Dynamical and Photometric modeling of Saturn’s Rings:

Heikki Salo (Dept. Physics, U. Oulu, Finland)

Kobe 4.11.2011
Why are Saturn’s rings interesting?

- Cassini Orbiting Tour
  Close range images during SOI (July 2004)
  Solar Equinox (August 2009)

- Rings = Orbital Laboratory
  Coolest disk in the universe? \( \nu / \sigma \sim 10^6 \)
  Many old ideas of disc galaxy dynamics manifest best in Saturn’s rings

Specific topic of this talk: Local Ring Thickness
relates to Self-Gravity wakes, Local Stability properties, Opposition Effect ...

Collaborators:
* Dynamics of dense rings/embedded moonlets: J.Schmidt, F. Spahn, M.Seiss (Potsdam), M.Sremcevic, M. Albers (Boulder)
* Modeling Voyager, HST, Arecibo, Cassini data: R. French (Wellesley), P. Nicholson (Cornell), R. Morishima,(JPL) K. Ohtsuki (Kobe)
THREE ’OLD’ OBSERVATIONAL PUZZLES CLOSELY RELATED TO LOCAL RING THICKNESS:

1. Opposition brightening

2. Azimuthal Brightness asymmetry

3. Wealth of unexplained radial structure
1. PRE-PRE-VOYAGER: RING OPPOSITION BRIGHTENING

Saturn and the "Seeliger Opposition Effect"
imaged by Geoff Chester, Alexandria, VA, USA

von Seeliger 1887: due to dissappearance of mutual shadows
(Maxwell’s Adams Prize Essay (1856): ring must compose of discrete particles)
2. PRE-VOYAGER: AZIMUTHAL BRIGHTNESS ASYMMETRY

(CAMichel 1958)

HST-profiles

(French et al. 2007)

INTRINSIC PROCESSES?

CONNECTED TO SATELLITE RESONANCES

OUTER B RING (6000 km)

OUTER A RING (3000 km)
Satellite Pan orbiting at Encke gap:
- sinusoidal gap inner edge
- kinematic wake of satellite
- Weak density waves at resonances
CASSINI: IRREGULAR(?) FINE-STRUCTURE

- Instability/overstability?

CASSINI ISS IMAGES: (Porco 2006)
- structures in km-scales
- bimodal jumps

CASSINI RSS OCCULTATION:
- Axisymmetric structures
  ⇒ act like “diffraction grating” for radiowaves emitted through rings

150 meter fine-structure (Thomsen 2007)
SOLAR EQUINOX IMAGES AUGUST 2009

- Low Solar illumination angle brought surprises:
  - Edge waves excited by Daphnis cast shadows
  - Thickness of perturbed regions several kilometers

Unexplained thickening of the inner edge of Cassini division
OVERVIEW OF SATURN’S RINGS

- $\sim 10^{16}$ METER-SIZED ICY PARTICLES
  - Keplerian differential rotation: $\Omega \propto a^{-1.5}$
  - Power-law size distribution: $dN/dr \propto r^{-3}$, $1cm < r < 10m$

- FREQUENT IMPACTS $\geq 10$ /per orbit
  - Local vertical thickness $< 100$ m (Ring diameter 270 000 km)
  - Impacts speeds $\sim 1cm/sec$ (orbital speed $V_{orb} \sim 20km/s$)

- DISSIPATIVE IMPACTS + CONSERVATION OF $I_z$
  - Rapid local vertical flattening: timescale a few days at most
  - Slow radial spreading: whole ring: timescale $> 10^8$ years
VERTICAL THICKNESS $H$: 

- Difficult to measure directly:
  - Ring plane crossing $\Rightarrow$ upper global limit $H < 2.4$ km (Dollfus, 1966) (HOWEVER: includes inclined F-ring, vertically extended ring edges etc.)
  - Sharpness of radial edges (Voyager) $\Rightarrow$ local thickness $H < 100$ m
  - UVIS occultation profiles $\Rightarrow$ at least some edges sharper than few meters (Albers et al. 2011 DPS)

- Photometric estimate:
  - Opposition effect $\Rightarrow$ volume density $D \sim 0.02$ (Lumme, Irvine, Esposito 1983), corresponds to $H = 50$ m assuming $r = 1$m particles

- Dynamical estimates:
  - Dissipative impacts $\Rightarrow$ flattening to $5 < H < 50$ m
  - Presence of selfgravity wakes $\Rightarrow$ $H \sim 10$ m
  - Similarly: axisymmetric oscillations suggest overstability (and thus flat ring)
DOES IT MATTER WHETHER $H = 10$ or $100$ m? YES

- Above limits = range of uncertainty in laboratory experiments
  - frosty ice ⇒ $H \sim 10$ m
  - smooth ice ⇒ $H \sim 100$ m

- Drastic effect on ring stability properties
  - $H \sim 10$ m ⇒ gravity wakes, overstable oscillations
  - $H \sim 100$ m ⇒ viscous instability

- Related to the time scale for ring radial spreading
  - $H \sim 10$ m ⇒ timescale $10^{10}$ yrs (viscous spreading of 10000 km wie zone)
  - $H \sim 100$ m ⇒ timescale $10^{8}$ yrs
MODELING DENSE SELF-GRAVITATING RINGS

- **INGREDIENTS**
  - impacts + selfgravity + differential rotation
  - external satellites, embedded moonlets and “icebergs”

- **METHODS**
  - **kinetic theory**: Goldreich-Tremaine-Borderies, Araki, Stewart, Hämeen-Anttila, Latter & Ogilve
  - **hydrodynamics** Schmit & Tscharnuter, Schmidt & Salo & Spahn
  - **N-body**: Trulsen, Brahic, Lukkari, Salo, Richardson, Mosqueira, Lewis, Daisaka, Ohtsuki; Charnoz
    - ⇒ Local simulation method (Wisdom & Tremaine ; Toomre & Kalnajs)
    - ⇒ combination with photometric simulations (Salo & French ; Porco & Richardson)
LOCAL SIMULATION METHOD

Equations:
- Co-moving coordinate system
- Linearized Hill-equations
- Periodic boundary conditions: ⇒ replicate particles

Collisions:
- Instantaneous impacts: $\epsilon_n, \epsilon_t, \gamma$
  or **Force-model** for impacts (Salo 1995)
  ⇒ modeling of gravity aggregates, adhesion

Gravity: (Note: compared to galaxy dynamics, need to be 'collisional')
- Nearby particles: PP forces ($\Delta < 0.5 \lambda_{cr}$) (Salo 1992)
- Intermed. range: 3D FFT in shearing coordinates (Salo 1995)
- Distant gravity: $F_z$ from infinite sheet

Tabulation:
  Position+velocity+spin snapshots
  Pressure tensor components $P_{ij}$
  Fourier components, autocorrelation etc
**Photometric Monte Carlo Ray-Trace Modeling**

Salo and Karjalainen (2003, Icarus):

- Particle field from dynamical simulations illuminated with large number of photons (sun/Saturn)

- Scattering: choose single photon, new direction from phase function with MC sampling, search new particle along the new direction

- Add contribution of each scattering to brightness in viewing direction

- Main interest to obtain $I/F$ as a function of $B_{obs}, \phi_{obs}, B_{sun}, \phi_{sun}$ for assumed particle phase-function, single-scattering albedo

- Can also make 'images' (next page)
Toy-model illustration of the vertical corrugation pattern found by Hedman et al. (2011)
LOCAL ENERGY BALANCE

\[ w_c(1 - \epsilon^2)c^2 = \nu(\partial \Omega / \partial r)^2 \]

VISCOSITY: (from \( P_{xy} \))

- momentum transfer via radial excursions (local viscosity; WT87 relates to \(< c_x c_y >\))
- transfer at physical impacts (nonlocal viscosity; WT87 \(< \Delta x c_y >_{\text{impacts}} / (N \Delta t)\))
- transfer via grav. forces (gravitational viscosity; Daisaka et al. 2001 \(< \int \Delta x F_y > / (N \Delta t)\))

⇒ TIME-SCALE OF LOCAL BALANCE: 10-100 impacts/particle

RANDOM VELOCITY, THICKNESS, VISCOSITY depend on:

- elasticity of impacts, friction
- optical depth \((w_c \propto \tau_{dyn})\)
- particle size distribution
- particles’ internal density (+distance via \(r_h \propto \rho^{1/3} a\))

⇒ VISCOSITY vs DENSITY RELATION

determines linear stability properties
long-timescale radial evolution
LINEAR STABILITY DEPENDS ON VISCOSITY vs. DENSITY RELATION

RADIAL MASS FLUX: $\tau u_r \sim -\partial \eta / \partial r$

INTERMEDIATE CASE

STABLE RING: $\partial \eta / \partial \tau > 0$

'THICK' RING

VISCOUS INSTABILITY: $\partial \eta / \partial \tau < 0$

FLAT RING + SELF GRAVITY

VISCOUS OVERSTABILITY: $\partial \eta / \partial \tau >> 0$
STABILITY SENSITIVE TO PARTICLE ELASTICITY

LABORATORY MEASUREMENTS OF ICE

'HOLLOW' RING:

⇒ VISCOSITY MAY DECREASE WITH DENSITY

'HOLLOW' RING:

⇒ VISCOSITY INCREASES WITH DENSITY

SELF-GRAVITY ENHANCES THIS TENDENCY

VERTICAL THICKNESS

DYNAMIC VISCOSITY \( \eta \)
SUMMARY: MICROPHYSICS ⇒ STABILITY PROPERTIES

- steady-state velocity dispersion determined via energy balance between collisional dissipation & viscous gain from differential rotation

- crucial role of particles’ poorly known elasticity:

  **Frost-covered particles** (Bridges et al. 1984 laboratory measurements)

  ⇒ flattened ring: $H \sim 10$ meters, susceptible to gravitational instability

  (also viscous overstability)

  **Smooth particles** (Hatzes et al. 1988 laboratory measurements)

  ⇒ “thick” multilayer ring $H \sim 100$ meter, gravitationally unresponsive

  (may lead to viscous instability)
SELFGRAVITY WAKES/ASYMMETRY
SELF-GRAVITY

- Gravitational collapse + dissipation + differential rotation
  \[ \Rightarrow \text{Self-regulation} \Rightarrow \text{minimum } Q_{\text{Toomre}} \sim 1 - 2 \text{ (corresponds to } h \sim 10 - 20m) \]


  \[ \text{radial scale: } \lambda_{cr} = 4\pi^2 G\Sigma/\Omega^2 \sim 10 - 100m \]

  \[ \text{pitch-angle: } \sim 20^0 \text{ (in Keplerian velocity field)} \]
HOW DO SG-WAKES MANIFEST IN SATURN RING OBSERVATIONS: AZIMUTHAL ASYMMETRY

Wakes unresolved, but have systematic \( \sim 20^\circ \) pitch angle \( \Rightarrow \)

Ring photometric properties should depend on ring longitude and elevation

(Salo et al. 2004)

Confirmed by HST comparisons:
(French, Salo et al. 2007)
INDICATIONS OF SELF-GRAVITY WAKES

- Ring’s Arecibo radar echo: (Nicholson et al. 2005)
- Saturn microwave radiation (Dunn et al. 2004, 2007)
- Cassini occultation experiments
  - UVIS: (Colwell et al. 2006, 2007)
  - VIMS: (Hedman et al. 2007)
  - RSS: (Marouf et al. 2006)
- Cassini CIRS: ring filling factor (Ferrari et al. 2009)
- Damping of satellite density waves (Tiscareno et al. 2008) consistent with gravitational viscosity (Daisaka et al. 2001)
- Strong peaking of asymmetry in the mid A-ring is a problem (wakes perhaps hidden by debris = free-floating regolith released in fast impacts? Salo et al. 2007 DPS)
SG-WAKES SENSITIVE TO VELOCITY DISPERSION

If impacts are able to maintain thickness which corresponds to $Q > 2$

$\Rightarrow$ wake structure would be absent

FROSTY ICE: SMOOTH ICE:

BRIDGES-ELASTICITY MODEL  HATZES-ELASTICITY MODEL

VERTICAL THICKNESS

H (m)

OPTICAL DEPTH

0.1 1.0 10.0

1

10 100
SG-WAKES AND SIZE DISTRIBUTION

Size distribution $\Rightarrow H_{small} > H_{large}$

SG-wakes weaker among small particles

(Salo, French 2004)
\( \lambda / R \) graininess, \( R_{\text{Hill}} / R \) pairwise sticking in tidal environment \( \Rightarrow \)

\[
\begin{array}{cccccccccc}
\tau & 0.25 & 0.49 & 0.57 & 0.66 & 0.74 & 0.82 & 0.90 & 0.98 & 1.07 & 1.15 & 1.23 \\
\alpha & 60 & 70 & 80 & 90 & 100 & 110 & 120 & 130 & 140 & 150 \\
\end{array}
\]

Salo et al. (2008); reproduced by Schmidt et al. 2009, Cuzzi et al. 2010

identical particles, \( \rho = 900 \text{ kg/m}^3 \), \( \epsilon = 0.5 \) \( 4\lambda_{cr} \times 4\lambda_{cr} \ N \propto \alpha^6 \tau^3 \)
**Approaching Roche Distance  ⇒  Accretion**

Karjalainen and Salo 2007

- Charnoz et al. 2010: viscous spreading spills rings over the Roche distance  
  ⇒  formation of small moons outside the main rings

Details depend on $\epsilon_n$, friction, size distribution
VISCOUS INSTABILITY
AND OVERSTABILITY
LINEAR STABILITY: DEPENDS ON $\eta(\tau)$

RADIAL MASS FLUX: $\tau u_r \sim -\partial \eta / \partial \tau$

INTERMEDIATE CASE

STABLE RING: $\partial \eta / \partial \tau > 0$

'THICK' RING

VISCIOUS INSTABILITY: $\partial \eta / \partial \tau < 0$

FLAT RING + SELF GRAVITY

VISCOUS OVERSTABILITY: $\partial \eta / \partial \tau >> 0$

radial direction

vertical direction

time
VISSCUS INSTABILITY

- Particle flux directed toward density maxima
  - Dense/cool ringlets
  - Hot/rarefied region \[ \Rightarrow \text{BIMODAL} \]

= Original explanation for “ringlet structure” discovered by Voyager, but later discarded

Hämeen-Anttila, Lukkari, Ward, Lin & Bodenheimer

Requires smooth elastic particles, inconsistent with gravity wakes.

- Size-dependent selective instability?
  works also between two dense regions!

Salo & Schmidt (Icarus, 2010)

However, requires rather specific \( \epsilon_n \) vs particle size dependence
OSCILLATORY INSTABILITY (VISCOUS OVERSTABILITY)

Upper left corner: weak gravity, high impact frequency $\Rightarrow$ axisymmetric oscillations superposed on inclined selfgravity wakes

Ring overshoots in smoothing density variations due to steep rise of viscosity with density.

Salo et al. 2001, Schmidt et al. 2001
OVERSTABILITY II

Oscillations with epicyclic frequency
Time-evolution over 1/2 periods

- Hydrodynamical stability analysis
  Schmit & Tscharnuter 1995, 1995
  predicted too easy onset of overstability

- Disagreement with N-body simulations
  (Salo et al. 2001)
  ⇒ improved hydrodynamic models

- proper kinetic treatment
  Latter & Ogilve 2006, 2007
OSCILLATORY INSTABILITY III

- Cause of the 150m oscillations in RSS occultations? (Thomsen et al 2007)
- UVIS occultations: axisymmetric structures (Colwell et al. 2007)

Matches the natural scale seen in simulations
VERTICAL SPLASHING - SHADOWS

B_{sun} = 25^\circ

Dense rings nearly incompressible
⇒ overstable oscillations associated with vertical 'splashing'
(Borderies, Goldreich, Tremaine 1984)

Effect strong enough to cause shadows (middle frame)
OBSERVABLE EFFECTS OF NON-RESOLVED SHADOWS?

TOY-model (true shadows non-resolved!)

Mean brightness as function of azimuth:
Even 10% systematic variations predicted
Salo & Schmidt 2011 DPS
OPPOSITION BRIGHTENING
RING FILLING FACTOR/PHOTOMETRY

**OPPOSITION BRIGHTENING**

Coherent backscattering at particle surface regolith
or
disappearance of mutual shadow between particles?
(Debated for over 50 years!)

Lumme et al. 1983: due mutual shadowing
⇒ filling factor 0.02

How to reconcile with dense rings?)
MECHANISMS FOR OPPOSITION BRIGHTENING
(Hapke, Irvine, Bobrov, Lumme, Esposito, Muinonen, Mischenko, Nelson ...)

- INTER-PARTICLE MUTUAL SHADOWING:
  Only illuminated surfaces visible $\alpha \rightarrow 0^\circ$
  HWHM $\propto R/L \propto D$ volume filling factor
  (R typical particle size, L separation)

- INTRINSIC BRIGHTENING OF PARTICLES
  - Shadow-hiding at particles’ surface regolith (SH)
    Basically same mechanism as interparticle shadowing
  - Coherent backscattering (CB)
    Constructive interference of incoming and outgoing photon in a medium made of wavelength sized grains
    HWHM $\propto \lambda/L_{tr}$ ($L_{tr}$ transport mean free path, depends on wavelength and grain size)
SATURN RING’S OPPOSITION EFFECT: INTRINSIC OR INTER-PARTICLE EFFECT?

- Inter-particle shadowing mechanism favored until late 1980’s
  - Lumme et al. 1983: $D \approx 0.02 \Rightarrow$ observed narrow peak
    for identical particles this corresponds to $H/R \sim 50$

- In 1990s intrinsic effect became more popular:
  - Elasticity measurements of frost-covered ice (Bridges et al. 1984)
    $\Rightarrow$ Dynamical models favor flattened rings ($D > 0.1$)
  - Laboratory measurements of intrinsic opposition peak

- Personal view: both effects MUST be present:
  - Simulations with size distribution $\Rightarrow$
    narrow inter-particle shadowing opposition peak unavoidable
  - Low optical depth C ring has strong opposition effect $\Rightarrow$
    particles must have a large intrinsic component
INTRINSIC AND INTRA-PARTICLE EFFECT DIFFICULT TO SEPARATE

- Theoretical formulae of CB and SH have nearly similar forms:

\[
B_p(g) = \frac{1}{2} \left( 1 + \tan \left( \frac{g}{2} \right) \right)
\]

\[
B_c(g) = \tan \left( \frac{g}{2} \right)
\]

Polarization measurements would be helpful

- CB: peak in linear and circular polarization ratios
INTER-PARTICLE SHADOWING DEPENDS STRONGLY ON $B$

width $\propto$ effective volume density $D$

- Large $B \Rightarrow$
  dense central layer
  wide OE

- Small $B \Rightarrow$
  low D 'envelope'
  narrow OE

- Effect depends on
  * optical depth
  * elasticity
  * size distribution

$\tau = 1.0$, Bridges-elasticity, $0.20m < r < 5m$
EXTENSIVE HST DATA SET

- French et al. since 1996: covers full Saturn Seasons
  Poulet, Cuzzi, French, Dones 2002 analysed phase curves, but only for $B = 10^\circ$

Cycle 13: “Saturn’s Rings at True Opposition” French et al. 2007
HST PROFILES AT TWO ELEVATIONS:

- Phase curve indeed steeper for smaller elevation!
  (from Salo & French, Icarus in press)
**HST Profiles at Two Elevations: Normalized to \( \alpha \approx 6^\circ \)**

![HST Profiles Diagram](image_url)
MODELING HST OBSERVATIONS

• Grid of dynamical/photometric simulations \((\tau, r_{\text{max}}/r_{\text{min}}, \epsilon_n)\)
  (MC method of Salo & Karjalainen 2003 (Icarus 164,428))

• Comparison to extensive HST observations \((\alpha, B_{\text{eff}}, \lambda)\)

• Match the elevation dependence of OE \(\Rightarrow\) best size distribution model
  \(\Rightarrow\) extract simulated inter-particle contribution from observations
  \(\Rightarrow\) what is left is intrinsic part

  single scattering enhancement due inter-particle shadowing:

  \[
  \text{optical depth } \tau = 0.1 - 2.0 \\
  \text{elevation } B = 4^\circ - 26^\circ \\
  \text{elevation } R_{\text{max}}/R_{\text{min}} \text{ varied}
  \]
Observed HST phase curves show elevation dependence!

⇒ Intrinsic and mutual shadowing can be separated! (Salo and French, Icarus 2010)

Narrow peak consistent with flat dense ring predicted by dynamics.
C-ring

C Ring 78-83 kkm  B=23°

C Ring 78-83 kkm  B=4.5°
SUMMARY: B-ring

B Ring 100-107 kkm  B=23°

B Ring 100-107 kkm  B=4.5°
SUMMARY

● SELF-GRAVITY WAKES CAN ACCOUNT FOR:
  A-ring and inner B ring asymmetry in HST observations
  Radar asymmetry
  Longitude and elevation angle dependent optical depth

● OVERSTABILITY:
  High density/weak gravity regime
  ⇒ 150 m oscillations, modulations(?)

● IMPLIED RING PARTICLE PROPERTIES:
  internal density $\sim 300 - 450 \text{ kg/m}^3$
  elasticity close to Bridges et al. 1984 ’frosty ice’

● STILL A PROBLEM: B-RING IRREGULAR VARIATIONS:
  Role of selective instabilities?, particle adhesion?
- Dense ring with vertical structure and size distribution can have narrow opposition peak
- Inter-particle and intraparticle effect separated by the elevation angle dependence
Saturn's rings at 30 cm resolution?

100,000 particles illuminated with $10^8$ photons
Thank You!
Final Disclaimer:

this seminar might have been unsuitable for children!

Mickey The Detective is following a thief to an observatory, and interviews the “astronomers” if anyone has seen anything unusual?
Which one is an imposter?

- “Not seen anything, have followed a supernova without a pause”
- “Too busy, estimating the thickness of Saturn’s rings”
- “No idea, have been staring a new black hole for hours”
- “No sign of thief, but have seen a two-tailed comet”
Can’t fool Mickey!
“There is no such thing as thickness of Saturn’s rings!
The one who claims to measure it is not a real astronomer!”