# The red giant structure and runaway accretion in gas giant planet formation

## The mechanism of runaway accretion

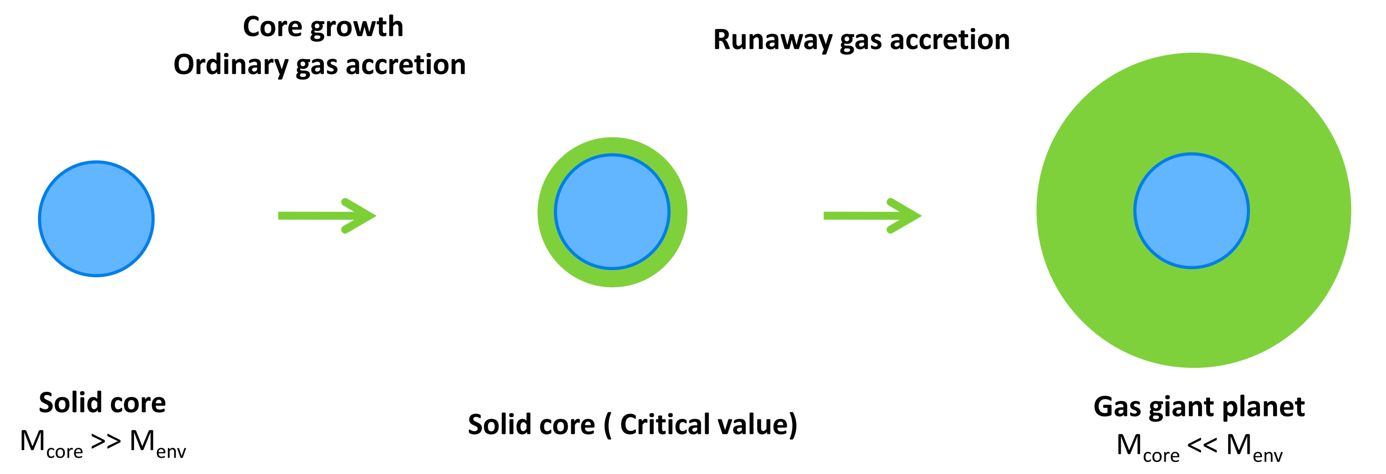
Kazuhiro KANAGAWA, Masayuki Y. FUJIMOTO (Hokkaido University)

## How are gas giant planets formed?

### ✓ Core Accretion Model

The presently most widely accepted scenario for the formation of gas giant planet.

In this model, proto-planets are composed by solid (ice/rock) core and gaseous envelope. In early phase of formation, proto-planet have only smaller envelope mass than that of solid core. Once the solid core has grown of critical core mass ( CCM), gas accretion to proto-planet increase rapidly, so-call Runaway accretion, and gaseous envelope of planet becomes massive, becoming gas giant planet.




### ✓ What factors cause the runaway accretion?

The fate of proto-planet depends on the onset of the runaway accretion, thus the conditions and process of it are important significantly. Many people believe that the CCMs are corresponding to an upper mass limit for static envelope above which no static solutions exist under thermal equilibrium assumed by earth other.

However, it isn't yet understood why the runaway accretion is caused when the solid core growth to the CCM.

### ✓ The aim of this study

From the discovery of many extra-solar planets by recent observations, the variety of extra-solar planets become clear and similarly proto-planetary disk, the site of gas planet formation, may have the variety. Thus it is important that the mechanism of runaway accretion becomes clear.

we consider the self-gravity system with two components, solid and gaseous component, under thermal equilibrium expressed by polytropic relation, and discuss conditions that the system has the CCM, and also the relationship of the runaway accretion and the CCM.

## Basic equations and Characteristic plane

### ✓ Basic equations

The solid component is highly centrally concentrated and immerses in a gaseous component, We assume the hydrostatic equilibrium in the spherical symmetry for the both components.

$$\frac{dP_{\rm solid}}{dr} = -\rho_{\rm solid}\frac{G(M_{\rm core,r}+M_{\rm gas,r})}{r^2}, \quad M_{\rm core,r} = \int_0^r 4\pi r^2 \rho_{\rm solid} dr;$$
$$\frac{dP_{\rm gas}}{dr} = -\rho_{\rm gas}\frac{G(M_{\rm core,r}+M_{\rm gas,r})}{r^2}, \quad M_{\rm gas,r} = \int_0^r 4\pi r^2 \rho_{\rm gas} dr, \quad \frac{N}{(N+1)} = \frac{d\log\rho/d\log r}{d\log P/d\log r},$$

For the solid component, we may well assume constant density for simplicity, the core radius, $R_{core}$, is defined as

$$R_{\rm core} = (3M_{\rm core}/4\pi\rho_{\rm solid})^{1/3}$$

### ✓ Boundary condition

As for the inner boundary conditions, we may adopt the same as for the stars, i.e., and P are finite at r = 0.

We may impose the same outer boundary conditions as the previous works

$$\rho_{\rm gas} = \rho_{\rm disk} \text{ and } P_{\rm gas} = P_{\rm disk} \quad \text{at } r = R_{\rm P} = \min(R_{\rm Bondi}, R_{\rm Hill}), \quad R_{\rm Bondi} = GM_{\rm P}/c_s^2, \quad R_{\rm Hill} = a_{\rm P}\left[M_{\rm P}/3(M_* + M_{\rm P})\right]^{1/3},$$

### ✓ Characteristic U-V plane

In studying the basic feature of the structures of gravitating systems in spherical symmetry, it is convenient to introduce the homology invariants, U and V , defined by

$$U \equiv \frac{d\log M_r}{d\log r} = \frac{4\pi r^3\rho}{M_r}, \quad V \equiv -\frac{d\log P}{d\log r} = \frac{GM_r\rho}{rP}, \quad \frac{d\log U}{d\log V} = -\frac{U+VN/(N+1)-3}{U+V/(N+1)-1},$$

The conditions at bottom of envelope (core surface) are,

$$V_{1,e}\, U_{1,e}{}^{1/N} = (3\rho_{\rm disk}/\rho_{\rm core})^{(3-N)/3N}(M_{\rm core}/M_0)^{2/3}. \quad M_0 = \left[(1/4\pi G^3)(P_{\rm disk}^3/\rho_{\rm disk}^4)\right]^{1/2}$$

### ✓ Classification of structure

The salient feature of these outer boundary conditions is that they pass through the lower domain below the critical line, differently from those of the stars.

Consequently, the planet formation necessarily starts by taking the configuration with a loop (or a part of loop ), similar to the red-giant structure (Core-Halo structure).

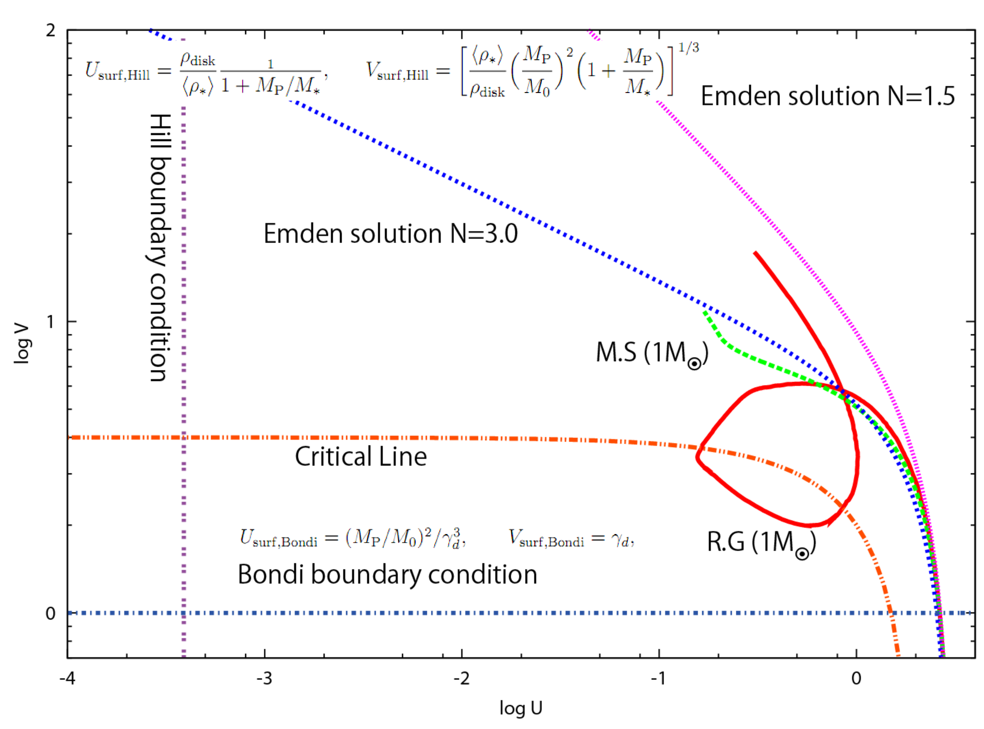



Fig. 1.— Examples of structure lines on log U – log V diagram for the main sequence stars and a red giant along with the Emden solutions of polytropic index N = 1.5 and 3. Also plotted is the critical line of 2U +V −4 = 0 and the horizontal and vertical lines denote the loci of the outer boundary conditions with the Bondi radius and the Hill radius, respectively.

And the structure of envelope must be so-call central condensed type solution.

### ✓ Characteristic lines

**• Critical line**

$$2U + V - 4 = 0.$$
$$d\log r = \frac{d\log M_r}{U} = -\frac{d\log P}{V} = \frac{d\log V - d\log U}{2U+V-4}.$$

**• Vertical line & Horizontal line**

the loci of points at which the tangent of structure lines is vertical or horizontal,.

$$U + NV/(N+1) - 3 = 0 \qquad \text{(vertical line)},$$
$$U + V/(N+1) - 1 = 0 \qquad \text{(horizontal line)}.$$

## Results with the single polytrope

If the entropy distribution is specified, the structure of hydrostatic equilibrium is determined. In this section, we assume the polytropic equation of state for the gaseous components

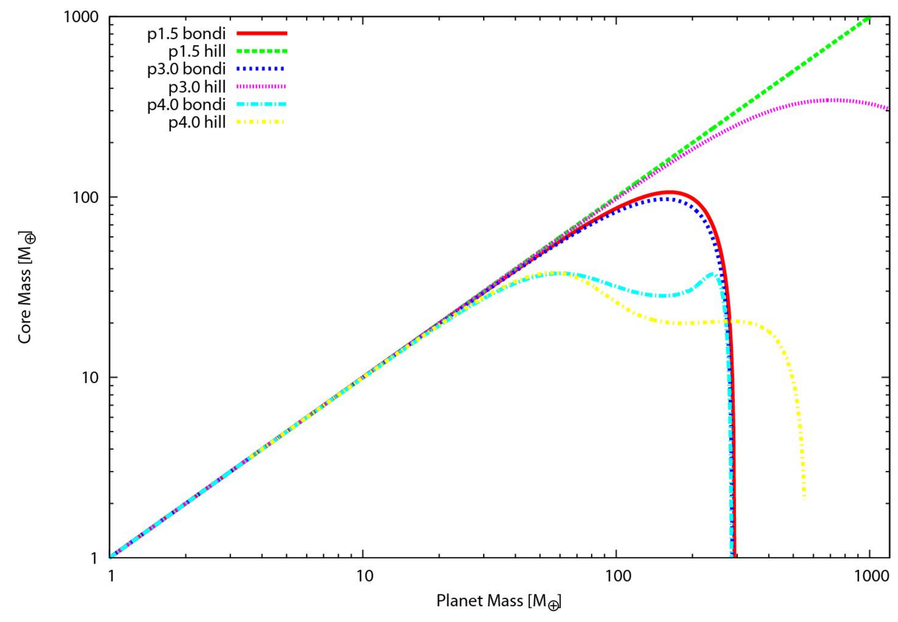



Fig. 2.— The relationship of the core mass, $M_{core}$, and the planet mass, MP, for the single polytrope models with the polytropic indices, N = 1.5, 3, and 4.0 (from the top to the bottom) and with the Bondi and Hill boundary conditions (solid and broken lines, respectively).

### Polytropic relation

$$P_{\rm gas} = K\rho_{\rm gas}{}^{1+1/N} = P_{\rm disk}(\rho_{\rm gas}/\rho_{\rm disk})^{1+1/N},$$

### Parameters

| Parameter | Value |
|---|---|
| $P_{\rm disk}$ | $5.5\times10^{-11}$g cm$^{-3}$ |
| $T_{\rm disk}$ | 150K |
| $\rho_{\rm solid}$ | 5.5g cm$^{-3}$ |
| $a_{\rm P}$ | 1AU (Hill) |
| $M_*$ | $1M_\odot$ (Hill) |
| $M_0$ | $182M_\oplus$ |
| $\langle\rho_*\rangle$ | $1.4\times10^{-7}$g cm$^{-3}$ |

### ✓ Structure lines

The single polytrope models demonstrate that the core mass has the maximum (or maxima) as a function of the total planet mass, the same properties as reported by the previous works with the realistic equation of states. However, the factor of feature can be classified as three cases; model of N>3, model of N≤3 with Bondi boundary and N≤3 with Hill boundary.

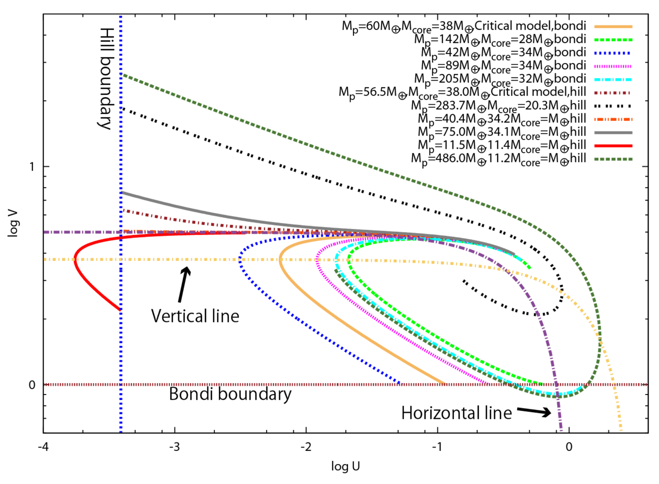
#### ✓ model of N>3

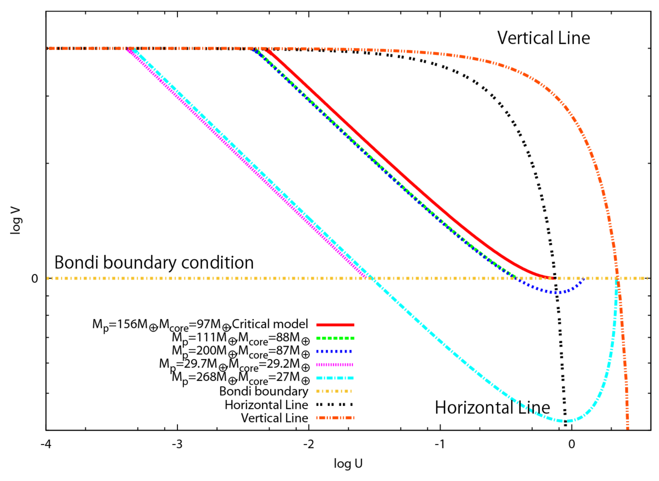
As the mass increases and the inner edge comes near to the singular point, the structure line starts to spiral down and steepens the gradient.

Eventually, the slope becomes stepper than the gradient, d log V/d log U = −1/N when the inner edge pass through the point of

$$U_{1,e} = (N-3)/(N-1), \quad \text{or} \quad \rho_{1,e} = \rho_{\rm solid}(N-3)/[3(N-1)]$$

Beyond this point, there is no solution because the structure line shifts downward while the jump condition moves upward for the model with a larger core mass.




#### ✓ model of N≤3 with Bondi boundary

In this case, There are two branches; one is that structure line connects from outer boundary to bottom of envelope directly, the other is that structure line go thought the region V<1.

The models of these two branches constitute a liner series with the total mass of planet as a parameter. the core mass takes the maximum value at the point where these two branches meet , i.e., where the structure line is tangent to the line of outer boundary;

$$U_{\rm surf} = 1 - \gamma_{\rm disk}/(N+1). \quad \text{or} \quad \bar{\rho}/\rho_{\rm disk} = 3/[1-\gamma_{\rm disk}/(N+1)],$$

#### ✓ model of N≤3 with Hill boundary

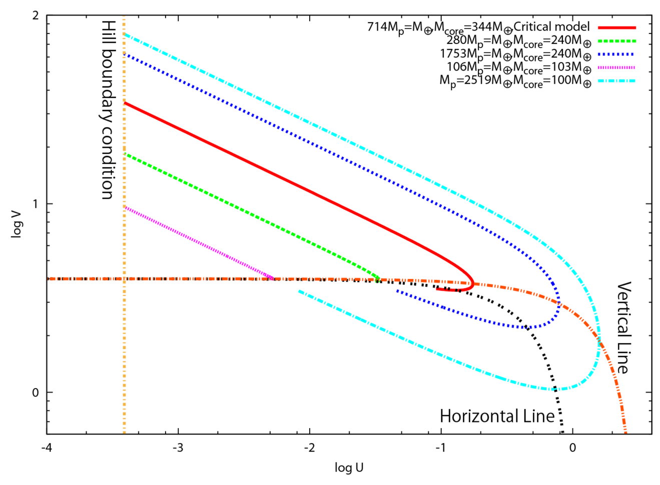
Since the Hill outer boundary condition allows a large value of $V_{surf}$ > N + 1, the centrally condensed-type solutions which run above the horizontal line are also applicable to the Hill models.

For a given core mass, the location of inner edge of the envelope may cut across the horizontal line at the gradient of

$$d\log V_{1,e}/d\log U_{1,e} = -1/N \text{ for } U_{1,e} < 1/(N+1)$$

since these two lines are tangent to each other .

For a given core mass, therefore, two solutions are possible with the inner edge of the envelope in either (upper or lower) side of the line. This may give rise to the bimodality of the configuration of planets.

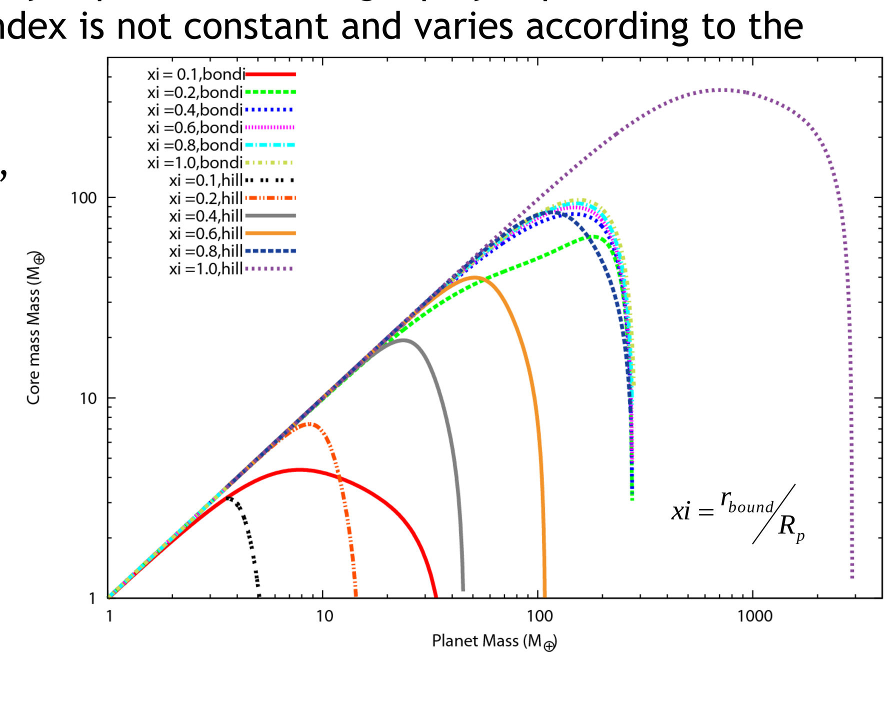



## The model with the energy source of core growth

In applying the actual systems, we should consider the thermal properties of envelope. Moreover, a realistic system may not be described by such polytrope model of single polytropic index. Under a realistic temperature gradient, the polytropic index is not constant and varies according to the local condition.

If we assume that energy generation of core growth is determined the temperature gradient, the upper layer of planet must be isothermal. Thus, it may be good approximation that the outer layer is isothermal and inner part is polytropic. The location of boundary of polytropic solution and isothermal layer is parameter depended on opacity and luminosity distribution.

As seen from the right figure, the critical core mass depends on depth of isothermal layer. The deeper the isothermal layer (smaller entropy), the smaller the critical core mass is.




## Conclusion

Based on the analysis of structure by using the characteristic variables to the theory of stellar structures, we have derived the conditions that causes the stability change to the accretion gaseous planets for the various situations including the different outer boundary conditions, which enables to elucidate the dependence on the model parameters of runaway accretion for the various situations.

1. We have shown that the structure change and the runaway accretion that are known to occur during the formation of gaseous giant planets are related to the red giant structure of stars.
2. We find that CCM is determined by three kind of mechanisms which divide the stable branch and unstable branch. The condition of CCM is determined by the density of gaseous component rather than the envelope mass and solid core mass.
3. The magnitude of CCM depends strongly on the entropy at bottom of envelope, the smaller the entropy, the smaller the critical core mass. It is shown that cooling and heating process in proto-planet are important factors to determinate CCM. The CCM become small by cooling.