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Nonlinear interaction between vortical flows and gravity waves in geophysical fluids

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1. Introduction

GW radiation from rotational flow (Spontaneous GW radiation)

Observational study

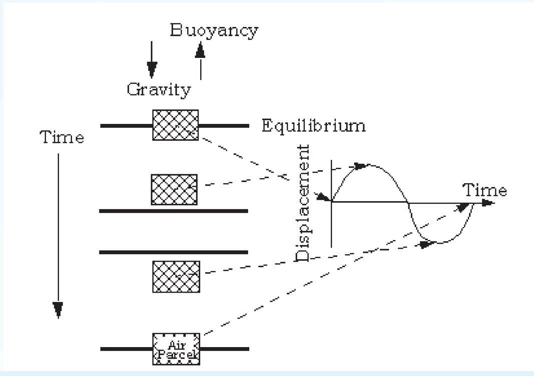
- Yoshiki and Sato(2000): polar night jet
- Kitamura and Hirota(1989): sub tropical jet
- Pfister et al.(1993): hurricane

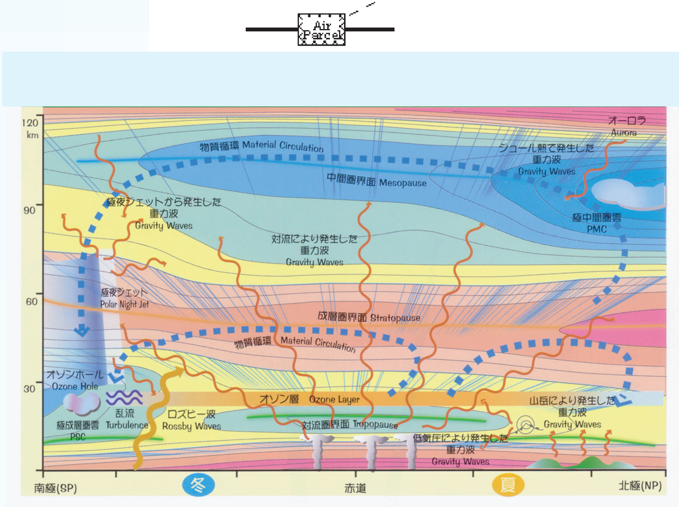
Experimental study

- Williams et al.(2005): 2-layer fluid in rotating annuls

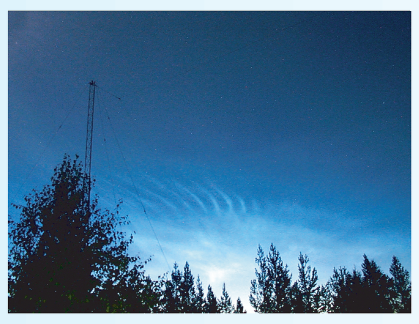
Numerical study (GCM, Meso scale model)

- O' Sullivan and Dunkerton(1995): sub tropical jet
- Zhang(2004): sub tropical jet
- Sato et al.(2005): polar night jet
- Plougonven and Snyder(2005): sub tropical jet

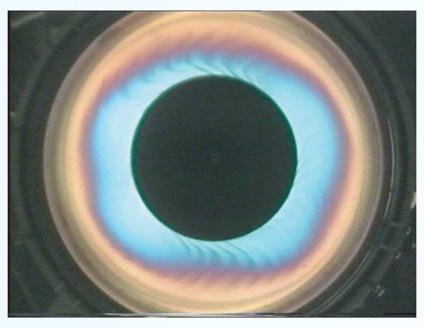




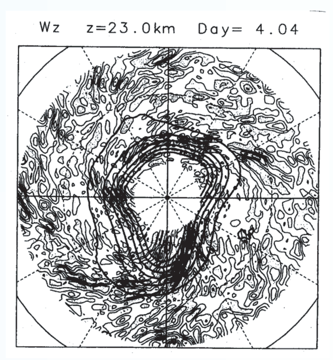
Pansy project (2005)



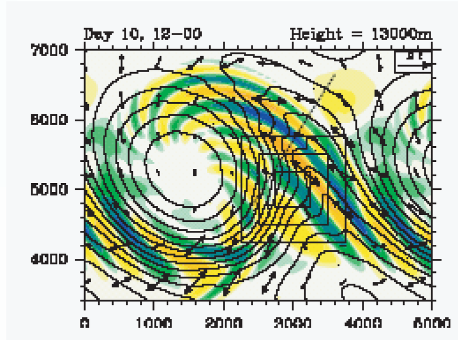
Dalin et al. (2004)



Williams et al. (2005)



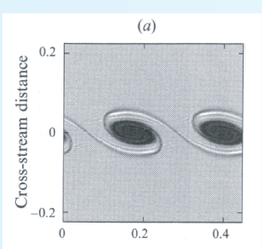
Sato et al. (2005)

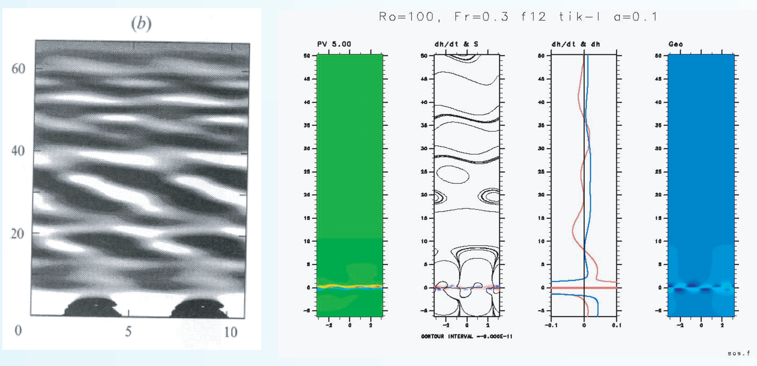


Plougonven et al. (2005)

Numerical study (simplified model=f-plane Shallow Water)

- Ford(1994): vorticity stripe, Lighthill analogy
- Sugimoto et al.(2005, 2007, 2008): unsteady jet with relaxation forcing





Ford (1994) Sugimoto et al. (2007)

Energy of gravity waves << Energy of rotational flows

We need a special numerical model!

Spectral scheme

- ◎high accuracy・high resolution
- △complicated・large memory・slow
- →not usable for parallel scalar machine

Finite difference scheme with high accuracy

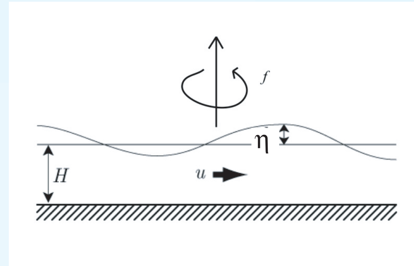
- Compact Difference scheme, CD
- Combined CD, CCD (Chu & Fan, 1998)
- →achieves high accuracy with few stencils

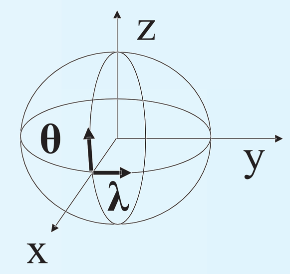
This study
We investigate spontaneous GW radiation in SH on a rotating sphere with high accuracy numerical model.

2. Experimental setup

Basic equation

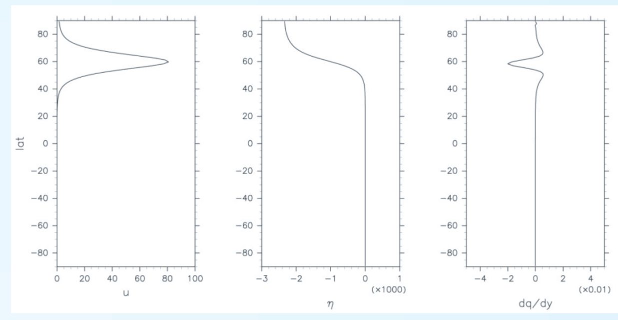
$$\frac{\partial u}{\partial t}+\mathbf{v}\cdot\nabla u-\left(f+\frac{u}{a}\tan\theta\right)v+\frac{g}{a\cos\theta}\frac{\partial h}{\partial\lambda}+\alpha(u-\bar{u})=0$$
$$\frac{\partial v}{\partial t}+\mathbf{v}\cdot\nabla v-\left(f+\frac{u}{a}\tan\theta\right)u+\frac{g}{a}\frac{\partial h}{\partial\theta}+\alpha v=0$$
$$\frac{\partial h}{\partial t}+\mathbf{v}\cdot\nabla h+\frac{h}{a\cos\theta}\left(\frac{\partial u}{\partial\lambda}+\frac{\partial v\cos\theta}{\partial\theta}\right)=0$$





Basic state

$$\bar{\eta}=-\frac{fBU_0}{g}\operatorname{atan}\left(e^{(2(\theta-\theta_j)/B)}\right)$$
$$\bar{u}=\frac{-\dfrac{af}{\tan\theta}\pm\sqrt{\left(\dfrac{af}{\tan\theta}\right)^2+\dfrac{8fBU_0}{\tan\theta}\left(\dfrac{e^{(2(\theta-\theta_j)/B)}}{1+e^{(2(\theta-\theta_j)/B)}}\right)+\dfrac{4fBU_0\cos\theta}{\tan\theta}\operatorname{atan}\left(e^{(2(\theta-\theta_j)/B)}\right)}}{2}$$



Experimental condition

Resolution : (λ, θ)=512×256grids

Boundary condition :

no grid at pole, periodic boundary

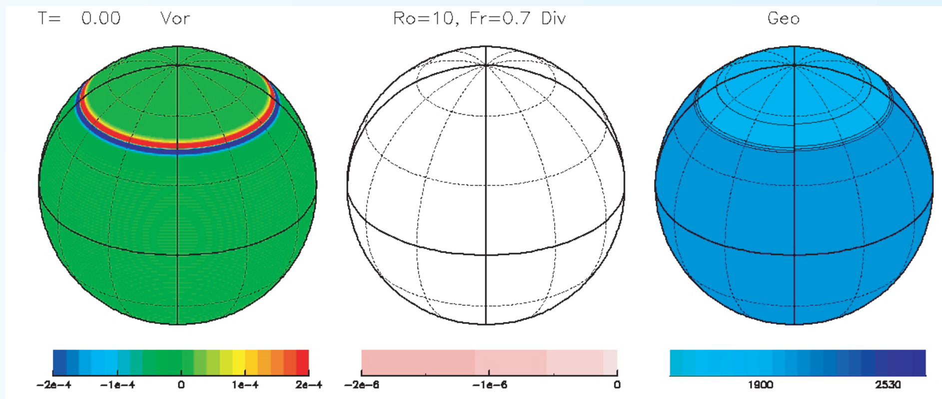
Numerical filter : low pass filter

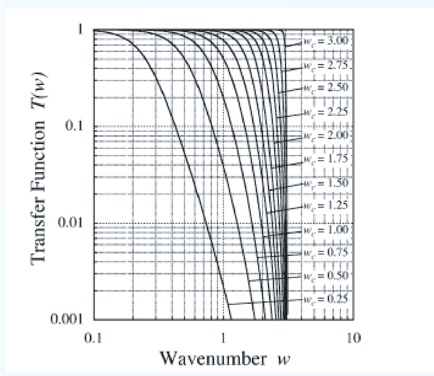
Time integration : 4th-order Runge-Kutta

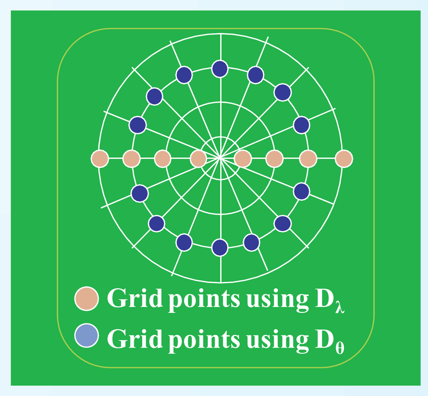
(full explicitly)

Experimental parameter :

$$f_{mid}=2\Omega\sin(\pi/4),\quad U_{max}=100$$
$$g=9.806,\quad a=6.317\times10^6,\quad B=2\times10^5,$$
$$Ro=\frac{U_{max}}{f_{mid}B}\approx1.5-30$$
$$Fr=\frac{U_{max}}{\sqrt{gH_0}}\approx0.7$$
$$\theta_j=11.25-78.75,$$

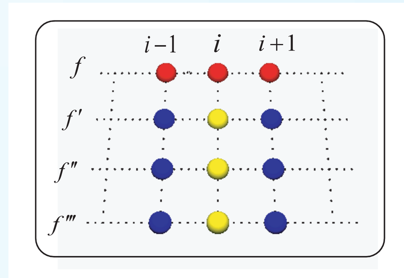






Combined Compact Difference (CCD) scheme

$$\begin{cases} f_i'+a_1\left(f_{i+1}'+f_{i-1}'\right)+b_1h\left(f_{i+1}''-f_{i-1}''\right)+c_1h^2\left(f_{i+1}'''+f_{i-1}'''\right)=\dfrac{d_1}{h}\left(f_{i+1}-f_{i-1}\right)\\ f_i''+\dfrac{a_2}{h}\left(f_{i+1}'-f_{i-1}'\right)+b_2\left(f_{i+1}''+f_{i+1}''\right)+c_2h\left(f_{i+1}'''-f_{i-1}'''\right)=\dfrac{d_2}{h^2}\left(f_{i+1}-2f_i+f_{i-1}\right)\\ f_i'''+\dfrac{a_3}{h^2}\left(f_{i+1}'+f_{i-1}'\right)+\dfrac{b_3}{h}\left(f_{i+1}''-f_{i-1}''\right)+c_3\left(f_{i+1}'''+f_{i-1}'''\right)=\dfrac{d_3}{h^3}\left(f_{i+1}-f_{i-1}\right)\end{cases}$$



●: known value
●: desired value
●: unknown value

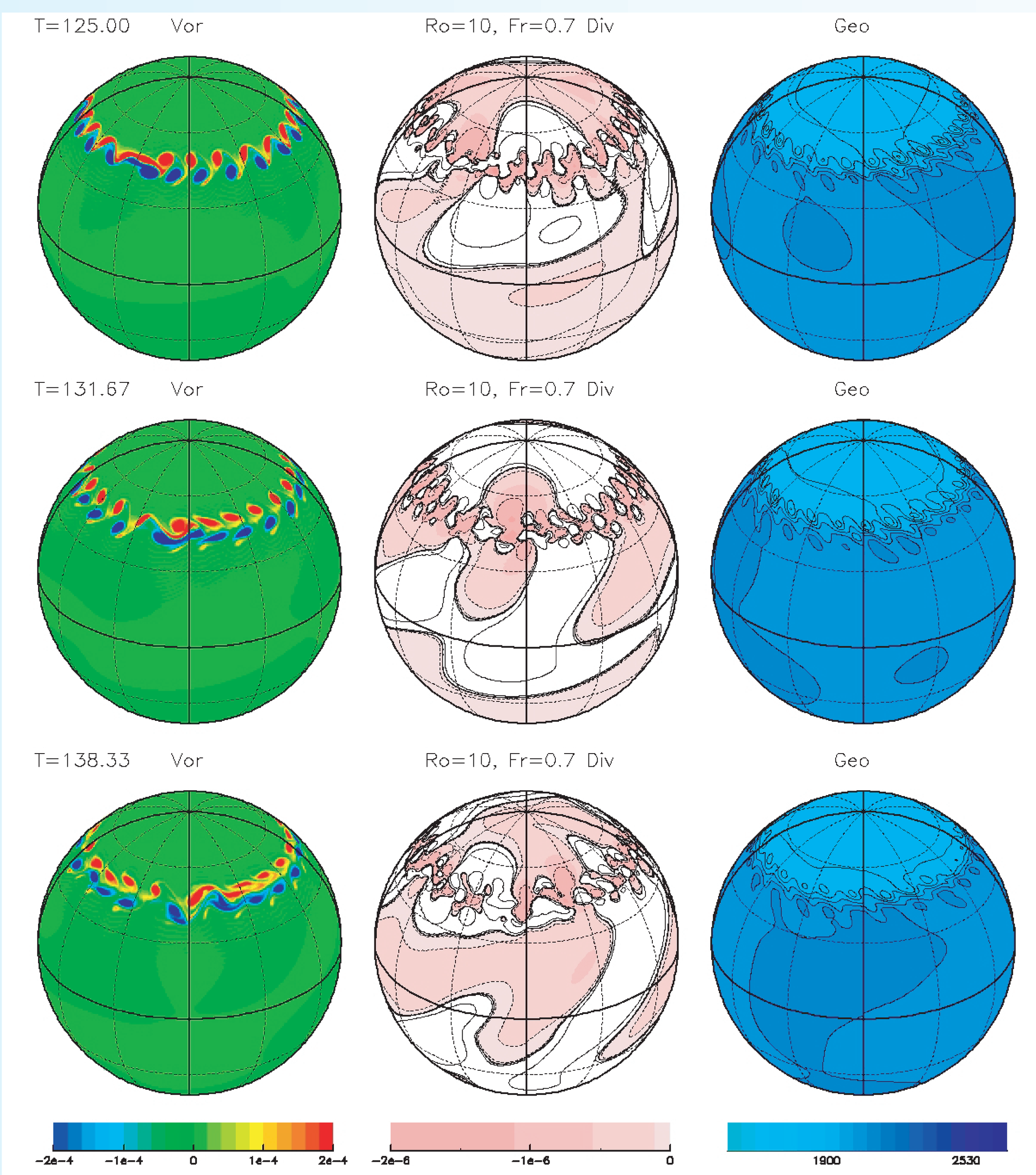
Spectral CD (Lele, 1992)

Spectral CCD (Niehi & Ishii, 2003)

Spectral high resolution and accuracy with few stencils

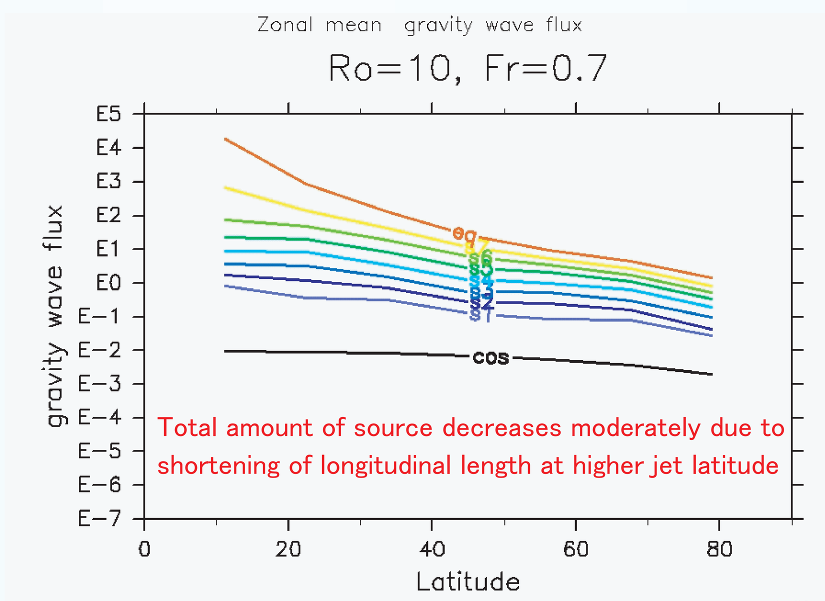
3. Results

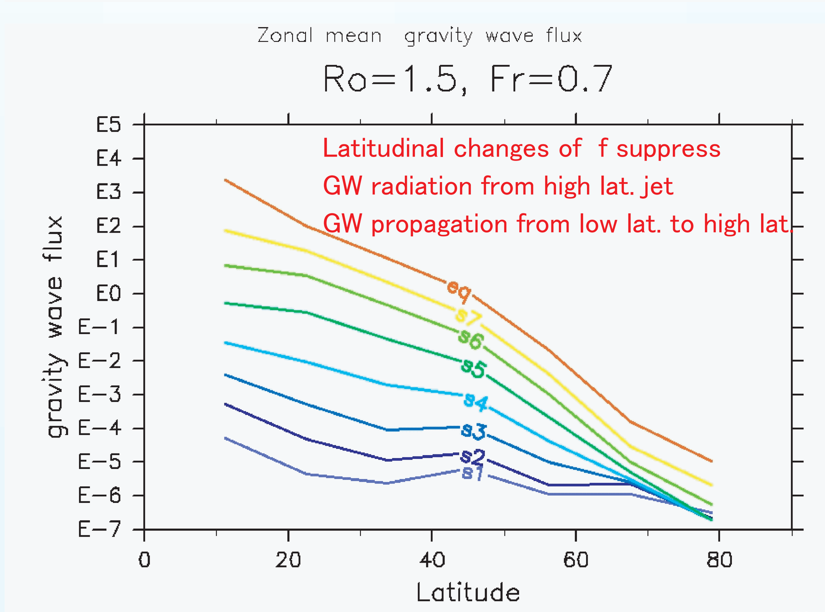
Time evolution of flow fields (Ro=10, Fr=0.7)



Gravity wave radiation from unsteady rotational flows

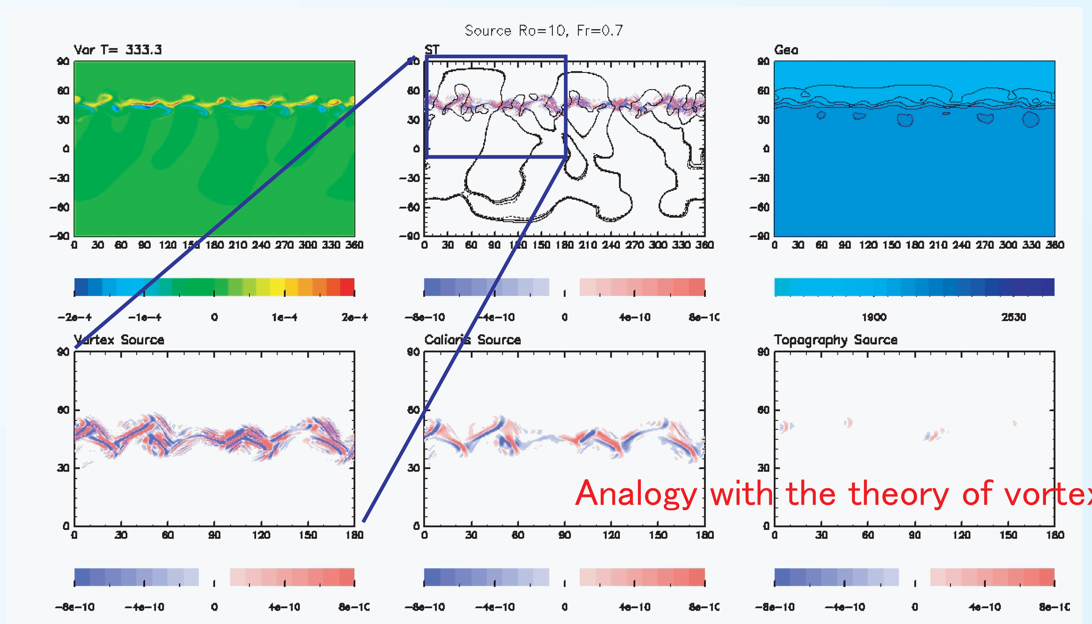
Latitudinal dependence of GW flux



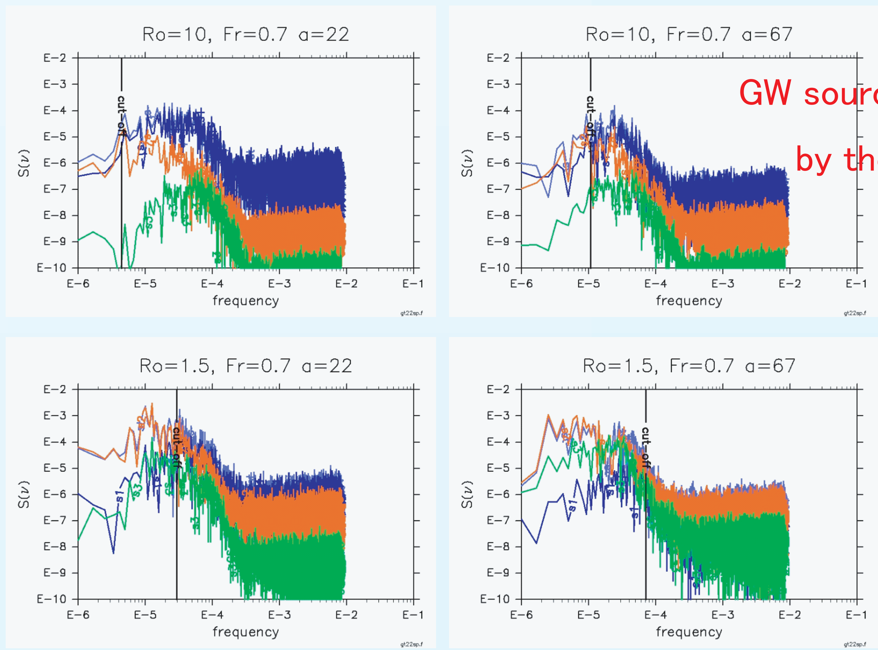


GW source with fixed f-plane approx.

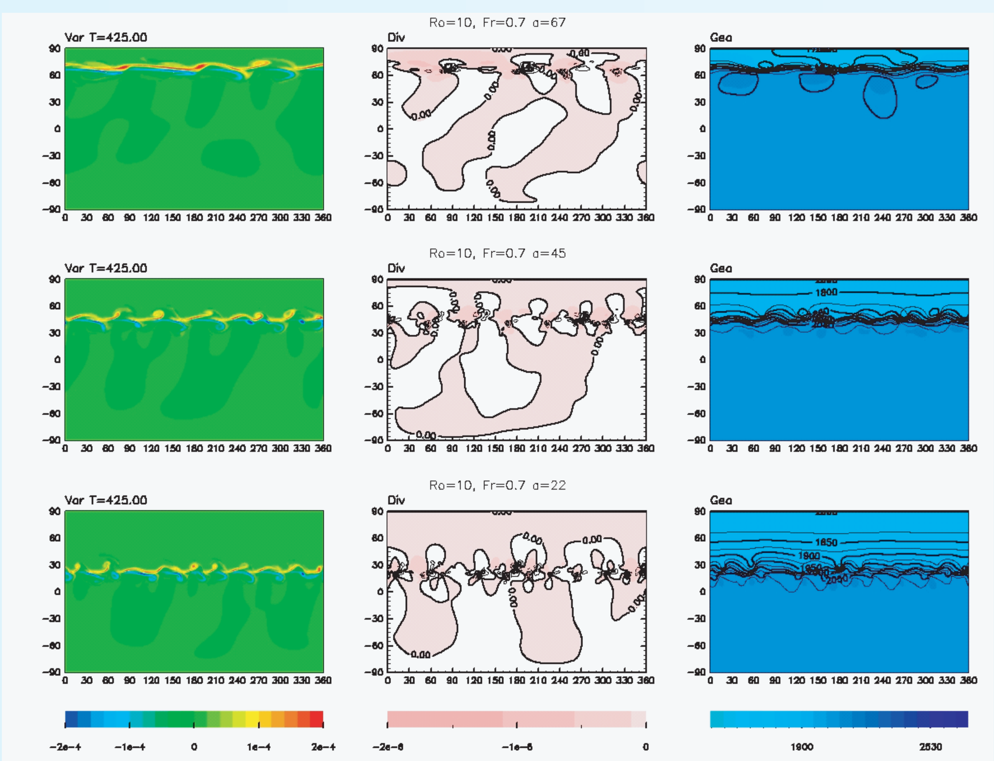
$$\underbrace{\left(\frac{\partial^2}{\partial t^2}+f^2-gH_0\Delta\right)\frac{\partial h}{\partial t}}_{\text{linear GW propagation}}=\underbrace{\frac{1}{a^2\cos\theta}\left[\frac{1}{\cos\theta}\frac{\partial^2}{\partial\lambda^2}\left(\frac{\partial(hu^2)}{\partial t}\right)+\frac{\partial^2}{\partial\lambda\partial\theta}\left(\frac{\partial(2huv)}{\partial t}\right)+\frac{\partial}{\partial\theta}\left(\cos\theta\frac{\partial}{\partial\theta}\left(\frac{\partial(hv^2)}{\partial t}\right)\right)\right]}_{\text{vortex source}}$$
$$+\underbrace{\frac{1}{a^2\cos\theta}\left[\frac{1}{\cos\theta}\frac{\partial^2}{\partial\lambda^2}(-fhuv)+\frac{\partial^2}{\partial\lambda\partial\theta}\left(fhv^2-fhu^2\right)+\frac{\partial}{\partial\theta}\left(\cos\theta\frac{\partial}{\partial\theta}(fhuv)\right)\right]}_{\text{Colioris source}}$$
$$+\underbrace{\frac{1}{a^2\cos\theta}\left[\frac{1}{\cos\theta}\frac{\partial^2}{\partial\lambda^2}\left(\frac{g}{2}\frac{\partial}{\partial t}(h-H_0)^2\right)+\frac{\partial}{\partial\theta}\left(\cos\theta\frac{\partial}{\partial\theta}\left(\frac{g}{2}\frac{\partial}{\partial t}(h-H_0)^2\right)\right)\right]}_{\text{topography source}}+F(\text{forcing term \& dumping})$$



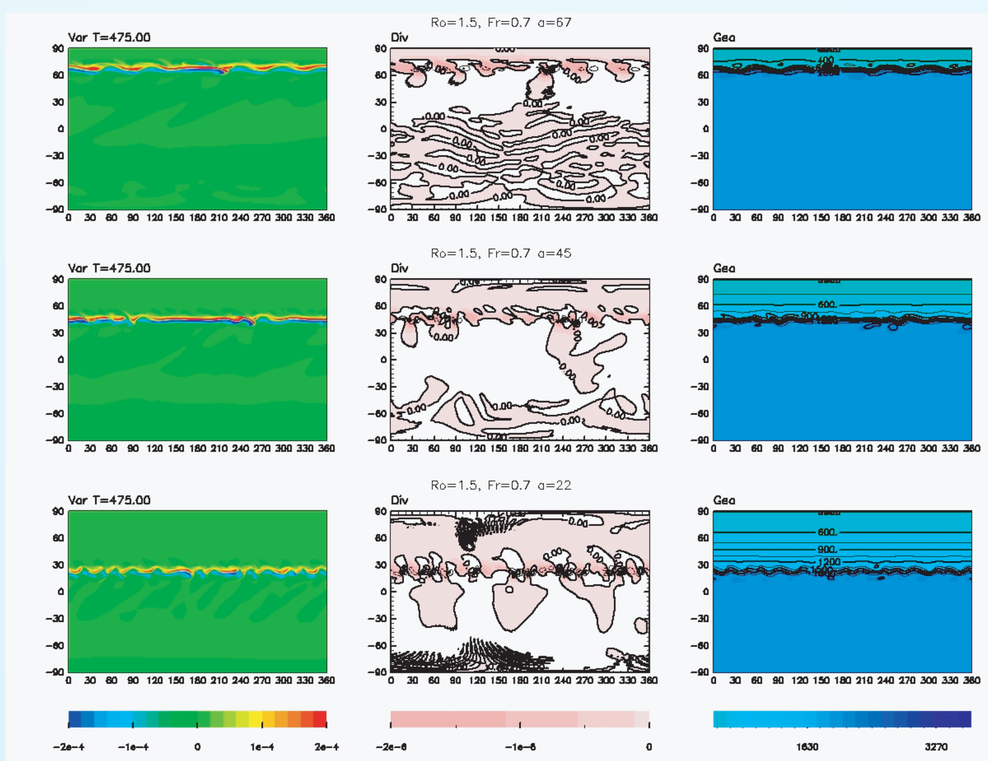
Frequency spectra of GW source



Latitudinal dependence of flow fields [Ro=10]



[Ro=1.5]



Significant change of GW propagation and radiation

5. Summary

- We investigate spontaneous GW radiation from unsteady jet flows in SH on a rotating sphere, using CCD scheme.
- GW flux depends on latitude of the jets, since the effects of the earth rotation and size are different.
- GW source and its analysis on the basis of f-plane approx. is useful to understand spontaneous GW radiation from rotational flows.