grazing collision \rightarrow charge separation cross section : $\sigma_{SL} = r_S r_L$ Burying collision \rightarrow neutralization cross section : $\sigma_{SL(N)} = r_L^2$ Relative velocity: v_{SL} Event rate for grazing collision: $\sigma_{SL} n_S n_L v_{SL} [\text{cm}^{-3} \text{ s}^{-1}]$



Dust-Dust Collisional Charging and Collisional Neutralization 2

Ice crystal surface charge density : $\sigma_{ch} = 6.2 \times 10^9$ [e cm⁻²] Charge exchange efficiency : $\eta_{ch} = 10^{-1}$

(c.f. Dash et.al. 2001)

Ice surface involved in contact : smaller of

An Ice-Dust Charge Model with Dust-Dust Collision Taken into Account



- Solve charge exchange equilibrium of four components : Large dust(L); Small dust(S); ions(i); electrons(e) in Local Box (zero-dimensional) model. Gas phase ionization, plasma absorption to dust, dust-dust collision are considered.
- Large dust has some mean motion to other components and carries electric current; plasma creates neutralizing current and Electric field is determined by Ohm's law.
- Dust space density is *N* times that of MMSN model :

$$ho_{\rm S}$$
 = N × 10⁻² $ho_{\rm g}$, $ho_{\rm L}$ = N × 10⁻³ $ho_{\rm g}$

Calculating Dust Charge with Dust-Dust Collision Taken into Account



- Parametes are: dust aggregate radius $r_{\rm S}$, $r_{\rm L}$; fractal dimension $D_{\rm S}$, $D_{\rm L}$; nondimensional dust density N $\rho_{\rm S} = N \times 10^{-2} \rho_{\rm g}$, $\rho_{\rm L} = N \times 10^{-3} \rho_{\rm g}$ N = 100: Equal mass of dust and gas per volume
- Unknowns are: charge density of each component $Q_{I}(r_{S},r_{L}, D_{S},D_{L};N)$ Electric field: $E(r_{S},r_{L}, D_{S},D_{L};N)$
- The First Question : How does charge distributions Q_i , Q_e , Q_S , Q_L , Electric field: *E*, change as dust number density is increased?

Charge Evolution Equation for Dust Plasma

$$\frac{dq_{L}}{dt} = -\Delta q \ n_{s}\sigma_{SL}\Delta v_{SL} + e(n_{i}\sigma_{iL}v_{i} - n_{e}\sigma_{eL}v_{e})$$
Dust-dust collision/neutralization

$$\frac{dq_{s}}{dt} = +\Delta q \ n_{L}\sigma_{SL}\Delta v_{SL} + e(n_{i}\sigma_{iS}v_{i} - n_{e}\sigma_{eS}v_{e})$$
Plasma absorption

$$\frac{dn_{i}}{dt} = \zeta n_{g} - (n_{i}n_{L}\sigma_{iL}v_{i} + n_{i}n_{S}\sigma_{iS}v_{i})$$
Calculate Equilibrium
Solutions $Q_{I}(r_{S}, r_{L}, D_{S}, D_{L}; N)$
and their N dependence

 $n_{\rm I}$: number densities (I = L, S, *i*, *e*, *g*)

 q_L, q_S : charge per patricle

 $\Delta q (q_L, q_S)$: amount of charge exchanged in a single dust-dust collision event ΔV_{LS} : relative velocity between large and small dust, default = 3400cm/s ζ : gas ionization ratio

N: nondimensional dust density : $\rho_{\rm S} = N \times 10^{-2} \rho_{\rm g}$, $\rho_{\rm L} = N \times 10^{-3} \rho_{\rm g}$

Charge distribution of dust plasma



Charge distribution of dust plasma has four phase



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 J_{ei}

Electric field generated by dust motion



The Second Question: $N_{\text{crit}}(r_{\text{S}}, r_{\text{L}}, D_{\text{S}}, D_{\text{L}})$?

- Lightning takes place if dust number density larger than $N_{\rm crit}$ (in unit of MMSN)
- The Second Problem: calculate $N_{\rm crit}$ as a function of dust radii $(r_{\rm S}$, $r_{\rm L})$ and fluffiness $(D_{\rm S}$, $D_{\rm L})$





$\log_{10}(N_{\rm crit})$					
0	1 2	3	4 5	6	
r _S [cm]	r _L [cm]	D _S	D _L	N _{crit}	
10-2	102	2.0	2.4	73	
10-4	10^{0}	2.0	3.0	90	
10-2	10 ²	2.0	2.6	26	

And how I made this simulation 100'000 times faster ... that's another story.

Solver	Number of cases	Time	Optimization
Direct integral, CPU ^a	9885	636714	1.0
Binary search, CPU ^b	19 200	977.973	1.3×10^{3}
Binary search, GPU ^c	19 200	6.987 05	1.8×10^{5}
Final problem ^d	364 325	226.469	1.0×10^{5}

analytic form for $N_{\rm crit}$ accurate & precise!





simulation results: colour map & dashed contour

 $= \eta_{crit}^{(c)}$

 $\eta_{\text{crit}} = \eta_{\text{crit}}^{(d)}$ if $\eta_{\text{crit}}^{(d)} > \eta^{(cd)}$,

otherwise.

analytic formulae: colour contour

Only 3 uncertain parameters in analytic formulae: fits 364'325 points with 9.2% precision!

$$= \eta^{(cd)} \text{ if } \eta^{(c)}_{crit} > \eta^{(cd)} > \eta^{(d)}_{crit},$$

$$= \eta^{(c)}_{crit} \text{ otherwise.}$$
(122)
$$\eta^{(i)}_{ort} = 1.1 \times 10^{3} \left(\frac{\Delta q_{A,C}}{62 \times 10^{2} e}\right)^{-\frac{1}{4}} \left(\frac{n_{E}}{4.7 \times 10^{13} cm^{-3}}\right)^{\frac{1}{4}} \left(\frac{n_{S}^{S}}{8.8 \times 10^{-1} cm^{-3}}\right)^{-\frac{1}{4}} \left(\frac{n_{S}^{S}}{1.0 \times 10^{-4} cm^{-3}}\right)^{-\frac{1}{4}} \left(\frac{n_{S}^{S}}{1.0 \times 10^{-4} cm^{-3}}\right)^{-\frac{1}{4}} \left(\frac{\Delta v_{L,S}}{1.0 \times 10^{-4} cm^{-3}}\right)^{-\frac{1}{4}} \left(\frac{\Delta v_{L,S}}{1.0 \times 10^{-18} cm^{-1}}\right)^{-\frac{1}{4}} \left(\frac{\Delta v_{L,S}}{1.0 \times 10^{-18} cm^{-1}}\right)^{-\frac{1}{4}} \left(\frac{\Delta w_{L,S}}{1.5.4 ev}\right)^{\frac{1}{2}} \left(\frac{T}{1.7 \times 10^{2} K}\right)^{-\frac{1}{2}},$$
(124)
$$\eta^{(cd)} = 3.3 \times 10^{2} \left(\frac{\Delta q_{A,C}}{6.2 \times 10^{2} e}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{4.7 \times 10^{13} cm^{-3}}\right)^{-\frac{1}{4}} \left(\frac{\pi s}{3.4 \times 10^{2} cm s^{-1}}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{1.0 \times 10^{-16} cm^{-3}}\right)^{-\frac{1}{2}} \left(\frac{\pi s}{1.0 \times 10^{-16} cm^{-3}}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{3.4 \times 10^{2} cm s^{-1}}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{3.4 \times 10^{2} cm s^{-1}}\right)^{-\frac{1}{2}},$$
(124)
$$\eta^{(cd)} = 3.3 \times 10^{2} \left(\frac{\Delta q_{A,C}}{6.2 \times 10^{2} e}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{4.7 \times 10^{13} cm^{-3}}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{3.4 \times 10^{2} cm s^{-1}}\right)^{-\frac{1}{2}},$$
(125)
$$\eta^{(cd)}_{cct} = 5.9 \times 10^{4} \left(\frac{\Delta q_{A,C}}{6.2 \times 10^{2} e}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{4.7 \times 10^{13} cm^{-3}}\right)^{\frac{1}{2}} \left(\frac{n_{E}}{8.8 \times 10^{-1} cm^{-3}}\right)^{-\frac{1}{2}} \left(\frac{n_{E}}{1.0 \times 10^{-18} cm^{-3}}\right)^{-\frac{1}{2}} \right)^{-\frac{1}{2}}$$

 $\left(\frac{r_{\rm S}}{1.0 \times 10^{-4}\,{\rm cm}}\right)^{-\frac{3}{2}} \left(\frac{r_{\rm L}}{1.0\,{\rm cm}}\right)^{\frac{1}{2}}$

 $\left(\frac{\Delta W_{\rm ion}}{15.4\,{\rm eV}}\right)^{\frac{1}{2}} \left(\frac{T}{1.7\times10^2\,{\rm K}}\right)^{-\frac{1}{2}}.$

 $\left(\frac{\zeta}{1.0 \times 10^{-18} \, \text{s}^{-1}}\right)^{\frac{1}{2}} \left(\frac{u_{\text{L}}}{3.4 \times 10^3 \, \text{cm s}^{-1}}\right)^{-\frac{1}{2}}$

(126)

Conclusions of Muranushi 2010

- Ice dust-dust collisional charging is considered, fluffiness of the dust aggregates, regions with higher dust number density than that of MMSN.
- 1. As dust number density *N* increase, the dust plasma experience four phase.
- The latter two phases are common, where dustdust collisional charging <u>needs</u> to be taken into account.
- 2. For typical fluffy dust, the lightning strikes at $N_{crit} = 10 \sim 100$. N = 100 is plausible when dust sedimentation or streaming instability takes place.
- It is <u>possible</u> that lightning do take place.
- We have an analytic solution of $N_{\text{crit}}(r_{\text{S}}, r_{\text{L}}, D_{\text{S}}, D_{\text{L}})$ for use in other disk environment.

-- Planned end of my talk --

Thank you for your attentions!!

Part X-1 An exercise An estimation for N_{crit} MMSN, r = 2.7AU, Gas : S : L = 1: 10⁻² N :10⁻³ N Let us consider the following pair of dust: Monomer Size : $r_m = 10^{-5} \text{ cm}$ $D_S = 2.0$ $n_S = 2.0$ $m_L = 10^2 \text{ cm}$ $m_L = 1.5 \times 10^2 \text{ g}$ $n_S = 4.0 \times 10^{-2} \times \eta \text{ cm}^{-3}$ $n_L = 1.0 \times 10^{-15} \times \eta \text{ cm}^{-3}$

Grazing cross section: $\sigma_{SL} = r_S r_L = 1.0 \text{ cm}^2$ relative velocity = icebreaking velocity: $v_{SL} = 3.4 \times 10^3 \text{ cm/s}$ (Wada et al.2008) Grazing collision event rate: $\sigma_{SL} n_S n_L v_{SL} = 1.4 \times 10^{-13} N^2 [\text{cm}^{-3} \text{ s}^{-1}]$



ice surface density : $\sigma_{ch} = 6.2 \times 10^9 \text{ e cm}^{-2}$ charge separation efficiency : $N_{ch} = 10^{-1}$

(e.g. Dash et.al. 2001)

Contact cross section is smaller of :

• $S_{\rm kiss} = r_{\rm S}^{-3/2} r_{\rm I}^{-1/2}$ dense kiss • $S_{\rm kiss} = r_{\rm m}^{2-{\rm DL}} r_{\rm S}^{5/2} r_{\rm L}^{-{\rm DL}-5/2}$ sparse kiss This case is sparse kiss: $S_{\rm kiss} = 6.3 \times 10^{-4} \,{\rm cm}^{-2}$ charge separation per collision event: $\Delta q = \sigma_{\rm ch} N_{\rm ch} S_{\rm kiss} = 4.0 \times 10^5 \,\mathrm{e}$ charge separation rate: $J_{\rm SL} = aq \sigma_{\rm SL} n_{\rm S} n_{\rm L} v_{\rm SL} = 5.5 \times 10^{-8} N^2 \text{ e cm}^{-3} \text{ s}^{-1}$ $J_{\rm ie} = e \zeta n_{\rm o} = 4.7 \times 10^{-5} \,\mathrm{e} \,\mathrm{cm}^{-3} \,\mathrm{s}^{-1}$ lightning critical point: $J_{\rm SL} = J_{\rm ie}$ $\therefore N_{\rm crit} = 29$

fluffiness of dust multi-component dust dust denser than MMSN are all important for lightning 35

Part X-2 Answers(?) to the problems

Answers to Three Problems against Chondrule Formation by Lightning

①Neutralization Problem (Gibbard ,Levy & Morfill 1997)

- Lightning do not occur. Since PPD is weakly ionized and conductive, and dust is few, generated E field is too weak. $E \propto n^2$ so you need unnaturally high dust number density.

②Energetics Problem (e.g. Weidenschilling 1997)

 If lightning do occur, it isn't energetic enough to melt all the chondrules observed today, because the budget is turbulent energy --- a little portion of total kinetic energy.

③Destruction Problem (Güttler+ 2008)

 If lightning do occur, experiments show that it destroys the dust aggregates rather than melts them.³⁷ ①Neutralization Problem (Gibbard ,Levy & Morfill 1997)

- Lightning do not occur. Since PPD is weakly ionized and conductive, and dust is few, generated E field is too weak. E∝n² so you need unnaturally high dust number density.
- A1. If dust aggregates are fluffy, you need much less dust. Moreover, E grows steeper than N^2



Figure 3. Amount of charge stored in each species, en_e , en_i , $|q_S| n_S$, and $|q_L| n_L$, as function of η . This figure is for ice dust-

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②Energetics Problem (e.g. Weidenschilling 1997)

- If lightning do occur, it isn't energetic enough to melt all the chondrules observed today, because the budget is turbulent energy --- a little portion of total kinetic energy.
- A2. Not only turbulent energy, but whole the dust migration energy, may be harnessed by the *E* field and converted to lightning, for certain choices of dust parameters.



③Destruction Problem (Güttler+ 2008)

- If lightning do occur, experiments show that it destroys the dust aggregates rather than melts them.
- A3. Lightning environment in protoplanetary disc is different from that used in the experiment (Güttler+ 2008.) I don't know if two has the same effect on dust aggregates.



New Problem I've Learned in this workshop

- Chondrule forming environment is considered to be reductive, rather than oxidative
- H_2O environment = oxidative.
- A con for ice-dust induced lightning as a heating mechanism for chondrule.

Part X-3 Observation

Lightning Mechanism is Too Uncertain

... in spite of that, you can estimate typical luminosity or duration of an lightning event fairly well.

mean free path : $l = 1.2 \times 10^{2}$ [cm] ionization energy : eE l = 15.4eV critical field strength: E= 4.3×10^{-4} [G] disk scaleheight : H = 2.4×10^{12} [cm] lightning energy per event: E²/8 $\pi \times$ H³ = 4.3×10^{29} [erg]

Possible signals of disc lightning

- γ -ray: 3 × 10¹¹ eV
 - Recently, it has been revealed that thunderclouds and lightning discharges may produce relativistic electrons and gamma-rays(\sim 10MeV). (Enoto et al. 2008)
- UV lines
 - Recombination lines from ionized molecules

 $H_2^+ + e^- \rightarrow H_2 + 15.4 eV$

- Obstacles: interstellar dust absorption
- Optical (Literally see the Lightning)
- IR imaging
 - Disc heating by lightning. If you want to distinguish from other sources, see variability/timing/correlation.
- ALF wave: wavelength $\sim 0.1 \text{AU}$
 - Earth thunderclouds emit ELF wave of its scale height (e.g.Koshak & Krider 1989)

Shape of a lightning bolt

- Terrestrial lightning bolts are known to have radii of ~5000 mfp. Borrowing this, astrophysical lightning is a cylinder:
 - length H = 2.4×10^{12} [cm]
 - radius 5000*l* = 6 × 10⁵[cm]
 - filled with 15.4eV electrons
 - amount to 4.3×10^{29} [erg]
 - duration 10⁴[sec]



- Energy Budget tells minimum interval: 10[sec]
- How can we observe this object?

Basic Energetics

$$\frac{4.3 \times 10^{29} [erg]}{10^{4} [sec]} \frac{10^{23}}{(100 pc)^2} = 1.4 Jy$$

- ALMA: 0.1mJy is available.
- Default band for lightning is UV, which is very hard to observe.
- Is there any mechanism to transfer a portion of the lightning energy to other bands?

Candidates

- Cyclotron radiation
- Planck radiation Tail
- Velocity dispersion of charged molecules
- Strange molecules formed by lightning
- Noise mining (nongaußianity)
- afterglow

Electron Cyclotron Radiation

$\omega = eB/mc$

- Typical MRI disk \rightarrow B = 6[G] \rightarrow 0.1GHz
- ALMA bands: 31.3GHz~950GHz
- Lightning equipartition magnetic field : 3500[G]
 → 50GHz
- Lightning leaders (charge packets at the front of lightning bolts) may cause more magnetic field?

Colder Tail of Planck Radiation

- If the lightning bolt cylinder is heated by electrons and cooled by Blackbody radiation, equilibrium T = 800K
- Looking at the cylinder @900GHz band

~10²⁰[erg/s]
$$\rightarrow$$
 3µJy

Velocity dispersion of charged molecules

- +1, -1 charged molecular species are also accelerated by lightning field. The acceleration and mean free paths are the same as electron, so the energy is: 15.4eV
- molecular weight 30 = 7km/s
- A velocity much greater than thermal speed is a smoking gun for an acceleration process.

Strange molecules formed by lightning

- Which molecules? Das wußte ich doch nicht.
- Dust struck by lightning?
- Amino acid?

\rightarrow protoplanetary disk = origin of life hypothesis

nongaußianity

- Draw information from protoplanetary disks imaging
- statistically detect that there are transient, bright spots in disks, of timescale ~1hr



afterglow

• afterglow of lightning

Part X-4 About the opening movie

Initial Conditions

- 2-D model
- φ -*z* slice of the disk
- gas+small dust (red), large dust (blue)
- gravity towards equatorial plane
- velocity shear given to cause KH instability (shear velocity is not selfmaintained...)



Resistive MHD with advected scalars

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho \mathbf{u} \right) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \bullet \left(\rho \mathbf{u} \bar{\mathbf{u}} - \mathbf{B} \bar{\mathbf{B}} \right) + \nabla P^* = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$

$$\frac{\partial E}{\partial t} + \nabla \bullet \left(\left(E + P^* \right) \mathbf{u} - \mathbf{B} \left(\mathbf{B} \bullet \mathbf{u} \right) \right) = 0$$

$$\frac{\partial S_i}{\partial t} + \nabla \bullet \left(S_i \mathbf{u} \right) = 0$$

— Independent Variables

density *ρ*, velocity **u**, magnetique field **B**, total energy *E* scalar fields S[i]

 $\nabla \bullet \mathbf{B} = 0$

Constraint -

Dependent Variables _____ current $\mathbf{j} = \nabla \times \mathbf{B}$ electric field $\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{j}$

Internal Energy $\mathcal{E} = E - \frac{1}{2} \rho u^2 - \frac{1}{2} B^2$ Gas pressure $P = (\gamma - 1)\mathcal{E}$ Total Pressure $P^* = P + \frac{1}{2} B^2$

2-Scalar Field Model

- S[0]... Number density of the Larger dust
- S[1]... Ionization degree

$$\frac{\partial S_0}{\partial t} + \nabla \bullet \left(S_0 \left(\mathbf{u} + \mathbf{u}_g(z) \right) \right) = 0$$

$$\frac{\partial S_1}{\partial t} + \nabla \bullet \left(S_1 \mathbf{u} - D_1 \nabla S_1 \right) + \frac{S_1 - S_{1,eq}}{\tau_1} = \delta_{discharge}$$

- Large dust advects while falling at the terminal velocity.
- Ionization suddenly rise at lightning path (discharge model; next slides.) then it advects, diffuses and rapidly decays to equilibrium value.

Discharge toy model

Solomon, Adamo, Baker 2002(discharge) & Muranushi(toy)



①Suppose the dust plasma is in (c)charge-up phase: Electric field is dominated by ηj term.

(2) Introduce a pair of 'Charge Packet' (q = ±1) at the cell Eesf > Ecrit
 (3) Each charge packet randomwalk as a chess king. They obey electric field and likes the highly ionized cell.

The probability that direction **n** is chosen

$$P(\mathbf{n}) \propto \operatorname{expone}\left(qS_1^{0.5} \frac{\mathbf{n} \bullet \mathbf{E}_{esf}}{|\mathbf{E}_{esf}|}\right)$$

if x > 0 expone(x) = x else expone(x) = exp(x)

Discharge toy model



④ Initially, charge packet has ionization degree S[1] = 1.0
 ⑤ At each cell charge packet ionization is updated as

$$S_1 \leftarrow S_1 \min\left(1.0, \frac{\left|\mathbf{E}_{esf}\right|}{0.1E_{crit}}\right)$$

charge packet can survive if Eesf > 0.1Ecrit

(not Eesf > Ecrit ;Avalanche discharge model)

6 If the packet ionization degree < cell ionization degree, remove the packet

O This happens in a instant (compared to hydro timescales)