How Does Nonhydrostaticity Affect the Determination of Icy Satellites' Moment of Inertia?

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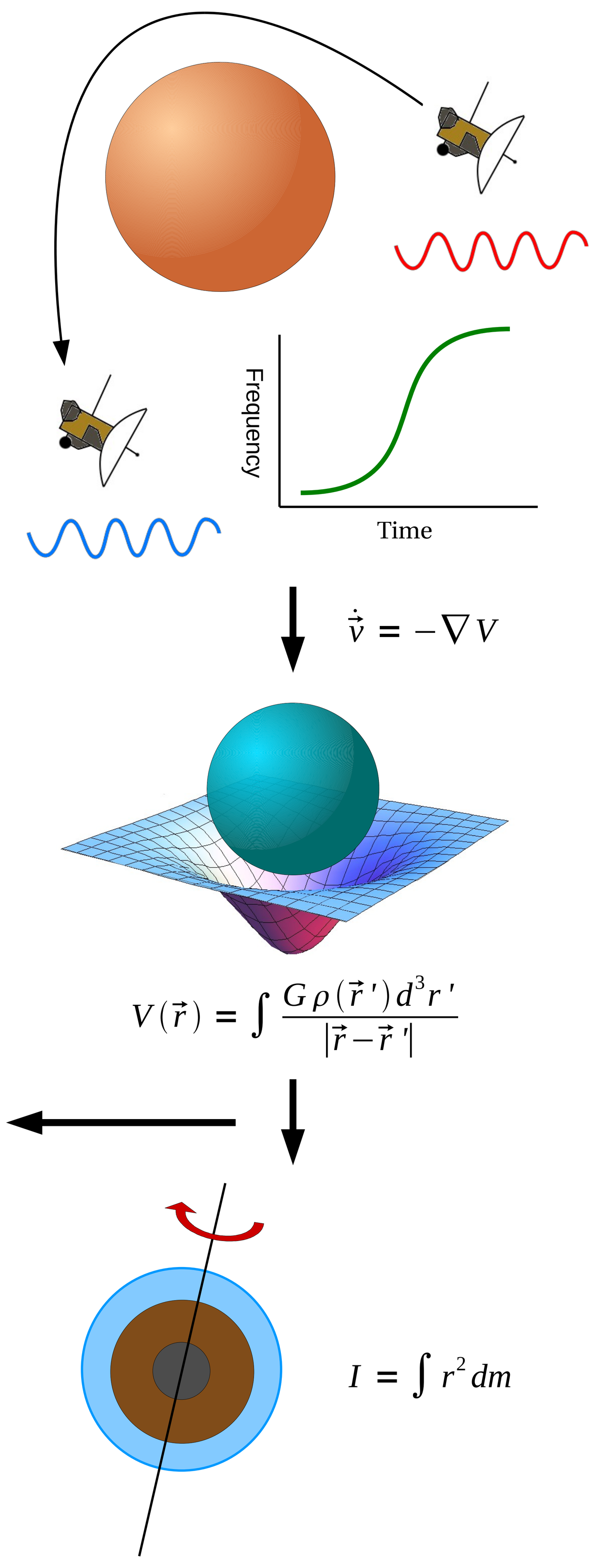
Abstract

Current models of icy satellite formation and evolution depend on the accuracy with which we determine their interior structures. These can be inferred from their moments of inertia (MoI), which can be estimated from *in situ* gravitational field measurements by spacecraft. The primary method of estimation is the Radau-Darwin approximation (RDA) [1], which relates the MoI to the degree 2 response of the body to rotation and tides J_2 and C_{22}. This method makes the assumption that the body is in hydrostatic equilibrium and also ignores the effects of large density variations. It has been applied to several large icy satellites in the outer Solar System, including Titan [2], Callisto [3], and Ganymede [4]. Interpretations of the correlation between their MoIs and orbital distances lend credence to the "gas starved disk" model [5]. However, hydrostatic equilibrium is not guaranteed ([2], [3], [4], [5]), so it is prudent to assess the impact of nonhydrostatic structures on the accuracy of RDA. We use a simple model to show that nonhydrostatic stresses of ~1 bar can result in errors of up to 10% in the RDA-calculated MoI, and faster-rotators require greater stresses in order to exhibit the same error, thus casting doubt on the accuracy of Titan and Callisto's MoIs.

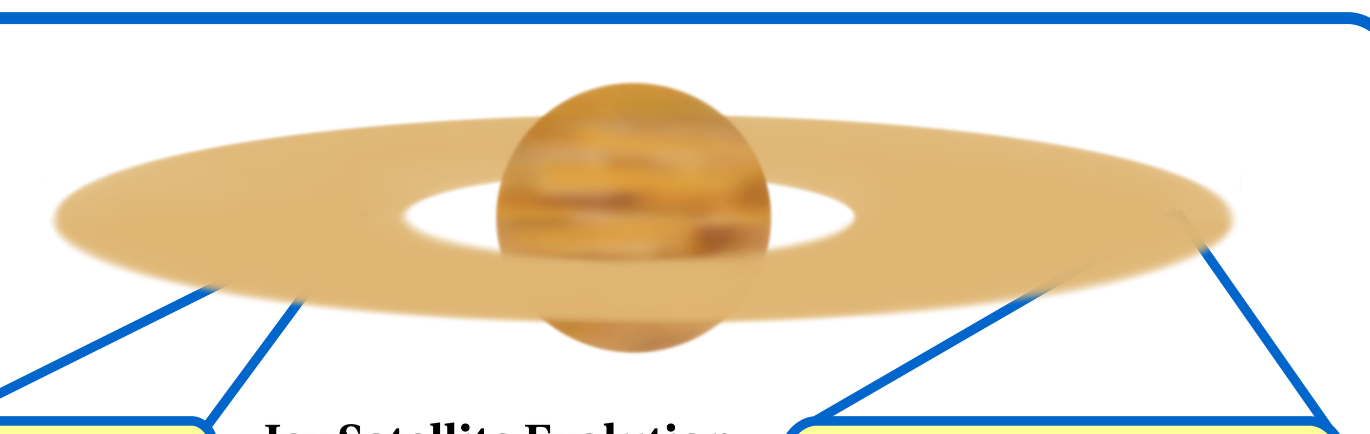


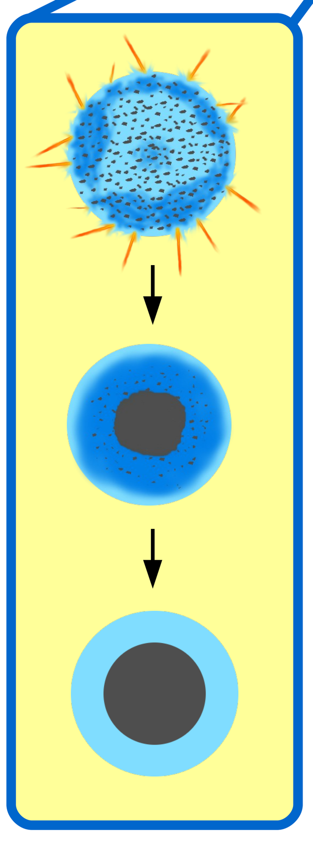
Radau-Darwin Approximation

The Radau-Darwin Approximation (RDA) is given by:

$$\frac{C}{Ma^2} = \frac{2}{3}\left(1-\frac{2}{5}\sqrt{\frac{5}{3\Lambda_{2,0}+1}-1}\right), \quad \Lambda_{2,0} = \frac{J_2}{q}, \quad q = \frac{\omega^2 a^3}{GM}$$

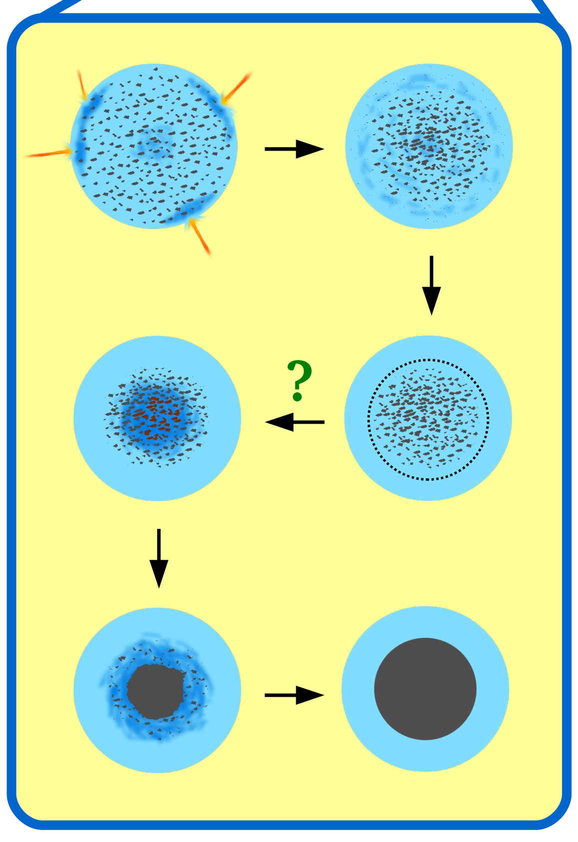
Where C is the polar MoI; M is the mass; a is the mean radius; ω is the angular rotation rate; and G is the gravitational constant. Every quantity found in $\Lambda_{2,0}$ can be measured, and thus RDA is the primary method for finding a body's MoI. RDA relies on two assumptions: 1). the body is devoid of large density variations and 2). the body is in hydrostatic equilibrium. Neither assumptions apply perfectly to Solar System bodies, and we hope to quantify the errors in MoI caused by deviations from these assumptions.



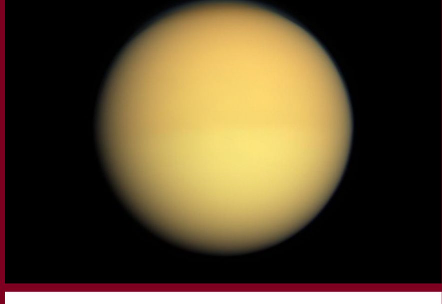
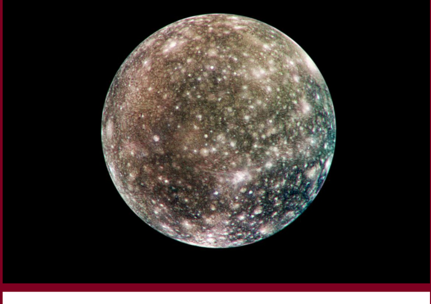
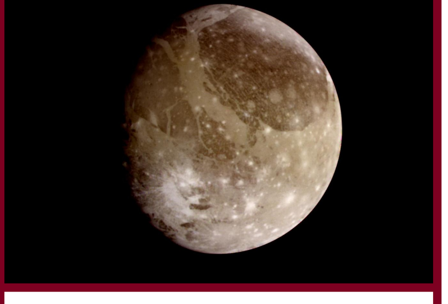
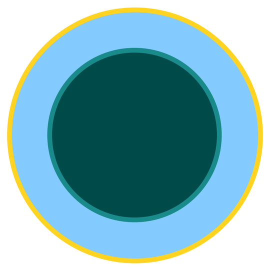
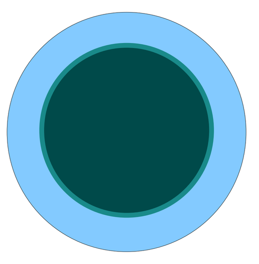
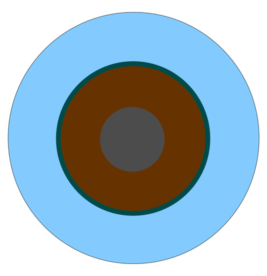


Icy Satellite Evolution

The currently-accepted theory by which icy satellites form and evolve is the "gas-starved disk" model [6]. It posits a subnebula around gas giants that decreases in density with distance from the planet, and with gas flowing in at some rate from the rest of the Solar Nebula. For satellite embryos close to the planet (left), accretion rates are high due to high density of material; this high rate of formation heats up the accreted ice and rock such that most of the ice melts, causing the rock to settle towards the core. This release of gravitational potential energy then causes runaway differentiation [7], forming satellites such as Ganymede.

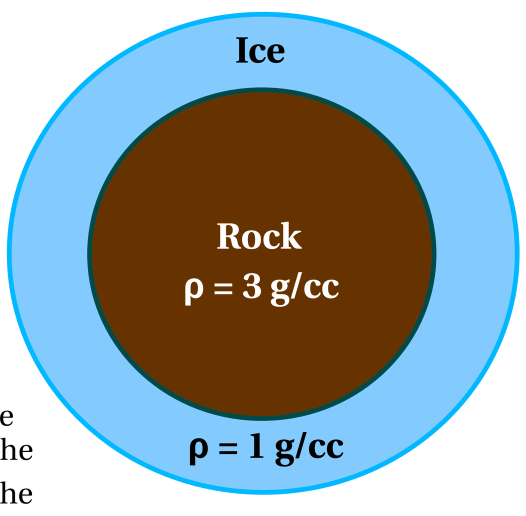


Meanwhile, satellite embryos farther away from the planet (right) will see lower accretion rates, causing only small amounts of ice to melt – not enough to cause runaway differentiation. This results in a partially-differentiated interior, as proposed for Titan and Callisto [8]. However, recent work by O'Rourke and Stevenson [9] shows that a compositional gradient within a partially-differentiated satellite will inhibit convection, causing build-up of heat emitted by long-lived radioactive isotopes. This heat will likely melt the ice near the center of the satellite, resulting in runaway differentiation. If this scenario is true, then both Titan and Callisto should be differentiated, which is not reflected in their MoIs. Is the theory wrong, or is it the method with which the MoIs were derived?

	Titan 	**Callisto** 	**Ganymede** 
Moment of Inertia	**0.3414 ± 0.0005 [2]**	**0.3549 ± 0.0042 [3]**	**0.3105 ± 0.0028 [4]**
Inferred Interior Structure	**Partially-Differentiated** 	**Partially-Differentiated** 	**Completely Differentiated** 
Comments	Titan can also be completely differentiated, but with a low-density, hydrated core [10].		Anderson et al. [4] also reported an intrinsic magnetic field, indicating the presence of an inner core of iron or iron sulfide; this further suggests complete differentiation.

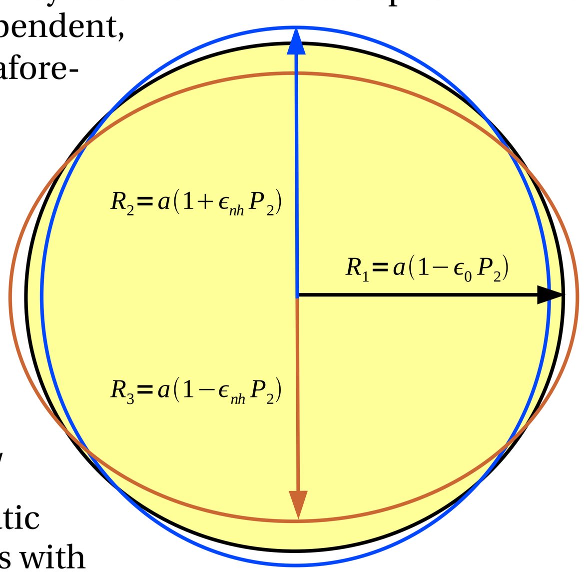
Model Icy Satellite

Our model consists of a layer of water ice overlaying a rock core, both having constant density, as shown on the right. This simple arrangement allows us to isolate the essential physics of the problem. We further assume that the only departure from sphericity is a rotational bulge, since the (larger) tidal distortion involves the same physics and can be readily superimposed. The degree 2 zonal distortions in the shape and gravitational field of the satellite is described by two oblateness factors, which are small for all bodies of interest. This simple model allows us to completely determine the oblateness factors and J_2 by assuming that both the satellite and core surfaces are equipotentials under the condition of hydrostatic equilibrium.



Introducing Nonhydrostaticity

Due to rotation, the hydrostatic shape of an icy satellite is an oblate spheroid. The radius of the satellite is then colatitude-dependent, as shown by R_1 to the right, where ε_0 is the aforementioned (hydrostatic) oblateness factor, and P_2 is the degree 2 Legendre polynomial with variable $cos\theta$, with θ being the colatitude. It can be shown that J_2 can be written in terms of ε_0, and thus the MoI calculated by RDA is dependent on it as well. To introduce nonhydrostaticity, we simply Taylor expand the RDA equation around ε_0 to allow for larger/smaller ε's that correspond to nonhydrostatic shapes, e.g. R_2 and R_3. This then provides us with corresponding nonhydrostatic MoIs, which we can compare to the hydrostatic value. This is done for both the satellite surface and the core surface. Note that the R_3 configuration is more likely, (i.e. the nonhydrostaticity results in a smaller measured MoI) as true polar wander would tend to maximize the polar MoI through reorientation.

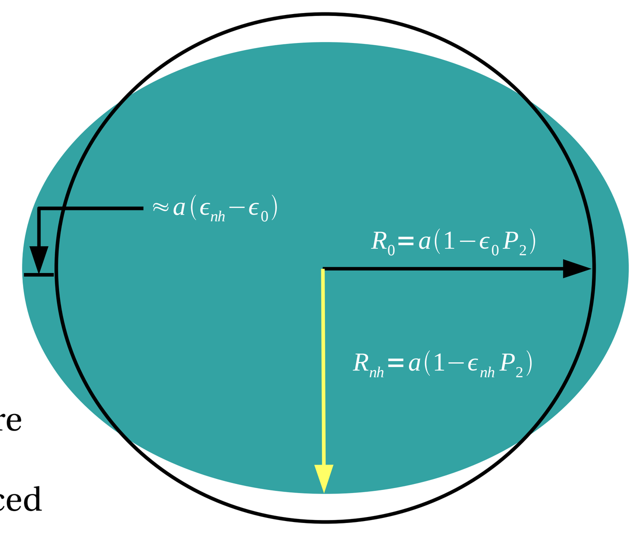


Nonhydrostatic Stress

Under the condition of hydrostatic equilibrium, the satellite and core surfaces are equipotentials, and thus experience force balance. By changing the oblateness into a nonhydrostatic shape, we introduce stresses at the satellite and core surfaces, as they are no longer equipotentials. We model the stress σ as simple loads on a hydrostatic surface:

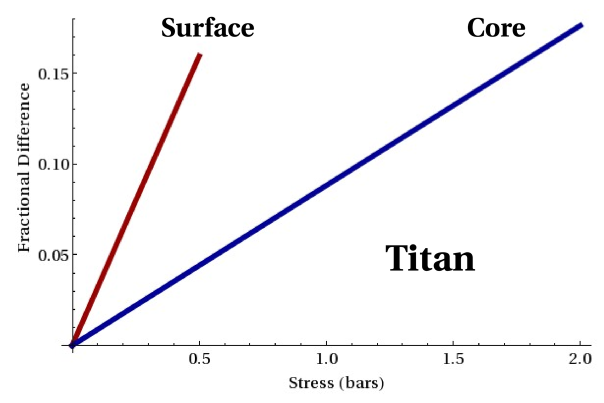
$$\sigma = \rho\, g\, |\epsilon_{nh} - \epsilon_0|\, a$$

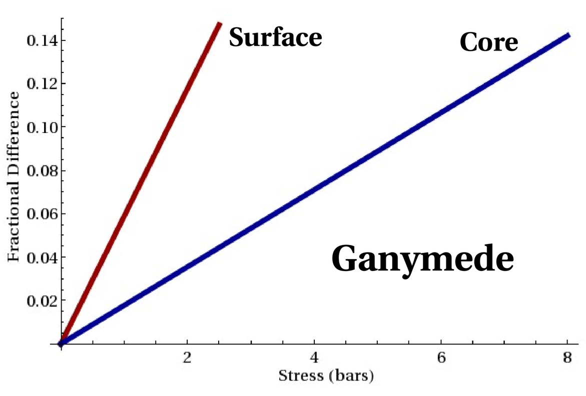
Where ρ is the surface (ice) density and g is the surface gravitational acceleration. An analogous equation exists for the core surface. This way, we can relate MoI deviations (derived from ε_{nh} - ε_0) to physical stress values, and assess whether the induced stresses are viable.

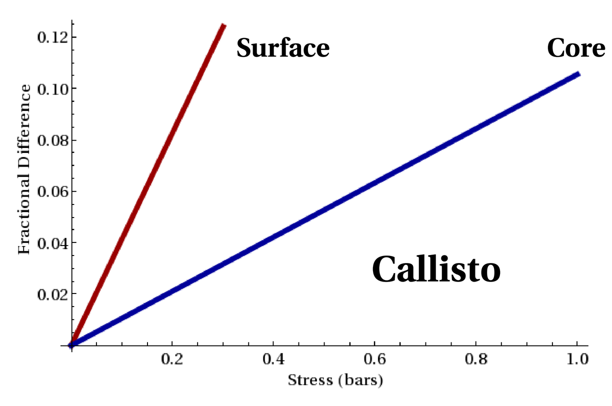


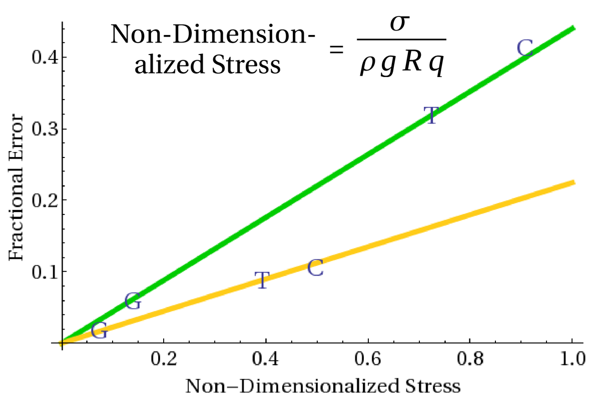
Results

For Titan, Callisto, and Ganymede, we plot the stresses needed on the surface (red lines) and core surface (blue lines) that will cause a fractional difference in MoI of ~10%. We see that the core requires greater stresses to result in the same fractional error, though this should be easier to accomplish, since the core would be stiffer than the ice mantle, and thus any deformations would survive for a longer time. We see that the stresses needed are on the order of 0.1-1 bar for Titan and Callisto, and 1-10 bars for Ganymede. In the lower right, we plot the fractional error in MOI vs. the non-dimensionalized surface (green line) and core (yellow line) stresses using Titan's model core fraction (similar to those of the other bodies in consideration). The position of the letters indicate the fractional errors caused by 1 bar of non-hydrostatic stress on the surface/core of Ganymede (G), Titan (T), and Callisto (C). These plots all show that 1). All the stresses are relatively small - easily accomplished by a load of a few tens of meters, and 2). Ganymede will see less of an effect than Titan and Callisto, which correlate with their MoIs, indicating that Titan and Callisto's MoIs may be "contaminated" by nonhydrostatic effects, while Ganymede's is "true", as the contamination is less severe.







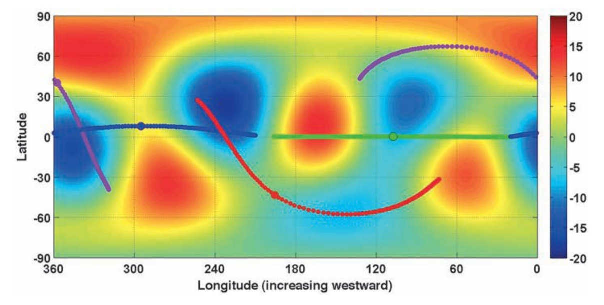


Summary

We have shown through a model icy satellite that the Radau-Darwin Approximation, a tool often used to calculate the moment of inertia of solid bodies, and that which assumes hydrostatic equilibrium, can be wrong by a factor of 10% if degree 2 nonhydrostatic stresses of 0.1-10 bars were present on the satellite surface and/or the core surface. We saw how slow rotators such as Titan and Callisto were affected by this more than faster rotators like Ganymede, with the effect being an order of magnitude more pronounced in the former than the latter. This may offer an explanation for the high moments of inertia of Titan and Callisto, in that the true values are actually lower (matching that of Ganymede's) and the currently-accepted values are wrong due to false assumptions of hydrostatic equilibrium.

Evidence of Nonhydrostaticity

Iess et al. [2] measured nonhydrostatic geoid height variations on Titan of up to ~20m (see below – note that the figure is taken from [2]), which is on the order needed to produce ~10% errors in its MoI. Mass anomalies have also beendetected on Ganymede [11].



Future Work

Future work on this project will involve the characterization of mechanisms that create the necessary nonhydrostatic structures, such as convection and possible primordial sources. We will also investigate the lifetime of these sourcesand determine the likelihood that the icy satellites discussed here currently exhibit them.

References

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